Audio System Engineering
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Lecture – 02
Fundamentals of Linear Vibrations

Good Morning. So, as I mentioned in the first class that I should cover that of the sound wave is generated and then sound transmission. So, next few lectures will be on sound generation and sound transmission. Now, if I say what is sound now, I have already said the sound is generated through a mechanical vibration of a medium, may be solid, may be liquid or may be air, I do not know, there any medium. So, any medium, if the medium is vibrating, a sound is generated. Now, sound has two properties you heard about that. Sound means which is, I perceived. May not be, human perception is 10 per for 20 hertz to 20 kilo hertz. If the sound wave frequency are lie between the 20 hertz to 20 kilo hertz then I said it is audible sound by human being; may be after 20 kilo hertz, sound may be perceive by some other animals, I do not know that, may be perceive by some other animals, yes, it is perceive by some other animals. So, that means, as a human being, I should it is audible sound if it is 20 hertz to 20 kilo hertz. So, here I know frequency of vibration, so that is I will come how do know the frequency of vibration.

Now, below the 20 hertz, sound wave after 20 hertz, 20 kilo hertz, what is that call, what is the sound wave call supersonic, not a supersonic actually, ultrasonic; that means, I cannot heard that sound, but sound is vibrating mechanical body is vibrating. So, whether it is audible or not, let us we do not care. We said any mechanical vibration create sound. It may be audible, if it is vibration frequency 20 hertz to 20 kilo hertz, it is audible. If it is above, it is not audible; if it is below, it is not audible, but sound always there and energy is transmitted. And then you said that that vibration creates the movement of the particle and that particle movement transmits that energy from one point to another point.

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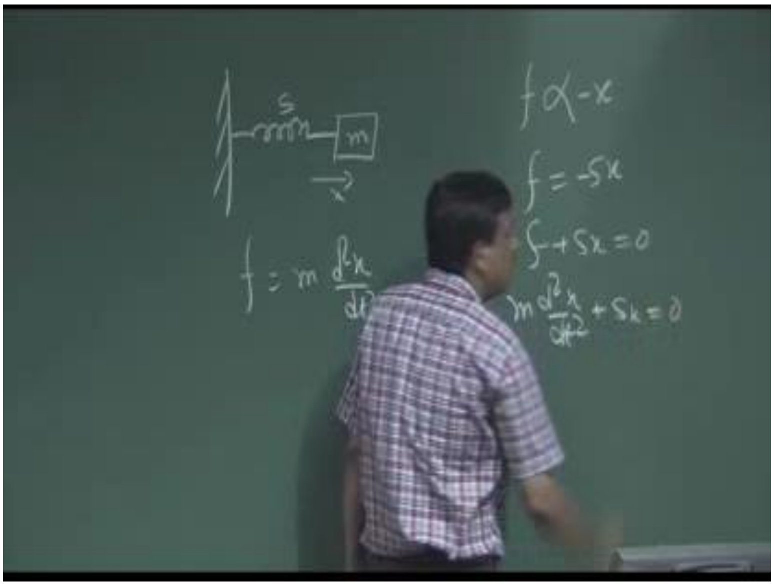

So, let us first start with that vibration. What is the vibration? We said, if I strike this board, the sound is generated. How it is generated because of mechanical vibration. So, what do we mean by mechanical vibration. You know simple harmonic motion everybody know the simple harmonic motion; that means, let us consider any mechanical things it is nothing but the spring mass system; that means, a spring connected through the mass m; s, s is stiffness of the spring constant of the spring, m is the mass attached with this spring. So, any vibration I can say is mechanical oscillation. A mechanical, a simple mechanical oscillator is nothing but a spring connected with the mass.

Now, as per the Hooke's law says that if I apply a force or if I deform the spring, spring started oscillation, so that force is nothing but the proportional to displacement of the mass; m is displaced due to this spring, so m is displaced if x is there. So, if I move that mass this side and release it, it will vibrate. So, mechanical vibration will be happen; that means, a mass will be displaced in this way. And Hooke's law said if there is a no external force applied in this system; that means, there is no friction, there is no damping, no friction means - no damping. So, there is no external force on that system then it is creating the oscillation will be sustained, this mechanical oscillator will be oscillated with constant amplitude with constant amplitude, and oscillation will not be (Refer Time: 05:04). So, this oscillation is called simple harmonic motion.

So, this x - the displacement of the mass follows a simple harmonic motion; that means, displacement of the mass should be in the form of a sinusoidal function, so cos omega t. What is the omega? Omega is nothing but a frequency so that is why it is called simple harmonic motion. So, the displacement, so they look likes a simple harmonic motion. So, it has a frequency and it has constant amplitude, there is no external force applied to the system. As per the Hooke's law, Hooke's law said force is proportional to displacement; and this is – opposite direction of the applied force so that is why it is f is proportional to minus x. So, we said f is equal to s x, so f plus s x should be 0.

