Fundamentals of MIMO Wireless Communication

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Lecture – 08
Small Scale Propagation Multipath Model

Welcome to the lecture on Fundamentals of MIMO Wireless Communications. Currently we are in the phase where we understand the wireless propagation channel. We have the briefly looked into the large scale propagation effects, currently we are going to start our journey into this small scale proportion models and we will start with the multipath propagation.

So, in small scale propagation model - that means, when we study multi path the reference book again that we need to follow, of course there are many other books which are very good, but the ones which we will closely be referring to in this particular section are Principles of Mobile Communications by Gordon Stuber and Microwave Mobile Communications by Jakes. Most of the things that we do can be found almost directly in these two books, but again there are many other references which are very good whichever you feel comfortable with feel free to follow those references they are all quite similar.

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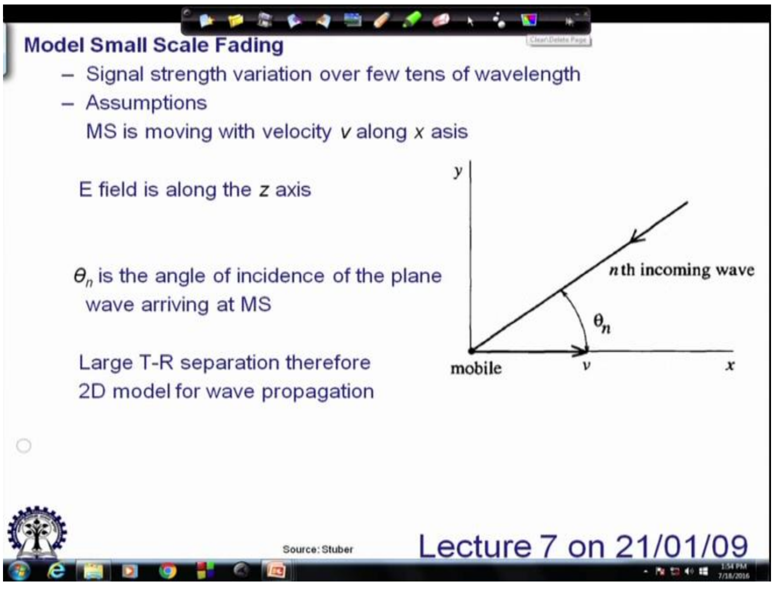

So, when we talk about propagation channel especially the multipath propagation channel, this is one of the most important parts of this particular subject. We need to understand the propagation thoroughly we need to understand the models because understanding of these channels is very, very critical towards design of transmitter and receiver. If you see how MIMO communications are evolved they have primarily become so useful namely because the channel has been well understood and one of the most important things is the multi path propagation channel. So, we need to understand it thoroughly so that we can design transmitter signals, as has been said earlier as well as, so that we can cancel the effects of the channel.

For this particular part will make some assumptions about the model that we considered, we would take a look at that part where the signal strength variation over few tens of wavelength that is the small scale propagation models. And the important assumption is that the mobile is moving with velocity v along the x axis as can be seen in this particular figure. So, we will be having the mobile moving along this axis this is the x y plane and the moved velocity is v is given by this the e field is along the z axis. So, that is coming out of the screen or out of the page on which we are writing and theta n is the angle of incidence.

So, we can look at it in this way that there is a mobile and there is a transmitter. So, mobile this is the y and x axis to be very clear, y axis this is the x axis and let us say this is the antennae. So, electric field is along this axis z axis mobile is moving with velocity v along this direction and theta n is the angle of incidence of the plane wave arriving. So, plane waves are arriving from the transmitter to the receiver and although in this particular figure we have the radiator almost in straight line it could be anywhere. So, we could have the receiver instead of this location, we could have receiver here. In that case this and then which this ray makes with this velocity is theta you can say. So, this is theta n in this particular case or if it is moving in this direction, if it is moving in this direction then in that case this would be the angle of incidence.

So, there could be reflections. So, this ray could be getting reflected it could be coming in this direction in this case; this is the angle of incidence with the mobile. So, that is the basic premise on which we are going to go ahead with the discussion, right. There is large transmitter receiver separation distance, so if the separation distance is large then we can make this 2D wave propagation assumption basically if this distance separation is large, as the wave propagates we almost have like plane wave arriving and we can do a top view, so we can have a top view of this whole plane this is the y axis, this is the x axis as if we can do 2D model for wave propagation of this.

