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Lecture – 12 Sound Transmission (Contd.)

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So, as per that we have discuss about the normal incident and double layer things.



Now, we will go for the oblique incident. If the pressure wave incident on a two boundary on just interface of the two boundary instead of normally there is a oblique. So, let us this is the medium one, and this is the medium two and characteristic string should be r 1 and r 2. Velocity of the sound in here is C 1 and velocity of the sound in here is C 2. Now, if the wave is incident not normally a oblique incident, so a ol is create a angle theta i which is the angle of incident of P I – and the incident wave. And there will be a reflected wave will be create the same angle theta r which is theta i is equal to theta r and this is P r – reflected wave. And there will be a transmitted wave which is P t, which is theta t – angle of transmission.

So, what is P i, P I is the incident wave, so it is so in the medium one, ok so let's pressure amplitude is P I e to the power J omega t minus what is there angle width theta with the normal. So, the wave is propagated in a x, y plane with the angle of theta i, so it is nothing but a minus k 1 wave number in the medium one is k 1 x cos theta minus k 1 y sin theta. Similarly, what is P r, wave is reflected wave, reflected again x, y plane e to the power J omega t minus, why it is minus it will be minus or plus, it will be plus because it is negative x-direction, but y direction is positive, so this is x-axis, and this is y-axis. So, I can write k 1 x cos theta is positive minus k 1 y sin theta. And what is P t, P t is nothing but let pressure amplitude P t e to the power j omega t minus the second medium, so it is k 2 x cos theta t so it is theta r, theta i, theta i, theta i, cos theta t minus k 2 minus sin theta t. So, this is the wave equation.

Now, the condition at the boundary, what I said at the boundary the pressure on both side will be equal. So, at x equal to 0, pressure on both side will be equal to same. So, at x equal to 0, the x term will be vanish, so at x equal to 0. So, I can write P I e to the power J omega t minus k 1 y sin theta i plus P r e to the power J omega t minus k 1 y sin theta r is equal to P t e to the power J omega t minus k 2 y sin theta t. So, e to the power j omega t, e to the power j omega t will not be there.

So, I can write p I e to the power minus J k 1 y sin theta i plus p r e to the power minus it will be minus j k 1 y sin theta r is equal to p t e to the power j minus j k 2 y sin theta t. This is the condition number one pressure on both side will be equal. Then normal component of the velocity will be equal. So, u cos theta i let us it is u i cos theta i plus u r sin theta r will be equal to u t cos theta r cos theta r will be equal to cos theta t, normal component of the particle velocity will be equal.

Now, from here, if I divide by that incident wave pressure, so it is 1 plus r is equal to t. Now, here u cos theta i plus u r cos theta r is equal to u t cos theta t. Now, you know that u is r p relationship between p and u is nothing but plus minus r into u and theta i is equal to theta r this two things I know angle of incident is equal to angle of reflection and p is equal to plus minus r by u. So, sin theta i is equal to theta i or cos theta i is equal to cos theta r sin theta i is equal to sin theta i. So, for using this equation one, this is equation two, this is equation one, I can get 1 minus r is equal to I can write r 1 cos theta t divided by r 2 cos theta t into T.

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So, from there, I can calculate I can find out the expression of r. You just simplify that things in yourself if you do not do then I can discuss in discussion forum how can simplify this equation and derived. Then I derived that equation r is equal to r 2 by r 1 minus cos theta t by cos theta i divided by r 2 by r 1 plus cos theta t by cos theta i, this is the R equation. So, once I get the R equation this is the reflection coefficient equation I can calculate the t equation 1 minus R is equal to or 1 plus R is equal to T. So, I can calculate the T. So, I can calculate 1 plus R is nothing but transmission coefficient pressure transmission coefficient.

And another equation the Snell's law says that sin theta i by c 1 velocity in the first medium of the sound is equal to sin theta t by c 2. So, from there I can say sin theta t is nothing but c 2 by c 1 into sin theta i this law Snell's law. So, if angle of incident is given and speed of the sound is given in the two medium then I can calculate sin theta t, I can calculate sin theta t angle of transmission.