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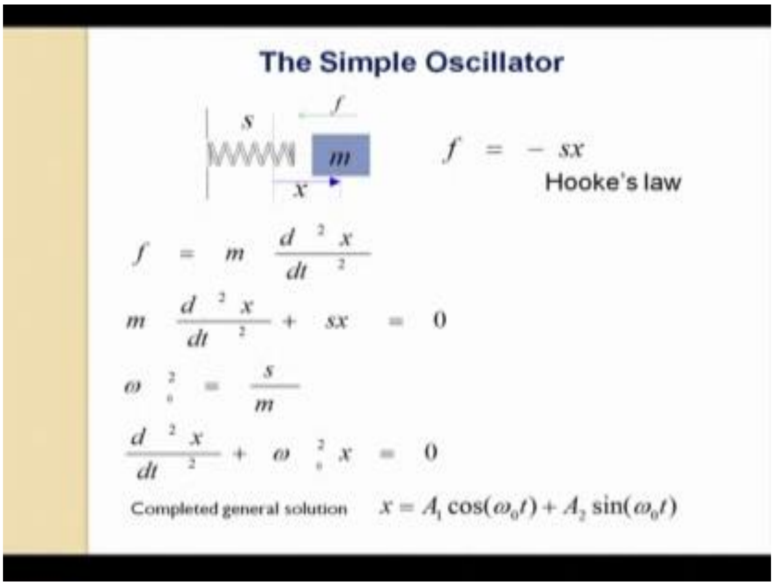

So, what is force? As per the Newton's second law of motion, a force is nothing but a mass into acceleration. So, what is acceleration, if it is x is a displacement, so acceleration is d 2 x by dt square. So, this equation become m d 2 x by dt square plus s x is equal to 0. So, d 2 x by dt square plus s by m x is equal to 0.

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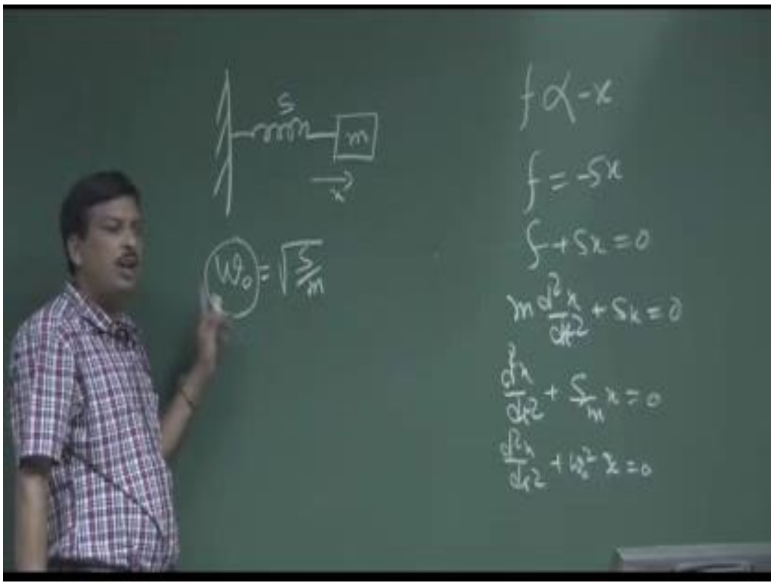

Now, if I consider that s by m lets omega 0 is equal to root over of s by m. Then this equation is nothing but d 2 x by dt square plus omega 0 square x is equal to 0. This omega 0 is called the natural resonance frequency of that simple oscillator that means, if I applied a force here if the oscillator on oscillated. And if the mass m and stiffness is s, natural resonance frequency of this mass system is root over of s by m. So, if I increase m, what is the physical significance, if I increase m, omega 0 will be decrease; if I increase s, omega 0 will be increase. So, if you see, if the tension of the spring is very hard or spring constant is very high, if you strike it, it produces a sound, which is in high frequency. Now, if you now increase the mass of the spring, then and there, it produces the low frequency.

Similarly, just think about the membrane vibration kind of things. If you see the drum – huge drum, if you strike the drum, and if you see the small drum, if you would strike the drum, if the tension on the both the drum is same, what will happen, large drum is produce a low frequency, small drum produce a high frequency. If you see the dig tabla, tabla is large membrane, dig is small dig is large membrane, tabla is small membrane. If you strike here, what frequency I heard, and if you strike here, if you strike here high frequency will be high because its area is very small very small and mass is small, and here mass is high, so a low frequency will high.

