Audio System Engineering
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Lecture – 04
Equivalent Electrical Circuits for Oscillators

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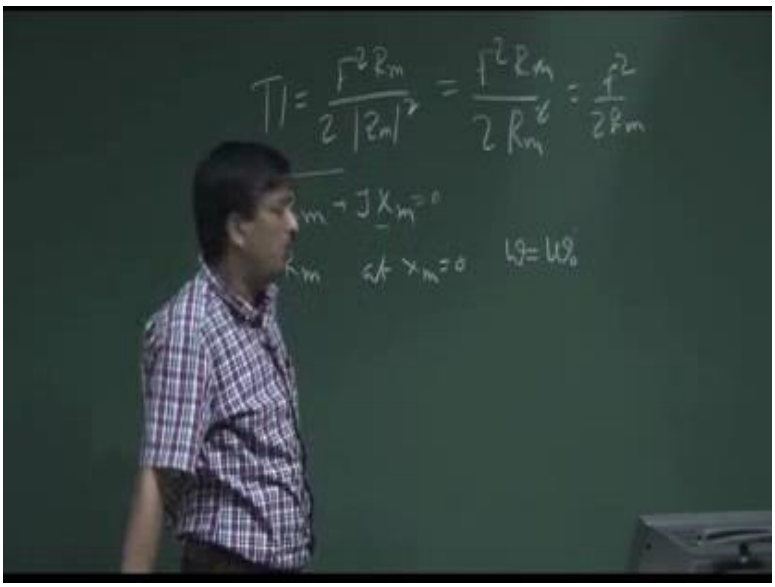

Morning, now last class we have said that average power for a forced oscillation we have derived that average power in case of force oscillation is nothing but a F square is the applied force divided by R m – mechanical resistance divided by two mod of Z n square, which is mechanical impedance. And we said that average power will be maximum when Z m is minimum. And Z m is nothing but a R m plus J X m, and Z m will be minimum, if X m is equal to 0, if X m is equal to 0, so; that means, Z m is equal to R m at X m is equal to 0. When X m is equal to 0, the X m will be zero, if the (Refer Time: 01:12) of the force frequency, if the force oscillation, so the force frequency is equal to the natural resonance frequency of the system, omega is equal to omega 0.

So, if the natural resonance frequency of the system is omega 0, and if the applied force is the omega, frequency is omega then X m will becomes zero, because we have done that in last class we prove that because X m is nothing but a omega m minus S by omega, you prove that. At omega equal to omega 0, X m is equal to 0, so in that case, the maximum power transfer will be happen. Now, if omega is equal to omega 0, this has a

another name, if omega is equal to omega 0 then the maximum power is nothing but F square by R m into 2 R m square, so R m square will be cancel, so F square by 2 R m is the maximum power. Now, when the omega, applied force frequency is equal to the natural resonance frequency, we said the mechanical resonance has happened, so that phenomena are called mechanical resonance.

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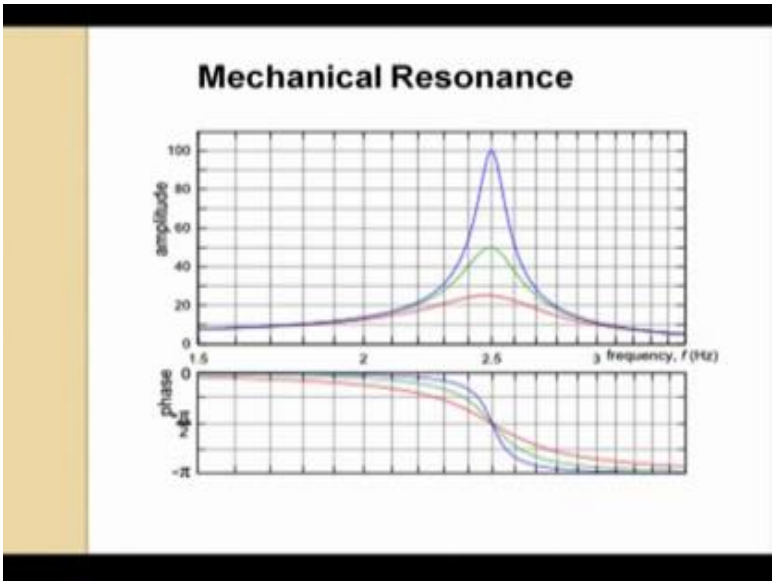

Now, if you see, if I draw, so I said the mechanical resonance maximum power transfer will be happen at omega equal to omega 0. Now, if I draw the curve maximum power transfer curve, if you see the curve in the power point, what is saying that if I take axis this axis is omega and this axis is the power.

