Fundamentals of MIMO Wireless Communication

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Lecture – 40
MIMO in Practice

Welcome to the lecture and Fundamentals of MIMO Wireless Communications. So, this is the concluding lecture of this particular course and till now we have covered various aspects starting from understanding the channel in quite details which gives us, which leads at to the basic assumptions which we use in derivation of the error probabilities and capacities and then we moved on to calculating the diversity gains divide by. We learnt that one could include the error probability then we moved on further and looked at the capacity gains of multiple antenna systems, we have seen multiple configurations.

We have also seen the effect of correlation and we have also seen the situation where there is no SNR, what would happen when there is high SNR as well as we have considered the cases when the channel is random and what happens to ergodic capacity, what is outreach capacity, what is a meaning what happens when number of antennas goes to a very, very large numbers, what we saw is the outreach capacity or p percent outreach capacity is almost equal to the medium value because the distribution becomes very, very, very, very narrow, it does not spread much. So, these are some of the things which we have seen which covers the basics of fundamentals of MIMO communications.

So, essentially we have covered the basic tools that are required to understand or analyse any MIMO communication system, which has been the objective of this particular course. So, our objective was to mainly give you a set of tools or the methods by which you could analyse MIMO systems. So, given at this point whatever we have covered you are equipped with basic set of expressions and methods through which you can understand any advance MIMO.

With a help of this you will be able to explore the further in the domain of MIMO, where they lies lot of things to explore, for example, space time coding there could be transceiver, architectures and multi user MIMO configurations and so on and so forth. So, with whatever we have covered I would like to summarize them in this particular lecture, move on to see how these things we have discussed would affect a typical MIMO communication system.

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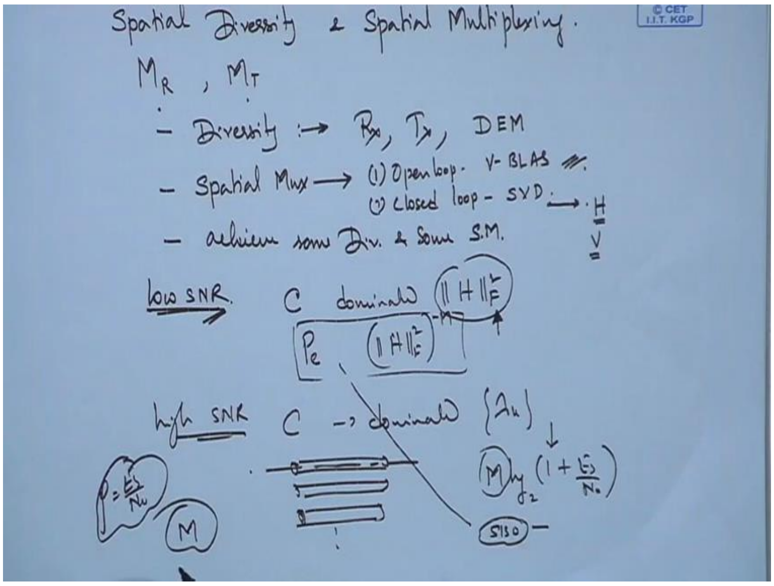

So, till now we have seen that we can achieve spatial multiplexing and spatial diversity gain. This in we can achieve spatial diversity gain and spatial multiplexing gain right. So, now, suppose you are given m r number of receive antennas and m t number of transmit antennas. The natural question that 1 can ask is, what you do with us, would you configure it to operate in diversity mode or would you configure it operate in a spatial multiplexing mode? So, let us see when do it what or what kind of combinations can be achieved.

So, if essentially with these m r and m t transmitting receiving transmitting antennas 1 can either use it for diversity. So, for diversity we would remember various modes you could achieve receive diversity, you could achieve transmit diversity and there was this mode which was discussed dominant eigen mode transmission which would use all the antennas at the receiver and at the transmitted and using as procedure into do certain encoding to achieve diversity gain. So, that uses the highest value eigen value to achieve that you could do spatial multiplexing which could be done in 2 options; 1 is open loop 1 example we gave was v blast and the other was closed loop in the closed loop system we had seen the SVD.

So, in this case it is an extreme form where signals are directly send from the transmitter to receiver that means, if there are three antennas, 1 4 antennas in this particular thing I would send symbol 1 symbol, 2 symbol, 3 and symbol 4 directly in to the system.

Where as in SVD based thing there is a pre-coding which is done before it radiates to the air. So, in there are extremely bit's of performance in SVD you would require the whole channel matrix to be fed back or the v matrix to be fed back. So, other than these modes you could actually achieve some diversity and some spatial like multiplexing. So, there is a detail theory into this diversity multiplexing trade of, but again in since 1 of the last lecture that we doing we will not go into the details as we done for the diversity and for multiplexing it is quite elaborate and you can easily go through it. Now, that you are trained in these diversity and multiplexing independently, but what will show at least how you can achieve both simultaneously when it is required.

So, you could achieve some aspects of diversity some aspects of multiplexing. So, we have also seen that when there is low SNR we have seen this in the previous lectures the capacity c is dominated by mod of h f squared and we have also seen that probability of error is also almost inversely proportional to h f squared. So, it is raised to the power of minus m in some form it is not exact form. So, when it is low this is in general true this is in general true not necessarily follow SNR what we said is probability of error is dominated is probably control by this parameter. So, what we have seen is that when SNR is low this place of eigen factor. So, this gives us hence that probability at low SNR we could improve this whereas, when there is high SNR we see that capacity is dominated by the eigen values this spread of eigen values we have seen this in the previous lecture.

