Audio System Engineering

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Lecture – 03
Damped Oscillation and Forced Oscillation

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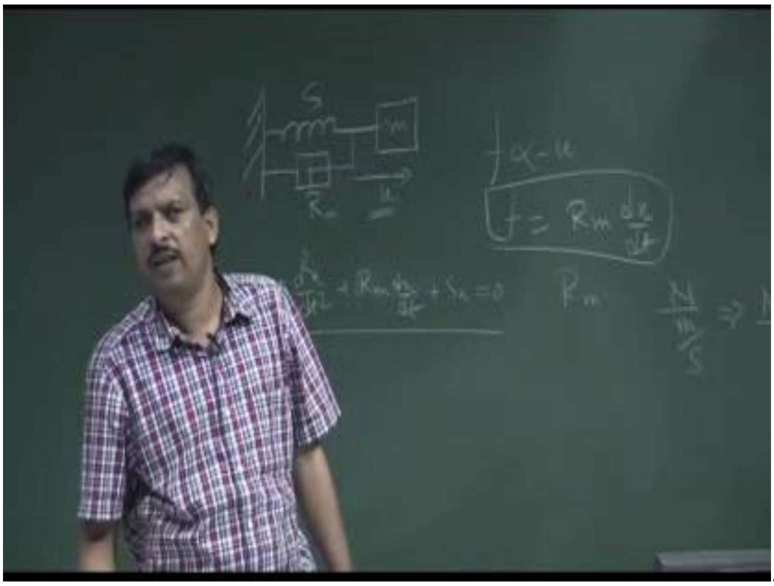

So, we have discussed about the simple harmonic motion is that means, there is no external force applied on that motion circuits. So, in that case, we draw the figure like this that I have a spring then I have a mass, and there is no external force is applied on that oscillation. Now, think about if that oscillator is oscillating, a friction force will acting on it. So, what will do, you say in simple harmonic motion, the amplitude of oscillation will remain constant throughout the oscillation.

Now, if you seen that a mass is oscillating, and if a friction force is applied then oscillation amplitude will gradually reducing, reducing, reducing and it will be stopped. So, that means, if I apply external force on that oscillation, I oppose the motion of the mass m, external force will be oppose the motion of the mass m. So, that means, that if this mass in motion is particle velocity is u, velocity is u then applied force it may be a friction force, it may be a viscous force for liquid, so whatever that applied force will be oppose the motion of the mass that is called damped oscillation. That means, as if I employing a mechanical resistance to that oscillation and mechanical resistance is called R m.

So, in this circuits, what I am doing, I oppose the oscillation; that means, I am apply as if I applying a mechanical resistance to that motion, so that that force which is applied to oppose the motion is directly stop that motion of that mass. So, that force is proportional to motion of the mass u, it will be negative, force is oppose the motion proportional to u. So, force that force is nothing but equal to mechanical resistance into the velocity. So, what is the velocity in term of displacement dx by dt.

So, now, earlier it is simple harmonic motion it is m into d 2 x by dt square plus S x is equal to 0. Now, I say there is a friction force acting on it and it creates a mechanical damping, so I put another term, which is R m d x dt plus S x is equal to 0. So, in case of damp oscillation m d 2 x by dt square plus R m into d x dt plus S x equal to 0, this kind of damping arrangement is use to oppose the motion. Can you give a practical real life example of this kind of things? Think about your suspension of your bike, why the bike required a suspension, why the car, if you ride on a car, every car has a suspension system why that suspension system is required, so why it is required.

Now, if you see that suppose you are travelling on a road, let us it is a horizontal force. Now suddenly road is not plain, so if there is any bump, it create a vertical force. So, if create a vertical force then what will happen whole bike system is a mechanical body, so it nothing but a spring mass system. So, if you feel a vertical direction oscillation, if I applied the force vertical direction and oscillation. So, if I feel a vertical direction oscillation what will happen by you will be move up and go down, move up and go down, so that is uncomfortable for your journey. So, what I required, I required a suspension system which can drive down that oscillation. So, mechanical damping that is why it is massive, suspension is nothing but mechanical damping that is why the picture is like that.

So, there may be a air suspension, there may be a spring suspension, there may be a liquid suspension. So, different kind of suspension system is there. Think about the landing of a plane, while the plane is landing on a runway, so plane is landing on a runway, once it touch the ground, it also get a reaction force towards the vertically up. So, plane is oscillating in the ground and vertically up, so that is why I required a real suspension system that can observe that vertical oscillation, so that is called plane landing suspension gear. If you see there is if the mass is high then the oscillation frequency will be high. So, you required to initially step down. So, your mass has to be the plane has huge mass; the suspension should have that capacity. Similarly, car similarly bike everybody has a suspension system, so that is nothing but mechanical damping.

Now, you have to know, so now equation of the motion of that oscillator, it is damped oscillator it changed, we have introduce another damp, which is called mechanical resistance which resist the motion of the mass. Now, if you see this is the equation, can you tell me what is the unit of the R m, what is the unit of R m, how do you express in R m, what is the unit of force – Newton, what is the unit of speed meter per second. So, it is nothing but a force by u. So, it is nothing but Newton by meter per second, so it is nothing but Newton second per meter. So, the unit of the R m mechanical resistance is nothing but Newton second per meter, so that will be unit.

