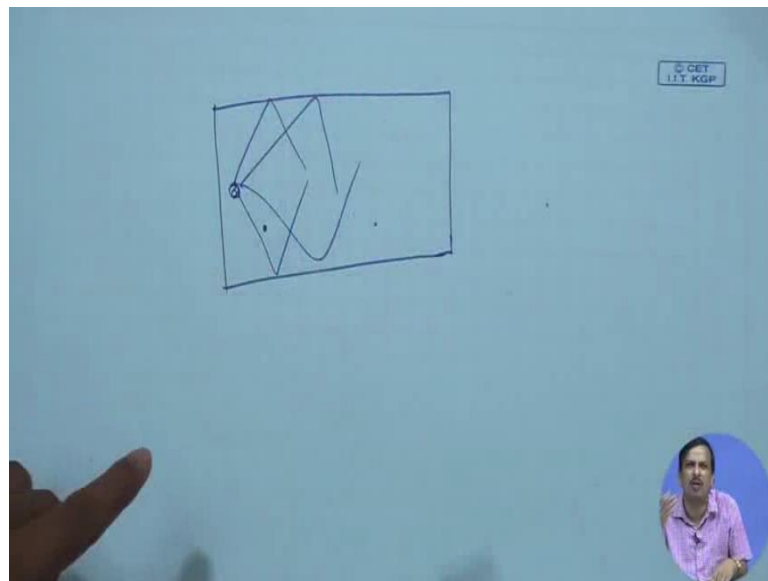


Audio System Engineering
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Lecture - 14
Room acoustics-I

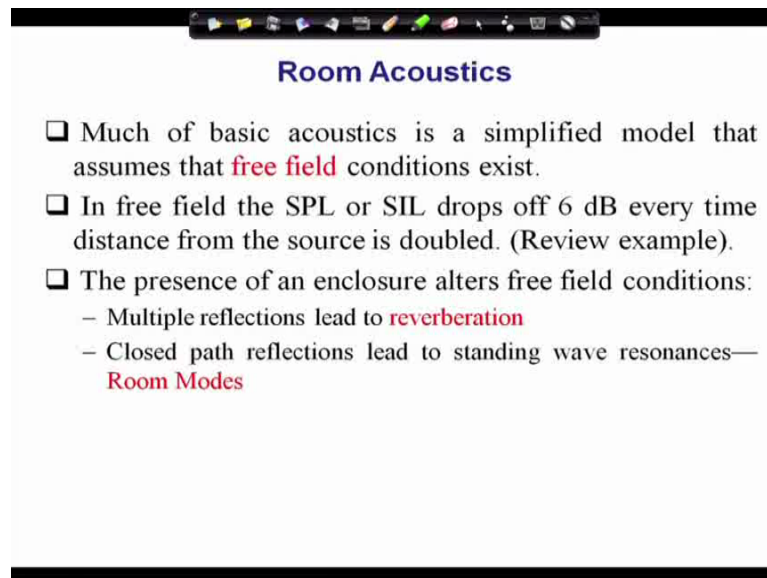
Now, we discuss about the room acoustics.

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Room acoustics means that if suppose, if I create a sound in a enclosure, this a enclose, so room acoustics means it can be a auditorium, it can be a class room, it can be a theatre room, it can be my drawing room whatever is the room acoustics. So, what is room acoustics and I said that the sound will be reflected from different surface. So, suppose I create a sound in here, so that will be reflected back from different, different surface from ceiling, from ground, from wall all, from door, from window, all way it will be reflected. Some portion will be pass back, same portion will be absorbed by the medium by the wall, so that kind of phenomena will be happen that is called room acoustics. So, how do you study the room acoustics, what are the parameter we have to define for room acoustics. Now, when you design a acoustics of an auditorium, what are the properties we should look for all has to be discussed in room acoustics.

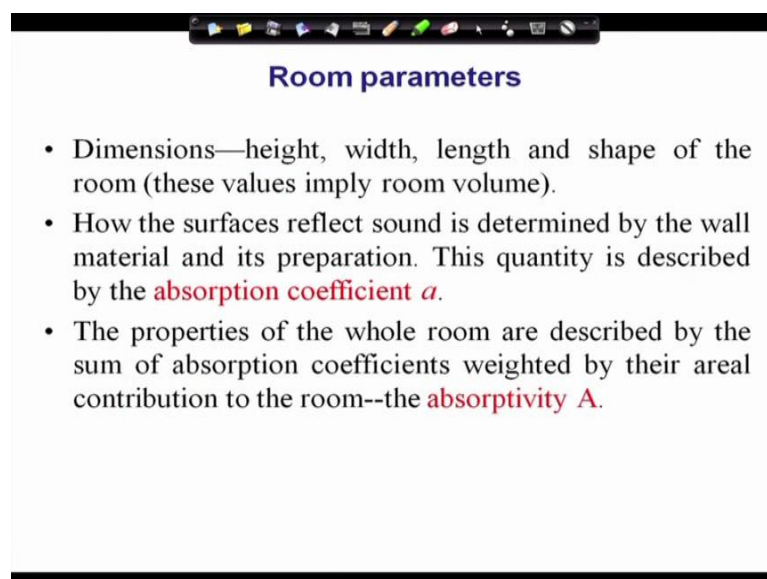
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Room Acoustics

