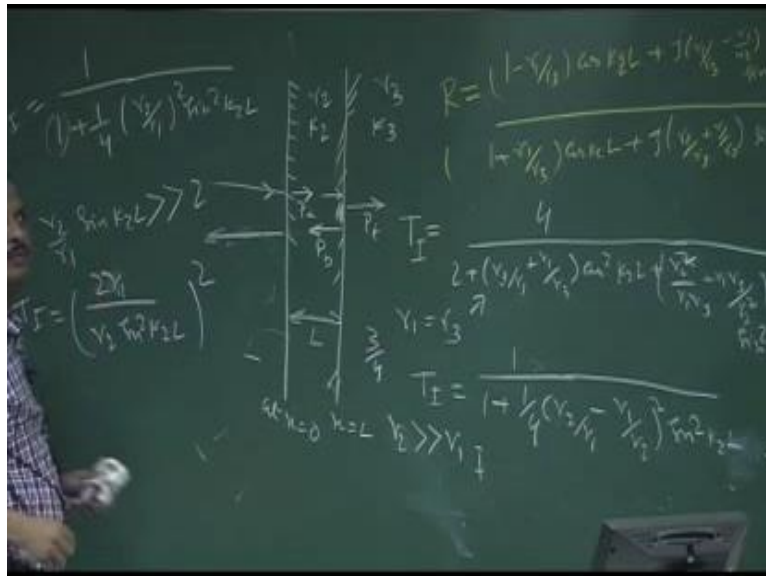


Audio System Engineering
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Lecture - 11
Sound Transmission

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So, now last class we have derived that R and expression T I expression T I expression transmit intensity coefficient for the sound transmission. Because what I want we want that suppose I play a box in this room and there is a wall of thickness L, then I want how much energy - acoustic energy from this room will be transmitted in the corridor; there is a corridor after the wall that is why. Similarly, I can say if there is some sound in the corridor of intensity I, how much intensity of that sound will be transmitted inside the studio if I know the thickness of the wall and characteristic impedance of the wall. So, in case of this things, the thickness of the wall is L then T I transmission coefficient is nothing but $4r_1r_3 / (r_1 + r_3)^2 \cos^2(k_2L) + (r_1 - r_3)^2 \sin^2(k_2L)$ that is the transmission equation.

Now, if I consider that r_1 and r_3 is equal, what is the meaning that means, sound is transmitted from air medium of inside this room to that wall and after the wall there is another air medium, lets the two air medium is same that means, r_1 is equal to r_3 . Then

T_I will be what will be T_I , I place r_1 is equal to r_3 . Then T_I will be $1 + 1 + 4r_2$ by r_1 minus r_1 by r_2 whole square $\sin^2 k_2 L$. I put r_1 is equal to r_3 in this equation. So, r_1 is equal to r_3 this is 1, this is again 1, $1 + 1$ again 2 , $2 \cos^2 k_2 L$ plus $r_1 r_3$ is same. So, this is r_1 square, this is r_1 square, r_1 square $r_2 r_2$ by $r_1 r_2$ by r_1 . So, r_2 by r_1 plus r_2 by r_2 , so $2r_2 r_1$ square. So, in that case, if I simplify that thing if r_1 is equal to r_3 I get $1 + 1 + 4r_2$ by r_1 minus r_1 by r_2 whole square whole square into $\sin^2 k_2 L$.

Now, in this equation if I say that r_2 is much, much greater than r_1 , what I said r_2 is much, much greater than r_1 what is the meaning physical meaning. That lets this is the air impedance of the acoustic and impedance of the air compare to the solid wall the solid wall is r_2 . So, solid wall is much, much higher than the acoustic impedance of the air. So, in that case r_2 is much, much greater than r_1 then again it will be simplified to, simplified to what again T_I will be simplified to T_I will be equal to $1 + 1 + 4r_2$ by r_1 whole square $\sin^2 k_2 L$. R_1 is much, much less than r_2 . This implied that the transmission of the sound form air in one room to the solid wall into the adjacent room. So, this is the transmitted coefficient T_I is $1 + 1 + 4r_2$ by r_1 whole square $\sin^2 k_2 L$.

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and algebraic manipulation yields the pressure reflection coefficient

$$R = \frac{(1 - r_1/r_3) \cos k_2 L + j(r_2/r_3 - r_1/r_2) \sin k_2 L}{(1 + r_1/r_3) \cos k_2 L + j(r_2/r_3 + r_1/r_2) \sin k_2 L}$$

$$T_I = \frac{4}{2 + (r_2/r_1 + r_1/r_3) \cos^2 k_2 L + (r_2^2/r_1 r_3 + r_1 r_3/r_2^2) \sin^2 k_2 L}$$

Special Forms of above equation which are of particular interest:

1. If the final fluid is same as the initial fluid, $r_1 = r_3$,

$$T_I = \frac{1}{1 + \frac{1}{4}(r_2/r_1 - r_1/r_2)^2 \sin^2 k_2 L}$$

If, in addition, $r_2 \gg r_1$,

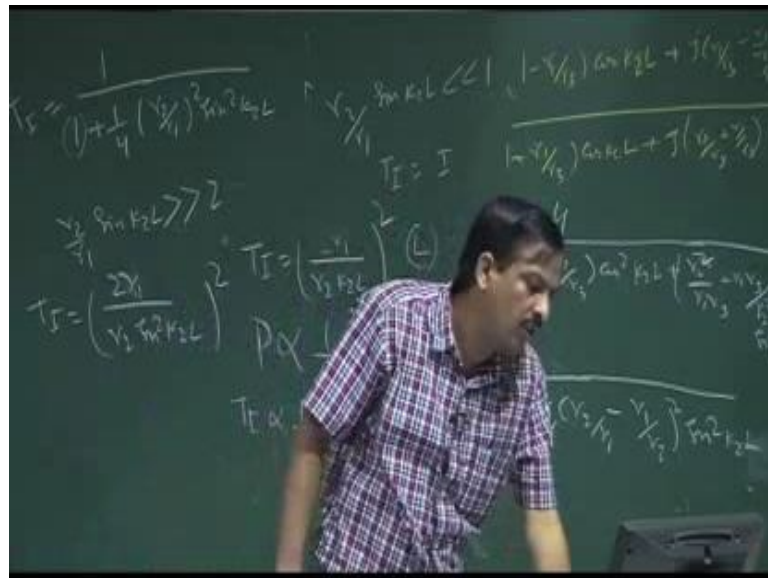
$$T_I = \frac{1}{1 + \frac{1}{4}(r_2/r_1)^2 \sin^2 k_2 L}$$

This is applied to the transmission of sound from air in one room through a solid wall into air in an adjacent room.

Now if I consider that r_2 by $r_1 \sin k_2 L$ is much, much greater than two then T_I will be $1 + 2r_1$ divided by r_2 into $\sin^2 k_2 L$ whole square; I consider that r_2 by $r_1 \sin$

$k^2 L$ is much greater than 2. In that case, this equation will be simplified to $2 r_1$ by r_2 into $\sin^2 k^2 L$ whole square, this 1 will be ignore and 4 will be, you understand. So, now, $2 r_1$ means $4 r_1$ square divided by $r_2 \sin^2 k^2 L$, I can ignore this 1. So, now, if $k^2 L$ now another simplification what I said that r_2 by $r_1 \sin k^2 L$ is much greater than 2, what is r_2 , r_2 is the characteristic impedance of wall, r_1 is characteristics impedance of the air into $\sin k^2 L$, L is the thickness of the wall.