So, when we divide the multiple channels or the multiple antenna links into spatial parallel links what we have seen is it becomes the capacity comes log base 2 1 plus e s by n naught; that means, n times capacity of in the same manner or in the same tone we should also say that these links have the channels strength which is similar to that of the SISO links. So, naturally this hints that the error probability of each of the links is also similar to that of the SISO link.

So, if I have m r receive an m t transmit antenna or for instant let us say we have m number of transmit and antennas in that case and there is certain value of SNR rho equals to e s by n naught then we are left to decide what to do now at this point we should note that whenever we have talked about capacity we have when we derived when we have actually explained intuitively what is the meaning of capacity or we used it we actually never touched upon the error term.

So, whereas, if you look at the channel coding theorem it basically tells us that we have to the capacity expression is the 1 which gives us the highest bits per second per hertz which could be transmitted from the transmitted to the receiver with arbitrary low probability of error; that means, we are talking of schemes by which error probability almost goes to 0.

So, even if your SNR is very, very low it such as that you can possible find very, very, very, very large code words through the use of which you could achieve nearly 0 error probability whereas, in practical systems it is far from what we have just discussed in practical systems your code lengths are limited you have to use the channel in finite duration of time your channel changes with time. So, all those assumptions or all those important are bases do not hold good any more. So, you are limited because of practical constraint. So, in practical situations we have to work with modulations and finite length code words or error corrections code lengths. So, when you are using modulations.

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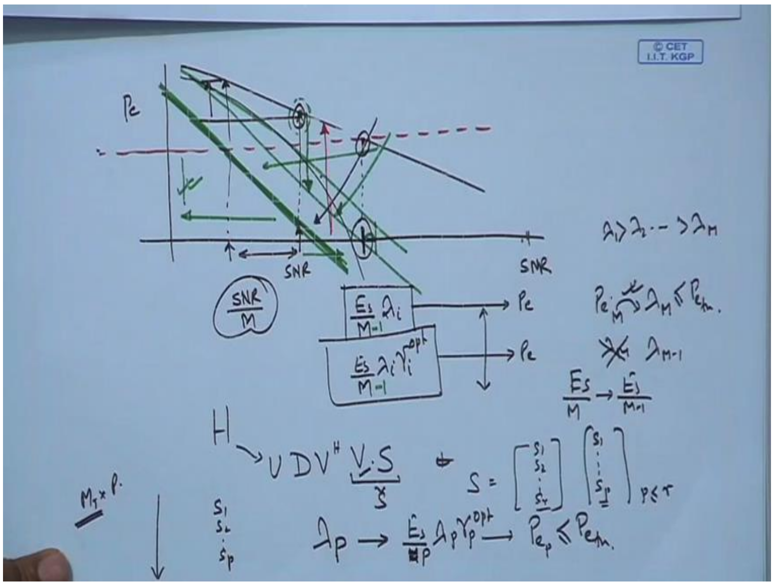

We always we always know that as SNR with the relationship of probability of error with SNR for a w channel is like water fall whereas, for Rayleigh channel is almost like a straight and as we increase the diversity order it becomes better. So, basically there is a primary loss in error probability when you go from e w g n to fading channel whereas, as we increase the diversity order you move more towards e w g n channel.

So, this tells us that if we are already at a low SNR situation our error probability is quite high. So, when our error probability is quite high although we have a large number of transmit and receive antennas we may not afford SNR upon n which is important because when we are doing this spatial multiplexing at each of the links we are basically given e s by m amount of power in case of no CSI at the transmitter it is simply getting the channel lambda and when there is CSI at the transmitter you can modify the powers you are actually putting in an extra amount of variation.

So, when we do this extra amount of power modulation we will know that this is going to have range which ever gamma is lambda is high we put more power to that which ever lambda is low we put less power to that so that means, each of the links are definitely having an SNR which is lower than the SNR of a SISO link so that means, that each of the link is going to experience even worse probability of error. So, it from here it moves up further. So, this tells us that probably it is not a good idea to go for a diversity to go for spatial multiplexing mode under this condition when SNR is so low that it does not even meet a certain threshold of error probability.

So, in that case if we use a diversity combining in that situation we might be able to bring this curve down to the situation here. If you are using it might be possible to bring it there and this error probability point might come down here. So, what we are suggesting is if we have m antennas or SNR is low we have low SNR condition. So, instead of going for spatial multiplexing it might be worth going for diversity definitely receive diversity in going to increase the array gain if you are increasing array gain instead of going down you are going on the right side of the SNR axis further if you are using dominant eigen mode then you can increase the SNR even mode because you are adding the average array gain at the transmitted. So, basically you are not only this line shifts bents downwards also because of array gain you kind of shift these lines in this direction.

So, over all we might be able to bring the error probability curve here from here. So, if we use diversity. So, if we are there then we can clearly from this scale diagram that when error probability was not getting satisfied we have now moved to the situation where error probability has become much better or the reliability of the channel has become quite good. So, whereas, if we are in the high SNR regime that means, we are somewhere here let us say we are in 30 d b or 40 d b SNR when SNR here if we distribute this SNR amongst multiple channels suppose I get the SNR the averages not per transmit antenna at this point. So, that the error probability is just satisfied below this red dash line; that means, each of this parallel links would be able to sustain the error probability.

