Fundamentals of MIMO Wireless Communication
Prof. Suvra Sekhar Das
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 09
Small Scale Propagation Frequency Flat Fading

Welcome to the course on Fundamentals of MIMO Wireless Communication. Now we are undergoing the discussion on multipath propagation effects, whatever we discussed in the previous lecture is one of the foundations on which the whole course is standing. So, we will take a little deeper look into the expression which we have developed towards the end of the last lecture.

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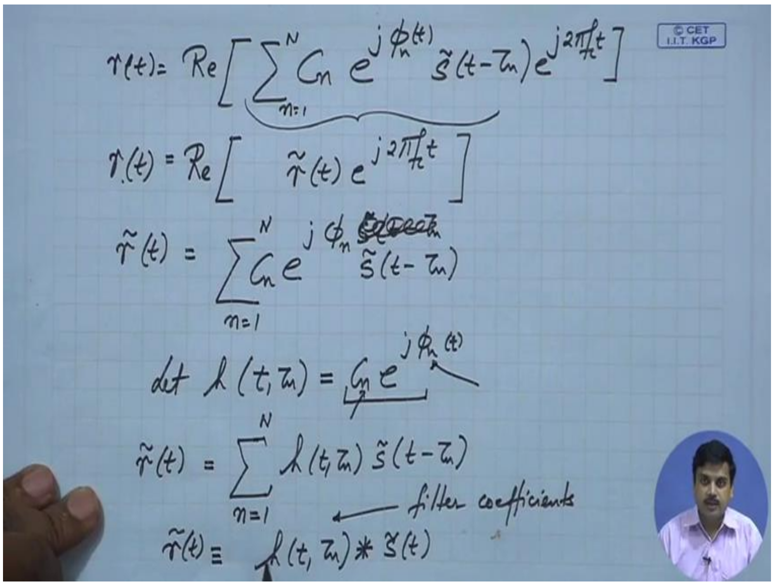

So, what we discussed in the last lecture is the complex envelope, how did we received baseband signal which can be represented in this form where h t comma tau n is the good product of the amplitude coefficient associated with the nth path and the phase associated with the nth path. And it could be written as convolution expression. So, what we will now try to do is take a detail look at how does this arise.

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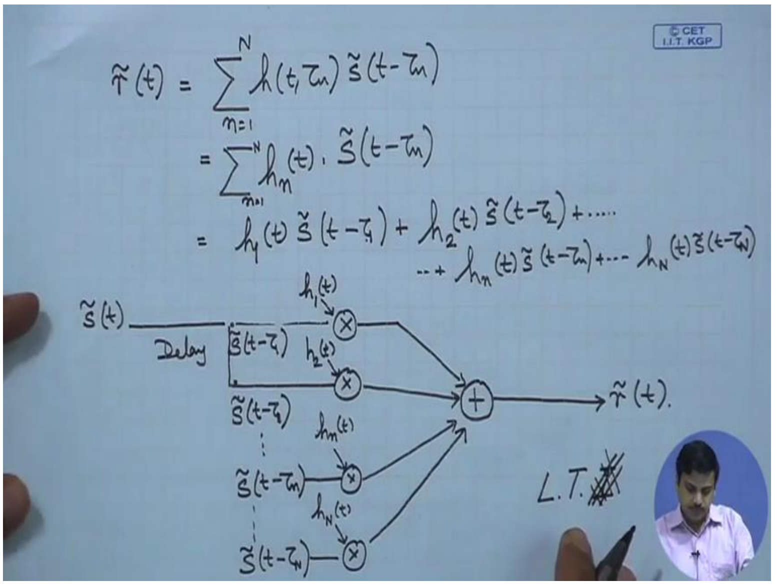

So, when we write that r tilde t is equal to n is equal to 1 to capital N h of t comma tau n s tilde t minus tau n, this we could also write as instead of having tau n written over their we could easily write it has h n of t because this is dependent on the nth path multiplied by s tilde t minus tau n. The n gets reflected in the phase where you have f c tau n. So, tau is getting multiplied in that case we are suffixing n to indicate the nth path.