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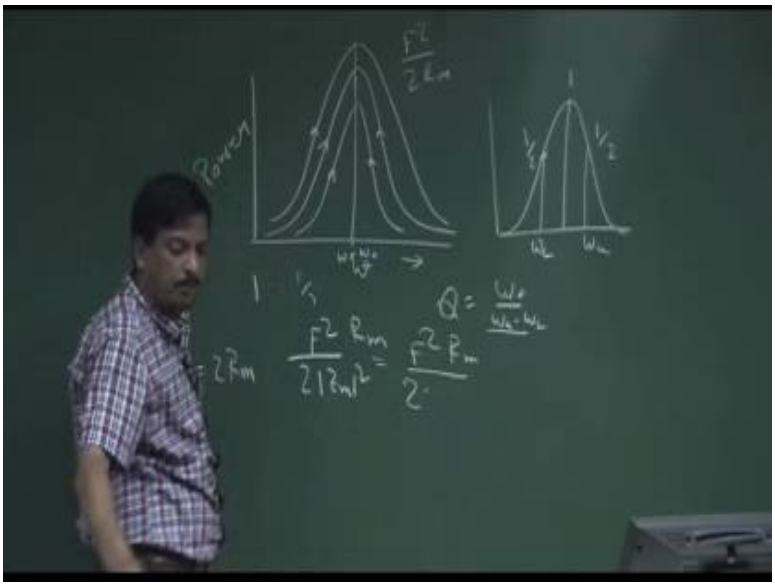

Then I said the maximum power will be happen at omega is equal to omega 0, maximum power transfer value. So, less than omega 0, this kind of curve I will get; so this is called mechanical, this is the power; mechanical resonance curve. So, mechanical resonance curve different kind of resonance curve will be I will get; so the maximum power is F square by 2 R m. Now, how we define this slope of the curve, who define, which parameter define this slope of the curve. So, how to find out those parameter which define the slope of the curve?

Now, you know that from electron is where the electronics background they know that that if I have a this kind of curve, if this is the maximum power then Q dB down, if the power expressing dB then Q dB down is half power frequency, this is called half power frequency. So, if this power is one, this will be half, this will be half that will (Refer Time: 04:28) Q dB down. So, if the lower half power frequency is omega L and upper half power frequency is omega U, upper half power frequency.

Then we know the quality factors of this resonance curve, how steep is the resonance curve is defined as omega 0 divided by omega U minus omega L that is called quality factor; that means, if this bandwidth omega U minus omega L is the bandwidth. If bandwidth is very small then the curve will be steeper; if the bandwidth is very large, curve will be broader, so that is the power transfer curve of the mechanical oscillator. So, the quality factor is depend on omega 0 divided by omega U minus omega L.

Now, how do we get the value of this parameter, omega L and omega U.? Now, think a mechanical system, we said it is half power, so if the if the power is maximum is the one then it is the power is half so what does the power, what is the power equation if average power is F square by 2 Z m square into R m this is the power equation. Now, if this is the power equation, when it will be half, when the Z m is equal to at Z m is equal to 2 Z m the power is if it is twice then the power will be half, if it is twice then the power will be half. So, I said that if Z m is equal to 2 R m because as the total power is F square by 2 R m, so this will be half, when Z m is equal to 2 R m. If you put that Z m is equal to 2 R m, you get this equation half of this, half of the total power. Because if it is nothing but a F square by R m divided by 2 into 4 R m square so half power will be happen.

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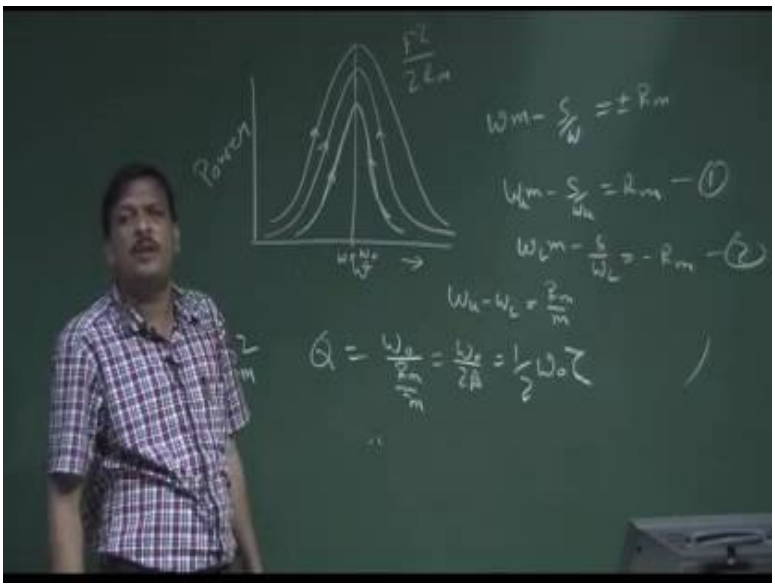