So, still I mean this is just an average indication what we actually need to do is work out these values if we are doing close look based close look based MIMO schemes or if it is open loop, we have to work out these values and see what is the probability of error corresponding to the modulation that we are choosing. So, if the probability of error for probability of error for the nth mode corresponding to the capital lambda m capital lambda m is the lowest eigen value because we have said that lambda 1 is greater than lambda 2 is greater than lambda capital m. So, if this probability of error is less than some probability of error threshold then we can choose this. If it is not true we have to drop lambda m and have to start using lambda m minus 1.

So, when we drop lambda m we are actually dropping 1 of these parallel modes and we are reducing the number of modes. So, when we have lesser number of modes we have certain amount of extra powers. So, in this case instead of e s by m, we now have e s by m minus 1. So, each of the branch are going to have extra power. So, instead of e s by m, we now have e s by m minus 1 right so that means, the average SNR of each has increased. So, even this point has shifted here and the error probability then might be satisfied. So, it will be using lesser number of streams than would be allowed by this m antenna branches.

So, basically there is a trade of which we can do. So, when we when we try to use this. So, basically we are talking about a situation where suppose this channel broken into u d and v hermitian and we said we have to use v with s at the transmitter side in order to send this. So, this was still the s tilde. So, instead of sending r streams that means, instead of s being equal to s 1, s 2 up to s r where r is the rank of the matrix we would rather send s 1 to s p where p is less than or equal to r. So, that the lowest eigen mode. So, that lambda p lets say lambda p ensures that whatever is the mode of lambda p we would have e s by let us say p number of antennas times lambda p times gamma p opt right this has a corresponding error probability or the pth index which is less than or equal to probability of error threshold with some modulation order.

So, if this is true then we are going to have p transmission mode. So, if we look at this now we are going to have s 1 s 2 up to s p and this v will have length of m p. So, this will m t cross p. So, m t cross p would mean it is m t cross p that is you are transmitting across on the t antennas m t antennas, but there are p parallel streams.

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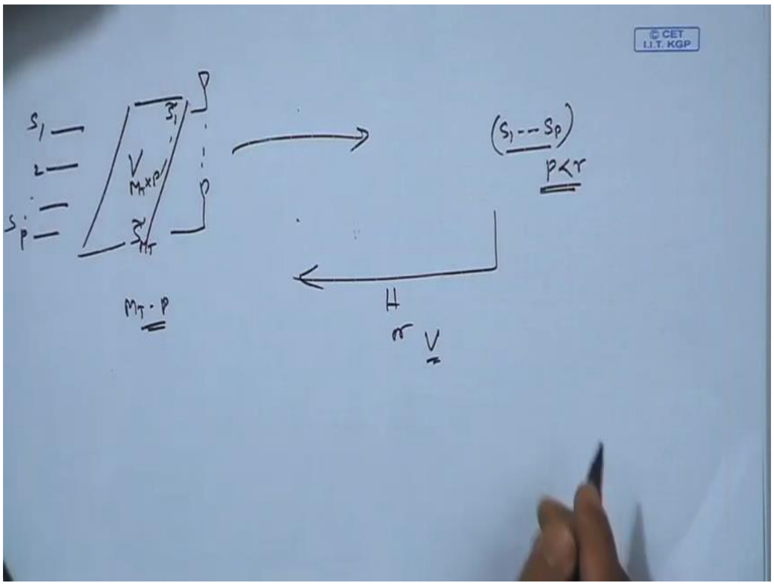

As if we can say that we are having a situation which is like this. So, there is some pre-coding and this is v. So, this is the 1, 2 up to p. So, and this is m t cross p. So, these p streams s 1 to s p are getting converted to s 1 tilde to s m t till the s p streams are getting converted to m t number of symbols and then they are been sent out into the air.

Similarly, at the receiver we are going to receive s p number of symbols and will decode them we have p which is less than the rank of the matrix number of spatial streams rest of the antennas m t minus p and on the other side m r minus p would be used for diversity gain. So, in this way we could achieve both multiplexing and diversity gain simultaneously where the number of streams that we are sending is less than the maximum theoretical number of streams possible.

So, that we can support the minimum error probability the number of streams which are less than the total number of antennas that are available. So, the excess number of antennas which are available could be used for diversity mode and this would increase the error probability as just as been explained. So, this particular scheme requires a feedback of either h or of v that is clear because we will be encoding with v right at the transmitter will be encoding with the matrix at the transmitter.

So, we need to talk about schemes where you do not have channel state information at the transmitter because that would allow use of practical schemes and that would be much, much beneficial than using channel state information at the transmitter.