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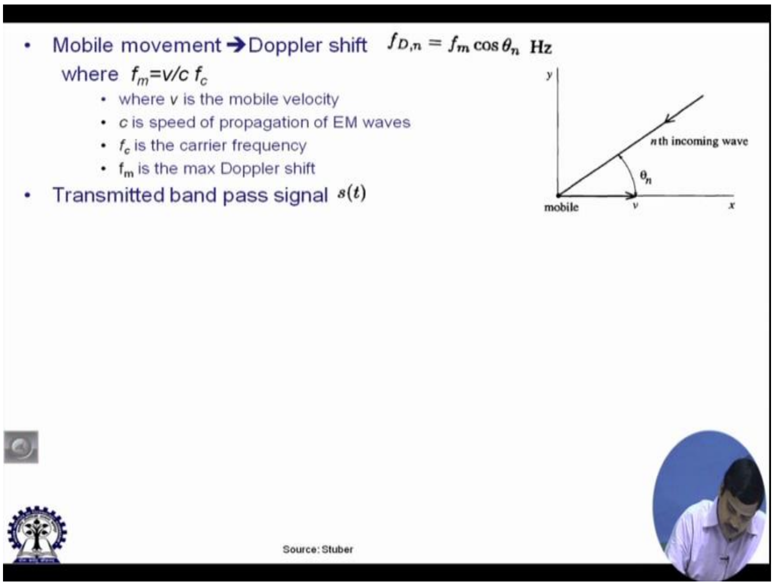

So, this is the typical pictures we make it small and we assume that since the mobile is moving in this particular direction with velocity v there is a Doppler which is introduced, now this is from classical physics that we consider Doppler. So, if f m is the maximum frequency of operation then f d that is the Doppler frequency from the nth path this is the nth path this sorry if it is the Doppler frequency from the nth path is equal to f n maximum frequency and the cos of theta n, that is the component of that frequency in the particular direction. So, where f m is v by c times f c now this is pretty standard; that means the maximum frequency of Doppler shift is v by c where c is the speed of the electromagnetic wave or speed of light f c is the carrier frequency.

So, what we can clearly see that as v that means, velocity increases f m increases. So, if f m increases for a given value of theta f d increases or the Doppler frequency increases and vice versa. The max and mean value being plus and minus v by c times f c. All other values would be between plus and minus f f max or f m.

Moving forward, we can now start taking a look at the transmitted signal we will consider the band pass signal.

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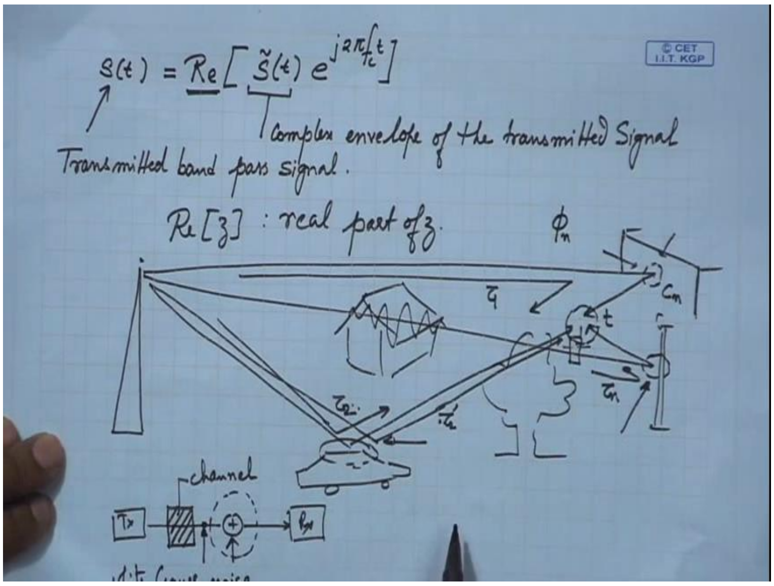

So, what we will do is we will write S of t as the transmitted band pass signal is equal to the real path of S tilde t which is the complex envelope of the transmitted signal - e the power of j 2 pi f c t where this is the complex envelope of the bypass of the transmitted signal.

So, this is the transmitted band pass signal and this Re is basically Re of let us say z is the real part of z. So, this is the basic model with which you will proceed in our calculations and we will of course, assume that there is a transmitter there is a band station and the signal propagates through multiple paths. This is one of the most important things that we will receive. So, there are more than one path through which the signal propagates and these paths could be because of reflection from building, could be because of reflection of moving vehicles, could be from lamp post or could be from trees, could be from houses, there could be blockage, severe blockage into a building into something present in a line of sight. So, end of day we are getting multiple signals arriving at the receiver and what we are going to study is what happens to the signal when we add up these signals at this point of the receiver.