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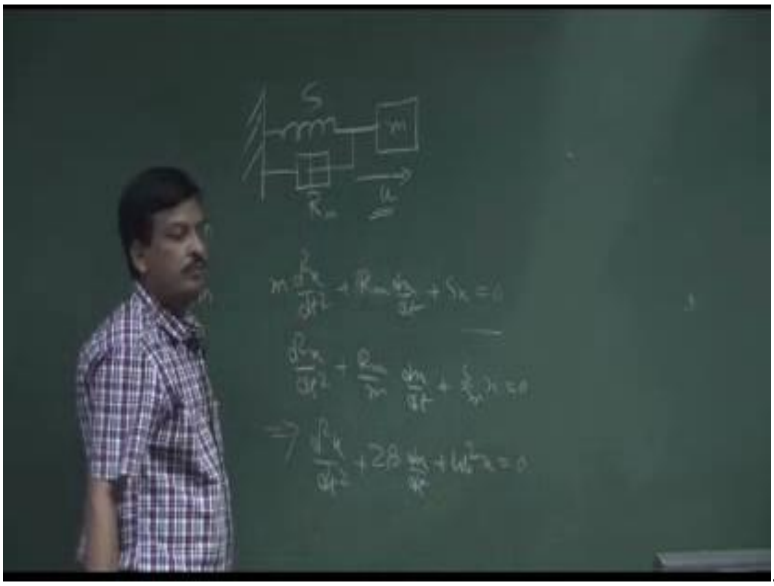

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Now, instead of same thing same solution principle I should apply, so it is also a second order differential equation. This damp oscillation is also a second order differential equation. Just first simplify it, so it is nothing but d 2 x by dt square plus R m by m dx dt plus S by m x is equal to 0. Same things I consider lets omega 0 is nothing but root over of S by m or S by m is nothing but omega 0 square. And beta another constant beta is nothing but R m by 2 m if I consider then this equation become d 2 x by dt square plus it is 2 beta R m by m is there. So, I can write 2 beta d x dt plus omega 0 square x is equal to 0. I just simply and I just put some constant. Now, this is also differential equation with the second order.

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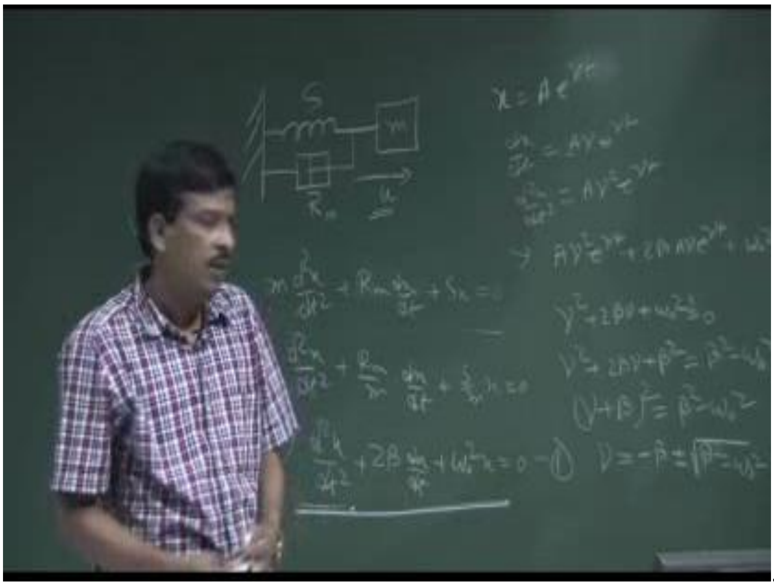

Let us the solution of the differential equation is x is nothing but A e to the power J omega 0 or let us not J omega 0 let us the solution is a exponential solution, the solution exist gamma t, this differential solution exist a solution which is x is nothing but A e to the power gamma t. So, I have to find out the value of gamma and A. Now, if this is the solution it supports this equation. So, what is d x dt A gamma e to the power gamma t? What is d 2 x dt square A gamma square e to the power gamma t. Now, if I put this equation this things in this equation - equation number 1, what I will get d 2 x by dt square. So, it is A gamma square e to the power gamma t 2 beta d x dt plus 2 beta A gamma e to the power gamma t plus omega 0 square x is nothing but A e to the power gamma t is equal to 0. So, A e to the power gamma t is common to all.

So, I can write gamma square plus 2 beta gamma plus omega 0 square is equal to 0. This term is 0, and then A e to the power gamma t is equal to 0. So, if this term is 0, what is the solution of the gamma, I can write gamma square plus 2 beta gamma plus beta square is equal to beta square minus omega 0 square, I add beta square this side, this side I add beta square and omega 0 is this side minus omega 0 this side. So, it is nothing but gamma plus beta whole square beta square minus omega 0 square. So, gamma is nothing but minus beta plus minus root over of beta square minus omega 0 square, simply, this is the gamma.

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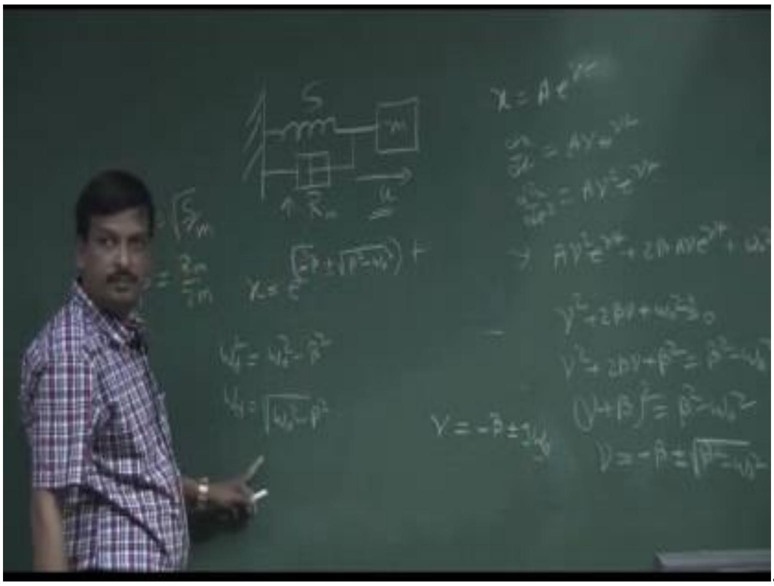.