If we expand this we could write this as h 1 of t multiplied by s tilde t minus tau 1, that means signal which started at a time tau 1 units of time before the current time has come through the first path which as a delay of tau 1 and there is a coefficient h 1 which includes the amplitude and the phase change associated with the first path; plus h 2 that is means a coefficient associated with the second path s tilde t minus tau 2 and so on the generic being h n of t s tilde t minus tau n up to h capital N of t s tilde t minus tau capital N.

So, if we see this expression we could pictorially write this expression as the signal s tilde t getting transmitted there is some propagation delay, and after which we have s tilde t minus tau 1 let us say the delay is tau 1 this is getting multiplied by the coefficient which is h 1 of t and then the signal goes out it gets added to s tilde t minus tau 2 that means this is delayed further through the another path there is multiplication by gain h 2 of t gets added to that so on, s tilde t minus tau n gets multiplied by the gain h n of t and so on up to s tilde t minus tau capital N which gets multiplied by h n of t gets added and what we get at the output is r tilde of t.

So, as if the signal after a delay is getting multiplied by one coefficient, further delayed multiplied by second coefficient, further delayed multiplied by third coefficient and so on. So, we could set this as almost finite impulse response filter for a transversal filter structure.

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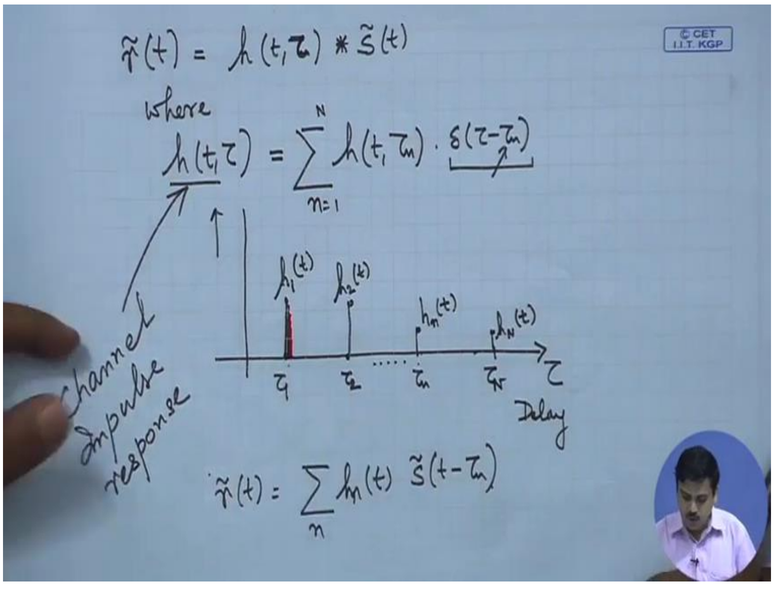

So, we could write these expressions in a better way and state that we have r tilde t is equal to filtering of s tilde t with h t of n, which we have as h t comma tau n convolved with s tilde t. Where, we could write h t comma tau is equal to the sequence n equals 1 to capital N h of t comma tau n is called convolution of h t comma tau times delta tau minus tau n.

Or in other words what we have we can pictorially if this is the delay axis we call this the delay axis and this is the gain axis, so at a delay of tau 1 what we have is h 1. And remember it is a function of time, at a delay of tau 2 we have h of 2 which is also a function of time. At a delay of tau n of course there in between we have h n t and so on. Finally, at the last tilde tau capital N we have h n of t. That means, this is what we can say is h t comma tau represents the channel impulse response, because we are having a convolution of the channel impulse response with the signal to get to received baseband signal or the received complex envelope. And the h values represent the different or coefficients of the impulse response or the filter tabs which themselves are function of time.