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Now, if $\sin k^2 L$ is less equal to $k^2 L$ $\sin \theta$ is equal to θ $\sin k^2 L$ is equal to $k^2 L$ $\sin \theta$ is almost equal to θ then I can say the $T I$ is nothing but a $2 r_1$ divided by $r_2 k^2 L$ whole square. So, transmitted pressure intensity is inversely proportional to the transmitted pressure intensity is varied with the L square. So, I can say that the transmission pressure is where is inversely proportional to the L thickness of the wall L . So, I can say P is pressure amplitude inversely proportional to the length of the thickness of the wall. So, if the thickness of the wall is increases, then the pressure transmitted pressure amplitude will be decreases; and $T I$ is nothing but proportional to 1 by L square, if the thickness of the wall is larger then there is a chance of less intensity of the sound will pass through the wall in the next room.

So, suppose I want to build a studio or auditorium if I do not want that outside sound should not go inside that auditorium, the thickness of the wall of the auditorium must be larger. If the thickness is increases, the outside sound intensity of the outside sound

which is entered in the auditorium will be less. Similarly, the sound which will be inside the auditorium will be going outside the auditorium will be very less. So, thickness of the wall is inversely proportional to the transmitted pressure amplitude.


For thin plane or low frequency if let that another condition, the condition is that if r_2 by r_1 into $\sin k_2 L$ is much, much less than 1, then T_I will be one T_I will be 1. If T_I is equal to 1 that means, if l is very thin l is very low such that r_2 by r_1 into $\sin k_2 L$ is much, much less than 1 then the partition that second medium r_2 does not affect the sound transmission, does not affect the sound transmission.

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2. The intermediate fluid has a larger characteristic impedance than either fluid 1 or fluid 2 but such a small thickness that

$$r_2 \sin k_2 L \ll 1 \text{ and } \cos k_2 L \approx 1.$$

$$T_I = \frac{4}{2 + (r_3/r_1 + r_1/r_3) \cos^2 k_2 L + (r_2^2/r_1 r_3 + r_1 r_3/r_2^2) \sin^2 k_2 L}$$



$$T_I = \frac{4r_2 r_1}{(r_3 + r_1)^2}$$

- A thin membrane of solid material of appropriate characteristic impedance may be used in
 - preventing two gases or two liquids from mixing and
 - yet not interfere with sound transmission between them.
- In particular, if $r_2 = r_3$ then
 - there is total transmission from fluid 1 to fluid 3 as if fluid 2 did not exist.

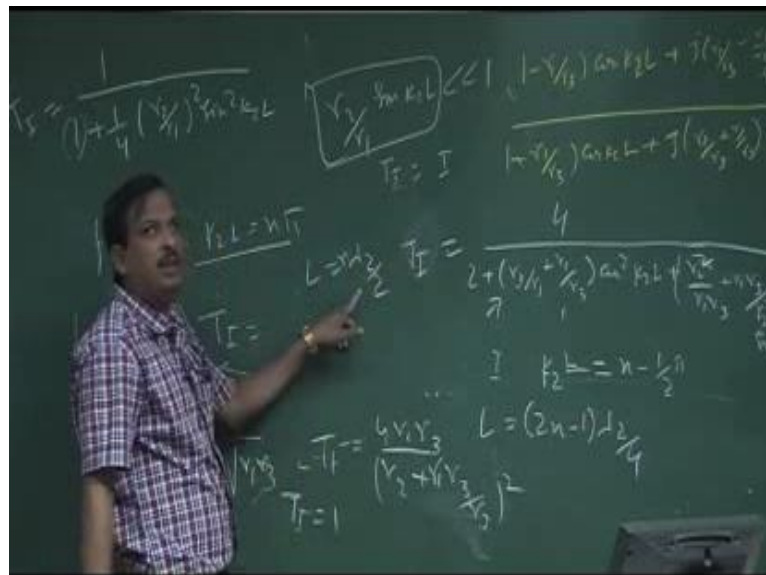
So, think about practically. There is a two phase separated by a very thin membrane which L is so low that r_2 by r_1 $\sin k_2 L$ is much, much less than 1 then the sound produce in the one fluid will completely transmitted to the next fluid. So, the separation does not affect the sound transmission separation does not affect the sound transmission. So, thin membrane I can separate the two fluids, but it does not affect the sound transmission the intermediate fluid has larger characteristic impedance then either fluid one by small size is it OK.

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$$\begin{aligned}
 3. \text{ If } k_2 L \approx n\pi \quad f = nc_2 / 2L \quad L = n\lambda_2 / 2 \\
 T_I = \frac{4}{2 + (r_3/r_1 + r_1/r_3) \cos^2 k_2 L + (r_1^2/r_1 r_3 + r_1 r_3/r_1^2) \sin^2 k_2 L} \\
 \text{Reduce to } T_I = \frac{4r_3 r_1}{(r_3 + r_1)^2} \\
 4. \text{ If } k_2 L \approx (n - \frac{1}{2})\pi \quad L = (2n - 1)\lambda_2 / 4 \\
 \cos k_2 L = 0 \quad \text{and} \quad \sin k_2 L = 1 \\
 T_I = \frac{4r_1 r_3}{(r_2 + r_1 r_3 / r_2)^2} \\
 T_I = 1 \quad \text{when} \quad r_2 = \sqrt{r_1 r_3}
 \end{aligned}$$

Now, I go for the other condition lets $k_2 L$, this is the T_I expression, expression of T_I is this one; this is nothing but T_I . Now, put the condition number three, what are the condition I prove the thickness of the intensity pressure is inversely proportional to that things I proved.

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Now, this is the T_I . The condition is that $k_2 L$ is nothing but $n\pi$, $k_2 L$ is nothing but a $\cos^2 k_2 L$ is nothing but a $n\pi$, so integer multiple of π $\cos^2 k_2 L$. So, then T_I will be what T_I will be $4r_3$ by r_1 divided by r_3 plus r_1 whole square. $k_2 L$ is

equal to $n\pi$ I put the $k_2 L$ in $n\pi$, then in this equation then I get that $T I$ is nothing but $4 r_3$ into r_1 divided by r_3 plus r_1 , because this term will be 0.

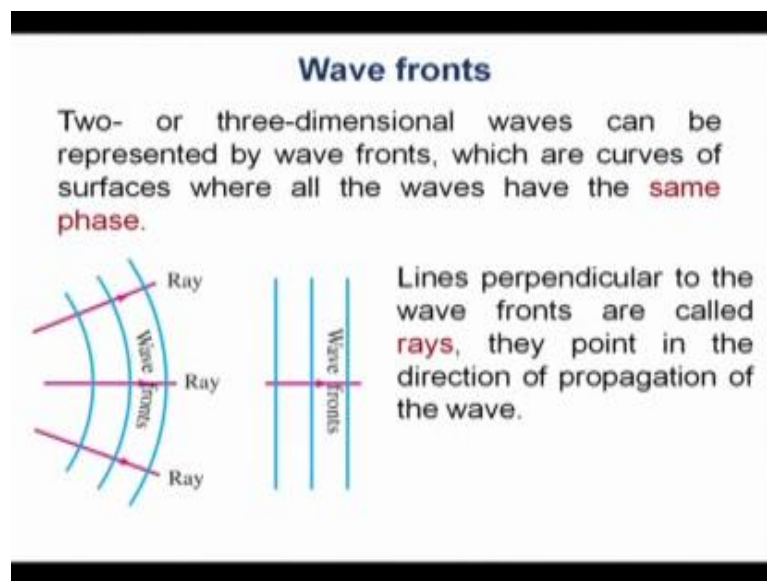
Now, if it is this is the condition, so what do you mean by $k_2 L$ is equal to $n\pi$. $k_2 L$ is equal to $n\pi$ then what is the frequency f , f is nothing but $\frac{n c_2}{2 L}$ what is C_2 , C_2 is the velocity of the sound in the second medium, n is the integer number and 2 into L is the length of wall. So, I can say l is nothing but $\frac{n \lambda_2}{2}$. So, I can say if the thickness l of the wall is the integer multiple of $\frac{\lambda_2}{2}$ that means, the wave length is λ_2 in those frequency whose frequency of half wave length is integer multiple of half of the wave length is equal to L . In case of those frequencies the $T I$ will be this 1, $T I$ will be $4 r_3$ into r_1 divided by r_3 plus r_1 whole square. If r_1 and r_3 are equal then it is nothing but this $4 r_1$ square divided by r_1 plus $2 r_1$. So, this is $4 r_1$ square it is nothing but 1. So, that frequency will be passed through the wall. If the length of the wall is the integer multiple of the half wave length of the frequency then that frequency will be pass of the wall.