So, if it is up power happen, then I said Z m square will be sorry Z m square will be 2 R m square. So, if it Z m square is equal to 2 R m square, then I said pi is equal to F square by F square into R m divided by 2 Z m square. So, I said F square by R m into 2 into 2 R m square, so R m R m - cancel. It is 1 by 2 into F square by 2 R m. So, this is half power happen. So, the condition is that if Z m square is equal to 2 R m square, then the half power is happen. So, this is the condition that when half power will be happen, when Z m square is equal to 2 R m square. What is Z m square? It is nothing but R m square plus X m square, nothing but a Z m square is equal to 2 R m square. Now, then X m square will be what plus minus will be one R m R m square, so X m will be plus minus R m.

So, if X m is plus minus R m that what is X m. Reactance X m is nothing but a omega m minus S by omega is equal to plus minus R m. So, I can say if it Z value is plus minus R m, so X m is plus R m and X m is equal to minus R m. So, at upper frequency value will be half – high, lower frequency the value will be low. So, in that case, the omega m minus S by omega thus omega is u S by omega u is equal to plus R m; and omega L m minus S by omega L equal to minus R m. Now, if I substitute these two equations in the S, this is the equation number one, equation number two. Just substitute the S, I get omega U minus omega L is equal to R m by m. So, then quality factor is nothing but a omega 0 divided by R m by m.

So, steepness of that curve, how narrow is the curve is defined by parameter R m by m – mechanical resistance divided by the mass of the system. So, quality factor is nothing but a omega 0 divided by R m by m. So, I can write it is omega 0 divided by two beta, because beta is equal to R m by 2 m. Similarly, 1 by beta is equal to tau, I can write half omega 0 tau. So, this is called mechanical resonance.

Now, why it is require? Suppose, I want to design this microphone, if I operate a microphone at resonance frequency, what will happen, the diaphragm will be broke down. So, I have to design that value below the resonance frequency, and I have to know what is the power transfer of that point. So, I have to draw the power curve of the diaphragm. So, I have to know these what is the quality factor of that power curve. So, I can easily calculate the quality factor of the power curve, so that is why this theory has to be known.

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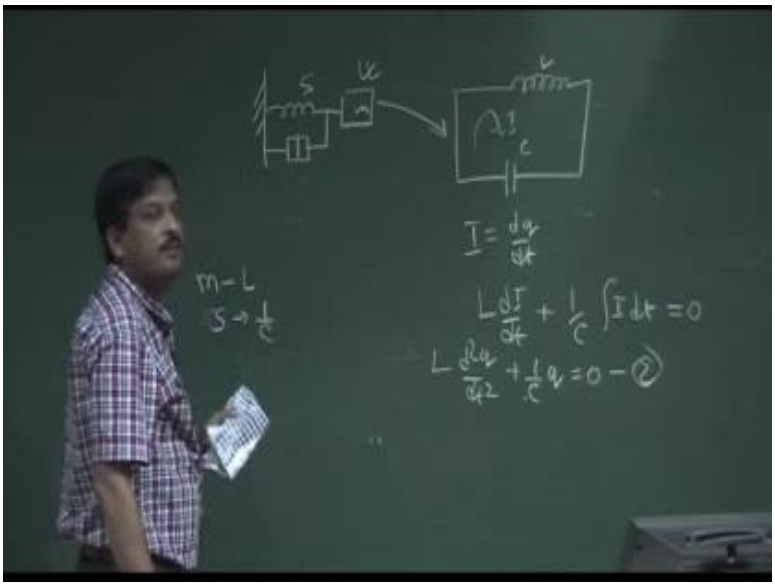

Now, I go for another chapter which is called mechanical to electrical conversion or equivalent circuits; it is not equal, it is equivalent circuits. Can I draw an equivalent electrical system for a mechanical system, yes, it is possible. If I start with the simple oscillator, you know that and mass, the simple oscillator mass and stiffness is S. What is the equation of motion, d 2 x by dt square plus S x is equal to 0.