So, I can say the gamma is in the form of gamma is nothing but minus beta. So, x is e to the power minus beta plus minus root over of beta square omega 0 square into t. Now, let us consider that omega d square is equal to omega 0 square minus beta square; that means, if I apply a damping circuit the modified natural frequency is natural and the natural resonance frequency the when the damping circuit is not there. So, omega 0 root, over of S by m where the damping circuit is not there; omega 0 is the natural angular frequency when the damping circuit is not there if omega d is the new damping new natural resonance frequency is nothing but old natural resonance frequency minus beta square. So, I can write omega d is nothing but root over of omega 0 square minus beta square. So, if I put the things in the gamma. So, gamma is nothing but minus beta plus minus J omega d minus sign will common. So, the root over of minus 1 means j, so plus minus J omega d.

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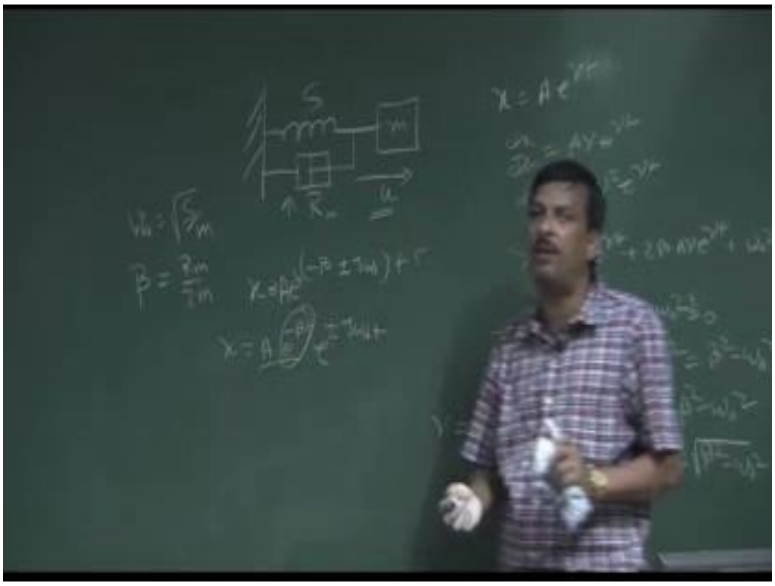

So, I can write that x is nothing but e to the power minus beta plus minus J omega d into t. So, A will be there, A is there put the gamma value. So, x is A into e to the power minus beta into e to the power plus minus J omega dt, think about that sorry beta t, see that amplitude part is multiplied by negative exponential term. So, what is that? Meaning if I negative exponential what is the negative exponential term; it will be gradually degrade down.

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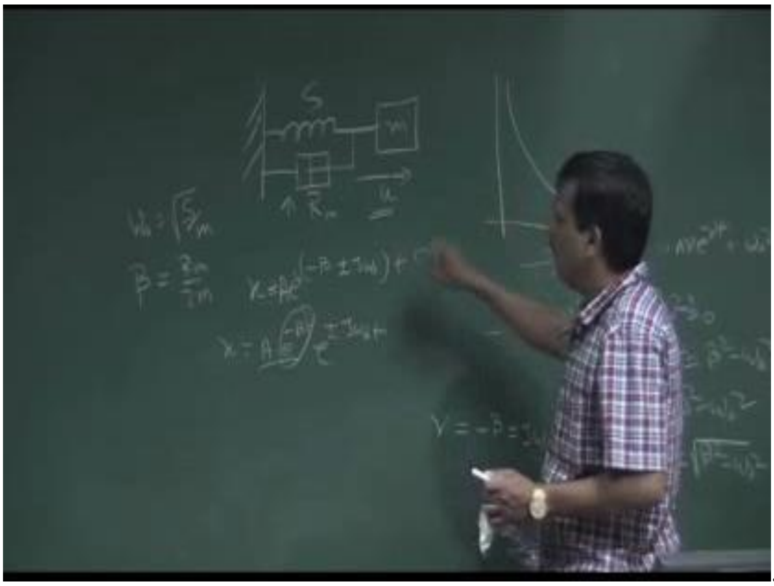

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The e to the power minus A, it is nothing but this one. So, this is t. So, if beta value depending on the beta value it will be gradually damping down. So, the amplitude will be reducing over the time. So, amplitude is not constant that is why it is called damped term oscillation. So, beta is nothing but damping factor depending on the beta how quickly it will amplitude will be dry down depend on that that things. So, now you have another term terminology is there which is called characteristics time. What is characteristic time it is defined that if an oscillation has amplitude A, damped solution initial amplitude is A, it is the time required to reduce that amplitude by 1 by e factors.

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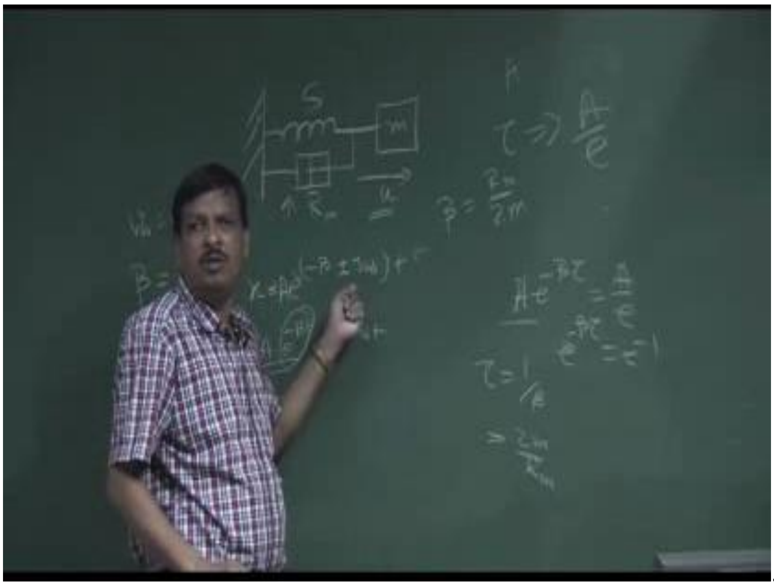
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So that means, if let us tau time is let us say I have an amplitude A and tau time is taken to reduce the amplitude by A by e that is tau is called characteristic time or relaxation time. Now, then if it is what is amplitude lets A e to the power minus t equal to tau, it is minus beta tau that means, if the amplitude is A this factor must be equal to A by as per the definition, so A, A cancelled. So, minus beta tau e to the minus power beta tau is nothing but e to the power minus 1. So, tau is nothing but 1 by; so characteristic time is defined as 1 by beta. Now if the beta is the beta is what beta is nothing but R m by 2 m. So, it is nothing but 2 m by R m.