Similarly, if the $k_2 L$ is equal to n minus half π or I can say the L is equal to $\frac{2 n - 1}{4} \lambda_2$ then also $r_2 T I$ will be $4 r_1 r_3$ divided by r_2 plus $r_1 r_3$ divided by r_2 whole square. In that case if the $k_2 L$ is equal to $(n - \frac{1}{2}) \pi$ integer multiply by 1 minus half π our L is equal to $\frac{2 n - 1}{4} \lambda_2$ then the $T I$ will be $4 r_1 r_3$ divided by r_2 plus $r_1 r_3$ divided by r_2 . If the r_2 is equal to root over of r_1 into r_3 the $T I$ will be 1. So, if the characteristic impedance of the second medium or intermediate medium is a geometric mean of the two separate two medium is then the transmission coefficient will be the 1.

So, if for those frequency for which is if L is equal to $\frac{2 n - 1}{4} \lambda_2$ wave length divided by 4 and the r_1 and r_2 is the geometric mean of r_1 and r_3 then the $T I$ will be 1. For that frequency also the transmission co efficiency will be 1. So, when I design that length of the wall and for a acoustic studio, I have to think about what is the operating frequency condition. For that frequency condition also I have to think about the L , I should not make the L such that, if you see that if L is the integer multiple of $\frac{\lambda_2}{2}$, integer multiple of λ_2 for those frequency for which the L is integer multiple of λ_2 the $T I$ will be 1. So, the transmitted wave will be passing through the wall. So, this is this is the thing will use when we design that acoustic studio.

Basically, thumb rule is that when you design a studio the length of the wall thickness of the wall must be such that the pressure wave you know that transmitted pressure wave amplitude will be inversely proportional to the length of the wall. So, if a thick wall it will reduce sound transmission from one room to another room or I can increase the r^2 . So, now so far we have discussed about that plain wave propagation and spherical wave propagation. Now, how that wave will be transmitted means that when the acoustic wave be generated and process to the medium and if say that if the wave is fallen to the one surface of another medium that the wave is transmitted in the air medium then it fall on the wall, there will be solid medium. So, how the sound will be transmitted that means, some portion of the sound will be reflected some position of the sound will be transmitted and there will be other phenomena like that sound diffraction and then interference, so those phenomena we will discuss in this class.

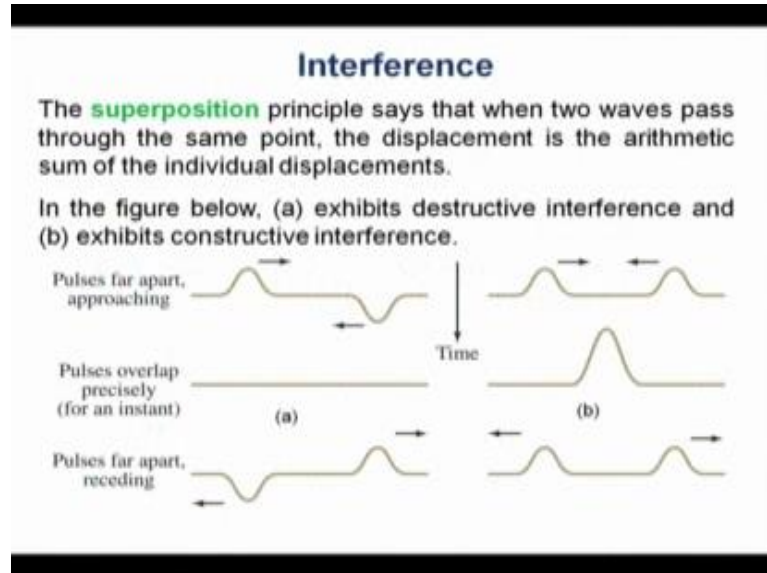
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So, that any sound or you can say acoustic wave has an wave font. What is wave font two or three-dimensional wave can be represented by wave fronts, which are curve of surface where all the waves have the same phase if you see the blue line all are the wave fronts. So, within the line, the wave has the same phase and line perpendicular to the wave font is called ray. As per the figure if you see that this magenta color line, line perpendicular to the wave is called ray. The ray direction is the direction of the propagation of the wave. So, if you thrown a stone on a pond, you see that there is a

wave will be generated, those wave is propagated towards the bank of the river or bank of the point, so that propagation direction of the propagation is the pointed by the ray.

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Now, if this is way the wave propagated then there will be interference.

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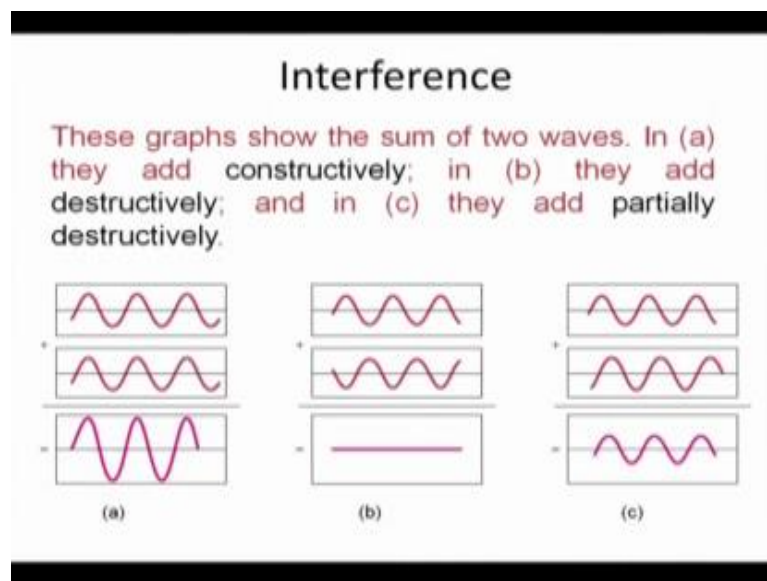


Suppose, I have a wave like this and another wave is approaching like this. So, when the two waves are passes in the same point there will be interference will be happen in the same point suppose this point then there will be interference will be happen between the two rays two waves. So, now, this interference support the super position principle

means at the point of interference the particle displacement to do the both wave is the arithmetic sum of the displacement of the first wave and the displacement of the second wave. So, you understand so; that means, at the points of interaction it support the superpositional principle that means, the displacement of the particle due to the first wave plus displacement of the particle due to the second wave the arithmetic sum of those two displacement is the resultant displacement at point of interaction.

So, if you see the figure in here lets one wave is going this direction and another wave is negative, negative phase going in reverse direction. So, if that if they meet at certain points when interact at the certain point then there will be a destructive interference because this amplitude and this amplitude is negative and this amplitude is positive. So, negative. So, let us this is a, this is minus a, a plus minus a is equal to 0. So, destructive interference will be happen.