What you may have encountered before is such a situation that means such a filtering linear transversal filters where these coefficients where not varying with time. These where constant with time and you had a linear time invariant system, but what we encounter here is a linear time variant system and this is fundamentally because the phase which is a function of time and that has happened because of the multipath propagation where we have mobility involved as part of relative distance between the transmitted and receiver or the points of reflection or the surface on which the waves in pinch.

So, we could also look at this expression, that means r tilde t is equal to if you look at this expression r tilde t which we have been writing is equal to sum over n h n of t multiplied by s tilde t minus tau n.

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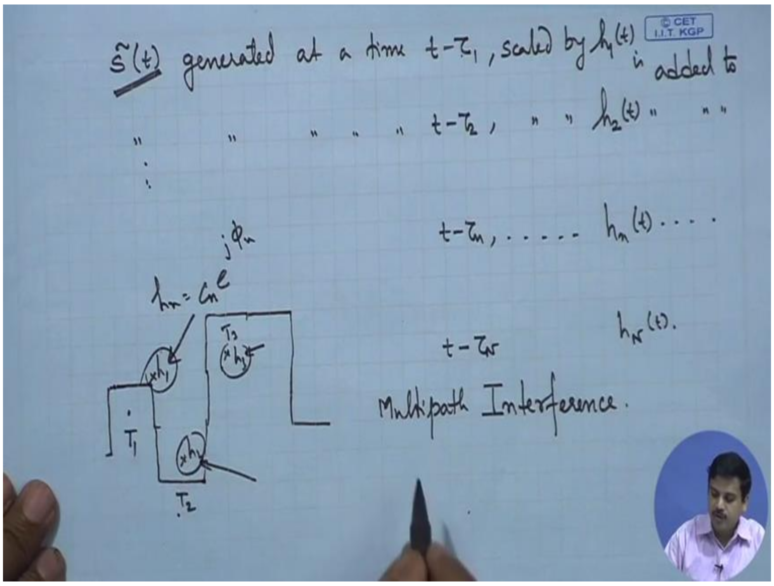

What we could also say is that as if we have the situation where the signal s tilde t which is generated at time t minus tau 1 and scaled by h 1 of t right that is what we are getting over here if you see all this picture generated at a time tau 1 before t scaled by h 1 of t is added, so let us write that it is added 2 s tilde t generated at a time t minus tau to units scaled by h 2 of t and is added to. So, this generated tau 1 units of time I had scaled by h 1 of t added to s tilde t generated at a time tau 2 units before scaled by h to 2 added to the others and so on and so forth it keeps we could write it has the one which is generated at a tau n times before multiplied by h n added to and it continuous to t minus t n h n of t.

So, what does it mean? It effectively means that s tilde t which is the basic signal or t he signal that is generated is getting added to something, so what we have is some signal generated at some earlier time scaled getting added to the same source, but generated at different time. Now these two signals just consider a digital communication system where at certain time t 1 let us signal like this we generated at another time t 2, let us say this signal is getting generated at a time t 3 if you might have a signal this generated at a time 4 it might be this generated and so on, it goes on.

So, what this statement tells us is that signal generated at time let us say t 1 so this value gets multiplied by let us say h 1, this gets added to the signals which is generated at time t 2, t 2 is equal to t minus tau 2 gets multiplied by h 2 plus the signals which got generated at time t 3 gets multiplied by h 3 and so on so forth.

So, clearly the received signal has multipath these at due to the paths because they are equal to c n e to the power of j phi n h n, this we have already said. Multipath and since they are all getting added as we see in this expression they are all getting added they are getting mixed up we used the term multipath interference. So, what we have is signals from different delays scaled by appropriate path coefficients are getting added together and it is mixed up. So, what we can see is that a clean signal which we have drawn over here will not appear in the same way at the receiver, we could draw another picture and try to look at it what would happen.