So, if I increase the mass, vibration frequency will be decreases; if I increase the stiffness, frequency will be increase, you know that things. So, that means, a huge microphone may not be able to produce the high frequency, so that will understand, that way that will say here why it is not produce the high frequency and what are the extra arrangements we should made, so those things will done in the microphone design. So, actually we should know physically meaning of that natural resonance frequency increase mass, natural frequency low increase stiffness frequency will high.

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Now, forget about this part. Now, third component here, now this is the second order differential equation. This is nothing but the second order differential equation. So, if I want to solve this second order differential equation, so I can say there is general solution of this differential equation for the displacement x is nothing but a A 1 cos omega 0 t plus A 2 sin omega 0 t. This is the complete solution of the differential equation. In cosine term, or A 1 and A 2 are the arbitrary constant, we do not know the value of A 1 and A 2 of the arbitrary constant. How do we get the arbitrary constant value by putting the initial condition? What is initial condition? I said at t equal to 0 x is nothing but a x 0 and u is nothing but a u 0. What is u, u 0 is the, u is the particle velocity. If displacement is x, d x d t is the particle velocity motion or velocity. So, velocity is nothing but a d x d t. So, at t equal to 0, I said in this system as initial displacement of x 0 and as initial speed is u 0.

Then if I put those things in this equation, what I will get. So, at t equal to 0, x is nothing but a x 0, so I can say x 0 is nothing but A 1. I put that t equal to 0 cos 0 - 1, sin 0 – 0, so A 1 is nothing but a x 0. Now, if I take the cos theta derivative of this equation, say u is nothing but a d x by d t. So, it is nothing but a A 1 sin omega 0 and omega 0 will be come here and it will be minus cos differentiation then A 2 omega 0 cos omega 0 t, differentiation of sin omega 0 t.

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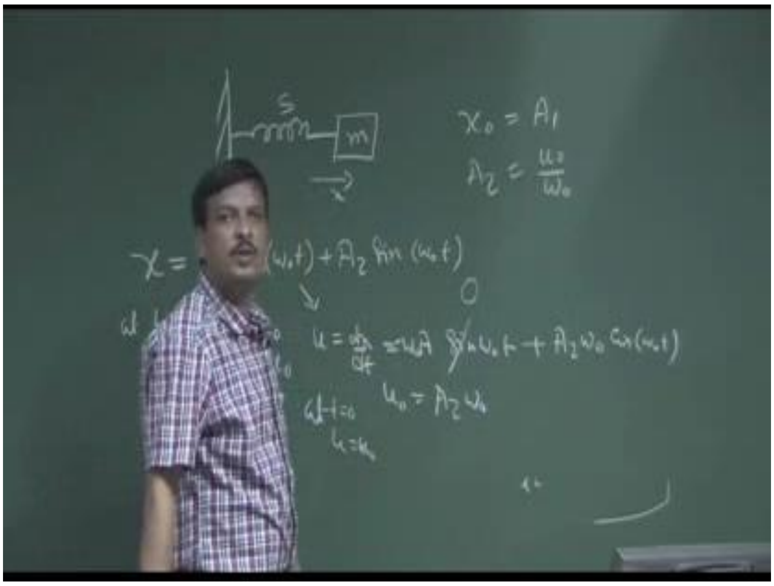

Now, at t equal to 0, u is nothing but a u 0; so if it is u 0 omega 0 t equal to 0, so this term will be 0, sin omega t zero, sin 0 – 0. And this term will be exist, so A 2 omega 0. So, A 2 is nothing but a u 0 by omega 0.

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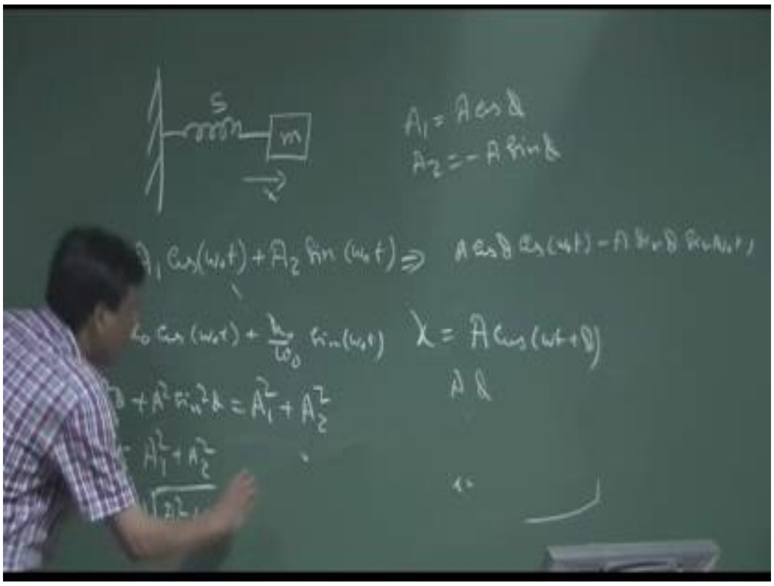