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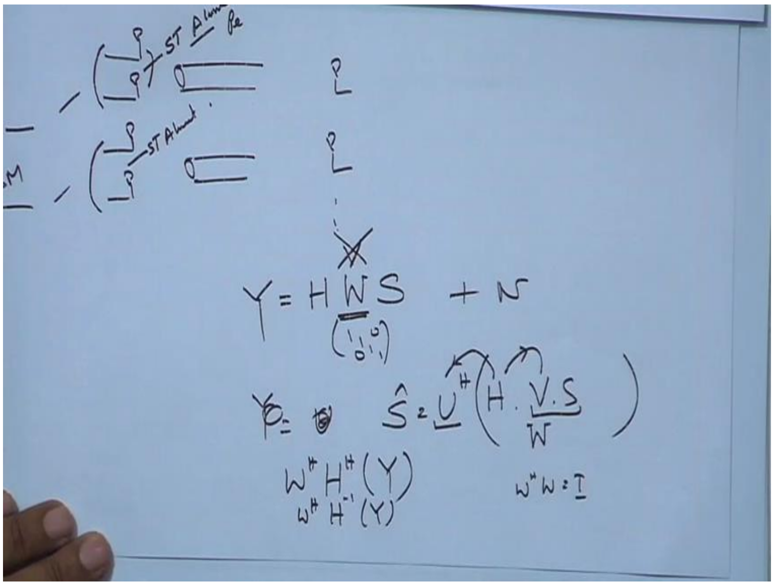

So, 1 of the simplest schemes that we can draw or we can talk about would be the 1 where 1 particular example I will just take, there could be many suppose I have situation which has four transmit and 2 receive antennas it can have more what we could do is we could club these 2 and we could club these 2 and say that I want to set up 2 spatial streams and in each of the spatial streams I could do space time coding let us say Alamouti coding in each of them.

So, in this way we achieve 2 orders of diversity at the transmitter in each of the links and we have 2 links. So, we would improve certain amount of probability of error here and we would achieve certain amount of spatial multiplexing. So, this is 1 possible way of doing it there are many different ways of doing it and this whole domain there is a huge amount of literature which is available which talks of space time code design and this is the pretty rich feel and this is huge amount of literature I would encourage you to explore this particular direction to find out how 1 could design a code words which could provide you trade of between a multiplexing and diversity what we have just discussed is 1 particular fixed scheme. So, there are many other schemes which are not necessary fixed as shown there, but you can really move around using the different possibilities.

So, we will quickly take a look at how things are done in case of LTE which is a commercial system which uses multiple antennas right.

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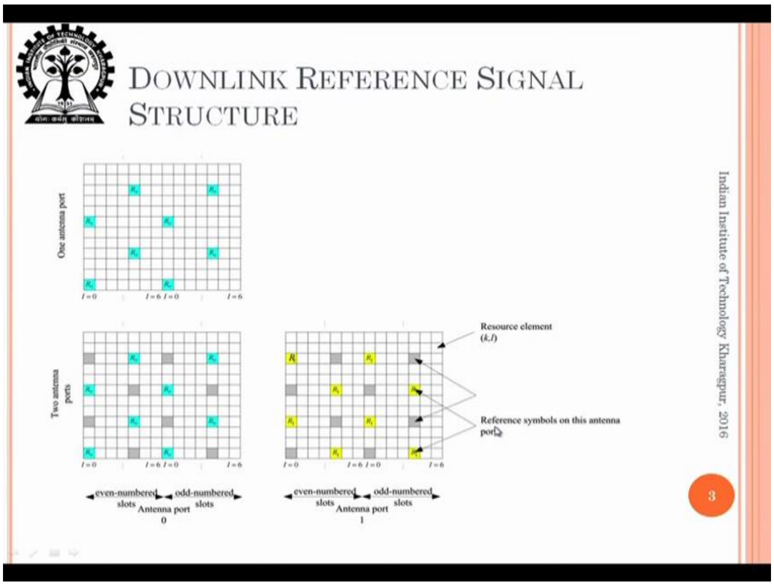

So, in LTE we are not going to talk about the o f d m.

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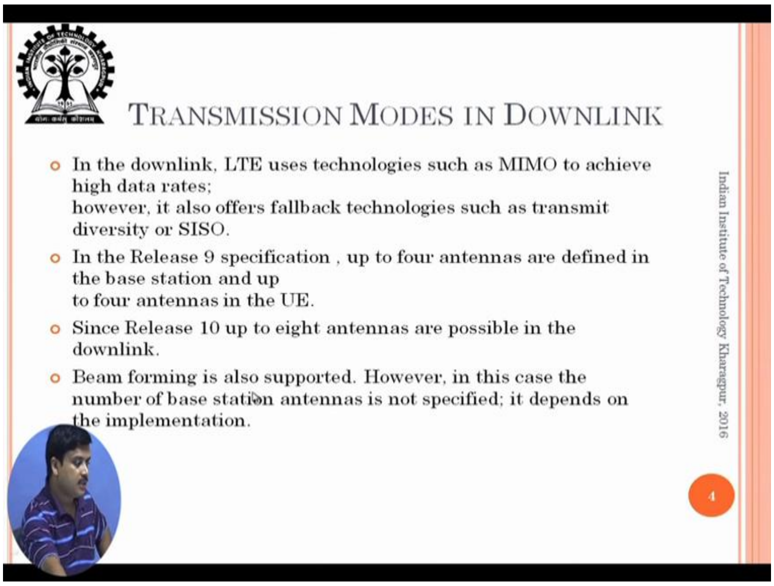

We will simply talk about the MIMO mode. So, we will skip some of the information which may be available here. So, in LTE there are many, many different modes which have been proposed because of the released version these versions have been improving from time to time and just 1 second. So, with these different versions or different releases as as been coming we have with time like release eight then there was 1 point release nine release ten release ten allows up to eight transmit antennas. So, basically it says it allows eight cross eight system. So, there are many, many versions which have been developed in LTE that is long term evolution.