- ❑ Much of basic acoustics is a simplified model that assumes that **free field** conditions exist.
- ❑ In free field the SPL or SIL drops off 6 dB every time distance from the source is doubled. (Review example).
- ❑ The presence of an enclosure alters free field conditions:
 - Multiple reflections lead to **reverberation**
 - Closed path reflections lead to standing wave resonances—**Room Modes**

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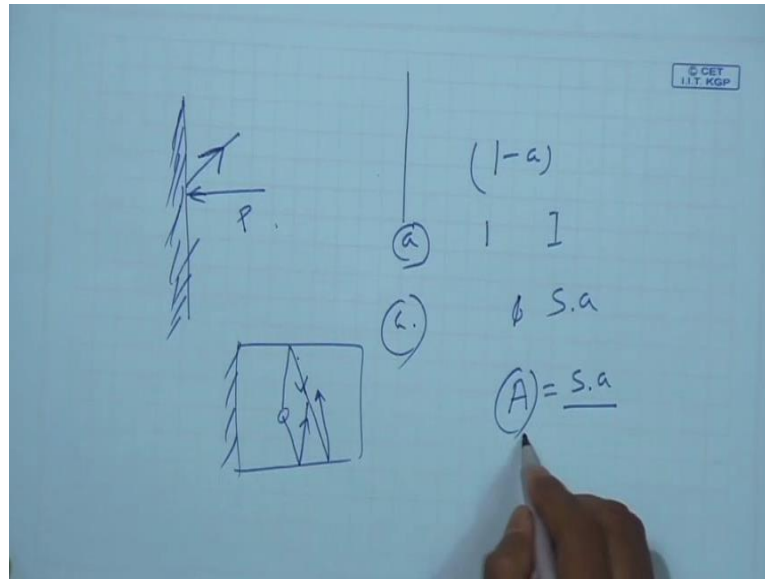


Room parameters

- Dimensions—height, width, length and shape of the room (these values imply room volume).
- How the surfaces reflect sound is determined by the wall material and its preparation. This quantity is described by the **absorption coefficient α** .
- The properties of the whole room are described by the sum of absorption coefficients weighted by their areal contribution to the room--the **absorptivity A** .

So, let us start with that multiple reflection, sorry I have already discussed this. So, room parameter what are the basic parameter available for a room, height, width length, and shape of the room. So, height, there is a height, length I can say shape of the room it may be a circular, so there may be radius it may be a hexagonal that kinds of things will be there. So, height, length, width and shape are there. How the surface reflect the sound is determine by the wall material and its preparation. This quantity is described by the absorption coefficient, now we discuss the absorption coefficient. So, what is absorption coefficient?

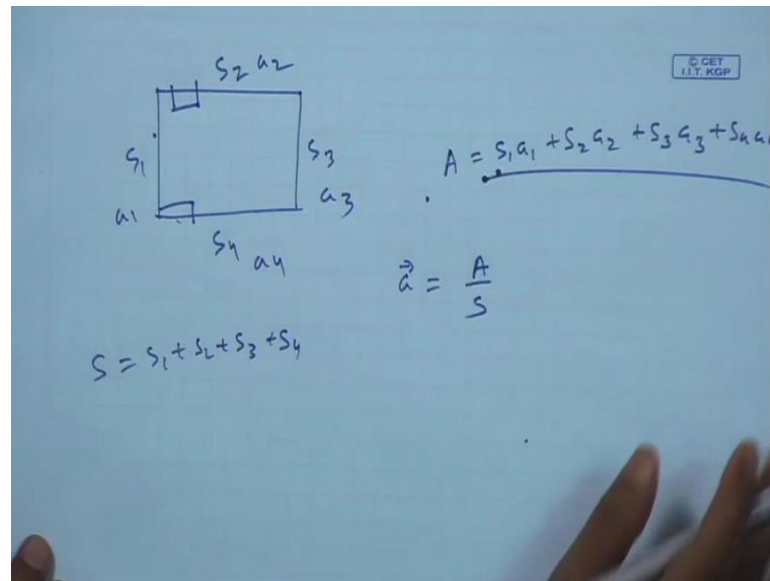
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Suppose, I have a wall lets sound is incident on the wall, so it will be reflected same portion will be reflected, same portion will be transmitted that along with transmitted there will be some absorbed. So, unless it supposed incident is p and same portion is reflected, so rest portion is absorbed by these wall, this wall is absorbed sound so that is sound absorption. So, in that case, so if a surface has an absorption coefficient a ; that means, if p is incident on that surface p into a will be absorbed by the surface. So, the reflected is one minus a . So, a is called absorption coefficient. So, if say I say the unit power unit pressure one unite pressure is the incident on that so it the a unit into a will be absorbed by the surface and one minus a will be reflected by the surface.

Now, in the room, if I create a sound, so it is a will be absorbent and one minus a be reflected. Similarly, it will fall on this wall, then this reflection fall on this wall, there will create this reflection. So, all reflection, it creates the sound pressure level you can say if I switch on the microphone switch on the loud speaker in this room for some time it require to reach the steady state level of that room sound field. Then if it is reverberant field or if it is semi reverberant field since there is absorption semi reverberant field so sound pressure level will reached in a steady state. So, if the absorption coefficient is a , then the absorptivity of the surface is defined by area of the surface let S multiply by a , which is called capital A , A equal to surface area multiply by absorption coefficient. A is measure in term of called Sabin; A is measured in term of Sabin.