Now, think about it. If mechanical resistance is increases tau will be decreases; that means, if my suspension system is very good then very quickly amplitude will be reduce. If my suspension system is not that good it requires time to reduce the amplitude. So, depending on the value of beta, beta is R m by 2 m e to the power minus beta t. So, if the beta value is very large quickly decrease, beta value is very small require time to reduce the amplitude. So, depending on the beta value, I have a condition whether it is over damped or under damped. So, there is condition called over damped, under damped and critical damped; you heard about that things I am not going details on that over damped, under damped and critically damped. Only in case of just physical meaning is that if depending on the beta value half the damp oscillation will be it depends. So, if the beta is very high, amplitude will be very quickly fall down; if beta is very low, it takes a longer time to fall the amplitude.

So, depending on the first one that is why BMW car and normal car suspension system is different that is why if you ride on a BMW car you cannot feel there is a road or there is a bump, because in that case the suspension system is very good. On general car, suspension system may not be that good that is why we feel jerking on that car. If you heard about the tractor, have you ride on the tractors there is no suspending system only the driver sheet suspending system, if you ride in a tractor jerk will be very high. I am not going that details of under damp and over damp condition.

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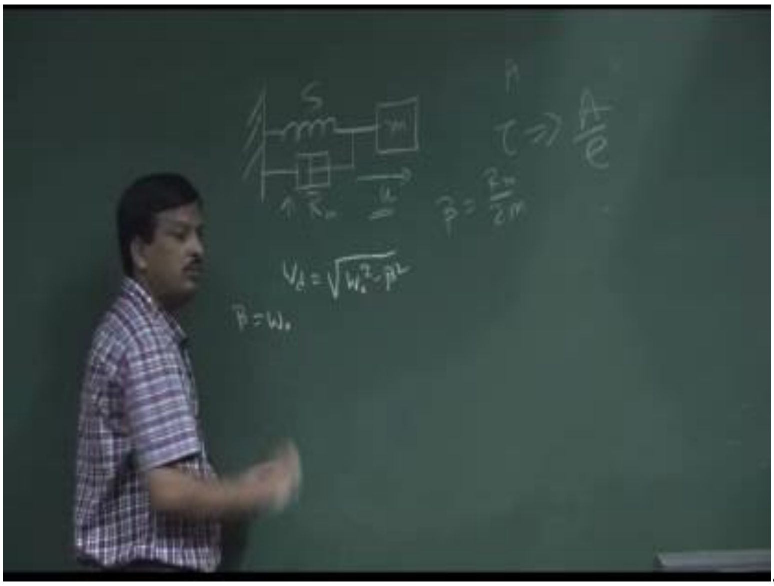

So, if I say the system is critically damped in that case if it is critically damped that means beta omega d is equal to root over of omega 0 square minus beta square. If I say it is a critically damped then the beta value is equal to or sorry omega d is equal to omega d is equal to sorry.

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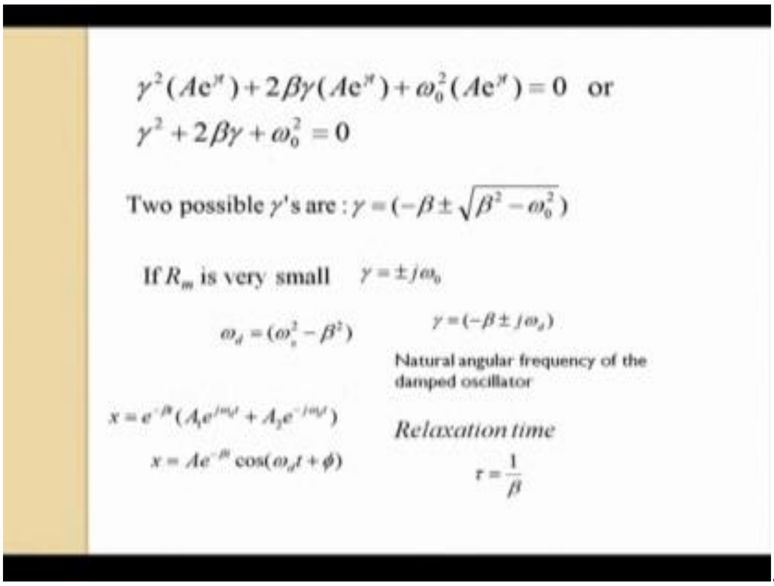

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So, in case of critically damped what should be the condition. We said the beta is equal to nothing but omega 0. Because in case of beta is equal to omega 0 then omega d is equal to 0. So, there is no damped oscillation, so that is why it is called critically damp, similarly there will be an over damp and there will be a under damp condition.