So, I get the value of A 1 and A 2. If I put that value in here, what I will get x is nothing but a x 0 cos omega 0 t plus u 0 by omega 0 sin omega 0 t. So, this is the solution of that equation. Any doubt? Now, I want to write these equation in a another form, because if I write A 1 and A two, we always heard about something A cos x 0 to A cos omega t something. So, I want to write that way that way in this equation. Let us consider A 1 is nothing but a A cos phi, A and phi are arbitrary constant, I can write. And A 2 is nothing but a minus A sin phi. So, I have to know only the A and phi from the initial condition. Same equation, I can write A cos phi cos omega 0 t minus A sin phi sin omega 0 t. I just put the A 1 and A 2 value - cos A cos B minus sin A sin B, cos of A plus B. So, I can write x is nothing but a A cos omega t plus phi or not, . Now, then from the initial condition, I know the value of A 1 and A 2 same initial condition, I should get the value of A and phi.

Now, if I see A 1 and A two, A cos phi A sin phi. Now, what is x 0, if I take the square, so I say A 1 square? So, if I want to say that let us I write A square cos square phi plus A square sin square phi is nothing but a A 1 square plus A 2 square; value of A 1, A 2, I take the square only. Now, if I say A square is nothing but a A 1 square plus A 2 square, because cos square theta plus sin square theta is equal to one. So, A is nothing but a root over of A 1 square plus A 2 square.

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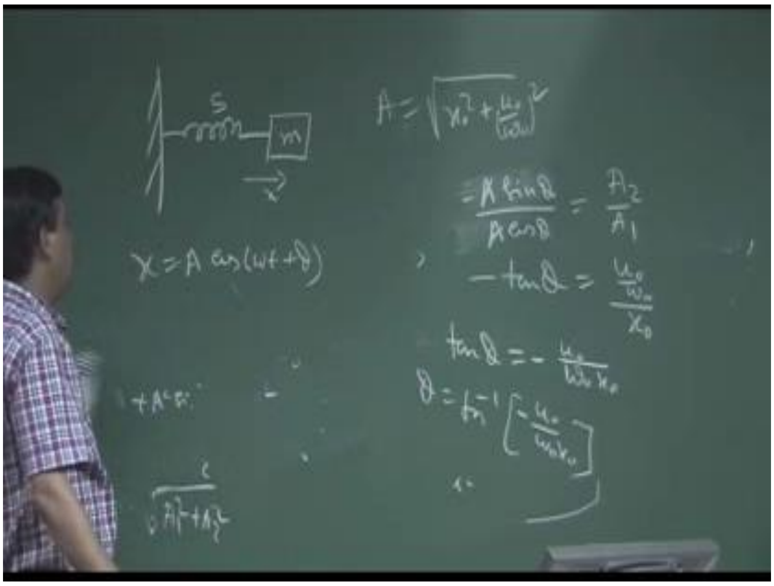

Now, what is the value of A 1 and A 2? A 1 is nothing but a x 0, so I can say A is nothing but a root over of x 0 square plus u 0 by omega 0 whole square, same A. Similarly, if I say A 1 is equal to A cos phi, so if I A cos phi divided by A or I write minus A sin phi divided by A cos phi is nothing but a A 2 by A 1. A 1 is equal to A cos phi and A 2 is equal to minus A sin phi. So, A, A cancel; so, minus tan phi is nothing but a A 2 by A 1. What is A 2 u 0? Omega 0 what is a 1 x 0 if it is x 0 then what is that tan phi tan phi is nothing but minus u 0 by omega 0 into x 0. Now, what is phi tan inverse minus u 0 by omega 0 x 0? So, I know the phi value, I know a value, I can write a x is nothing but a A cos omega t cos phi. So, omega 0 t t plus phi, so omega 0 is the natural angular frequency of that motion understand.