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Next, I told you that suppose I have room, let's this area of this surface is S_1 , area of this S_2 , let's area of this surface different S_3 and this is S_4 , and absorption coefficient A_1 , A_2 , A_3 , A_4 . So, total absorptivity of the whole surface is nothing but A is equal to $S_1 A_1$ plus $S_2 A_2$ plus $S_3 A_3$ plus $S_4 A_4$. So, there may be a window, absorption coefficient is different; there may be a door, absorption area of the door multiply with the absorption coefficient. And total Sabin is A is nothing but the surface area multiply by the absorption coefficient and all sum is called total Sabin.

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Absorptivity

Absorptivity formula

$$A = a_1 S_1 + a_2 S_2 + a_3 S_3$$

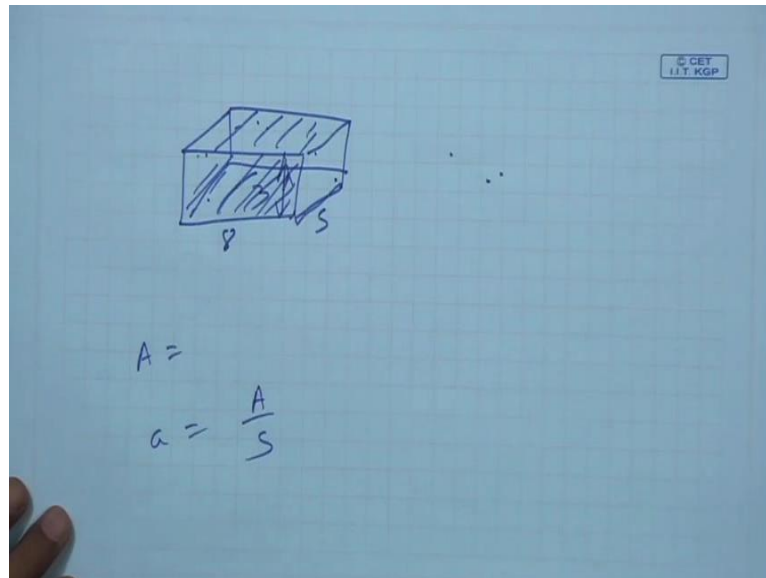
Average Absorptivity $\bar{a} = \frac{A}{S}$

Example: Room 3 m tall with floor and ceiling 8 m x 5 m. $a_{\text{ceiling}} = 0.3$, $a_{\text{floor}} = 0.6$, $a_{\text{walls}} = 0.12$.

- What is A ?
- What is weighted average a ?
- Remember a depends on frequency.

Now, if I say what is average absorptivity, so average absorptivity a is nothing but A by total surface area. So, what is total surface area, S is nothing but S_1 plus S_2 plus S_3 S_4 divided by A by S . Let us do the mathematics then you understand what is this; is that. A room 3 meter tall with floor and ceiling is 8 meter into 5 meter.

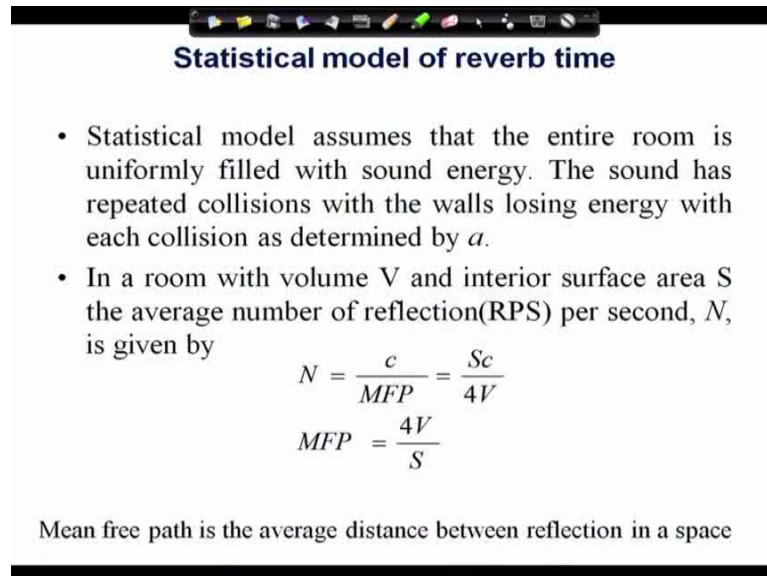
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So, I can say I have a room, 8 by 5; that means, this is length is 8, 5 is width and 3 meter is the height, height is 3 meter. So, I said that ceiling has absorptivity different floor has a wall has. Now, what is A ? So A is nothing but this surface multiplies by the wall, these four walls are there. So, I can calculate the area of the four wall multiply by the wall absorption coefficient plus area of the ceiling multiply the ceiling absorption coefficient, area of the floor multiply the floor absorption coefficient then I get A .

Now, what is an average absorptivity, a is nothing but capital A by total surface area. But this absorptivity is depends on frequency. If you see in practical, I show you also that once you buy a material acoustics treated material, the absorption coefficient is retain with the frequency range, for this frequency, it is this; so up to this range, it works; after that this frequency may not work, absorptivity will totally change. So, absorption coefficient is frequency dependent.