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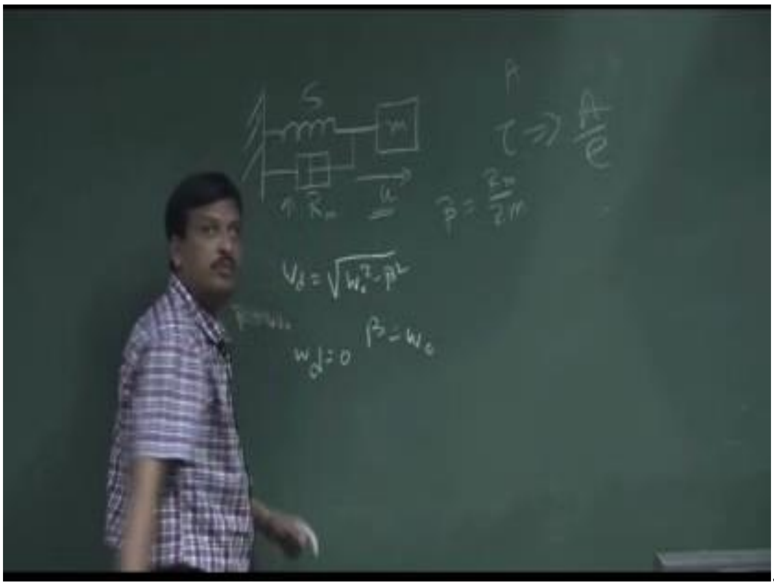

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Now, we go for another thing this is the damped oscillation this is now think about that our requirement any system any system mechanical system is not a very good very simple harmonic system it has a damping. So, I have a damping oscillation system, how it will be oscillated by external force, think about the loudspeaker. Loudspeaker has, you see the loudspeaker has a membrane, the loudspeaker has a diaphragm is nothing but the mechanical system, it is suspended on a gage, and it has a tension, it has a mass and it is the damping factor also.

Now, if I apply electrical circuit I wants that membrane should be vibrate as per the electrical signal frequency, so that is called host oscillation. I have a mechanical oscillatory circuit; I apply a force on that oscillation, so that that oscillation can give me a constant oscillation as per the force frequency. So, if you see really in this host oscillation, we have two part; initially there is no external force, no oscillation system in rest condition. When I apply a force then in there it cannot be oscillate as per the force frequency.

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So, there should be a transitory portion from rest position to where it will be steady state then act force frequency it will be oscillate. So, to reach that steady state I required the transitory portion. So, transitory portion force oscillation is two parts - transitory portion and steady state portion. So, at steady state portion, when I drive the mechanical loudspeaker, loud speaker is in the rest condition; initially then in there it cannot go to that oscillation which is has applied force oscillation. So, it required some time to reach the steady state. Once it reaches the steady state, the oscillation of that mechanical system will be act for the frequency of the applied force that is I want. So, force oscillation has a two solution one is called transitory part another one is called steady state part.

So, now, what kind of transitory part the transitory part is the same solution which I applied in that force the force is not there, and I get that solution. Now, think about the stead state which is called force oscillation that means if I want a diaphragm should oscillate as per my applied electrical signal frequency. So, what I said in force oscillation that this oscillator at steady state will be oscillate as per the applied force frequency.

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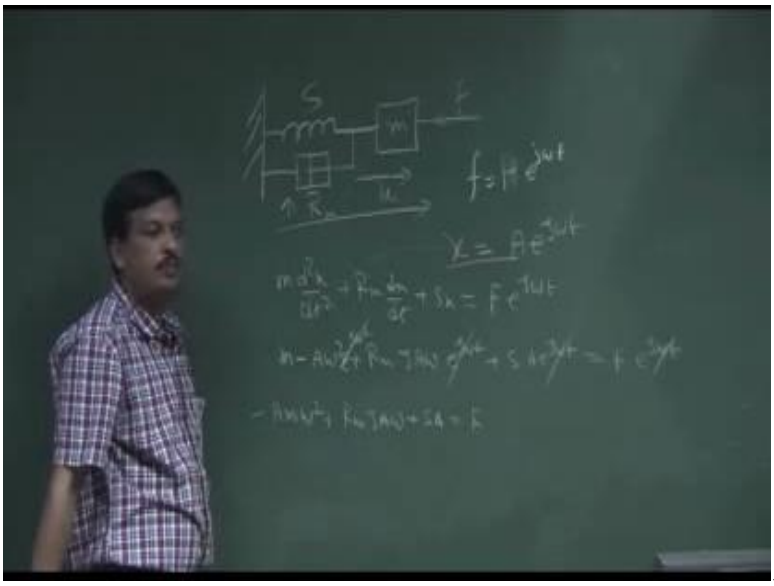

So, let force F is nothing but A e to the power J omega t applied force I can say or instead of A I write capital F, capital F is the amplitude e to the power J omega t; omega is the frequency of the applied force. I said in steady state, the oscillator also oscillate in the same frequency. So, I can write the x is nothing but A e to the power J omega t; A is the displacement amplitude, and omega is the frequency of the oscillation. Now, what is the equation of oscillation circuits? It is nothing but m d 2 x by dt square plus R m dx dt plus S x is nothing but equal to the applied force F e to the power J omega t on that steady state condition.

Now, if I put the value of x in here, so it is nothing but m minus a omega square d 2 x dt square I can calculate from here plus R m J A omega sorry into e to the power J omega t e to the power J omega t plus S into A e to the power J omega t is equal to F into e to the power J omega t it is ok or not, ok. Now e to the power J omega t, e to the power J omega t will be cancelled, all are there. So, it is minus A m omega square plus R m J a omega plus S a is equal to F, it is nothing but F.

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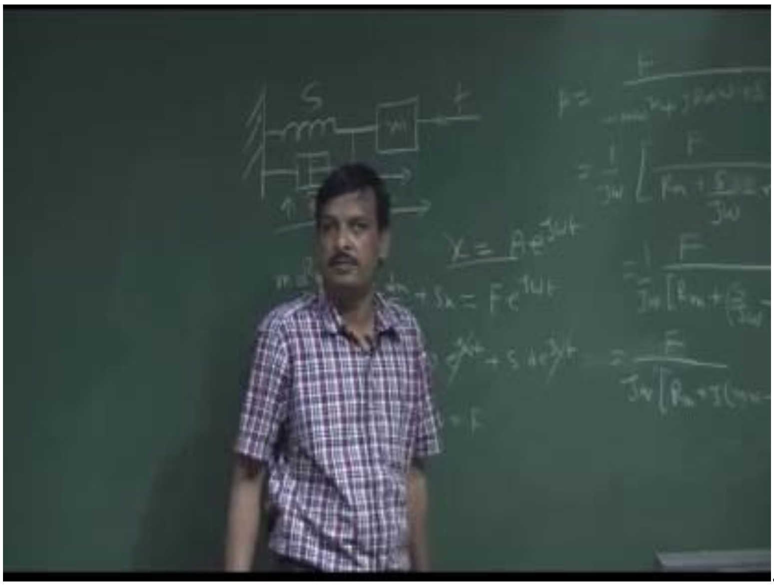

So, if it is that then what is the value of A, A will be F by minus m omega square minus m omega square plus J R m omega J R m into omega plus S is it plus S. Now, if I simplify that thing let us take the J omega is outside then it will be F by I taken J omega outside. So, it will be R m will be J omega plus S will be S J omega S into or S by J omega S by sorry it will be S by J omega minus m by 1 omega square m minus square by J omega.