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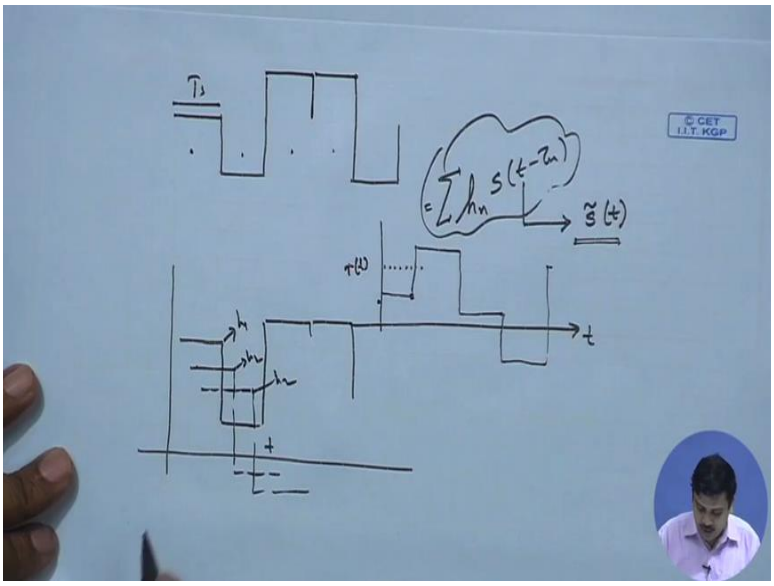

Suppose, I have a signal sequence which goes like this, so at the receiver at this instant of time what do I accept; what I accept is the signal which was transmitted here gets scaled by h 1 plus the signals which was transmitted here gets scaled by h 2 plus what was transmitted here gets scaled by h 3 and so on and finally we sit over here.

So, if I would have the received axis on time and let us say the received signal on this axis we can never sure what is the value of the signal that we get. If there was no such multiplicative factor for instance, if we did not have any such things suppose we had all of this as zeros and this has 1 we would have only the delayed version of the original signal. That signal scaled by some arbitrary value added to another signals scaled by an arbitrary value we would not be able to draw the signal that we get over here.

So, the signal amplitude would be here and the next time instance signal amplitude could be anywhere, we are not sure where the signal amplitude will lie. Because that would depend upon the coefficients h n and there is a summation of all of this coefficients, so we have an arbitrary sequence which comes at the receiver. Now, if we are able to understand this thoroughly then when we built the receiver from this combination of signals we would like to extract s tilde t so that the signal which was originally transmitted is reconstructed at the receiver without any effect of the propagation channel.

Now let us go ahead a little bit further and what we would assume is that this delays that we are suggesting over here are negligible compared to the symbol duration. So, what we mean is if the symbol duration is this much this is the symbol duration the delays are very very small. So, what we would have seen, in this case if you have to reconstruct the signal. The signal transmitted at a certain delay, so basically they would be a delay signal generated, added to the replica of the signal delayed, but this getting scaled by h 1 this getting scaled by h 2, again there is a delay scaled by h 3 all of this added together. What we are now making the assumption is these delays are so small that the negligible compared to the symbol duration.

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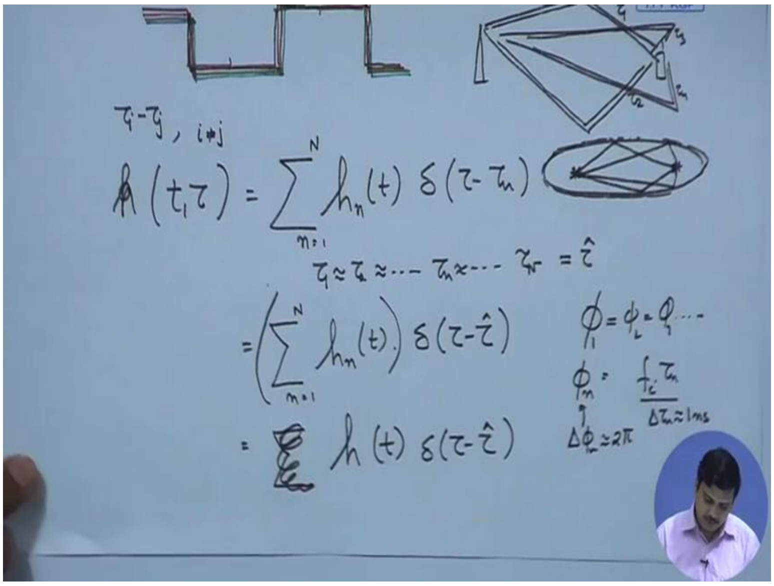