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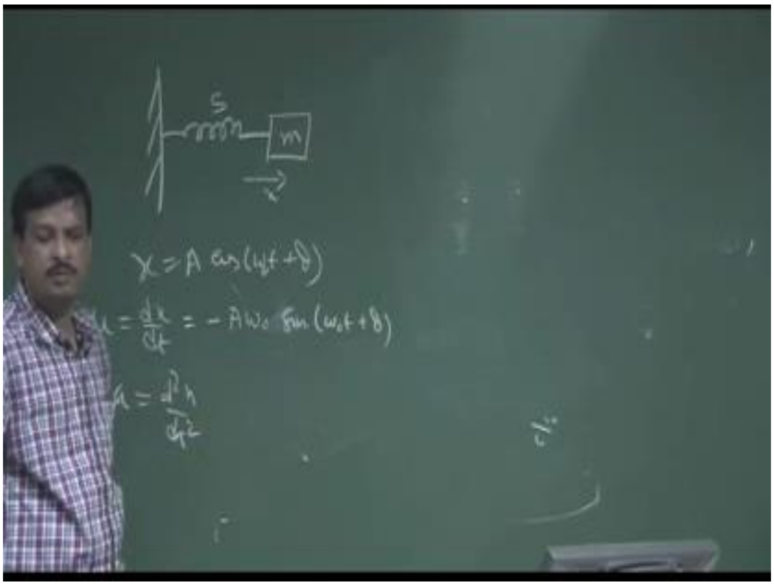

Now, same thing then what should the u this is the x then what is u, I will use these things in later on also. So, if I want to find out the u, it is nothing but dx by dt it is nothing but u so speed is a minus a omega 0 sine omega 0 t plus phi. Let us write it, I can find out a acceleration d 2 x by dt square that way I can find out acceleration also. So, I can find out acceleration see that this is cos function this is sin function so they are different they did not in same place so that I will discuss later.

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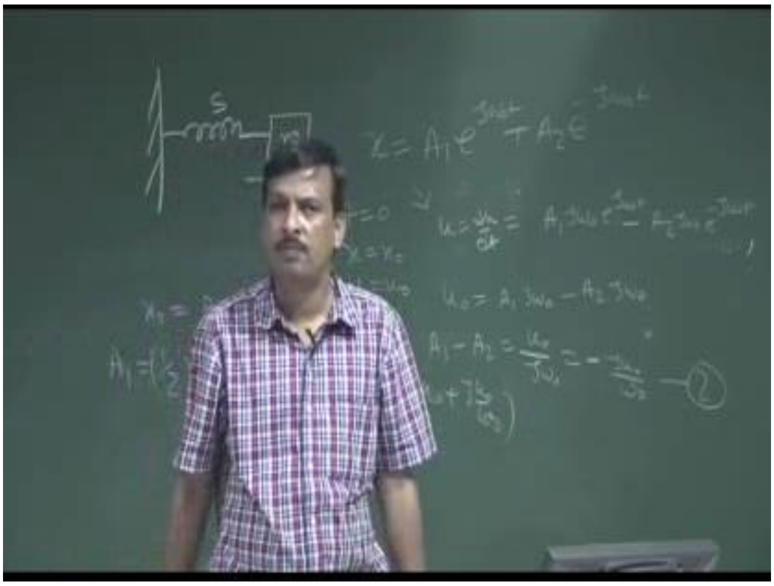

Now, same solution this is called sinusoidal solution similar can be x find out from a exponential solution so instead write x is equal to a cos theta plus b sin theta. I can write same solution from that different second order differential equation x may be a 1 e to the power j omega 0 t plus a 2 e to the power minus j omega 0 t complete solution is in exponential form instead of cos or sin. I can write e to the power now a 1 a 2 is the arbitrary constant. Now, if I apply the same initial at t equal to 0 x 0 x is nothing but a x 0 and u is nothing but a u 0 so at t equal to 0. If I put here, so x 0 is nothing but a 1 plus a 2 a t equal to 0, I putting here.

Now, if I say the cos theta derivative u is nothing but a d x d t so it is nothing but a a 1 j omega 0 e to the power j omega 0 t minus a 2 j omega 0 e to the power minus j omega 0 t I take the form differentiation. Now, I put t equal to 0 so u 0 is nothing but a a 1 j omega 0 minus a 2 j omega 0 or not. Now if I say a 1 minus a 2 is nothing but u 0 by j omega 0 j omega 0 is common so u 0 by j omega 0. Now, if I put the j in other side so it will be minus j u 0 divided by omega 0 multiplied by j lower side j square is minus 1, so minus j u 0 by omega 0. So, this is equation number two and this is equation number one. Now, using the one and two, I can find out the value of a 1 and a 2. So, what is the value of a 1, a 1 would be half of x 0 minus j u 0 by omega 0 just and a 2 will be half of x 0 plus j u 0 by omega 0 so a 1 a 2 value I got I got the value of a 1 a 2.