Now, come to the situation lets go to the some situation. So, now if it is that if the sin theta t, I said sin theta t is nothing but sin theta t angle of transmission can be c 2 by c 1 into sin theta i. Now, if c 1 is greater than c 2 then this ratio will be less than 1. So, in that case theta t will be always smaller than the theta i. So, angle of incident angle of transmission always will be lower than the angle of incident. So, what you mean by c 1 is greater than c 2, velocity of the sound in medium one is higher than the medium two. A practical example, if a sound transmitted from a water to air, so in water sound velocity is c 1 and air is c 2. So, in c 1 is greater than c 2. So, if the sound transmitted from water to air angle of transmission always will be less than the angle of incident. So, I can say if this is a water and this is air then I can say that transmitted ray should be bend towards the normal or angle of incident. So, this bends towards the normal.

Now, other case, if c 2 is greater than c 1, so this ratio will be always greater than 1; c 2 is greater than c 1. So, the angle of incident and now I can say theta t may be greater than theta i. So, theta t is greater than theta i; that means, that angle of transmitted wave will be away from the normal transmitted wave will be go away from the normal. So, if it is go away from the normal maximum can it goes to 90 degree. So, when if the angle of incident is such that the angle of transmission is 90 degree then that incident angle is called critical angle, that incident angle is called critical angle.

So, I can say I want to say the theta i is critical angle then I can say c 2 by c 1 into sin theta c the angle of incident for which the angle of transmission is 90 degree is equal to 1 or sin theta c is equal to c 1 by c 2. Where the wave is transmitted from it lower wave velocity of the wave in c 1 in medium one which is incident wave in c 1 and this is c 2 so that means, if a wave pressure wave come from air to water. If this is air and this is water air to water so velocity of the sound in air is c one and velocity c two is greater than so c 2 is greater than c 1. So, in that case the angle of transmission will be away from the normal and the critical angle is defined by c 1 by c 2. So, theta c the critical angle for which the transmitted angle is 90 degree is theta c critical angle is c 1 by c 2.

So, if I give you the velocity of the sound in two medium and I told you find out the critical angle then you can say the critical angle theta c is nothing but sin inverse c 1 by c 2. So, theta c is nothing but sin inverse c 1 by c 2. Now what will happen if the wave is incident in that condition where c 2 is greater than c 1 and angle of incident theta i is greater than theta c greater than critical angle. If it is that condition that means, the wave pressure wave is transmitted from air to lets water and angle of incident is greater than the critical angle then what will happen.

So, I will derive the mathematical expression what will happen in that case what will happen to the pressure wave is it come back in the first medium as you know the light total internal reflection, it is come back in the first medium or what will happen. So, I have to find mathematically what will happen. So, we will start from that if you say the sin square theta or you say cos square theta i plus sin square theta t or I can say cos square theta t plus sin square theta t is equal to cos square theta plus sin square theta is equal to 1. So, I can say the cos theta t cos theta t is nothing but root bar of one minus sin square theta t. Now, what is sin theta t sin theta t is nothing but c 2 by c 1 sin theta i. So, I can say it is nothing but root bar of 1 minus c 2 by c 1 sin square theta i.

Now, if c 2 is greater than c 1 then cos theta t sin theta i is real and cos theta is purely imaginary if this product of this thing is greater than 1, then this cos theta t becomes imaginary. So, I can write cos theta t. So, angle of incident is greater than the critical angle such that this becomes greater than 1. If it is greater than 1, then cos theta t is nothing but minus j root over of c 2 by c 1

sin square theta t sin square theta i minus 1. I can write this way minus j root over of c 2 by c 1 sin square theta t minus 1.