Or, in other words if a transmitted sequence is this I am drawing a random transmitted sequence which probably different from the previous transmitted sequence. The delays are so small that the received signal would be hardly much delay amongst the path. If this is the signal that has come to the first path, of course we are not making assumption on the coefficients, what has come through the second path, what has come to the third path, and of course there are associated scaling coefficients. So, this is what is the assumption that we are making at the receiver front.

So, in other words we are saying that this tau i minus tau j for i not equal to j is very very small. In other words when we are writing the coefficient that means when we are writing the channel impulse response we would look at this particular expression. So what we could write is the h of t comma tau which was equal to sum over h n of t delta tau minus tau n, n equals to one two capital N. Now, since we have written that let tau 1 is almost equal to tau 2 is almost equal to tau n is almost equal to tau capital N. And let that be equal to sum tau cap, then you could write this has equal to h n of t multiplied by delta tau minus tau cap.

So, what we have is effectively all this delays there was different previously now they are the same. So, what we are saying is just try to use a previous picture, so I have it here its good. I have this picture. Previously because of this expression delta these delays that means, h 1 at tau 1 because of the delta function it is shifted at tau 1, because of the delta function h at 2 is shifted to tau 2, because of tau n it is shifted at tau n. Now this tau n is very close to each other. So we will probably have h 2 over here, h 3 over here, h 4 over here h 5 over here h 6 over here.

As if the delays are very very close. So, what we mean a pictorially is in this we could draw, is that transmitter should not violate our earlier picture receiver signal propagates to path tau 1 signals propagates to path tau 2 signals propagates to path tau 3 signals propagates to path tau 4 and so on. All we make the assumption is that this propagation delay is negligibly small difference between them; there is negligible small difference in the delay.

Now this can happen if the transmitter and the receiver are at the two focal points on an ellipse, in that case all the path lengths are exactly the same. We are not saying that they are exactly the same buy they are very very close to each other so as if all the reflectors factorise everything are located in such a way that the propagation delays very very similar. Now this could give raise to the situation where phi 1 is equal to phi 2 is equal to phi 3 and so on and that would destroy the whole structure.

However, we remember the expression of phi n is contains f c times tau n, and there we have seen that a gap of delta tau n is almost equal to 1 nanoseconds causes delta phi n to be changing by nearly to phi. So, we make the assumptions that all they are very close that means this tau n are not separable, but phi ns are still quantified; that is a very very strong assumption that we make.

So, with this what we have is this expression is this tau cap is common for all. Therefore we could add up everything and we need not have the summation sign any more so what we could write this as h of t, because all h n have added up now because this tau caps have gone previously they where a different delays so they were not getting added up they were at separate gaps. But now since they have almost come to each other very close to each other we are as if they are at the same time location approximately this is a model, so this is an approximation.

So, we are able to add them up and we are having a single effective h of t times delta tau minus tau cap. So, what does it affect in our diagram. In our diagram this particular figures all this tau n's are getting modified with the same value of tau.