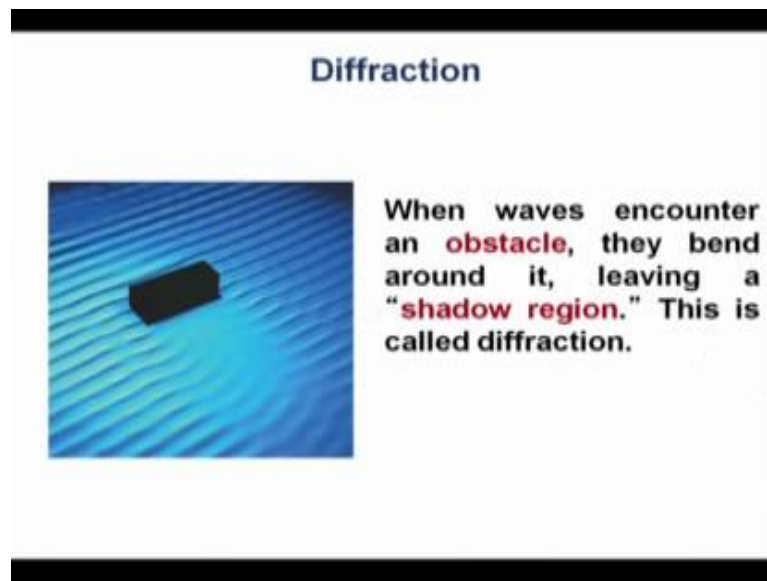
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Similarly, if both waves are in the same phase at the point of interaction then the arithmetic sum will be double the first individual wave. Similarly, when the two waves are interference then slight bit of phase change then different kind of amplitude will be happen. Let us see the figure like this way. See that two waves let us see sinusoidal waves or if they are same phase and interference in one point then the amplitude will be added up. If the two waves are in opposite phase interference then the amplitude will be destructive, so zero. If there is a slight change of phase then there will be a resultant

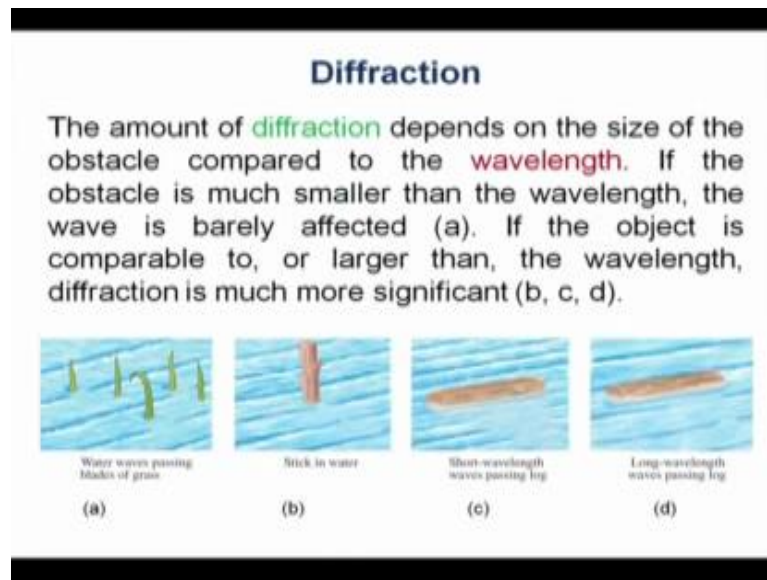
wave will be generated based on the arithmetic sum of the two waves. So, this is called wave interference, you know that. If the sine wave is process another sin wave will be add together if they are same phase the amplitude will be double; if they are opposite phase amplitude will be 0. If they are slight phase different then the any point this will be the arithmetic sum of the other two waves.

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Similarly, there will be diffraction when wave encounter an obstacle if you see if you see in this pond there is a wave they put a lock wooden lock wooden lock there you see there is some portion behind the lock there will be no interference there is no wave. So, this is called diffraction, the wave is diffracted by the log. So, when the waves encounter an obstacle the bend around the obstacle and create a shadow region this region is called this phenomena is called diffraction.

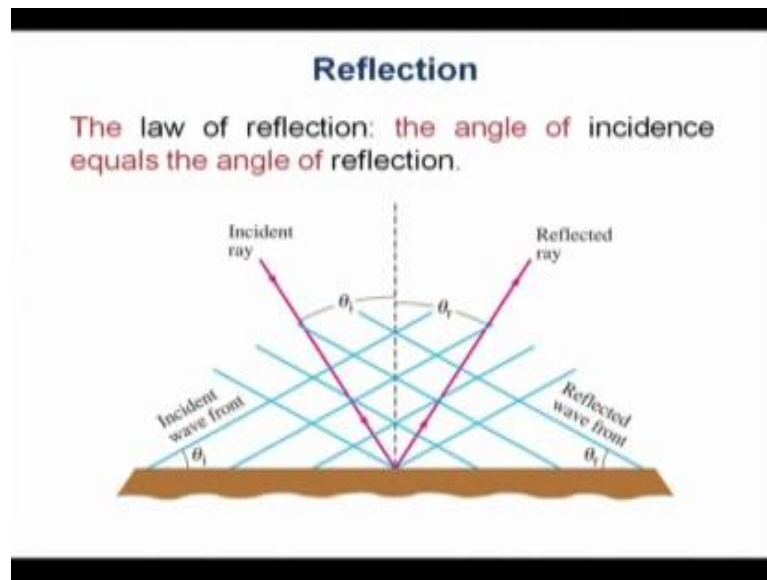
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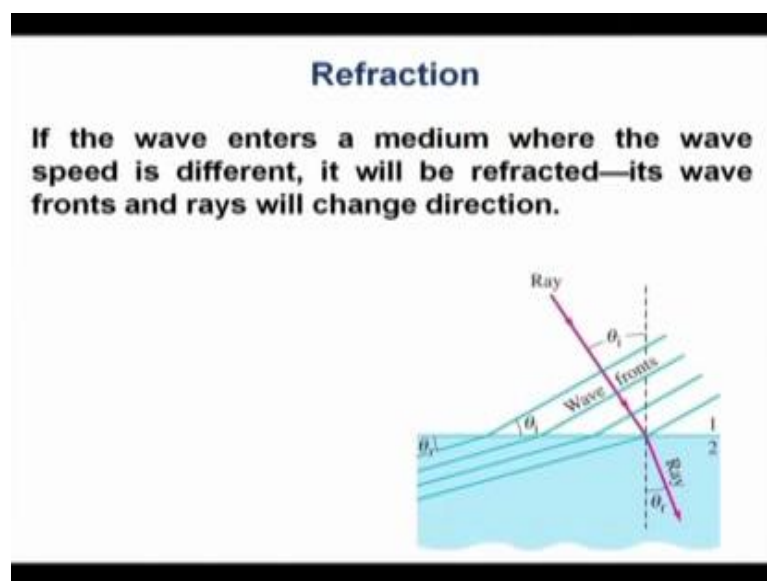
So, how diffraction defined, the diffraction depend on the size of the obstacle. If the size of the obstacle compare to the wave length is very less or called negligible then you see there is no diffraction. If you see the picture a there is no diffraction not at all a diffraction, because the wave length of the wave front wave and the length of the obstacle is very less length of the obstacle is very less compare to the wave length. So, that is why the diffraction is not happened.

Now, if the length of the obstacle is larger, then the diffraction will be larger. So, that is called sound diffraction then there will be a reflection phenomena suppose an acoustic wave propagated in this room and just incited on the wall. So, what will happen some portion of the acoustic wave refracted that from the wall? So, that is called the wave reflection and reflection is also following the same principle as the light; that means, the incited an incited angle is equal to the reflected angle. So, angle of incited will be same as the angle of reflection, so that is same as the light wave.

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Similarly, there will be a refraction also, some portion of the acoustics wave transmitted inside the wall, so that is called refraction. So, this two phenomena will details analyze in this lecture will be or we can say the details we discussed the mathematics of that transmission and reflection. So, that we can calculate the intensity suppose I play this in practical example suppose I want to produce the sound of sixty d p in this room and I do not want to that sound should go in outside the room. So, what should be the wall thickness that we have to derive how I should treat the wall, so that the sound produce inside the room that does not pass the next room? So, those kinds of phenomena that is

sound transmission, so sound transmission will be discussed in this class, details of the sound transmission.