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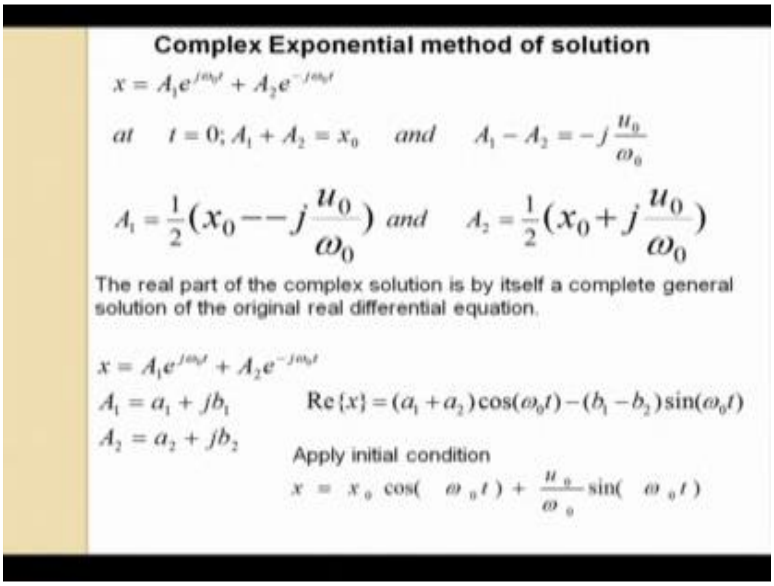

If I put the value of a 1 a 2 I should get the solution I should get the solution now interestingly if you see a 1 and a 2 both are complex both are in complex form a 1 and a 2 both are in complex form. So, now let us consider let us consider that a 1 is nothing but a complex form A plus J u. Let us consider A 1 is nothing but a a 1 plus J b 1 and A 2 is nothing but a a 2 plus J b 2. I can write down that a and b are complex number. So, any complex number can be represented by a plus J, a plus J b, now we represent that way.

Now, if I do that A 1 is like that and x is equal to A 1 e to the power J omega 0 t J omega 0 plus A 2 e to the power minus J omega 0 t. Now, if I put the value of A 1 n here and A 2 n here, a 1 plus J b 1 into e to the power minus J omega 0 t plus a 2 plus J b 2 into e to the power minus J omega 0 t. Now, if I find out what is the real part of this x? I want to find out the real part of x. How do we get it, e to the power J theta cos theta plus J sin theta, e to the power minus J theta cos theta minus J sin theta. If I put that thing and then multiply I will get - a 1 plus a 2 into cos omega 0 t minus b 1 minus b 2 into sin omega 0 t; if I explain that things; and find out the part of that things then you get this thing; now, real part of x, what is the real part x, a 1 plus a two. So, if I put the value of a 1 plus a two, so what is A 1 A one is nothing but a x 0 minus J omega 0 u 0 by omega 0. So, this is a 1 is nothing but a x 0 whole the half, half is there. A 1 is nothing but a I write A 1 is a 1 plus J b one, small a one, so small a 1 is nothing but a x 0.

What is small a 2, A 2 is half of x 0 plus J u 0 by omega 0. So, a 2 – small a 2 is also x 0, not x 0 half of the x 0, here also half of x 0. a 1 plus a 2 half of x 0 plus half of x 0 is nothing but a x 0, so it is nothing but a x 0 cos omega 0 t plus minus is there. What is b 1? Minus J u 0 by omega 0 half. What is b 2? Is minus half of u 0 by omega 0 J. So, if I add them it is nothing but a minus J u 0 by omega zero, minus J u 0 by omega 0. If I put the value in here, minus into minus a plus, so I get here also I get x 0 by omega 0 sin omega zero, u 0 by omega 0 sin omega 0 2.

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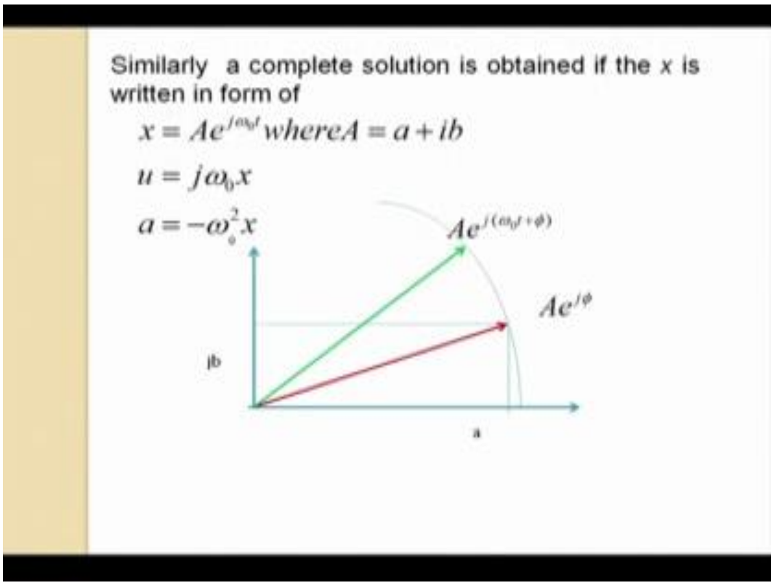

So, this solution is same as we have derived in initial. So, I can say a real part of x give me the complete solution of that same differential equation. So, real part also give me if the real part give me the solution, the complete solution is obtained. If x s is written in the form of it can easily put that complete solution can be obtained if the x is written in the form of x is equal to A e to the power j omega 0 t. So, if I write x is equal to A e to the power j omega 0 t is also a complex solution, where A is a complex number.