So, one omega will be cancelled. So, it is nothing but a F by lets I have not a J omega R m plus S by J omega minus m omega by J ok or not because 1 omega is cancelled. So, A omega will be there by J will be there. Now, since the J, I have to if I take that J is outside, so it is nothing but F by J omega R m plus lets J is outside. In case of J is outside, so I multiply both in j. So, J square will be minus 1 and here this going to be minus 1. So, it will be plus 1. So, in case of plus 1, so it is m omega minus S by multiplying J and this J square J, J square is nothing but minus 1. So, it is F by J omega R m plus J omega into m.

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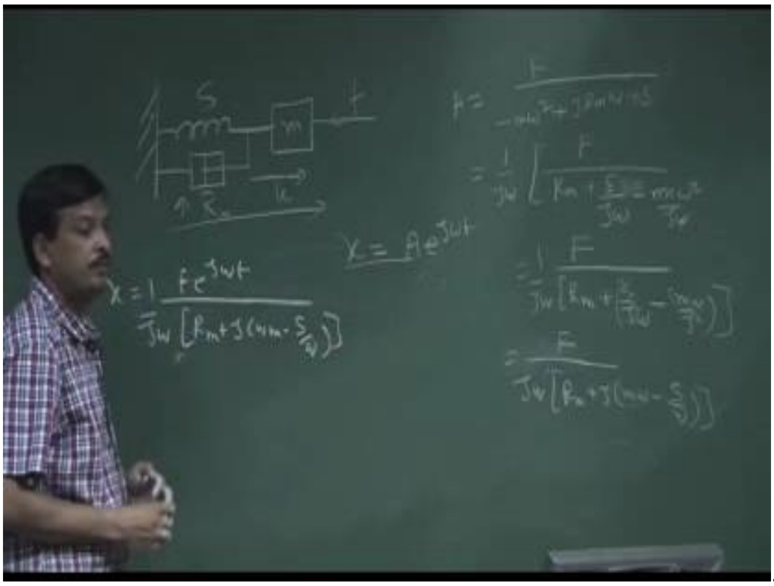

Now, what is the x then, so x is nothing but A into e to the power J omega t. So, x is F into e to the power J omega t divided by 1 by J omega into R m plus J omega m minus S by omega.

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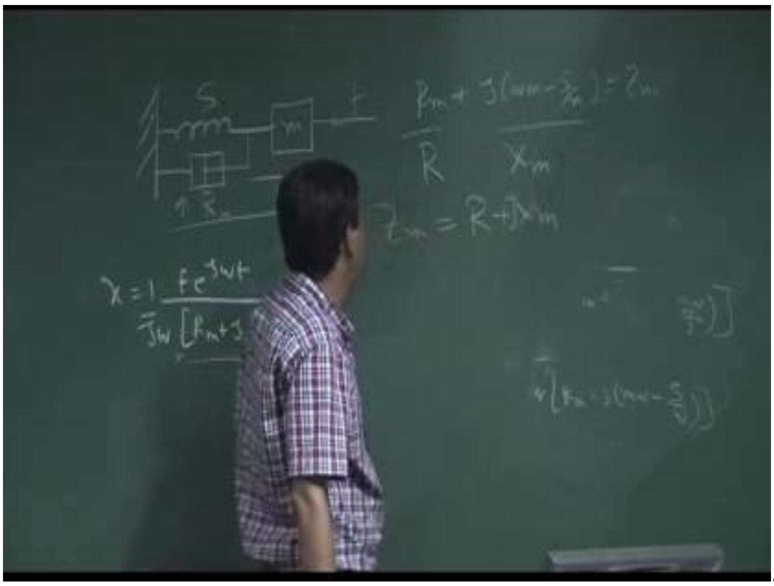

Let consider this one, let consider this part, this R m plus this part. So, this is R m plus J omega m minus S by omega let consider this is nothing but Z m which is called mechanical impedance. Z m is nothing but mechanical impedance which has resistive term mechanical resistance and has a mechanical resistive term mechanical resistance or mechanical reactions. So, total impedance resistance plus J term. So, resistance is real part, it is the imaginary part. So, I can say this is lets R and this is X m. So, I can write Z m is nothing but R plus J X m, where X m is nothing but m omega minus S by omega is it ok, now this is the Z m.

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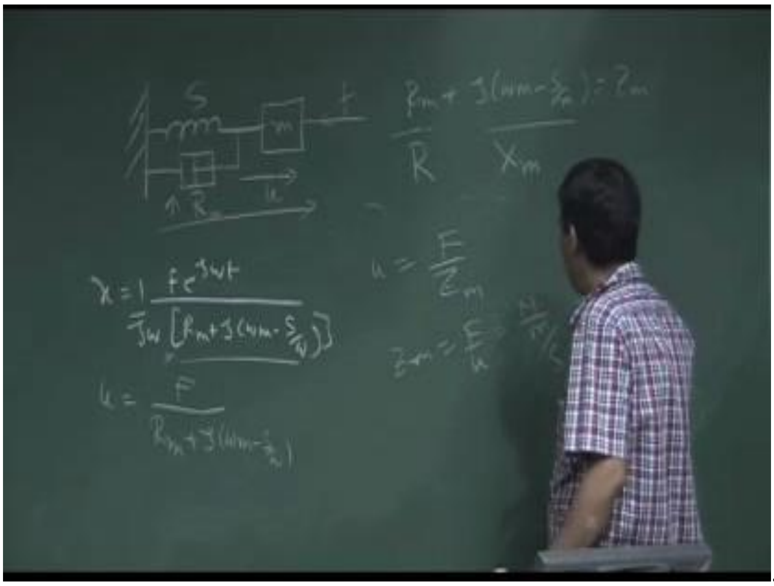.