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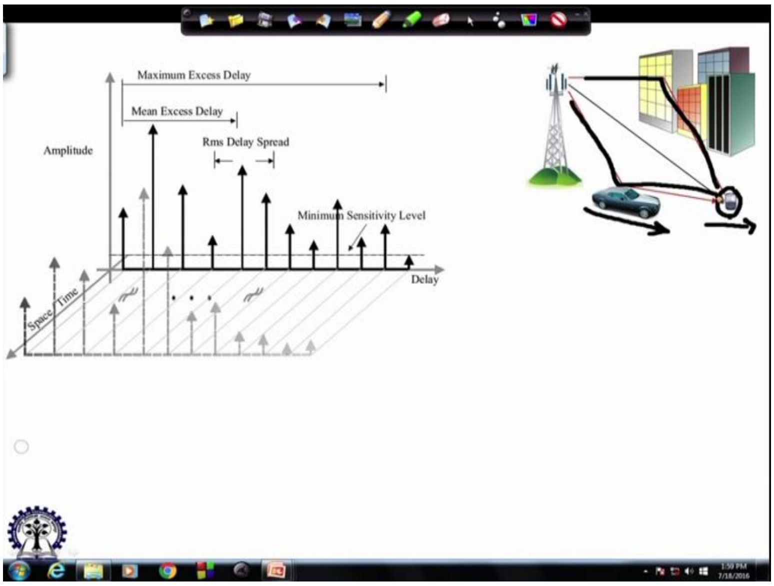

So, taking a careful look at the picture this is particular figure roughly explains what is going to happen this picture what is happening. So, from the transmitter as we have drawn there could be path from the vehicle there could be path from the building this mobile could be fixed or it itself could be moving. Now if the mobile is fixed and this reflector is moving still we are going to experience Doppler effect because of the relative motion n between the reflector and the mobile unit.

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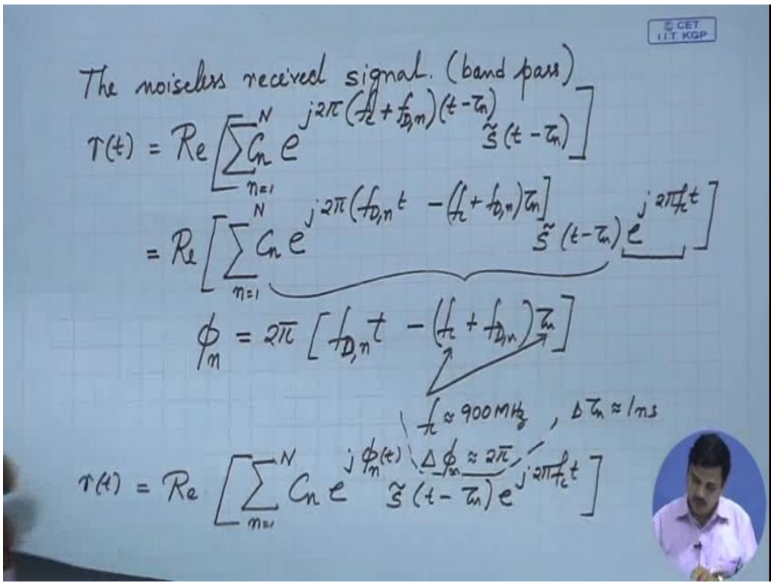

So, with this model we go ahead and say that the received signal, the noiseless received signal. Typically in a communication system you may have come across additive white Gaussian noise channel where we have the transmitter signal going through an ideal channel nothing is happening and there is only addition of noise. So, this is additive noise, it could be white, it could be Gaussian and then there is a receiver. Just to summarize we usually consider this noise is being added because of analog components and of the receiver front end typically and also due to conversion from the analog to the digital domain. Whereas, in the current model that we are considering we are just trying to see what happens to the signal in this point where it has almost not yet gone into the receiver you can say.

These noise effects can be studies separately. So, as of now we are interested only in the channel effects where there is a channel which lies between the transmitter and the receiver. So, there is a channel between the transmitter and the receiver and we are interested in the channel. We are basically trying to basically look at what happens to the signal at this point or what happens if there is no noise present later on we can always consider the effect of noise on top of these effects and we can study the combined cumulative effect.