So, what is the complex, why I write this equation because in later on when you go for the design and acoustic wave propagation, you always say the displacement is in the form of A e to the power j omega 0 t that is why I want to derive this equation. So, if this is the complex solution and then A is nothing but a complex number, so I can write a plus small a plus J small b, any complex number can be written as r theta component. So, r e to the power J theta, r is the amplitude and J theta is the direction. So, where r is equal to nothing but root over of a a square plus b square; and theta is nothing but a tan inverse b by a, I can write.

So, instead of A, I can write amplitude multiplied by the phase. So, I can small amplitude e to the power J phi, yes phi is the angle, e to the power J omega 0 t, so it is nothing but a mod of A e to the power J omega 0 t plus phi. So, what is the real part, e to the power J theta cos theta plus J sin theta, so it is nothing but a amplitude into cos omega 0 t plus phi for real x; same expression as we derived from the cosine transform – cosine part.

Similarly, I can find out if x is in this form, I can find out u d x d t, I can find out acceleration d x square d t square. So, both ways I can find out.

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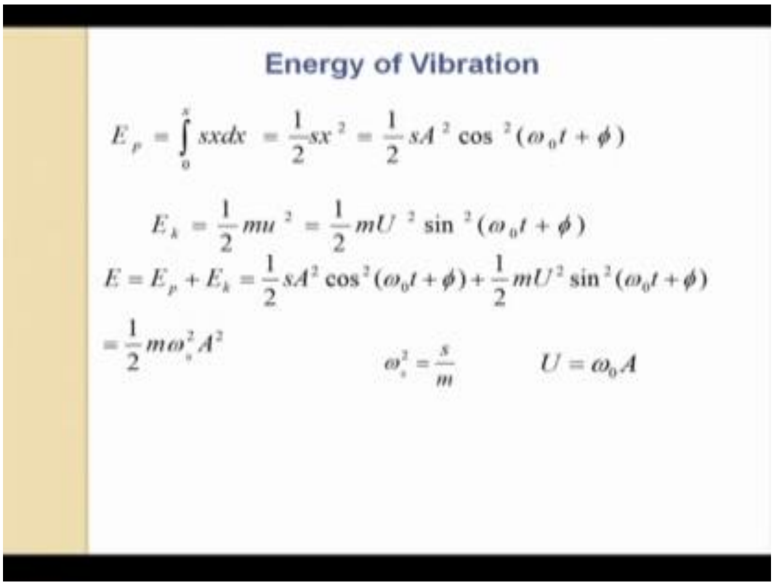

Now, when the energy of this, this is the equation of motion and equation of velocity I understand. Now, what is energy exist in this equation in this oscillation. What is the how much energy exists in this oscillation? So, now if this is the oscillation, I know any simple harmonic motion is spring mass motion.

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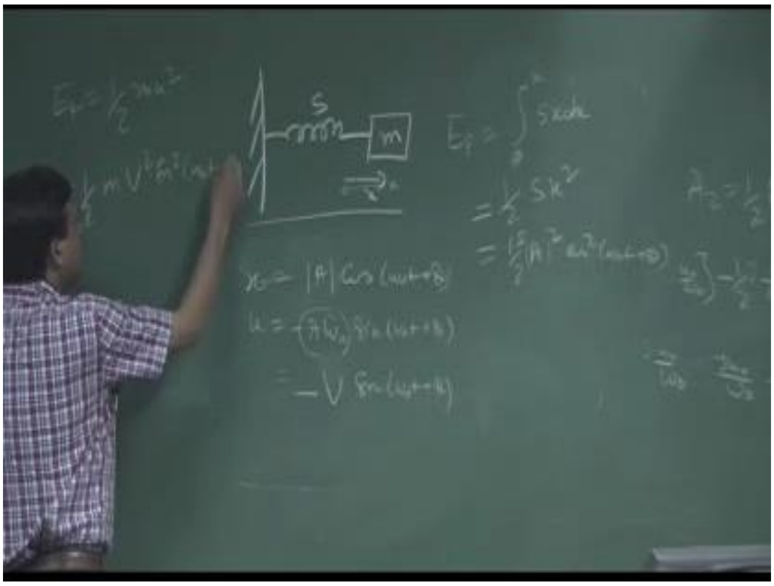