Now, what is p t, p t is p t amplitude e to the power j omega t minus k 2 cost theta t minus k 2 y sin theta t that is why I calculate the cos theta t. So, cos theta t is nothing but this one. So, I can write this is nothing but p t into e to the power minus minus j; j, j will be cancelled. So, if I say this is minus lets gamma x into e to the power j omega t minus k 2 y sin theta t. So, j cos theta t, I put the value of cos theta t in this equation. So, gamma is nothing but k 2 root bar of c 2 by c 1 sin square theta i minus 1. So, this term into k 2 term I said gamma. So, e to the power minus gamma x e to the power j omega t minus k 2 y sin theta t.

Now, if you say the transmitted wave which propagated along which direction only y-direction, because y is only there, x is not there. So, I can say the transmitted wave propagated along the y-direction and amplitude of the wave if you see the amplitude of the wave will be decreased as per the x axis. As x is increased, so wave is transmitted like this wave axis x is increased the amplitude of the pressure will be decreased. Now, if I say if it is positive, if it is negative then what will happen if gamma is negative then what will happen the pressure will be increases. If gamma is negative, then it is e to the power p into e to the power plus gamma x which means the pressure is exponentially increases as x increases, which is impossible. It cannot be possible in physical reality. So, only possibility is that if the angle of incident in case of oblique incidence is greater than the critical angle then the wave will be travel along the y-axis, and its amplitude will be decreases. So, if x equal to 0 pressure way amplitude is maximum then at x equal to if I increase then it will be decreases pressure way the amplitude will be decreases.

So, as normal light it is came back in the first medium, it is not came back in the first medium in case of pressure wave. So, I can say if it is a pressure wave propagate along the x, y plane, now what will happen that instant of oblique incidence in x y plane, then now if it is critical greater than critical angle pressure wave moves towards the through y-axis and amplitude of that pressure wave will be decreases as you move along the x-axis. So, this is the incident that is the oblique incident case.

But normal incident that means, the wave is propagated along the x-direction only y direction is 0, because normal incident. So, normal incident we have derive their things and we said the transmission coefficient is inversely proportional to the L, pressure transmission coefficient is inversely proportional to the L. And if the intensity transmission coefficient is inversely proportional to the L square. In case of oblique incident we said that yes that critical angle we have calculate, we know the theta t we can calculate, if you know the c 1, c 2, we can calculate the reflection coefficient and we can calculate the transmission coefficient both we can do. So, oblique also we can do.

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Now I am not going this thing that power transmission coefficient. I am not going details because there is a not that much of scope in this course to going all details. You can see that power transmission coefficient also.

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Refraction

Refraction of an earthquake wave.

An earthquake P wave passes across a boundary in rock where its velocity increases from 6.5 km/s to 8.0 km/s. If it strikes this boundary at 30°, what is the angle of refraction?

I will come there are the simple problem is given. It is said then an earthquake P wave passes across a boundary in rock, where its velocity increases from 6.5 kilometer per second to 8 kilometer per second. If it is strike the boundary at a 30 degree, what is the angle of refraction or angle of transmission. So, what I said let a pressure wave incident on a this is rock boundary this is a c 2 and this is c 1. So, from here it is incident with the 30 degree angle. So, what will be the theta t, I have to calculate the theta t. So, what I said sin theta i divided by c 1 is equal to sin theta t divided by c 2. So, sin theta t is nothing but c 2 by c 1 into sin theta i. So, c 2 is given c 1 is given theta is given I can calculate the theta t.

Even if I told you calculate the amount if the power is given, lets the adequate power is given that pressure power is given. So, what amount of the power will be reflected back or intensity of the earthquake is given I said the what amount of intensity wave will be transmitted the rock. So, I know the T, if I know the T, from there I can calculate the T I. Once I calculate the T I, I know that how much amount of transmission multiply by the incident intensity multiply transmission coefficient I get the transmission power, so that way you can know.