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Sound Transmission

P_t → transmitted wave Pressure
 P_i → incident wave Pressure
 P_r → reflection wave Pressure

A. Pressure transmission and reflection coefficients

$$T = P_t / P_i \quad R = P_r / P_i$$

B. Intensity transmission and reflection coefficients $I = P^2 / 2\rho$

$$R_I = I_r / I_i = |R|^2 \quad T_I = I_t / I_i = (\rho_1 / \rho_2) |T|^2$$

C. Power transmission and reflection coefficients

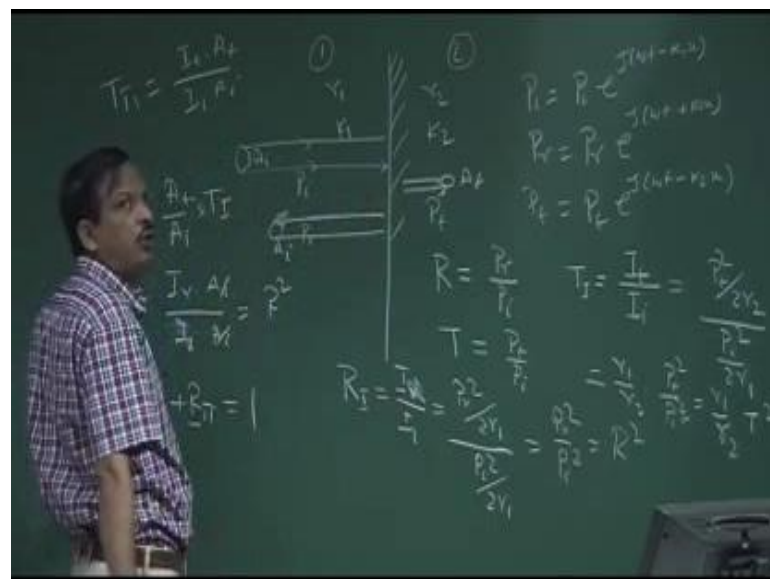
$$T_p = (A_t / A_i) = (A_t / A_i)(\rho_1 / \rho_2) |T|^2 \quad R_p = R_i = |R|^2$$

$$R_p + T_p = 1$$

- All the ratios depend on
 - o the characteristic acoustic impedances,
 - o speed of sound in the two media and
 - o on the angle of incident wave makes with the interface.

So, let us forget about that things lets we think that all the theory which will we discuss here all are derived based on the fluid, but same phenomena will exist for solid and air also.

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So, let us there is a two fluids this is fluid number one and this is the fluid number two or you can say this is the medium one and this is the medium one. So, one wave lets

acoustic wave normally incident in the boundary of the two medium, so that is called incident wave. So, this is pressure is P_i and the same if it normally incident that reflection also will be normally. So, it is p_r it is the reflected wave and some portion of the wave will be transmitted in the second medium that is P_t , so that is called transmitted wave transmitted pressure wave.

So, this is the pressure wave P_i . So, what is the expression of P_i , P_i is nothing but a lets amplitude of the $P_i e$ to the power $j \omega t - k_1 x$. So, let this medium is acoustic impedance is r_1 and this medium has r_2 . What is r_1 , r_1 is nothing but a $\rho_1 c_1$; and r_2 is nothing but $\rho_2 c_2$. So, since the velocity of the sound in the two medium are different, the wave number also the frequency, frequency of the incident wave pressure wave and frequency of the transmitted pressure wave will be the same, but since the k is nothing but ω by c , this is called wave number k is nothing but ω by c .

So, since ωc is different, the wave number will be different that is why if I say that the wave number of this medium is k_1 and wave number of this medium is k_2 then I can say P_i is nothing but $P_i e$ to the power $j \omega t - k_1 x$. Similarly, what is p_r p_r is amplitude e to the power $j \omega t + k_1 x$. Why, it is plus $k_1 x$ because the direction of the propagation of the reflected wave, it is reverse from the incident wave that is why it is plus $k_1 x$. Similarly, what is P_t , P_t is nothing but P_t amplitude e to the power $j \omega t - k_2 x$ because medium is different that is why wave number is different.

Now, if I want to know what is the task what is the reflection of the coefficient reflection coefficient of this incident. So, reflection coefficient r is nothing but a reflected pressure wave divided by incited pressure wave reflected pressure wave amplitude divided by incited pressure amplitude. So, what is the transmission coefficient T is nothing but a P_t by P_i - transmitted pressure wave divided by incident pressure wave is transmission coefficient. Now, reflection coefficient transmitted coefficient for the pressure. Now, if I say what is the what amount of intensity will be transmitted an amount of intensity will be reflected. So, I want to find out the intensity coefficient of the of the incident wave.

So, if I say T_I is the transmitted intensity coefficient it is nothing but a I_t - transmitted intensity divided by incident intensity. What is the transmitted intensity you know the intensity I is nothing but p^2 by $2 \rho c^2 r$ p^2 by $2 r$. So, I can say I_t is

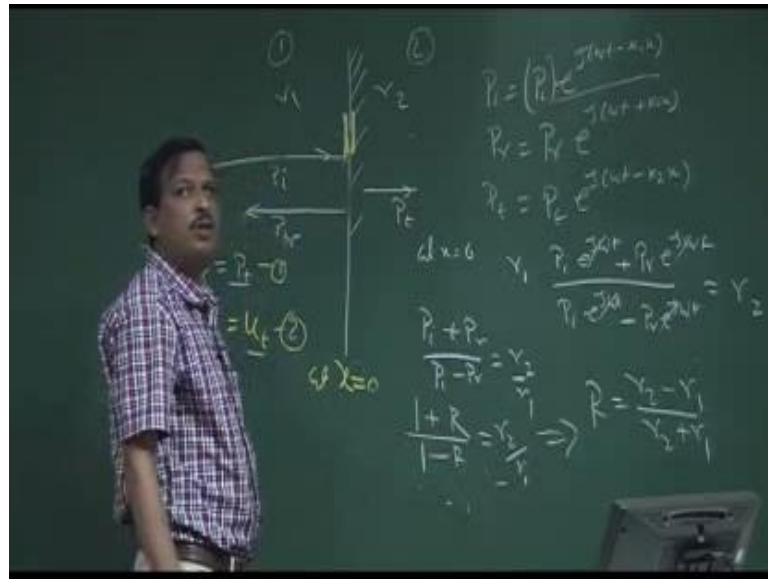
nothing but P_t^2 divided by $2r^2$ divided by P_i^2 divided by $2r^1$. So, this is nothing but a r^1 by r^2 into P_t^2 divided by P_i^2 . So, it is nothing but a r^1 by r^2 into t^2 t is the pressure coefficient r^1 by r^2 into square.

Similarly, what is reflected intensity coefficient? So, r_i is nothing but I_r divided by I_i , I_r divided by I_r . So, so what is I_r , I_r is nothing but a P_r^2 divided by $2r^1$; similarly, what is I_i , it is nothing but P_i^2 divided by $2r^1$. So, it is nothing but P_r^2 divided by P_i^2 which is nothing but a r^2 square. So, R_i is nothing but a r^2 square and T_i is nothing but a r^1 by r^2 square into t^2 , but T is the pressure coefficient and R is the pressure coefficient for reflected coefficient. So, T is the transmitted coefficient of the pressure r is the reflected coefficient of the pressure.