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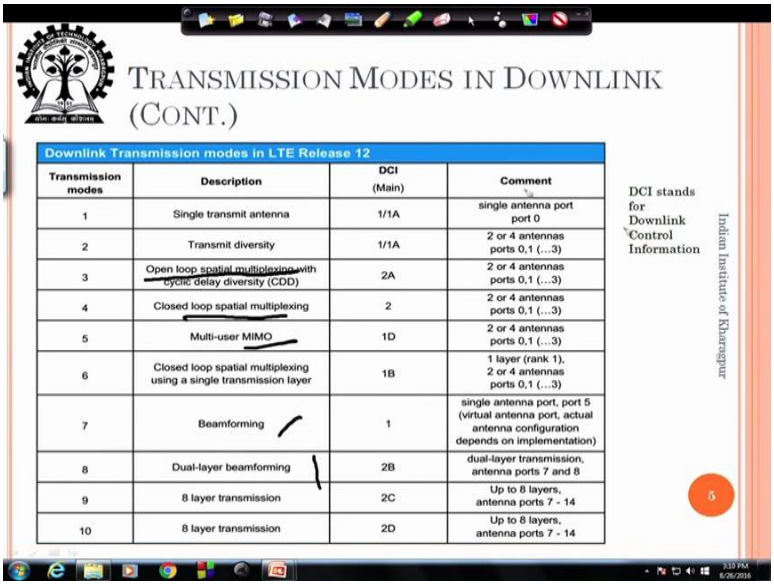

So, this summarises some of the MIMO modes there is single antenna transmission transmit diversity there is open loop spatial multiplexing which we have discussed and then there is closed loop spatial multiplexing there is multi user MIMO which is possible beam forming and so on so forth. So, all this based on fundamentally code book based MIMO communication. So, there are code books which are used and we are going to shortly describe some of the code books which are used in this particular system.

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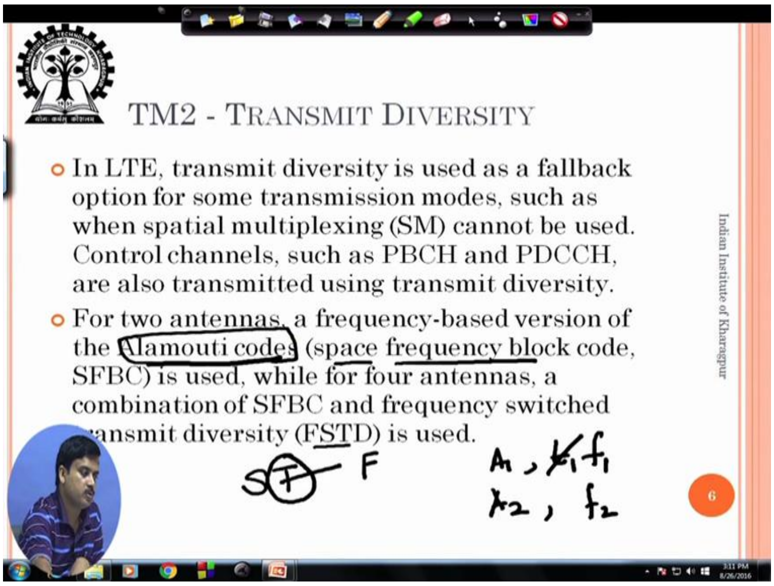

So, it allows transmit diversity. So, what this particulars slide tells you have to run quickly through this is basically, we are studied Alamouti in the time domain right. So, we have studied at in the space time block coding, whereas, here it is used in the space frequency block code the difference here is this time that we have studied is simply replaced by frequency.

So, if we have studied antenna 1 at time instant 1 that gets replaced by antenna 1 sub carrier index 1 and antenna 2 and sub carrier index 2 usually these are neighbouring of carriers or neighbouring physical resource blocks. So, that is the difference plus there is frequency switched transmit diversity is an additional on top of it, but fundamentally this is the Alamouti code which we have studied which is also used in LTE.

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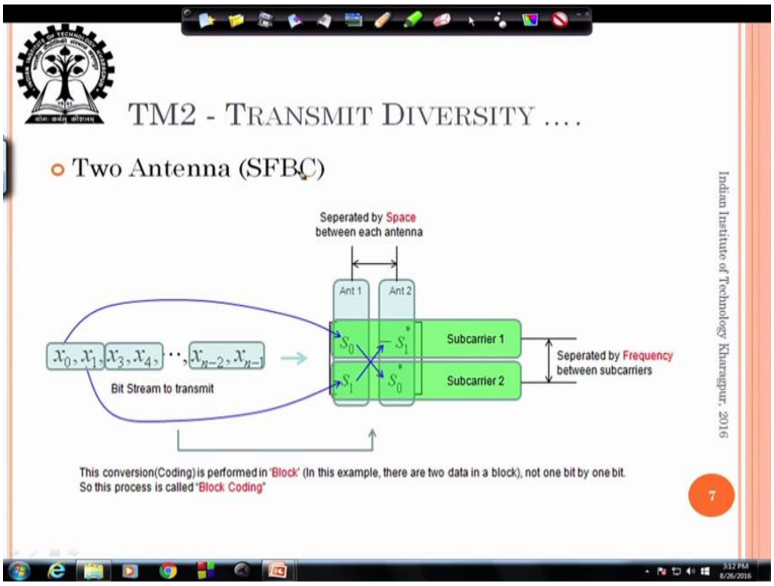

You can always use m r c at the receiver that is always possible. So, this particular slide gives you brief overview how is done. So, here you see clearly there is antenna 1 antenna two. So, here is sub carrier 1 sub carrier two, but we have studied time 1 and time 2, right. So, we move forward.