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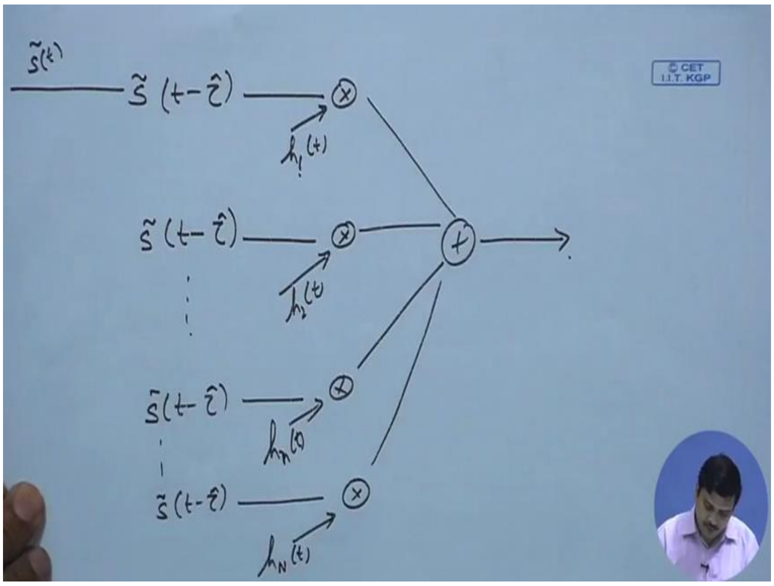

So, what we have now is s tilde t minus tau cap getting multiplied by h 1 of t is of course s tilde t coming s tilde again look at this t minus tau cap, because the delays are the same. But getting multiplied with h 2 of t although the delays are same still the coefficients are same this is very very important. So, we will have s tilde t minus tau cap for all because delays are same, but we are going to have h n of t and we are going to have multiplication by h capital N of t and they will all be added together and sent out.

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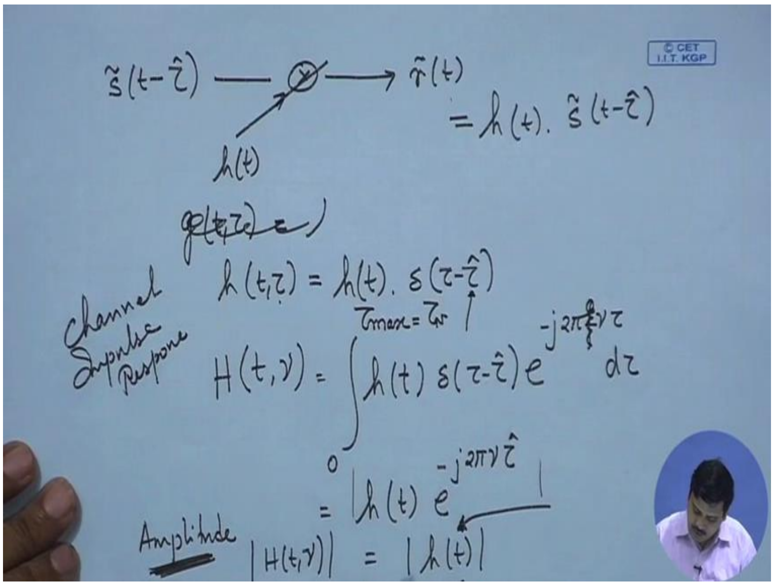

Now, since all this delays are same we could draw this example as if there is s tilde t minus tau cap coming from the source and it is getting multiplied by a single h of t as we have draw earlier and it is going out to give r tilde t. Or, in other words we could write r tilde t is equal to h of t multiplied by s tilde t minus tau cap. Now this is because we have g of t comma tau is equal to sorry, we have h of t comma tau is equal to h of t multiplied by delta tau minus tau cap. That means there is a propagation delay corresponding to tau cap and there is a coefficient.

Now when we study a filter we study channel impulse response as well as the frequency transform function. To study the frequency transform function we need to take the Fourier transform across the delay domain, so that means we have to take the Fourier transform across this axis. So, if we take the Fourier transform across this axis we could get h of t, because we are not taking Fourier transform in the t axis we are taking Fourier transform across this axis. Let us put nu which is equal to integration from 0 to tau max.