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Statistical model of reverb time

- Statistical model assumes that the entire room is uniformly filled with sound energy. The sound has repeated collisions with the walls losing energy with each collision as determined by a .
- In a room with volume V and interior surface area S the average number of reflection(RPS) per second, N , is given by

$$N = \frac{c}{MFP} = \frac{Sc}{4V}$$
$$MFP = \frac{4V}{S}$$

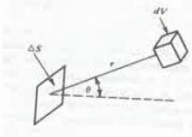
Mean free path is the average distance between reflection in a space

Now, I come how the sound field is growth, growth of the sound field. Suppose, I have a room multiple reflection is come. So, number of reflection, this lot of reflection, and if I say the number of reflection is statistically significant such that at any point of time average number any point that room, the average number of reflection is same. So, statistically mode assumes that the entire room is uniformly sound energy. The sound has a repeated collisions with the wall losing energy with each collision as determined by a , the room volume is V , and the number of reflection statically so large that at any point in the room the average number of reflection will be same. So, this is called statistical model then I can apply statistical model reverberation time. Forget about the mean free path and this N , I will come later on.

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A Simple Model for the growth of sound in a room

When a source of sound is started in a live room, reflection at the walls produce a sound energy distribution that become more and more uniform with increasing time. Ultimately except close to the source or to the absorbing surface, this energy distribution may be assumed to be completely diffuse.



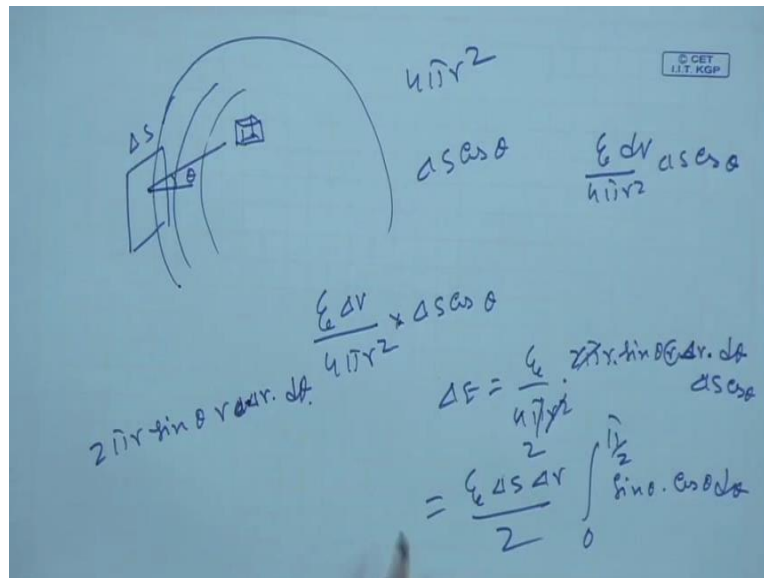
Acoustic energy in dV ξdV Where ξ is the energy density

The amount of energy strike Δs $\frac{\xi dV}{4\pi r^2} \Delta s \cos \theta$

So, how the sound is growth, simple model for growth of sound room, let us drive the mathematics how the sound is growth in that room. Now, let us consider if you see let us consider area which is called delta s. When a source of sound is started in live room, reflection at the walls produces a sound energy distribution that become more and more uniform with increasing time. So, what I do I play a loudspeaker, and wait for some time, let sound settle down means it reach the steady state. Ultimately, except close to the source or the absorbing surface, the energy distribution may be assumed to be completely diffuse; that means, equal, number of reflection at any point is equal.

Now, consider let us there is wall in any surface, let us there is a surface area which is delta s. Lets I put a source whose energy density is psi and lets there is volume of sound d v. That means what which in this room let us say consider small area delta s wall and I let say a cube of sound volume d V is strike on that surface with a angle theta, then it will be reflected. So, acoustics energy in d V in this volume is nothing but energy density psi multiply by the d V. So, volumes multiply by energy density total energy. So, within this volume, the total acoustics energy is psi d V, and that type the surface of delta s with angle of theta. The amount of energy strikes in delta s is nothing but total energy divided by four pi r square is the total hemisphere surface multiply by the delta s cos theta, are you understand this.

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I said I have a small surface Δs . A energy, a volume of energy you can say packet of energy strikes on the surface with angle of θ . Since, the acoustics energy is flowing through spherical in nature what is the total energy when it is strike in the wall, it is the nothing but $4\pi r^2$. So, what is the projection of the surface in that sphere is nothing but $\Delta s \cos \theta$. So, I can say $\xi \Delta V$ divided by $4\pi r^2$ is the unit area multiply by the effective area, is the energy it should be strike in that wall.