Now, then what is u this is x, then what is u, u is nothing but dx by dt. If I dx by dt J omega will be come here. So, J omega J omega will be cancelled. So, it is nothing but F by R m plus J into omega m minus S by omega these is, or not think about it what is this. So, u is nothing but F by it is nothing but Z m F by Z m. So, Z m is nothing but a so if it is mechanical impedance, what is the unit of the mechanical impedance F by u force by force by velocity. So, it is nothing but a Newton by meter per second. So, mechanical impedance is nothing but applied force divided by the particle velocity, velocity or motion of that oscillator.

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Now, I can write Z in different form also that is should have understand, I can in the Z in different form. So, I can write if Z is nothing but R plus J x m R plus J x m. So, instead of that I can write there is a amplitude part mod Z which is nothing but root over of R square plus x m square and or my Z m I should write Z m, mode Z m. And angle theta lets I have an angle theta. So, theta is nothing but a tan inverse x m by R, R is nothing but R m. So, it is nothing but tan inverse x m is omega m minus S by omega divided by R m and it is nothing but a R m square plus m omega minus S by omega whole square. So, if I know a mechanical system R m, S and m, I can find out the mechanical impedance amplitude of the mechanical impedance, again I can find out the angle of the mechanical impedance. So, angle means force and velocity has an angle which is theta.

So, theta is nothing but a tan inverse this thing. So, I can write Z m instead of writing Z m is mod of Z m into e to the power e to the power J theta e to the power theta is nothing but a tan inverse x m by R m x m by R m. Now, if it is that then what is x I said x is nothing but a F by 1 by J omega Z m into e to the power J omega t this is x 1 by J omega F into e to the power J omega t divided by Z m. So, instead of Z m, I can write J omega F e to the power J omega t divided by mod Z m into e to the power J theta mod of Z m e to the power J theta. So, this is nothing but 1 by J omega e to the power F into e to the power J omega sorry e to the power J omega t minus theta divided by mod of Z m. Now, e to the power J theta cos theta plus J sin theta, I can express this term 1 by J omega into F into cos omega t minus theta plus J sin omega t minus theta divided by mod of Z m.

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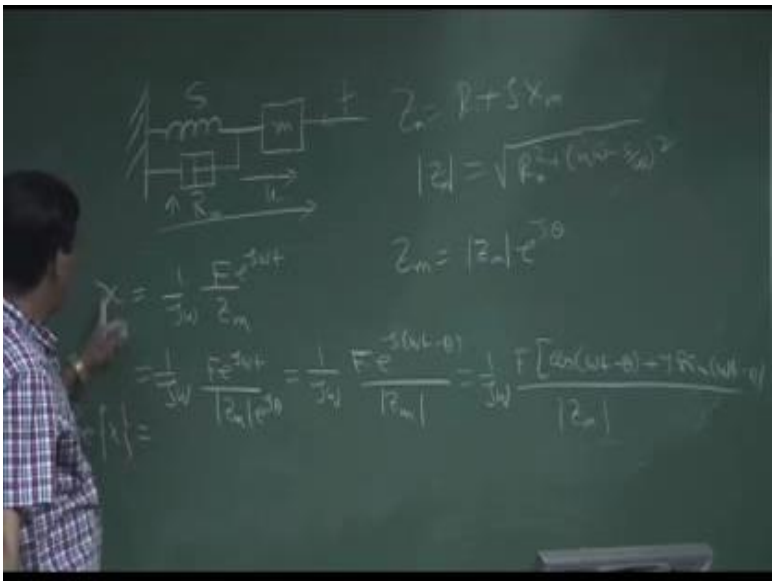

Now, if I say what is the real part of x. Real part of x, if I want to find out real part of x this will be imaginary part then and the second sin term will be the real part J, J cancelled sin term will be real part. So, real part is nothing but F by mod of Z m into omega 1 by J omega J, J cancelled omega will be there, so into omega sin omega t minus beta. So, this is the real part of x. So, what is the real part of the u? Similarly, real part of the u I can also find out real part of u from the u equation is nothing but F by mod of Z m into cos omega t minus theta. So, I have I can take the u, u equation I take the real part e to the power omega t minus theta. So, sin displacement is in sin function velocity is in cos function. Now, if I want to find out what is my intension is to find out the power if it is force oscillation what is the what power it is delivered. So, if I applied a force F, how much power it is delivered by the oscillator that is why this is force oscillation.

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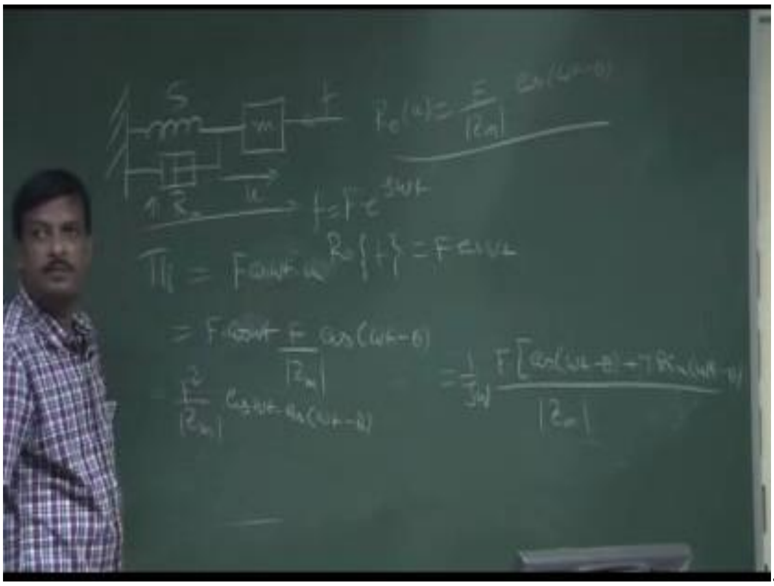