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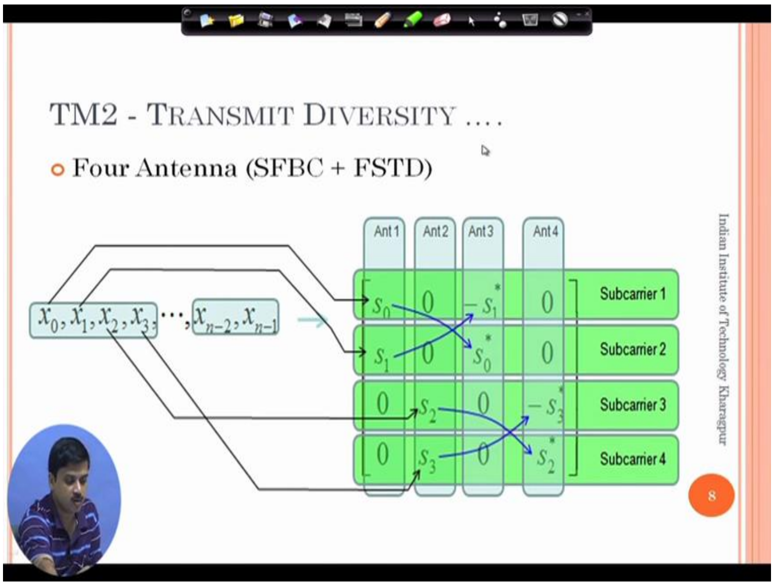

So, this is the four antenna based Alamouti scheme, which is basically similar. So, will not take a look this we can always look at these things.

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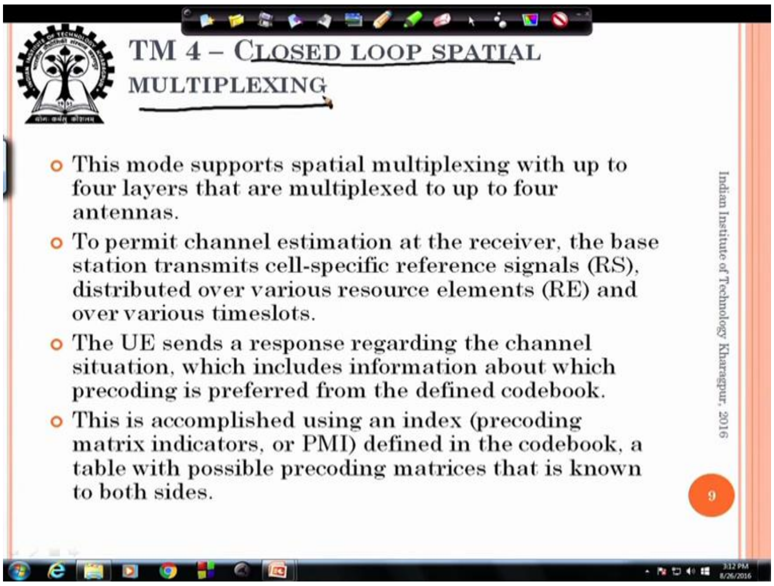

And then there is closed loop spatial multiplexing. So, what we have studied is SVD based closed book spatial multiplexing, but here instant it is code book based spatial multiplexing will explain shortly, what is the meaning of code book based spatial multiplexing?

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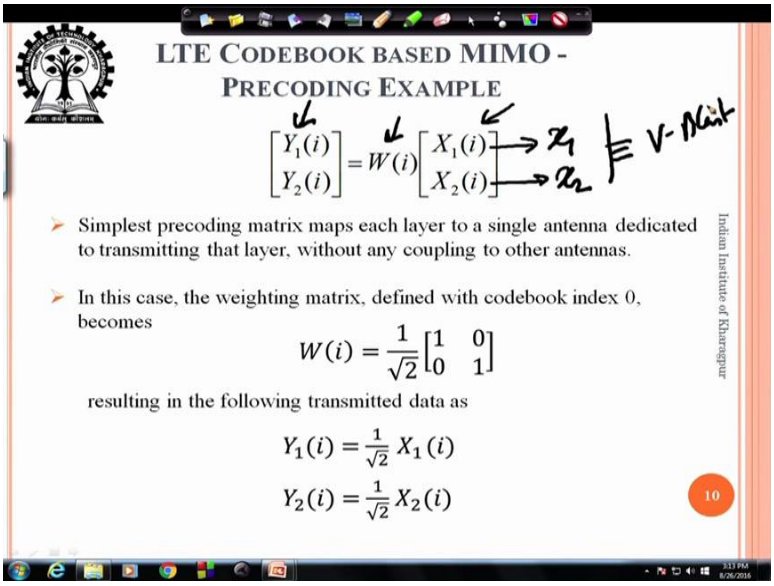

So, amongst the different code book 1 of the simplest code books that is used is the case where there is you can see that this is the transmitted symbol this is the code book this is the received symbol. So, code book is an identity matrix which means that x 1 the transmitted symbol goes to the antenna as x 1 and x 2 goes to the antenna as x 2 and you could clearly see that this is almost v 1 to the v blast technique that we used. So, this is done in general because if we indicate with the code book w then for all schemes will be using w in some cases we will be using identity matrix which would be like indicating a v blast in other cases will be using spatial matrix. So, the expression of writing would remain this same.