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Let dV be part of a hemispherical shell of thickness Δr and radius r centered on Δs . the acoustic energy ΔE contributed to Δs by this entire shell is obtained by assuming that energy arrived from any direction with equal probability. Integrating over the hemisphere with $dV = 2\pi r \sin \theta r \Delta r d\theta$

$$\Delta E = \frac{\xi \Delta s \Delta r}{2} \int_0^{\pi/2} \sin \theta \cos \theta d\theta = \frac{\xi \Delta s \Delta r}{4}$$

The energy arrived during time interval $\Delta t = \Delta r/c$ $\frac{\Delta E}{\Delta t} = \frac{\xi c \Delta s}{4}$

The rate of energy falls on a unit area $\frac{\Delta E}{\Delta t} = \frac{\xi c}{4}$

Now, consider this picture. So, let δv is the part of a hemispherical shell of thickness δr and radial distance r centre on s . So, what I said let this volume is the part of the hemispherical shell, whose thickness is δr , and its radius radial distance is r from s . The acoustic energy δE contribute to δs by this entire shell is obtained by assuming that energy arrived from any direction with equal probability; that means, if I say this wall is straight, next one wall is come from the what direction is straight. So, in different direction different sound packet or energy packet sound energy packet is strike on the δs surface that is nothing but a hemisphere shell is arriving at δs .

So, how would I get the total, how do you get the total energy δE contributing δs , because it is not once I want the sound it is not that sound will go for angle θ only on that δs , then different θ angle is possible on different direction, it is coming, so different θ is possible. So, I can say like a hemispherical shell is strike on the δs surface so how much total energy due to the all kind of the reflection is contribute to the δs . How do you get it δE what is the δV of the hemisphere is the nothing but the $2\pi r \delta v$ is $2\pi r \sin \theta r \delta r$ in to $r \delta \theta$ $2\pi r$ is the simple arithmetic simple geometric $2\pi r$. What is 2π is the circumference multiply by the $\sin \theta r \delta r$ into $d\theta$.

Now, what is total energy δE , δE is nothing but volume packet. What is the packet, packet is $\sigma \delta v$ by $2\pi r^2 \delta s \cos \theta$. So, it is the amount of energy strike on δs which is $\sigma \delta v$ divided by $4\pi r^2 \delta s \cos \theta$ for single packet. Now, it is entire hemisphere is striking, whose volume is this one then total energy is nothing but integrating from 0 to π by 2 on this. So, δv replace σ by $4\pi r^2$ in to δv , which is $2\pi r \sin \theta r \delta r \delta \theta$ or $d r \delta r$ in to $d\theta$ in to $\delta s \cos \theta$. So, 2π , 2π cancelled, π , π cancelled, r this r is create r^2 cancelled, and so 2 by 2 , it is nothing but σ into δs , sorry ψ into δs into δr 0 to π by 2 $\sin \theta \cos \theta d\theta$ divided by 2 .

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Handwritten derivation on a whiteboard:

$$\Delta E = \frac{\rho \Delta s \Delta r}{4}$$

$$= \frac{\rho \Delta s \cdot \Delta t \cdot c}{4}$$

$$\frac{\Delta E}{\Delta t} = \frac{\rho \Delta s c}{4}$$

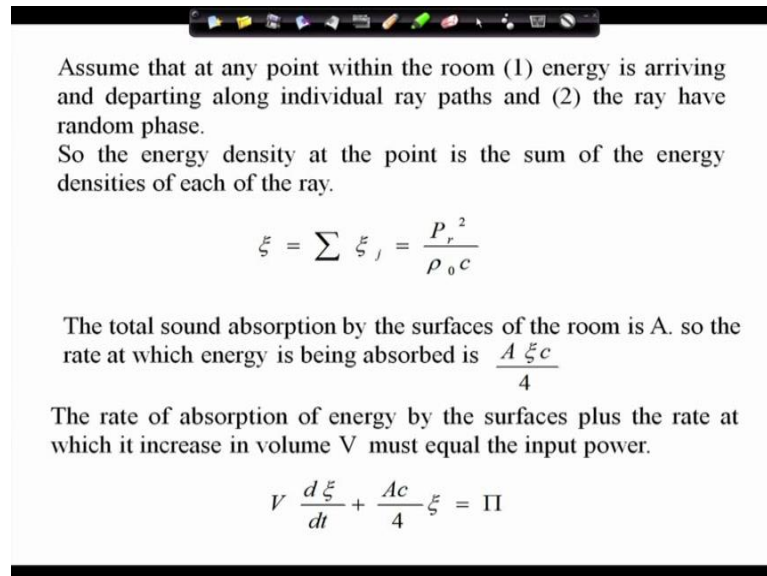
$$\rho = \frac{\rho_r}{\rho_e} = \frac{\rho_r^2}{\rho_e}$$

Other notes on the board include:

- $\Delta r = \frac{\Delta r}{c}$
- $\Delta r = c \cdot \Delta t$
- A diagram showing a square area Δs with a distance Δr and a pressure ρ_r .

Now, if I do this integration, what I will get, I will get $\rho \Delta s \Delta r$ divided by 4, if I do this integration. So, the energy arrived during the time interval. So, what I get, I get ΔE . I get ΔE energy contributing on the surface Δs to ΔE by the all reflection is nothing but a $\rho \Delta s \Delta r$ divided by 4. Now what is Δr , Δr is the distance. So, I can know that Δt time how much time is taken cause the Δr nothing but Δr by c velocity of the sound. So, I can say Δr nothing but c in to Δt . So, I can say nothing $\rho \Delta s$ in to Δt in to c . I can say $\rho \Delta s$ by Δt is nothing but $\rho \Delta s$ in to c divided by 4. So, $\rho \Delta s$ by Δt so rate of energy fall on the unit area if I say rate of energy fall on surface area is Δs so rate of energy $\rho \Delta s$ by Δt which is fall on that surface. So, in unit area Δs while be not there. So, the amount of energy unit area is nothing but ρc sorry ρc divided by 4. Now, ρc divided by 4 is the rate of energy fall on the unit area of that surface, it is the rate of energy fall on the unit area surface.