Similarly, if I draw a simple LC circuits, let us L is inductance, and there is a capacitance, simple L and C circuits. Now, if I say if the current is I varying in this circuit then what is the equation of the charge flow in this circuit? So, what is current, I is nothing but a charge per unit time, so I can write d q by dt. Now, what is the electrical equation of this circuit, total equation, it is nothing but a voltage across the L plus voltage across the C will be equal to 0. So, voltage across the L what is the voltage across the L, L d I dt plus voltage across the C 1 by C integration of I dt that should be equal to 0. There is no external voltage source, so that is will be equal to 0 that is called Kirchhoff's loop rule.

So, inductance is if there is no variation in the current, inductance across the voltage of the inductance is 0. See if there is variation in the current, then there is voltage across the inductance, capacitance – it store the charges, so it is I integration of I dt. Now, you just replace, what is I, d q dt, so it is nothing but a L d 2 q by dt square plus 1 by C d q dt

integration over the dt, so integration and differentiation will be cancel, so it is nothing but a q equal to 0.

So, if you see this equation, equation number two and equation number one, they are equivalent equation; m d 2 x by dt square plus S x is equal to 0. Here, L d 2 q by dt square plus 1 by C q into 0. So, I can say the m – mass is analogous to electrical inductance and or stiffness of the spring is analogous to inverse of the electrical capacitance. So, mass is analogous to L and capacitance is inverse of that stiffness or I can say C is 1 by S or S is 1 by C. So, any mechanical circuits if it is given, so if this is the mechanical circuits, I can say the electrical equivalent circuit is this one.

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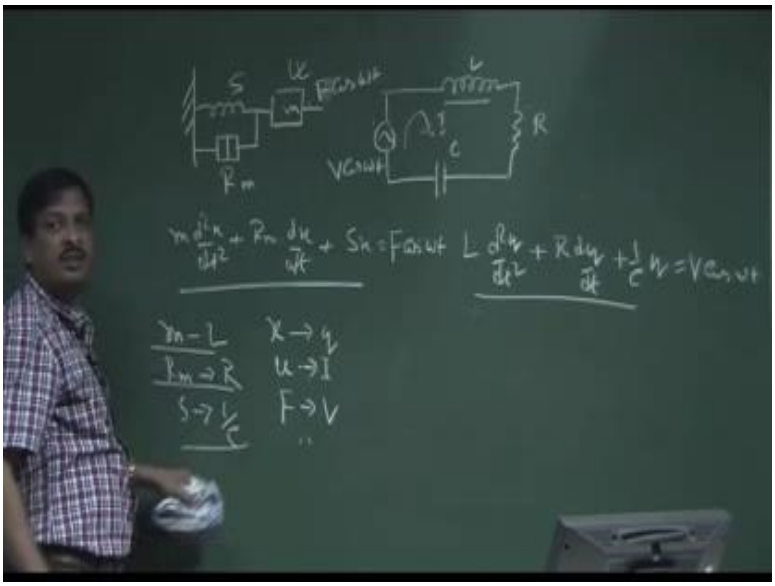

Now, if I add in the mechanical circuit mechanical damping. I just add a mechanical damping in that, which is R m. Now, if I apply a voltage source F in here, then the equation of motion, if the force is not there then the total equation of motion is m d 2 x by dt square plus R m into d x dt plus S x is equal to 0, equation of the motion. Similarly, just add a resistance here, I add a resistance R in here, electrical resistance, then what is the equation of this electrical circuits, it is nothing but a voltage across the L, which L d 2 q dt square we have already proved plus across the resistance R into I, I means dq dt plus one by C into q is equal to 0.

Now, see this equation and this equation are the identical equation, where m is again equal to L and R m is equal to R and S is equal to one by C – equivalent to one by C.

And x displacement in case of mechanical system is equivalent to charge in electrical system. Similarly, vertical velocity in mechanical system is equal to nothing but a d x dt, so this is equal to d q dt, so it is nothing but a current. Now, if I apply a voltage source here, sinusoidal voltage source here, and here apply a sinusoidal force here, so it is nothing but a sinusoidal voltage source let us V cos omega t, it is F cos omega t. So, this is equal to F cos omega t and this is equal to V cos omega t. So, force here, mechanical force is nothing but electrical voltage. So, equivalent electrical circuit for this mechanical circuit is this one. So, I can easily draw any given mechanical circuits, I can easily draw its equivalent electrical circuits.