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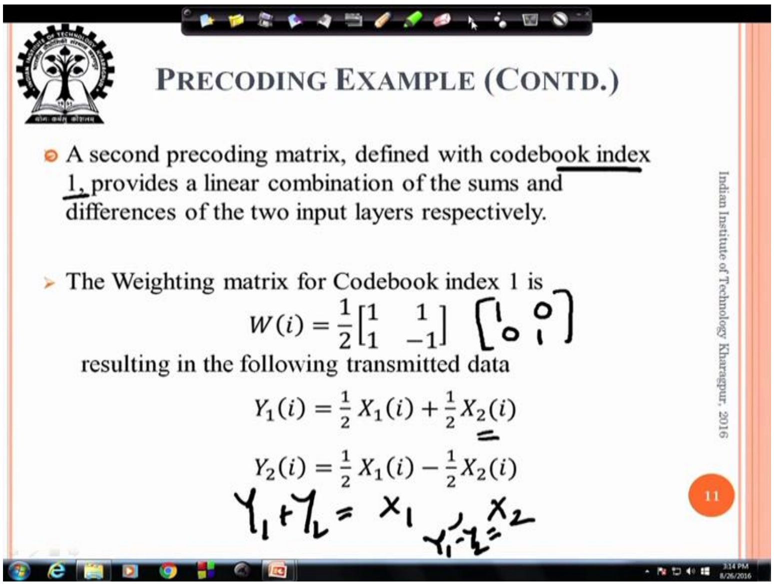

So, basically we would write as s times w times h is what is being transmitted is been received along with noise. So, this can be used to provide an identity matrix to indicate blast or any other combination to provide a particular pre coding moving forward there is another pre coding which is 1 of the simplest pre coding as shown over here and this is the code book index 1 here the code book matrix is like 1 minus 1. So, in the previous case it was 1001.

So, the received signal as you can see is x 1 plus x 2 and the received signal y 2 is x 1 minus x 2. So, if I add y 1 and y 2 I would get x 1 and if I subtract y 1 from y 2 what I will be getting is x of 2. So, basically y 1 plus y 2 is going to give x 1 and y 1 minus y 2 is going to give x 2. So, in this way also we are going to get 2 schemes 2 signals the advantage that we have over here is that we are basically sending the signal x 1 over the different links and over x 2 over the different links

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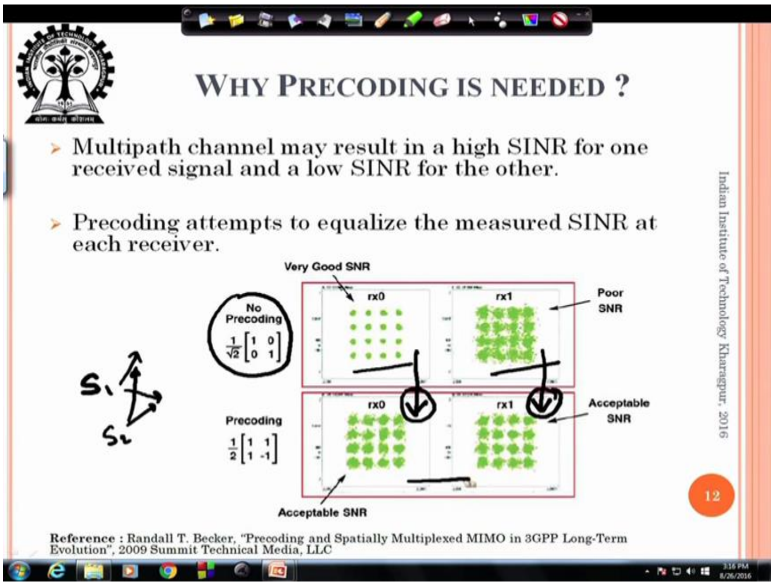

So, that we are able to get some better signal quality and reduce the variability of the channel moving forward this particular slide gives a brief in intuitive explanation that why we are going for pre coding basically by pre coding we are sending this signal x 1 over multiple paths and s 2 over multiple paths just like similar to diversity.

So, when we use this 1 the identity matrix that means, this v blast kind of thing it might be that 1 of the channel provides you with better signal quality the other channel is having poor signal quality whereas, when you send signal across the both the branches then there is an average in effect this becomes a little bit worse this becomes a little bit better, but this degradation that you are seeing over here does not affect error probability so much, whereas this improves the error probability significantly. So, basically outreach is what we would always like to minimize. So, what we see is that by virtue of averaging we could improve the overall performance of the system.