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Assume that at any point within the room (1) energy is arriving and departing along individual ray paths and (2) the rays have random phase.

So the energy density at the point is the sum of the energy densities of each of the rays.

$$\xi = \sum \xi_j = \frac{P_r^2}{\rho_0 c}$$

The total sound absorption by the surfaces of the room is A . So the rate at which energy is being absorbed is $\frac{A \xi c}{4}$.

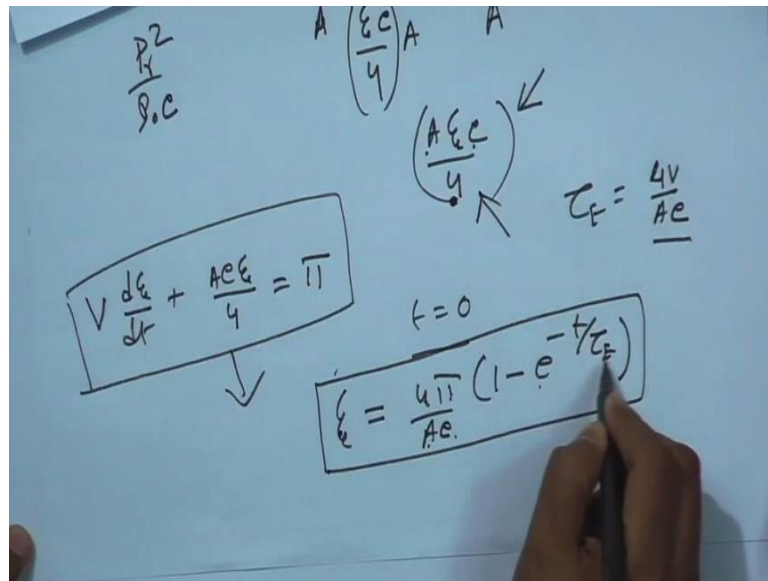
The rate of absorption of energy by the surfaces plus the rate at which it increases in volume V must equal the input power.

$$V \frac{d\xi}{dt} + \frac{Ac}{4} \xi = \Pi$$

So, I know that what is the rate of energy incident on the rate of surface unit area on the surface that I know, assume that at one part of the incident energy is absorbed by the surface and one part will be reflected back. So, energy arriving and departing that incident and reflected are individual ray paths. The ray has random in phase. I know that it is random. So, energy density at the point is the sum of the energy density of each ray. So, what I say that that suppose I consider this area Δs area in the wall, and I said from direction in the pocket of sound energy come and I know the rate of incident energy are rate per unit area on that surface.

Now, I same things I can consider lets different pressure energy ray consider the ray. So, different I can let say the Δs . So, different pressure ray or incident on that surface and it reflected back. So, what is the total reflected energy, energy density at each point the sum of the energy. So, each reflected energy is P_r lets not energy let pressure reflected pressure is P_r . So, energy is nothing but sum of each energy for each of the ray each of the reflected sound. So, $\sigma_i \psi_i$, so it is nothing but a P_r^2 square divided by $\rho_0 c$ what is P_r reflected sound lets total reflected sound pressure is P_r , it is nothing but P_r^2 square by $\rho_0 c$.

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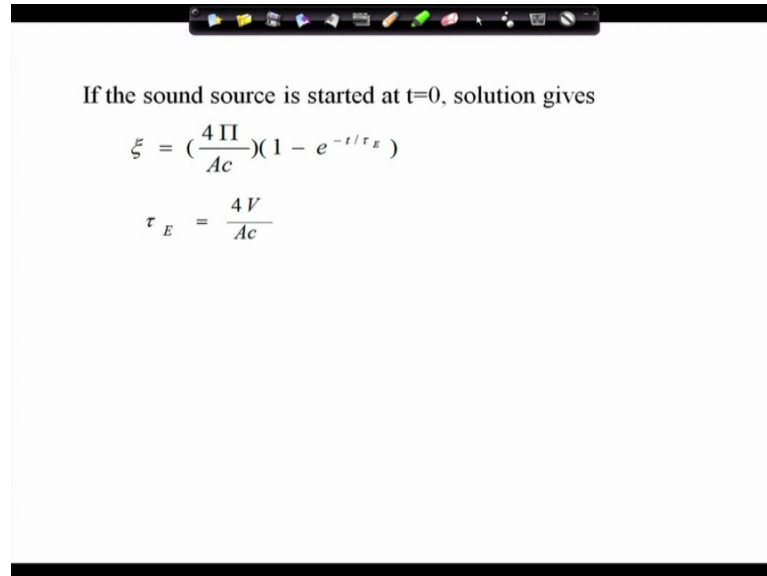
Now, total reflected sound pressure is nothing but a p_r square by $\rho_0 c$, p_r is the reflected sound pressure. And the energy incident on the rate energy incident on the unit area of the surface is A , it is σc by $4 \sigma c$ by 4 . Let us the absorptivity of the surface a . So, how much energy will be absorb by unit area of the surface the multiply by the absorptivity. So, it is $A \psi c$ divided by 4 , total sound which is absorbed by the wall total sound incident energy incident σc by 4 , and these amount this is absorbed. So, energy being absorbed by the wall unit area of the wall is $a \psi c$ divided by 4 ; understand or not.