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Now, there is a important physical properties in acoustics wave transmission which is mostly used in architectural acoustics. Generally, if you see in a factory, you see there is a lot of machine in a floor and there is a room, which is a separated by a glass partition. Once you inside the room the sound which is created by the machine is reduce drastically, but you have studied that the in transmission coefficient is parser transmission is inversely proportional to L, the length of the partition is very thin glass partition very thin glass partition. So, how the sound is blocked, this is contradictory. If you see I said the length of the transmission coefficient depends on inversely proportional to length. But in a factory if you see there is a lot of machinery in the floor and there is a some enclosure which is made up a glass partition or some gypsum board partition, and inside that partition the sound intensity is drastically reduce - machine noise is deistical radius how it is do that. So, that is called mass law and a physical property

So, that the portion the partition is open material whose motion is normal to the interface regardless of the angle of incidence of the sound and thickness much smaller than the wavelength for the frequency range of interest. So, you know that the thickness of that partition has very, very low and the partition are made) much such material whose motion is normal to the interface regardless of the angle of the incident.

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Mass Law If the intervening layer 2 is thin and completely flexible with surface density ρ_5 , the layer can be treated as an interface possessing mass so that the difference in pressures across the interface equals the product of the surface density ρ_5 of the interface and its acceleration. $\mathbf{p}_i + \mathbf{p}_r - \mathbf{p}_i = j\omega\rho_5 \mathbf{u}_i \cos \theta$ $\mathbf{u}_i + \mathbf{u}_r = \mathbf{u}_i$ $\mathbf{1} - \mathbf{R} = \mathbf{T}$ $\mathbf{u}_i + \mathbf{u}_r = \mathbf{u}_i$ $\mathbf{1} + \mathbf{R} = \mathbf{T} + j \frac{\omega\rho_5}{r_i} \mathbf{T} \cos \theta$ $\mathbf{T}_{\Pi}(\theta) = |\mathbf{T}(\theta)|^2 = \frac{1}{1 + [(\omega\rho_5/2r_1)\cos \theta]^2}$ For an incident wave failing normally on the surface, reduces to the equivalent case given by $\mathbf{T}_i = \frac{1}{1 + \frac{1}{4}(r_2/r_1)^2 \sin^2 k_2 L}$ With fluids 1 and 3 having the same characteristics impedances and thin layer with $r_2 \gg r_1$.

Now, I go for the mathematics what is there if the intervening layer that means, it is nothing but a. So, I can say suppose I have a factory and I make a glass partition or gypsum board partition for office room. I have a factory whole in air. So, this is the partition wall less this is a partition wall, whose thickness as is L, which is very low. Now, this side is air in factory air and this side also air. So, if I say the intervening layer 2 is the intervening layer. So, this is my an enclosure room. So, this is the intervening layer. So, once the acoustic wave fall on this intervening layer completely flexible with surface density flex surface density of this layer is rho into s, surface density surface area multiply by the density is the surface density treated as the interface possessing mass. So, that the difference in pressure across the interface equal to the product of the surface density and its acceleration that means, once the wave intensity are fall pressure intensity fall in here then the particles are move.

So, difference of pressure, so this is P t this is P r and P i. P r will be this direction, P i will be this direction this is P t. So, I say P i plus P r minus P t difference of pressure is nothing but a surface density multiply by the acceleration. So, what is surface density rho into s is the surface density what is acceleration. So, normal component of the velocity here this side is u i cos theta plus u r cos theta is equal to u t cos theta, let us theta is the any angle of incident theta is angle of incident. So, if it is that then for the normal I said that acceleration surface density. So, the velocity is nothing but u i cos theta is the velocity or sorry transmitted wave u t cos theta is the

velocity, surface density this side. So, if particle velocity of this position, this thin layer inside this thin layer is P t. So, in that case, the surface density surface velocity particle velocity is u t cos theta. So, acceleration is nothing but a derivative of the velocity. So, derivate is written at j omega d y by d t is nothing but J omega into u t cos theta. So, J omega is nothing but a d by dt. We said that in past lecture also d y by d t will always treated as j omega j omega p s j omega u t cos theta.