Similarly, if I want to find out what is the power coefficient transmitted power coefficient and reflected power coefficient. What is transmitted power coefficient T_{pi} nothing but a power is nothing but a intensity multiply by area cross sectional area. Let us think about incident ray has an area like this kind of beam channel is incident on the surface. So, incident ray area is A_i same area will be reflected. So, this area of the reflected coefficient will be also A_i , but transmitted coefficient transmitted pressure area may be different lets it is A_t , so it is A_t . So, I can say what is T_i transmitted power divided by the incident power. What is transmitted power it is nothing but I_t into A_t divided by I_i into A_i . So, it is nothing but a A_t by A_i into I_t by I_i , I_t by I_i means nothing but a T_i . So, it is nothing but A into T_i .

Similarly, what is r_{pi} , r_{pi} is nothing but a nothing but a I_r into A_i divided by reflected power divided by the incited pressure I_i into A_i . So, A_i , A_i cancelled it is nothing but a I_r by I_i which is nothing but a r^2 square which is nothing but a r^2 square. So, but if you see the T_{pi} plus R_{pi} total power total incited power is nothing but reflected power plus transmitted power. So, T_{pi} power plus R_{pi} coefficient must be equal to 1. So, total power is equal. So, this power is nothing but this power plus this power. So, power coefficient for reflected coefficient and transmission coefficient sum must be equal to 1. So, you understand this thing.

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Now, consider what will happen in case of P r. Now I have to find out how much intensity of the value of that all coefficients transmitted coefficient and reflected coefficient how to I find out the value. Lets I say that a ray of P i pressure ray is P i incident normally on the boundary. So, there will be a reflected wave, which is also P r is normal to the boundary and there will be a transmitted wave P t. So, I know P i, P r and P t. Now, what is the boundary condition at the boundary the pressure of this side of the boundary must be equal to the pressure on this side of the boundary have you understand or not pressure on this side will be equal to pressure on this side. So, both side will be pressure will be equal when the both side will be pressure will be equal.

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Boundary conditions:

1. The acoustic pressure on both sides of the boundary must be equal
2. The normal components of the particle velocities on both sides of the boundary must be equal.

$$\begin{aligned}
 p_i + p_r &= p_t & \text{at } x = 0 \\
 u_i + u_r &= u_t & \text{at } x = 0
 \end{aligned}
 \quad \longrightarrow \quad
 \begin{aligned}
 \frac{p_i + p_r}{u_i + u_r} &= \frac{p_t}{u_t} & \text{at } x = 0
 \end{aligned}$$

Since a plane wave has $p/u = \pm r$,

$$r_1 \frac{p_i + p_r}{p_i - p_r} = r_2 \quad \text{1+R=T}$$

$$R = \frac{r_2 - r_1}{r_2 + r_1} = \frac{r_2/r_1 - 1}{r_2/r_1 + 1} \quad T = \frac{2r_2}{r_2 + r_1} = \frac{2r_2/r_1}{r_2/r_1 + 1}$$

$$R_1 = \left(\frac{r_2 - r_1}{r_2 + r_1} \right)^2 = \left(\frac{r_2/r_1 - 1}{r_2/r_1 + 1} \right)^2 \quad T_1 = \frac{4r_2 r_1}{(r_2 + r_1)^2} = \frac{4r_2/r_1}{(r_2/r_1 + 1)^2}$$

So, if I write the expression if the pressure is equal it is the condition number one boundary condition one pressure on both side will be equal the acoustic pressure on both side of the boundary must be equal. So, acoustic pressure on this side is nothing but a $p_i + p_r$ is equal to must be p_t this is the condition number one and the condition number two is the normal component if you see the normal component of the particle velocity on both of the boundary must be equal. So, normal component of the particle velocity on both side of the boundary must be equal. So, I can write $u_i + u_r$ should be equal to u_t . So, this is the equation number one and this is the equation number 2 where the boundary is here that is x is equal to boundary is at x is equal to 0. So, at x is equal to 0 $p_i + p_r$ is equal to p_t and $u_i + u_r$ is equal to u_t put the both the equation.

So, now I divide it equation number one by equation number 2. So, I can write $p_i + p_r$ divided by $u_i + u_r$ sorry $p_i + p_r$ divided by $u_i + u_r$ is nothing but p_t by u_t equation number one divided by equation number two. Now, what is the relation between the p and u p is nothing but plus minus r u r is the acoustic impedance p is nothing but a plus minus r u . So, $p_i + p_r$ divided by what is u_i , u_i is nothing but a p_i divided by r_1 same medium this is the r_1 and this is r_2 reflected wave direction of the propagation is different, 180 degree phase shift. So, minus p_r by r_1 is equal to p_t by p_t by r_2

So, I can write from there that $p_i - p_r$ divided by $p_i + p_r$ is equal to $\frac{Z_2 - Z_1}{Z_2 + Z_1}$. So, it is nothing but r . So, at x is equal to 0 at x is equal to 0 p_i is nothing but only amplitude at x is equal to 0. So, p_i is nothing but a at x is equal to 0 at x is equal to 0 at x is equal to zero. So, p_i is r into p_i into $e^{j\omega t}$ plus p_r into $e^{j\omega t}$ divided by p_i into $e^{j\omega t}$ minus p_r into $e^{j\omega t}$ is equal to r . So, $j\omega t$ will be cancelled. So, I can say that $r = \frac{p_i - p_r}{p_i + p_r}$ is nothing but $\frac{Z_2 - Z_1}{Z_2 + Z_1}$. So, if I divided by p_i . So, $1 + r = \frac{p_i + p_r}{p_i}$ is nothing but $1 + r = \frac{Z_2 + Z_1}{Z_1}$. So, I can find out from here r is nothing but $\frac{Z_2 - Z_1}{Z_2 + Z_1}$ or not from here I can find out r is nothing but $\frac{Z_2 - Z_1}{Z_2 + Z_1}$.

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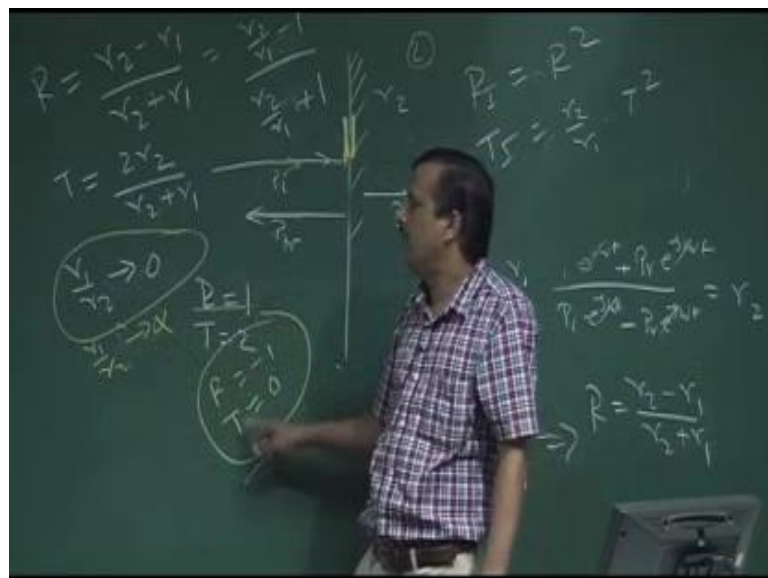


So, reflection coefficient r for this normal incident is nothing but $\frac{Z_2 - Z_1}{Z_2 + Z_1}$. What is transmitted coefficient, what is equation number one $p_i + p_r = p_t$. So, if I divided by p_i , so it is nothing but $1 + r = T$. So, if I do that then T is equal to $1 + \frac{Z_2 - Z_1}{Z_2 + Z_1}$. So, it is nothing but $\frac{2Z_2}{Z_2 + Z_1}$. So, transmitted coefficient T is nothing but $\frac{2Z_2}{Z_2 + Z_1}$. Now, what is $R I$, $R I$ is nothing but R^2 , what is $T I$ is nothing but r into T square. So, I know the T value, I know the R value, I can find out the $R I$ intensity of the coefficient.