So, now if I say this is this much energy absorbs. So, rest of the energy will be reflected back, so that reflected back energy will developed the sound field in this in this room it added with direct sound and develop the sound field. That means, let us switch on the microphone my loud speaker for same time and then switch of then what will happen different direction multiple reflections will come and that create that developed sound pressure level.

Now, energy, which is absorbed plus the rate at which the energy develop in this room is equal to the total energy. So, I can say the rate of the absorption energy by the surface plus rate at which increase the volume v must be equal to the input power. So, I say this volume is b room volume is $v \Delta \psi$ by Δt plus $a \psi c$ divided by 4 is equal to total energy understand or not total energy. So, this is the total energy equation. So, a enclose

room if I switch on a loud speaker multiple reflection will developed and the equation of the sound total energy is this total energy input power.

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If the sound source is started at $t=0$, solution gives

$$\xi = \left(\frac{4P}{Ac} \right) (1 - e^{-t/\tau_E})$$

$$\tau_E = \frac{4V}{Ac}$$

Now, if sound source is started at t equal 0, I can get a solute t equal 0 no energy before t equal to no energy is there to start from the 0. So, I can get the solution with initial condition at t equal to 0, then the solution will be ξ is equal to $\frac{4P}{Ac}$ in to one minus e to the power minus t by τ_E τ_E is nothing but $\frac{4V}{Ac}$. So, this is the ξ energy density expression, this the energy density expression is $\frac{4P}{Ac}$ 1 minus e to the power t by τ_E this along the time the sound energy density will be increase.

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Reverb time definition

Assume that in a live room a sound source has been turned on long enough to establish a steady state energy density ξ_0 and it then turned off at time $t=0$

$$\xi = \xi_0 e^{-t/\tau_E}$$

Reverb time, RT_{60} , is defined as the time for the sound energy to drop by 60dB. Thus

$$RT_{60} = 13.82\tau_E = \frac{55.3V}{Ac}$$

If V is in cubic meters and A is in metric sabin and $c=343\text{m/s}$ then

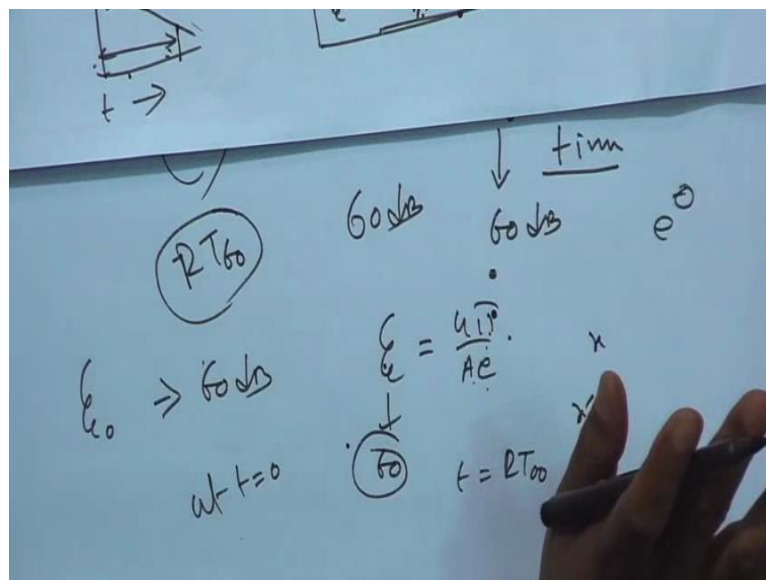
$$RT_{60} = \frac{0.161V}{A}$$

If V is in cubic feet and A is in english sabin and $c=1125\text{ft/s}$ then

$$RT_{60} = \frac{0.049V}{A}$$

Now, what is the reverberation time, you heard about reverberation time. Let us assume that in live room a sound force will be turn on long enough turn on for long enough to establish the steady state density.

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Lets steady state density is ξ_0 . Let this room switch on the loud speaker and allow to on the loud speaker I switch on the loud speaker to such a long time for time for particular take some time. So, that the steady state energy is developed in this room which is ξ_0 . Now, a t equal to 0, I switch of the force what will happen sound energy

is developed by multiple reflections. Now t equal to 0, I switch of the source then the this ψ_0 energy density in this room not 0 in that within that time then in that energy density cannot be 0, because the sound which is started from here it will be reflected from the wall come back. So, it has required some time to ψ_0 becomes to 0 required some time.

So, reverberation time is define reverberation time or RT 60 is define that time for the sound energy to drop by 60 dB. So, at t equal to 0, room has sound energy ψ_0 to drop that ψ_9 to the 60 dB, the required time is called reverberation time am I explain clearly. Suppose, I switch on a loud speaker in here for a shortened time then lets at steady state energy density on the this room is ψ_0 is developed. Now, I switch of the loud speaker then in that cannot be 0. So, it require a sometime to drop the sound down level is 0, now reverberation time is defined as if I switch of the loud speaker t equal to 0 go to that t by the time that the energy density of the this room is drop to 60 dB. So, if it is ψ_0 , then it is drop to 60 dB, so RT 60.