So, the noiseless received band pass signal, we are still considering the band pass signal can be written as r of t is equal to again the real part of we have this and if we say that we are looking at this particular propagation we will assume that each of the path has a propagation delay. So, let these paths have a propagation delay tau 1, this particular path can have propagation delay of tau 2 and so on, and this particular path in general has a propagation delay of tau n. In that case the signal that is received from the nth path would be the one which was the complex basement signal that was transmitted tau n units of time before the current time because this is the propagation delay through which it has come.

This signal is scaled by C n which is the amplitude associated with the nth path; that means, if the signal has started to propagate in this particular direction then because of reflection at this point there is a certain change in amplitude depending upon the reflection coefficient. If it has gone through this particular direction because there is an obstruction due to certain amount of attenuation through which the signal may have gone through, again because of this particular surface the kind of reflection happening here would be different. If there is diffraction or scattering there would be corresponding coefficient corresponding to the radar cross section and so many other things.

So, in this model we abstract the exact happenings at these points whatever is happening and we just model that there is some change in amplitude that has happened because of the different phenomena in the multiple paths. This is the abstraction that we make. Instead of this, if we were doing deterministic model - in a deterministic model we would have specified exactly what kind of surface is it on which this wave is infringing what is the frequency, what is the irregularity and what is the exact coefficient of reflections that happens, if there is a diffraction we would try to calculate what percentage of power comes in this direction, what is the phase, if there is a refraction accordingly we take everything into account and solve the equation in order to get it.

So, here we will we will try to do an abstraction and put C n for the particular amplitude associated with the nth path. Now with respect to the carrier what we have is e to the power of j two pi f c is what we had when we transmitted the signal. So, the transmitted signal was S t goes to real part of S tilde t e to the power of j two pi f c t. What we have seen at the receiver it should be S tilde t minus tau n because signal takes tau 1 time to precede it takes tau n time to proceed. So, at any instant of time if we are I am from seeing the signal and that signal has come by the nth path it must have started at a time which is t minus tau n and it must have undergone a multiplicative gain which is c n. So, that is what is reflected in this expression the signal in nth part must have started how many unit's of time before in that particular signal and this gain contributes to the gain of associated with that path there is f c is the carrier frequency.

Now, the received signal is not only the carrier frequency, but also there is the Doppler due to the nth path associated with that path. Now we instead of having j two pi f c since the time it has propagated through is tau n we have e to the power of j 2 pi f c t minus tau n. So, this is the expression of the pass band received signal when it has come to the nth path again if we go back and check this expression or check this situation we have just written the signal which has come to the nth path. What we get over here is signals which have come to the first path second path third path and so on for all possible paths. So, we modify this expression and we put a summation sign to indicate sum of n equals to 1 to capital n indicating as if there are capital n number of multiple paths through which the signal has come at the receiver.

At this point we have the complete expression and we would like to explore this expression a little bit more in order to see what lies in details. So, we could look at it further and we could say this is a real part of summation remains n equals to 1 capital n, C n e to the power of j 2 pi. Now you would separate the carrier from the Doppler because whenever we are studying communication system we have indicated before that we would like to study systems independent of the carrier frequency. So, we will just bring these terms separate so that we can concentrate on the complex envelope of the signal which is our main interest.

So, if we collect these terms we would have f d n of t and minus f c plus f d n of tau n s tilde t minus tau n e to the power of j 2 pi f c t. So, we have is e to the power of j 2 pi f c t is separated. So, if we try to compare the expression with which we started what we had here is S t equal to real part of s tilde t e to the power of j 2 pi f c t what we have here is some signal here some signal and e to the power of j 2 pi f c t. So, if we have to study the complex base band or if you have to study the base band then we move this part and we can focus on this part of the signal and study what has happened to S tilde t that got transmitted from the base station for example, or at the transmitter and has undergone multiple path reflection, reflection is scattering and there is arrived at the receiver after this propagation effect.

Now, in order to write it in a better way we would define phi n as 2 pi f d n that is the frequency due to the Doppler in the nth path minus f c plus f d n tau n. So, this is basically the phase associated with the nth path and we have this tau n term over here. So, if we look at this particular term what we see is that there is a multiplication of f c along with tau n. Now this is very very important for us to see tau n is the propagation delay of the nth path and f c is the carrier frequency. So, if we say that let f c is somewhere around 900 mega hertz say approximately equal to 900 mega hertz and we have a certain propagation delay. So, basically we have a certain propagation delay. Now what we are trying to consider is the situation that this mobile has moved a little bit or if we take this path this vehicle has moved a little bit in that case the reflected path would be this second one and we can call it t two prime.