So, I can say lets instantaneous power is pi i any movement power is nothing but a force multiply by the speed is nothing but a displacement per unit time. So, any movement what is the force acting on the oscillator force multiply by displacement work done power is force into the displacement of power. So, it is speed multiply by the applied force. So, I can say that it is nothing but force F e to the power J omega t multiply by u - speed. So, what is speed, F e to the power J omega t, what is the real part of the speed F by mod of Z m cos omega t minus theta. So, it is nothing but F square or what is the real part of the force instead of e to the power F is e to the power J omega t F force is nothing but what is the real part of F real part of F is nothing but F cos omega t. So, instead of complex part I should write cos omega t into u. So, it will be cos omega t.

Real force multiply by the real speed is the power - instantaneous power. So, power instantaneous power is F square by Z m into cos omega t into cos omega t minus theta minus theta. So, this is the instantaneous power.

Now, instantaneous power in real life application, we are not talk about the instantaneous power, instantaneous power I do not know in which time at anytime the instantaneous power. So, we said the term called average power, we said the term called average power. Then what is the average power how to calculate the average power. What do you mean by average power? Suppose, I have a sin wave, I want to calculate the average energy. So, I said since it is a periodic within a one period how much power is there divided by the length of the period is the average power.

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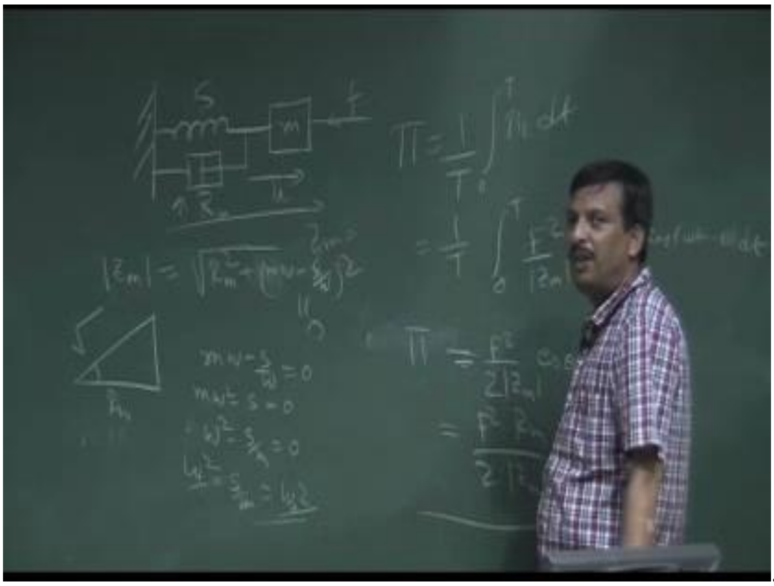

So, I can say that the average power pi is nothing but a integration over 0 to T is the period divided by 1 by T integration over the period of what of the instantaneous power pi i dt. So, if it is instantaneous energy is pi i then average energy is nothing but integration of that pi i over a time period of 0 to T divided by the total time period. So, if I do that integration putting the value 1 by T 0 to T F square by mod of Z m cos omega t into cos omega t minus theta dt. If I do this integration, I will get F square by 2 Z m cos theta cos omega t minus theta cos a minus cos b expand it and do the integration. you will get F square by 2 Z m mod of Z m cos theta. Same as if you see if I apply a voltage the current, total power is V I cos theta theta is the cos theta is the power factor angle between the voltage and current.

Here also if I apply a force F power is force multiply by the velocity, and power is nothing but V I same F into u into cos theta, cos theta is nothing but a as a power factor. So, theta is the angle between the applied force and the velocity in case of current theta angle between the voltage and current. Now, if it is cos theta then what is the value of the cos theta, if you know what is theta, theta is nothing but a tan inverse x m by that things. So, I can say that if it is mod Z m is nothing but a root over of R m square plus that x m square we have x m is nothing but a omega m minus S by omega, and theta is tan inverse that things. So, theta is nothing but a angle between the force and velocity and which has a hypotenuse that hypotenuse is this one and base is nothing but a R m. So, cos theta is nothing but a R m by mod of Z m R m by mod of Z m. So, if it is R m by mod of Z m then power total power average power is equal to F square by 2 Z m cos theta. So, I can put the value of cos theta F square by F square into R m by 2 mod of Z m square R m by Z m mod of Z m square.

Now, see that equation, F square into R m by 2 Z m square is the equation of average power. When will be the average power maximum, when Z m will be minimum. If Z m is minimum, average power will be maximum or mod of Z m will be minimum then the evaluate power will be maximum. What is mod of Z m R m square minus m omega minus S by omega whole square? So, this term I cannot minimize this term. So, if this term is equal to 0 then I can say Z m is minimized is the term of omega. So, a certain omega if this term is equal to 0 then I can say Z m is minimize. So, Z m will be minimum and this term is equal to 0. When this term will be 0, m by lets m omega minus S by omega is equal to 0 whether the condition I want. So, it is nothing but m omega square minus S is equal to 0 or a omega square minus S by m is equal to 0 or omega square is equal to S by m which is nothing but a omega 0 square.

So, if the applied force frequency is equal to the resonance frequency of the circuits then the resonance will be happen and power will be maximum. So, Z m will be maximum, Z m will be minimum when x m is equal to 0. So, when x m will be 0, at if the applied force frequency is coinciding with the resonance frequency of the natural resonance frequency of circuits then it delivers the maximum power. So, I get the average power will be maximum, this is called mechanical resonance, which will be discuss in the next class what is mechanical resonance.