And I know the equivalence of mass is equal to electrical inductance, equivalence of mechanical resistance, electrical resistance equivalent to stiffness is inverse to the capacitance; x equivalence to q is the charge; u equivalent to I - current, and force is equivalent to voltage. Similarly, Z is nothing but a force by velocity, force by u; so, in case of electrical, impedance is nothing but a voltage by current, V by I. So, impedance is nothing so mechanical impedance is nothing but equivalent to electrical impedance, so is it know that.

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Now, cascading, another point is that suppose I make two spring together above to that or I can replace one single spring by this kind of cascading or I can replace this spring by a parallel cascading. So, this two kind of cascading I can do, two kind of cascading I can

do, one is called parallel cascading, another is called series cascading. Now, in case of series, so a force is apply on spring one let say it is spring two – S 2 and this is spring one. So, force is apply on a spring two and it is extended, displace by x 1 or sorry x 2, let say it is x 2. And S 1 spring displace is x 1. So, total displacement is nothing but a x 1 plus x 2. Now, what is x 1, x 1 is nothing but a force by spring one stiffness; and x 2 is nothing but a force by spring two stiffness x 2.

So, if I put that this is nothing but a F by S 1 plus F by S 2. So, 1 by S 1, 1 by S 2, so it is nothing but a F into 1 by S 1 plus 1 by S 2. Now, if I put in F into C 1 equivalence, 1 by S 1 means C 1 and 1 by S 2 means, so it is nothing but a C 1 plus C 2. So, when it will be C, if I have 2 capacitance, one is C 1 and another is C 2. How it will be connected to get the equivalent capacitance, which is C 1 plus C 2. So, in that case, if it is connected in parallel, what will happen let say the 2 C 1 and C 2 are connected in parallel. What is the equivalent capacitance, C effective is C 1 plus C 2 because impedance is 1 by C 1 it 1 by C 2, so it will be 1 by C 1 by 1 by C 2. So, effective will be C 1 plus C 2. So, in that case, this is equivalent to this one. So, I can say if the two springs are connected in a series that means, it is nothing but a two equivalent circuit will be two electrical capacitance connected in series.

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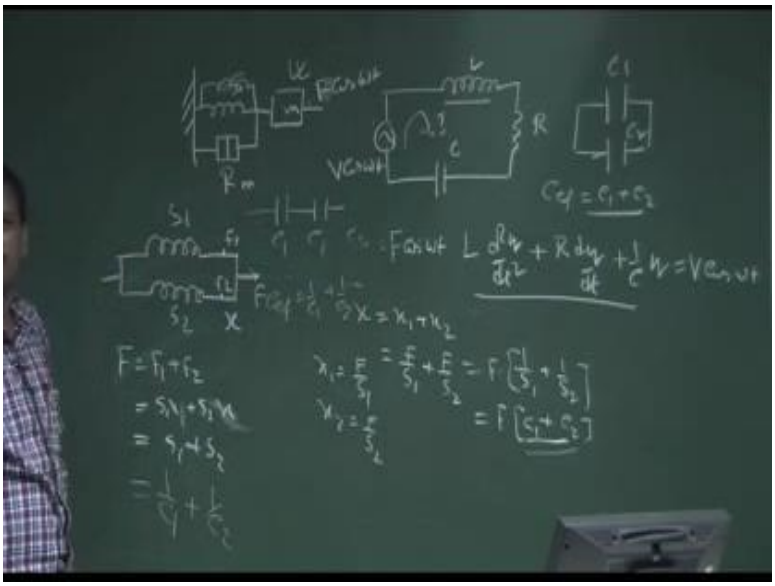

Similarly, if the two springs connected in parallel, this is the one spring and this is another spring connected in parallel. This is S 1; this is S 2; force is applied F. Then what

will happen, in that case, let us F is applied and displacement is x, total displacement is x. So, the total force is acting here is force in here plus force in here. So, if it is F 1 and if it is F 2, so F is equal to nothing but a F 1 plus F 2. Now, what is F 1? Spring into displacement, so it is nothing but S 1 x 1 plus S 2 x 2, it is nothing but S 1 x 1 sorry displacement is x, so x and x. So, it is nothing but a S 1 plus S 2; displacement will be x, so it is nothing but a S 1 plus S 2. Now, it is equivalent to 1 by C 1 plus 1 by C 2, because S is 1 by C, so 1 by C 1 plus 1 by C 2. So, when will be that capacitance 1 by C 1 plus 1 plus C 2, when they are connected in series? If it is C 1 and C 2, C effective is 1 plus C 1 plus 1 by C 2, so when they are connected in series.