Now, if I put the condition, now let see what is the condition available. The first condition is that if r_1 is less than r_2 , what you mean by r_1 is less than characteristic impedance of the first media is less than the characteristic impedance of the second medium. So, example is that like a wave is transmitted from air to a concrete wall. So, air characteristic will be $\rho_1 c_1$ is less than then the characteristic between the concrete walls. So, a wave is transmitted from air to solid wall in that case that R is always positive r will be always positive and T will be always positive. So, I can get the value of R and T if I know the r_1 and r_2 values. But if you consider where r_2 is less than r_1 or r_1 is greater than r_2 ; that means a wave is transmitted from the solid wall to air wave is transmitted from solid wall to air. In that case R will be reflection coefficient will be negative r_1 is greater than, so R will be negative.

Similarly, if r_2 is equal to r_1 means two media characteristic impedance is same then reflected coefficient r is equal to 0 r_2 is equal to r_1 . So, r_1 minus r_2 is equal to 0 and transmitted coefficient t is equal to one so; that means, if the two medium is same then the total transmission will be happen there will be no reflection. So, this is three conditions if the wave is passing only in the air. So, there is total transmission in the wave. So, there is no reflection by the air, if the characteristic impedance of the whole medium is same.

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Now what is the implication this phenomena. Now, if I consider mathematically that r_1 by r_2 is tends to 0, what will happen. If r_1 by r_2 I can write down this things it is nothing but r_2 by r_1 minus 1 divided by r_2 by r_1 plus 1; r_1 by r_2 tends to 0. So, the wave is reflected with no reduction in amplitude no then r will be 1 and t will be 2. So, what is the meaning, meaning is that the wave is reflected with no reduction in amplitude r is 1 and no change in phase. So, pressure coefficient r_1 means the reflected pressure wave, and incident pressure wave will be the same amplitude and no phase change because r is 1 plus 1 that means, there is no phase change is happen.

Now, if r_1 by r_2 , it tends to infinity, then what will happen? Then again r will be minus 1. So, what is meaning of the minus 1. So, in meaning of the minus 1 is that that amplitude is same, but phase is 180 degree shifted, 180 degree phase shift is happen. So, in that pressure wave amplitude is same but the compare to incident wave, the pressure wave has 180 degree phase shift. And what is t , in case of that, t is equal to 0. What is the meaning t is equal to 0, the transmitted wave has a pressure amplitude which is equal to 0; transmitted wave at pressure amplitude p_t is equal to 0. So, that that why this condition is called pressure release condition, pressure release boundary and last condition is called rigid boundary last condition is called rigid boundary and second condition is called pressure release boundary. So, you understand.

So, that means, if a wave is transmitted from one medium to another medium, if r_1 by r_2 is tends to infinity, that means, that r_2 is very small compare to r_1 , r_1 is very high; that means, r_1 is very high means the wave is transmitted from a dense medium to lighter medium. In that case, if r_1 , r_2 is equal to infinity in that case the boundary is called pressure release boundary; if r_1 by r_2 is equal to 0, the boundary is called rigid boundary.

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Normal Incidence

Transmission through a fluid

$$P_i = P_i e^{j(\omega t - k_1 x)}$$

$$P_r = P_r e^{j(\omega t + k_1 x)}$$

$$P_a = A e^{j(\omega t - k_2 x)}$$

$$P_b = B e^{j(\omega t + k_2 x)}$$

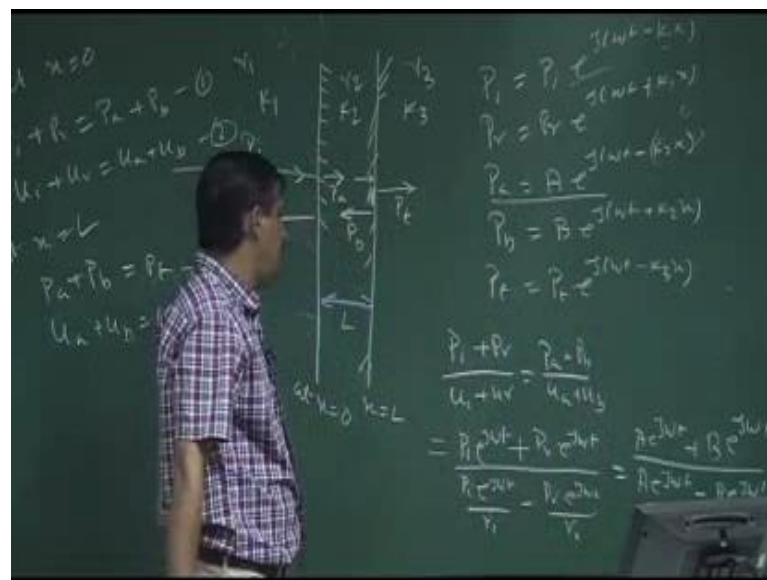
$$P_t = P_t e^{j(\omega t - k_3 x)}$$

Continuity of the normal specific acoustic impedance at $x = 0$ and at $x = L$ gives

$$\frac{P_i + P_r}{P_i - P_r} = \frac{r_2}{r_1} \frac{A + B}{A - B} \quad \frac{A e^{-j k_2 L} + B e^{j k_2 L}}{A e^{-j k_2 L} - B e^{j k_2 L}} = \frac{r_3}{r_2}$$

Now, think about, lets I have another problem. The problem is that suppose a acoustic wave is generated in this room and acoustic wave is generated in this room in the air and that wave is incited on the wall. Now, walls have a thickness.

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So, I said that I have a boundary this is the room number one. So, medium r 1 room number air acoustic wave generated in the room in air, so that will fall in the boundary of the wall lets this is the wall static boundary. So, at x is equal to 0. Now, I have a wall of thickness L, so this is at x is equal to 1. So, thickness of the wall is L. Then some portion

of the incident this is P_i , some portion of the incident wave will be transmitted in the second boundary inside the wall that also incident it in the so this is r_2 . Let us after the wall there is another boundary; lets this is the room acoustic wave generated in this room fallen to the wall and there is a corridor also there is air. So, after the wall there is a corridor air, so that wave will be transmitted in the corridor.

Let us the corridor air is r_3 . So, if p_i is incident air there will be some transmitted wave which is P_a , there is P_a transmitted wave. And then that will fall on this wall, P_a will fall on this wall, and will created a reflected wave is P_b . And some portion will be transmission that side which is P_t . Now find out the reflection coefficient of the pressure wave and transmitted coefficient of the pressure wave and some portion of here will be reflected which is P_r . Now, find out the r and t for this condition. That means, I want to find out in practical cases that if the thickness of the wall is defined then if I say that in this room I produce a 60 dB intensity sound, how much intensity of the sound will be released in the next room or next corridor that I want to find out.

So, if I want to get the transmission coefficient then I can easily find out how much energy will be transmitted in the next room after the wall. So, I want to find out that thing. So, I have to find out T and R . So, what is T , I have to find out T and T , then I want to find out transmitted intensity coefficient and reflective intensity coefficient then I can find out transmitted power coefficient reflected power coefficient all this thing I can find out. And I can find out what is the effect of the L th. If L is increased is the power is reduced transmitted power is reduced, if L is decreases is the transmitted power is increases. So, what kind of effect of this L has on the sound transmission that I have to find out, so that is called the three boundary transmission? So, there are two boundaries in this room, air to solid boundary then solid to air boundary. Next boundary, we have the solid boundary.