So, by tau max we mean the maximum value or tau n in this expression so here tau max could be equal to tau n in our expression. And since it is continuous domain that we are doing now it will be tau max, so we could have h of t delta tau minus tau cap e to the power of minus j 2 pi f sorry, we should have nu tau d tau. This would result in h of t because this is not depended on any such expression. And this integration would result in e to the power of minus j 2 phi nu tau cap is whole function in shifted by the delta function.

If you look at the amplitude of this that is the mod of h if t comma nu what you would get is the modulus of h of t, because this modulus is unity. So, what we see is that the amplitude response of this particular expression is not dependent on nu, because when we take the modulus, this is not dependent on nu; this is dependent on nu whereas this is becoming unity so this is independent of the frequency. However, it varies with time.

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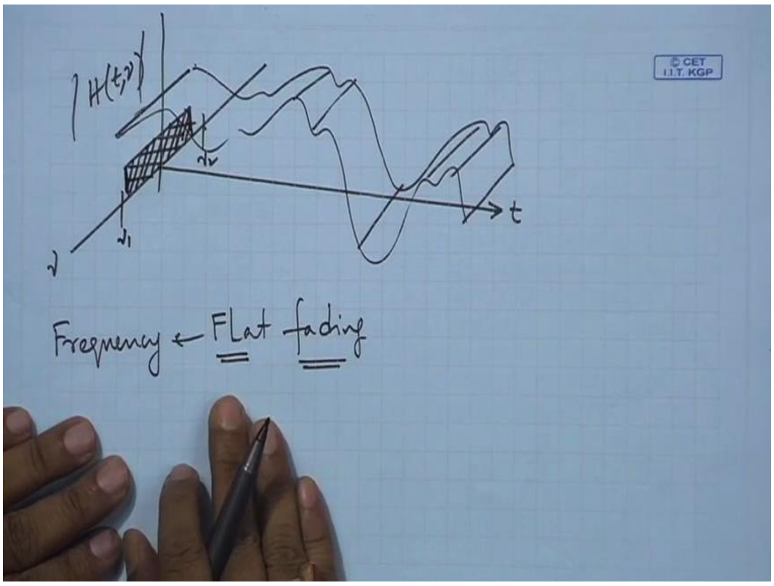

So, if we are a plotting the frequency response and let us say this is the nu axis, this is the gain axis, what we will get is across our interest of frequency zone we going to get a flatted response. It is not dependent on frequency; it is same value across different values in frequency. However, if we draw the time axis what we will see is that this value is fluctuating with time, so that means across time across this whole set of bandwidth the magnitude is remaining the same; there is no change in magnitude across the bandwidth. However, that same magnitude is fluctuating across time in as depicted in this figure.

Now since across the bandwidth there is no fluctuation in the amplitude this is (Refer Time: 28:16) known as frequency flat fading. Why this fading, because there is time domain fluctuation. Why this flat, because in the frequency domain there is flat. And why is it flat as we have clearly seen if we take the Fourier transform of this channel impulse response we get a situation which is independent of frequency. If you take the amplitude of it the reasoning being the channel impulse response is simply one delta function which is scaled by a value or in other words if we look at the channel impulse response it is just one impulse. And if you remember the Fourier transform of an impulse is a flat across the entire range of frequencies so that is the final consequence that we have. That if these delays or this path differences is negligible we let you to each other we have all the paths arriving at the same time in that case the frequency selectivity of the channel is flat. That means it is not fluctuating with frequency, but it is fluctuating with time.

The important consequence of this is that if I have a signal which spans between the frequencies nu 1 and nu 2 in that case the entire bandwidth of the signal is not going to experience variation of k n. The entire bandwidth if the signal is going to experience the same value of the channel amplitude. However, that is going to fluctuate with time.

In this course we will use this model almost entirely. This is a very important part of the lecture and I would recommend you to go through this as many time as possible so that you get an understanding of how we have arrived the expression and what is the meaning of this frequency flat fading.

Thank you.