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So, if the two springs are parallel, its equivalent circuit is two capacitances are in series which is C 1 and C 2. So, I can draw easily for any mechanical circuits to its electrical equivalent circuits. So, if I have a mechanical circuit is like this, let us do some exercise for that. Let us I have a mechanical circuit, so there is a mass, spring constant S, mechanical resistance R m and the force is applied in here. What is an equivalent electrical circuit? Always look for the u, total current in here – u, u is nothing but a current, current accurse mass, you have to look out that then though the equivalent circuits. So, I can see one or two exercise from the book, then I solve in here, so that you can easily conversion with that things.

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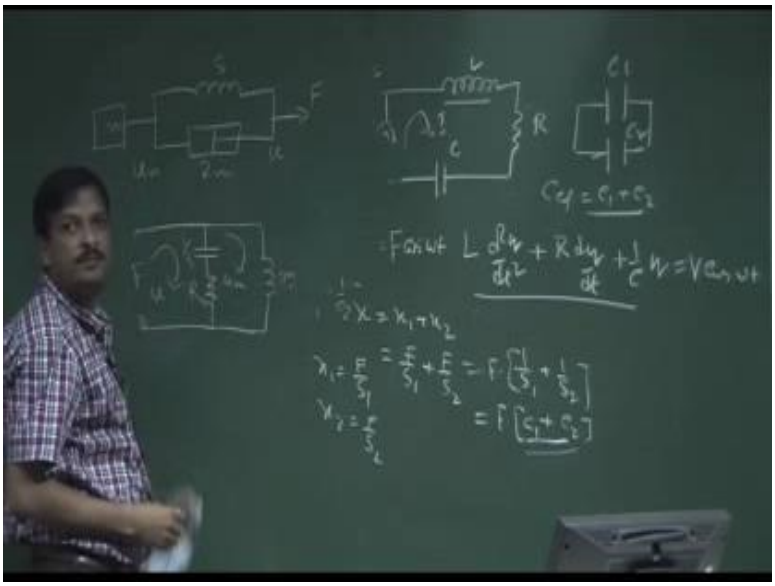

So, this is simple, which I have drawn; all are in series, all will be in series – resistance, capacitance then inductance then the voltage source. Now, instead of that if I give you the circuits' mechanical circuits like that let us m is here and it is connected to a mechanical resistance and a spring S, R m and a force is applied in here. Then what is the equivalent electrical circuit, if I told you to draw the equivalent electrical circuits, so that means, if the force is applied here, so the velocity in here is u, and velocity in here is u m. If I pull this circuit, so velocity on here it is u, and velocity on here is mass velocity, if the mass is acting on the velocity. The mass in motion, if the mass is motion, the motion of the mass is the particle velocity here.

So, now if I said this is my force; these two points I apply this force F between these two points. So, here I will go for a current u, particle velocity u, the current will flow like this way, so which will be expressed to the u, S and R m. So, what is S - capacitance, what is R m - resistance, and I have another current is u m here. So, u is what is u; m, L inductance. So, it is 1 by S, it is R m, it is m, so equivalent electrical circuits. Similarly, suppose, I have circuits - mechanical circuits like this, mass, spring, force – force is applied here. Similarly, I will say this is u, and this is u m, so this is the force within the two point force is applied here, so who see the u current, this is the u current spring see the u current; spring is nothing but a capacitance; and u m see will by the mass. So, this is m, this is 1 by S.

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Similarly, let us another example, another example is like this. Let us I have a ground, spring, then I have a mass m, this is S. This is S, this is m, and then I have mechanical impedance and I apply a force in here. So, if I see, the two system, let us this is one system and this is another system, so this will be u in here and this system let us u m is here. So, I can let us again you write the force, F then this is the u will be here acquires what resistance, draw. Then I get u m in two circuits, so there will be a capacitance and inductance L or m, 1 by S, R m, here is u will be implying, because this side is grounded so these two is one combined together.

Earlier, we said mass is different because mass is not supported by the ground, this side it is zero, because this is the grounded, so this total circuits is exposed to the u m current, so it is u m and this is u. So, this way you can solve any mechanical circuits into its equivalent electrical circuits. Similarly, if this is very easy to prove that the power average power, total power in the mechanical oscillation, which is e, it is nothing but a potential energy plus kinetic energy, which is same expression will come with the current problem V into I, same expression will come. So, power will also be the same.