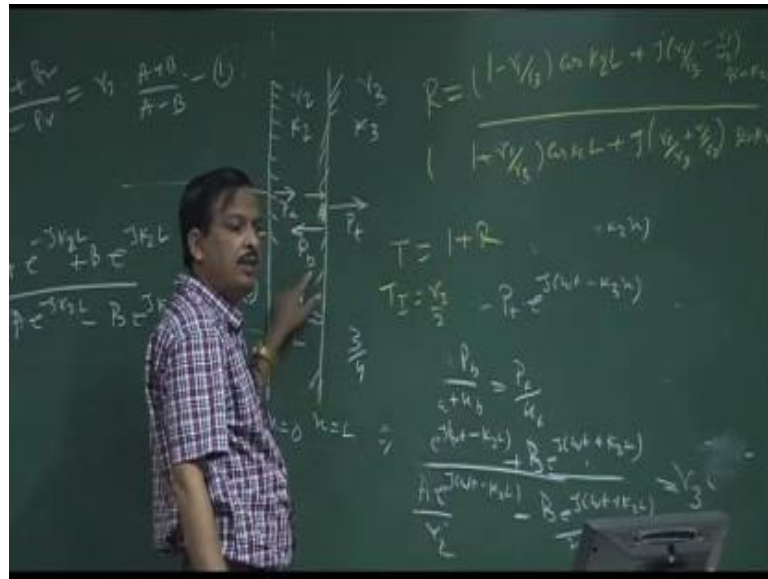
Now, find out the equation, first what is the equation. You know what are those equations the equations are p_i is nothing but amplitude $p_i e^{-j\omega t - k_1 x}$ to the power $j\omega t - k_1 x$ minus if this is k_1 the wave number in this medium is k_1 , wave number in this medium is k_2 , wave number is k_3 . So, it is nothing but $k_1 x$ what is p_r , p_r is nothing but $p_r e^{-j\omega t - k_1 x}$ to the power $j\omega t - k_1 x$ plus $k_1 x$ because it is in the reverse direction that is way I say this is plus $k_1 x$. Similarly, if I say what is p_a p_a is nothing but let amplitude is $a e^{-j\omega t - k_2 x}$ what is p_b again it is reverse direction. So, it is

nothing but $b e^{j\omega t + kx}$. Similarly, what is p_t , p_t is nothing but amplitude p_t into $e^{j\omega t - kx}$. So, those are the equation for the p_i, p_r, p_a, p_b, p_t .

Now, put the boundary condition. What are the boundary condition, when a two boundary condition at x is equal to 0, the pressure on this side which is $p_i + p_r$ is equal to pressure on that side which is $p_a + p_b$, at x is equal to 0 pressure on this side is equal to pressure on this side. And particle velocity normal component of the particle velocity $u_i + u_r$ is equal to nothing but $u_a + u_b$ ok or not, ok. But at x is equal to 1, what is the p_a total pressure in this side $p_a + p_b$ is equal to p_t and $u_a + u_b$ is equal to u_t this is the boundary condition this is the equation number 1, 2, 3 and 4.

Now, if I divide the equation number one by two. So, I can get $p_i + p_r$ divided by $u_i + u_r$ is equal to $p_a + p_b$ divided by $u_a + u_b$. So, at x is equal to 0 means, what is p_i p_i is nothing but $p_i e^{j\omega t + kx}$ plus p_r into $e^{j\omega t - kx}$ divided by what is u_i , u_i is nothing but $a p_i$ by r . So, I can write $p_i e^{j\omega t + kx}$ divided by r minus p_r into $e^{j\omega t - kx}$ divided by r is equal to what is p_a , p_a is nothing but this one - x is equal to 0 means this term will be 0. So, this is nothing but $a e^{j\omega t + kx}$ plus $b p_b e^{j\omega t - kx}$ divided by $u_a + u_b$. So, it is nothing but $a e^{j\omega t + kx}$ minus $b e^{j\omega t - kx}$ this is divided by r^2 this is divided by r^2 p is equal to u is equal to p by r . So, from there I can get what I get, I get I will just rub here.

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So, what I get from there from this equation what will get $p_i + p_r$ divided by e to the power $j\omega t$ cancelled for all the things divided by $p_i - p_r$ into r_1 is equal to r_2 into $a + b$ divided by $a - b$ plus b into $a - b$. Now, if I say at x is equal to L , what I will get at x is equal to L that I said equation number three divided by four. So, I get $p_a + p_b$ divided by $u_a + u_b$ is equal to p_t by u_t . So, it is nothing but p_a is nothing but a what is p_a at x is equal to 1 p_a is nothing but p_a into e to the power $j\omega t - k_2 L$ plus p_b . What is p_b , p_b into e to the power or a sorry it is A , amplitude is A , amplitude is B .

So, B into e to the power $j\omega t + k_2 L$ at x is equal to L divided by what is u_a , A e to the power $j\omega t - k_2 L$ minus B e to the power $j\omega t + k_2 L$ and whole this is divided by r_2 . And this is divided by r_2 is equal to nothing but a p_t divided by p_t by r_3 . So, now, from there I can get from there what I will get I will get that A e to the power $j\omega t$ will be cancelled r_3 into p is nothing but amplitude will be cancelled e to the power $j\omega t - k_2 L$ at x is equal to 1 what is p_t e to the power $j\omega t - k_2 L$ minus p_t , p_t cancelled. So, it is nothing but r_3 this is r_3 .

Now, what will I get, I will get A into e to the power minus $j k_2 L$ plus B into e to the power plus $j k_2 L$ it is $k_2 L$ divided by a e to the power minus $j k_2 L$ minus b e to the power $j k_2 L$ is equal to r_3 by r_2 . So, this two equations if I evaluate equation number one and equation number two, if I evaluate A and B what I will get I will get the

expression of R. R is nothing but a reflection coefficient. If I evaluate this equation number one and this equation number two or A, B I replace I will get R reflection coefficient. R is equal to $1 - r_1/r_3$ minus r_1/r_3 by r_2/r_3 into $\cos k_2 L$ plus r_1/r_3 into r_2/r_3 plus r_1/r_3 by r_2/r_3 minus r_1/r_3 by r_2/r_3 sin $k_2 L$ divided by $1 + r_1/r_3$ cos $k_2 L$ plus r_1/r_3 into r_2/r_3 plus r_1/r_3 by r_2/r_3 sin $k_2 L$ that I will get. So, I will get this expression.

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and algebraic manipulation yields the pressure reflection coefficient

$$R = \frac{(1 - r_1/r_3) \cos k_2 L + j(r_2/r_3 - r_1/r_2) \sin k_2 L}{(1 + r_1/r_3) \cos k_2 L + j(r_2/r_3 + r_1/r_2) \sin k_2 L}$$

$$T_t = \frac{4}{2 + (r_3/r_1 + r_1/r_3) \cos^2 k_2 L + (r_3^2/r_1 r_3 + r_1 r_3/r_3^2) \sin^2 k_2 L}$$

Special Forms of above equation which are of particular interest:

1. If the final fluid is same as the initial fluid, $r_1 = r_3$.

$$T_t = \frac{1}{1 + \frac{1}{4}(r_2/r_1 - r_1/r_2)^2 \sin^2 k_2 L}$$

If, in addition, $r_2 \gg r_1$,

$$T_t = \frac{1}{1 + \frac{1}{4}(r_2/r_1)^2 \sin^2 k_2 L}$$

This is applied to the transmission of sound from air in one room through a solid wall into air in an adjacent room.

If you see this, I will get a R expression; and from R expression, I can get the T expression - transmitted coefficient expression. So, what is T, T is nothing but a 1 plus R. So, I can get 1 plus R I can get T. And what is T I, T I is nothing but a T I is nothing but a r_3/r_1 by 2 into that I will get I will get the T i. So, then I get the T I expression from this I get the T I expression. So, once I get the T I expression then I can calculate I can impose the restriction in the next class.