Then I can say 1 plus R is equal to T transmitted coefficient plus J omega P s divided by r 1 t cos theta. Why r 1 because u p e is equal to plus minus u u into r. So, P i divided by p i. So, P i is can be converted p I can be u t we can converted by r or P i converted into u i. So, it is nothing but a r 1 T cos theta. Now, from here if I convert into the P, u is converted into the P, P by r 1. So, I can say u i cos theta is nothing but a P i cos theta by r 1 plus it will be reflected as it will be negative. So, it is P i this p I cos theta P r cos theta divided by r 1 is equal to p t cos theta divided by r 2 or I can say 1 minus R is equal to T from this equation putting p is equal to plus minus u r.

Now, forget about this equation. So, 1 plus R T is equal to this things. So, from there, 1 minus R is equal to T using this equation this is one, and this is two I can calculate T. And then I can calculate T I, which is nothing but it T square which is nothing but 1 by if you calculate it plus omega rho is s divided by 2 r 1 will be plus cos theta whole square. T I will be T square, which is 1 by 1 plus omega p s divided by 2 r 1 cos theta whole square this is the transmission coefficient in the second room, this is the transmission coefficient.

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So, if it is I then will be I multiply by T I. So, T I is 1 by 1 plus omega p s 2 r 1 cos theta. Now, if you think in the previous case when we said that intervene layer length is L, this is r 1, this is r 2, this is r 3. So, here r 1 is equal to r 3. So, for the normal incident case in the fluid, so that in that case the t I is nothing but a 1 by 1 plus 1 by 4 r 2 by r 1 whole square sin square k t l that is the normal case. Now, it is T I, in this case T I this one.

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Now, if this is my transmission coefficient then what is the relation the what is the transmission coefficient in a practical cases. So, let I have a office. Most practical situation in the air of in a factory if you see the factory both inside the enclosure room and factory room both are air and the enclosure room is separated by a thin partition of gypsum board or lets partition in gypsum board, let us plywood or gypsum board. And surface density is 10 kg per meter square. So, at frequency lets f is above 60 hertz if I calculate omega rho s divided by r 1 which will be greater than 9. Then I can say t I or if I ignore this one this term is very large than it is nothing but a 2 r 1 y omega rho s cos theta whole square whole square.

Now, if I increase the surface density by double if I increase the instead of 20 kg per meter square if I use a material which surface density 10 kg, it is 20 kg per meter square; that means, rho into s is increases 2 rho into s, then transmission coefficient will be reduces by one-fourth. So, every time if I double the surface density transmission coefficient will be reduces by one fourth. So, instead of plywood, if I use the glass lets the surface density is double then it will be reduces by one-fourth that is why if you see in a factory layout the working place in a factory is separated by a very thin partition which is create the mass law effect. So, that factory noise cannot go inside that room. So, there is a mass flow is created.

Sometime if you see that in acoustics isolation, when you make a door instead of solid door we make a glass door the two glass in between there will be a air. So, it will be reduced. So, first plus this is a studio this is a glass door in one layer, so there will be a inverse mass law calculating. So, if it is glass, then there will be air, again I have a glass, then the outside noise cannot go inside because if it is there will be a mass flow effecting and there will be also mass flow effecting the two wall. So the inside the inside that auditorium if the two separated glass doors are used very low, low noise will be outside noise will be entered, so that's in acoustics architectural acoustics this is mostly used.

So, next class, we will discuss about the sound absorption and then sound field means reverberation field. So, now we will for the details for the practical application, what is sound field, then how to design the studio that part will be going on the next week. So, from the next week, this theory is over now. Next week, we go for the how design of auditorium. So, those things will be consult when we discuss about those things. So, this theory will be I will use this

theory, when we discuss about that why there will be a door of like this, why there will be a wall of like this, why there will be absorption coefficient is equal like this, how this is achieved that kind of things.

So thank you.