So, what is ψ_0 is nothing but a 4π by $A C$ 1 minus e to the power t by τE . So, at t equal to 0, what is ψ_0 is nothing but a 4π by $A C$, this is the discharge equation this like a capacitor if you charge a capacitor then you switch of the power switch of the remove the supply is the capacitor then and there discharge. It requires some time to discharge; this is the discharge equation, in that rate energy will be discharged. So, that discharge equation I said that lets this is discharge like this way I said this is time I said the if the discharge level is this then this time is called reverberation time then I can calculate this time by discharging that. So, t equal to 0, ψ_0 is nothing but a 4π by $A C$, when the sound is steady state. So, energy density on the room is four π by a c RT equal to 0 e to the power 4π by $A C$ in that case it 4π by 0 or not 4 by $A C$.

So, now, I said the time ψ_0 has to be drop by 60 dB, 60 dB drop. So, if it ψ_0 is lets x then I required I have the x dot is the next energy which is drop by 60 dB. So, I can relate that equation and calculate at 60 that I put the t equal to RT 60. So, RT 60 I put in here RT 60 then I calculate the time and find out. So, RT 60 will be 13.82 τE . So, ψ equal to $\psi_0 e$ power to the power minus t by τE that is this way it will be discharge what is that discharge equation is that once the source is of then ψ_0 will be discharge ψ_0 in to e to the power minus t by τE .

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Handwritten notes on a blue background:

$$C = C_0 e^{-t/\tau_E} \quad \tau_E = \frac{4V}{Ac}$$

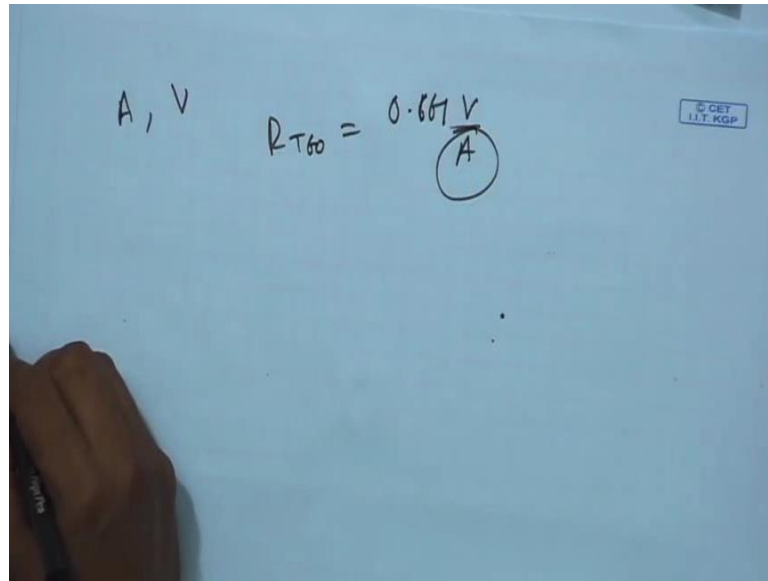
$$20 \log \frac{C_0}{C} = 10 \log \frac{C_0}{C} \Rightarrow \text{at } t = RT_{60}$$

$$RT_{60} = 13.82 \tau_E = 13.82 \frac{4V}{Ac}$$

$$RT_{60} = \frac{0.16V}{A} \quad e \rightarrow$$

Now, I said the time taken to drop it 60 dB, time taken to drop it 60 dB. So, psi 0 in dB let us reference dB is r. So, psi 0 in 10 log psi 0 by psi r let different is r this and then 10 log lets it drop to 60 dB. So, I can calculate that that things and equate with that RT equal to RT 60 lets psi is nothing but psi R capital R. So, psi R and psi difference is 60 dB, psi minus psi R is nothing but 60 dB. So, if it is that then I can calculate RT 60 is nothing but a 13.2 tau E. If it is 132 tau E, what is tau e tau e is nothing but a four v by a c four v by a c. So, it is nothing but 13.82 4 V by A C. I will show you that next slide how this comes 4 V by A C. Now, if it is c is in lets c is equal to 343 meter per second then a is metric in Sabin that means, in matrix system. Then I put the value c here and I get RT 60 RT 60 is nothing but a 0.161 V by A. V is the volume of the room; and a is the absorptivity of the room in Sabin.

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$$A, V \quad R_{T60} = 0.161 \frac{V}{A}$$

So, suppose I told you to calculate the reverberation time of an auditorium. So, if I know A and if I know the volume of the auditorium, I can say RT_{60} is nothing but a $0.161 V$ by A , or if I know the RT_{60} then the absorption time or volume then I can say the volume what absorptivity required. So, somebody told you design auditorium whose reverberation time will be lets 1.2 second. So, I put RT_{60} is 1.2 second, I know the volume of the auditorium from the civil specification, and then I know what should be will be the total absorptivity A in Sabin.

Now, I know the surface area, I treated with different material to reach that absorptivity. Once I reach the absorptivity, reverberation time is 1.2 second, so that way we will design that thing. If it is feet then it is $0.49 V$ by A . Now, once I know the RT_{60} , if you see I said from any point, there will be different number of reflection will be there summation of the reflection. So, can I calculate that mean free path that what should be the mean free part of all the reflection, what is the expression for a mean free path of all the reflection. Let us it will be done in the next class.

Thank you.