So, what we would like to study is the impact of the change in path length on the phase f of the nth path. So, to do that if we say that let delta tau n that is the small change is approximately equal to 1 nanosecond. In that case what we would find is the change in phi n is approximately equal to two pi; that means, it is undergoing full phase rotation because of just 1 nanosecond of change in path length. This is very very important to capture because a tiny change in the path causes a huge change in the phase a huge change in the phase would cause the signal strength to fluctuate significantly.

Now, when we have all these different paths coming to the receiver slightest change in any one of the path is going to cause a huge change in the signal phase which is coming at the receiver we are adding up all of these signals. So, what we are ending up is with a great change in the sum total effect of this signals coming from different paths.

So, if we see over here we should be able to write the received signal as real part of sum of n equals to n to capital N C n e to the power of j phi n which is to the power of time S tilde t minus tau n e to the power of j t pi f c t. So, here we have captured the phases the amplitudes the phases and the expression looks a little bit simpler compared to the above expression.

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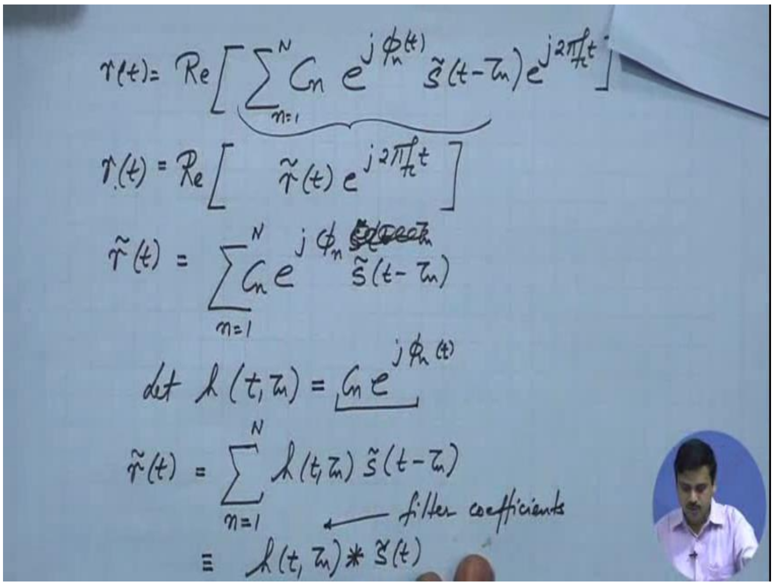

So, we would rewrite this expression as we are continuing on a new page. So, what we have is r of t is equal to real part of summation n equal n capital N C n e to the power of j phi n f t s tilde t minus tau n e to the power of j 2 pi f c t. This further you could write as real part of this whole thing if you write it as r tilde t times e to the power of j 2 pi f c t where r tilde t is the complex base band signal or the complex envelope of the received signal for r of t. So, if r of t is the pass band signal r tilde t is the complex envelope and the relationship between r t and r tilde t is real part of r tilde t e to the power of j 2 pi f c t is r of t.

We had the same expression for s of t. So, if we look at these two expressions they look very very similar this is the transmitted signal this is the received complex signal. So, now, since we have separated out the carrier we should be able to concentrate on what happens to the complex signal that was transmitted at the receiver.

We would of course, will define r tilde t is equal to sum over n equals to 1 to capital N C n e the power of j phi n s tilde t minus sorry this will s tilde t minus tau n. We could also say that let h of t comma tau n be equal to C n e to the power of j phi n; that means, we are taking these two terms the amplitude and the phase of a particular path is being assigned to the variable h which is a function of time as well as tau n which is the delay where this phase term contains the time and the delay effects.

So, if we take this we could write r tilde t is equal to sum over n is equal to 1 capital n h of t comma tau n multiplied by s tilde t minus tau n. If you look at this particular expression it could also be written as in equivalent terms as if h t comma tau n having convolution with s tilde t where h t comma tau n are the filter coefficients or in other words what we have is when the signal propagates through such a multi path as been shown in the figure here with at each of the reflecting surface or where the wave hit's we have an amplitude C n and because of the propagation we have a phase phi n what we end up is as if there is some convolution with some sequence given by h t comma tau n along with the signal that comes to the particular paths.

We this what we see is that the multipath propagation where at each surface where the incoming wave hit's there is a change in amplitude and there is a change in phase what we can effectively write is that the received signal in the complex form is the same as the as the transmitted signal in the complex form. However, it is convolved with some coefficient which is capturing the effects of the channels.

In the next lecture we shall see some more details about what happens to signal and what is the typical behavior of such phenomena when observed at the receiver for a particular transmitted signal.

Thank you.