There is two kind of energy, one is called E p, and another is called E k. What is E p, E p is nothing but a potential energy; and E k is nothing but a kinetic energy, so any simple harmonic as motion at any point as a two kinds of energy potential energy and kinetic energy. Just think about what is potential energy, what is the potential energy of these things. The potential energy E p is nothing but a integration of 0 to x, x is the displacement, lets rest position it is 0, it is displace to x. So, if it is displace to x, so due to the displacement to the x, the energy is stored. So, what is the energy, integration displacement from 0 to x, the energy will stored spring constant into x into dx.

If I have a spring, if I displace, nothing displace – no potential energy; if it is try to displace, there will be due to the displacement of spring, I store the mechanical energy or deformation, displacement create a deformation. So, I store the potential energy, so that potential energy, how do you find out integration from 0 to x, x, x, dx. So, if it is that if I do the integration, what is the integration half of S x square. Now, what I said x is nothing but a real part of x is nothing but a mod of A - amplitude into cos omega 0 t cos phi, so that is the amplitude cos omega 0 t plus phi. So, if I put that things half of A square – mod of A square into cos square omega 0 t plus phi, S will be there, half of S into A square cos square omega 0 t plus phi that is the potential energy.

Now, what is the kinetic energy? Let us kinetic energy E k is nothing but a half m into u square. So, if it is x, what is u, u is nothing but a omega 0 minus sin omega 0 t plus phi. So, let us A omega 0 t sin omega 0 t plus phi. Let us this motion has capital U, so I can write minus capital U, sin omega 0 t plus phi. So, it is nothing but a half m into square U square sin square omega 0 t plus phi.

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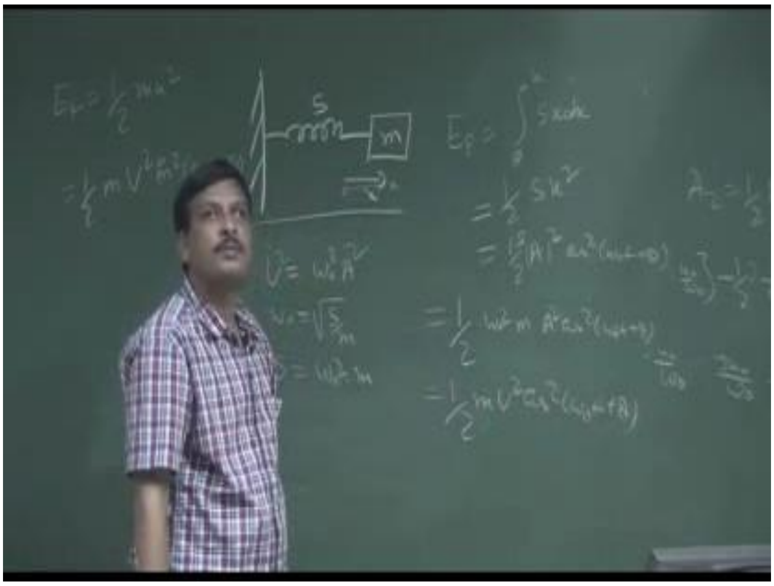

Now, if I convert, what are the conversions? What is U; U is nothing but a omega 0 into A, Capital A. Now, what is omega zero? Omega 0 is nothing but a root over of S by m, S by m. Then what is S? I can write S is nothing but a omega 0 square into m. So, I put that value in here, so half S means omega 0 square into m let us I write A square cos square omega 0 t plus phi. So, I can write let us that U - omega 0 A, so U square – omega 0 square A square, so omega 0 square A square, I can write half m U square cos square omega 0 t plus phi.

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Now, what is total energy, total energy it is E, E is the total energy, it is nothing but a potential energy plus kinetic energy. So, what is potential energy, half m U square cos square omega 0 t plus phi plus half by the kinetic energy m U square sin square omega t plus phi. Now, if I take the common half m U square – capital U square, then it is cos square theta plus sin square theta is equal to one, so it is nothing but a half m into capital U square. It can be proved that any point that the maximum potential energy that is E p max is equal to the E k max, we prove it. It can be proved from that equation.

So, I can say any point of time, the energy stored in the oscillation system is nothing but a potential energy plus kinetic energy, which is in the form of total energy is half m into capital U square, U is nothing but a into omega 0. But E p max – maximum when the potential energy maximum that times the kinetic energy will be zero. And when the kinetic energy is maximum the potential energy would that time will be zero. So, E p max is equal to E k max is equal to total energy. You can prove it, prove that or we will discuss in tutorial how it is proved.