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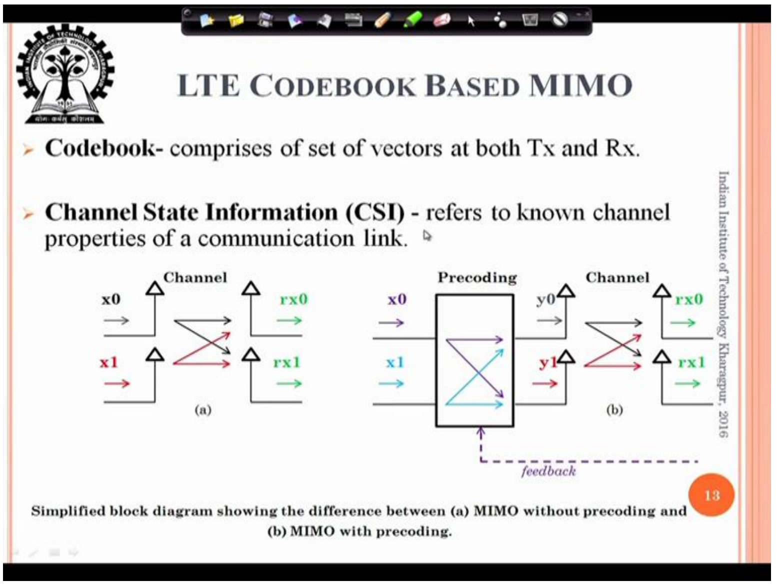

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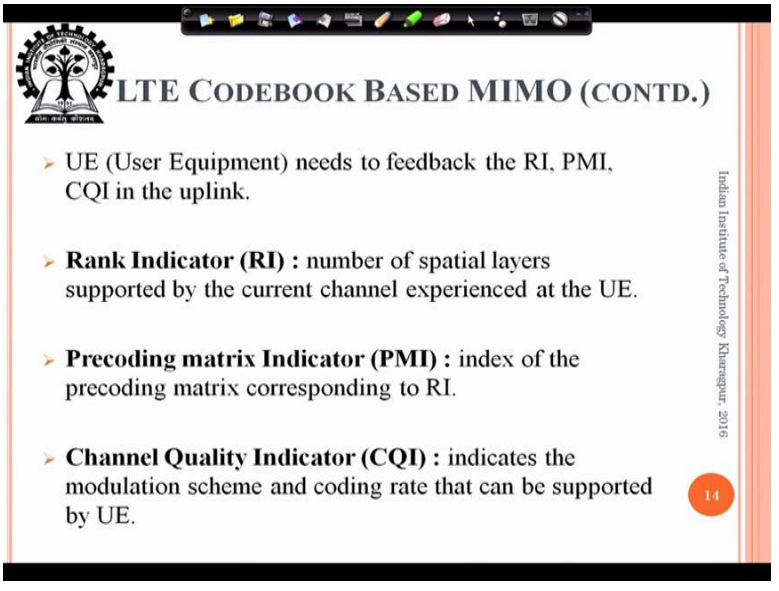

As we look forward in these schemes, when as we have just explained that one has to identify the number of spatial streams same as applying is applied over here.

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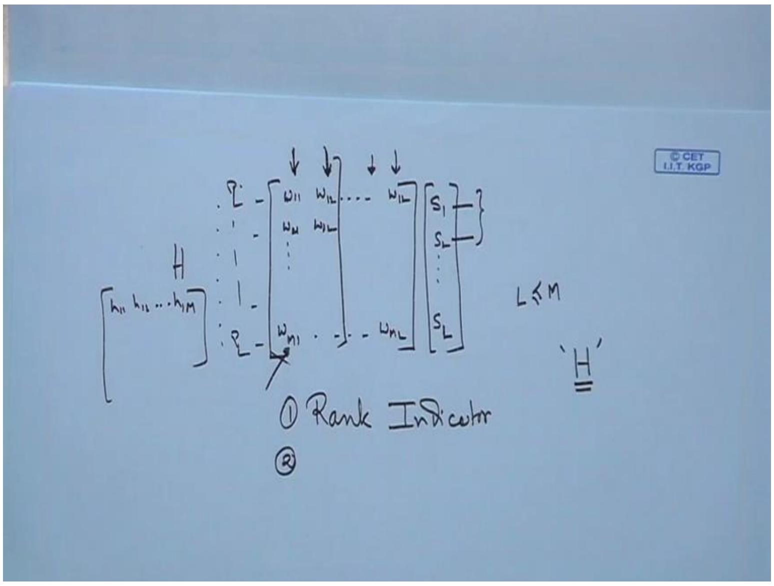

So, in this particular case as we have said we could write it as s 1, s 2 up to s l lets say and this has to be multiplied by a matrix right. So, definitely it needs l columns for this matrix operation to be held right. So, basically we can have w 1 1 w 1 2 up to w 1 l then w 2 1 up to w n 1 and w n l. So, basically the number of rows that we have over here would basically mean the number of transmit antennas that we have and the number of columns that we have would indicate the number of spatial streams that we would like to transmit. So, what we have seen is that we could play around with this matrix and use all the transmit antennas because you have h getting multiplied h is h 1 1, h 1 2 up to h 1 m. So, all the transmit antennas getting multiplied. So, if there are l if it is l is less than or equal to m then we could reduce the number of streams.

So, if my rank is low rank channel then I could use only 2 streams let us say and I would choose only 2 columns, any 2 columns. So, that my signals are spread across all the antennas, but there is only 2 signals that means, s 1 times w 1 1 plus s 2 times w 1 2 which goes to the first antenna w 2 1 times s 1 plus w 2 2 times s 2 goes to the second antenna and so on. So, we are sending s 1 and s 2 over several multiple paths. So, basically there are 2 spatial streams, whereas these different paths provide 2 diversity gains to the system.

So, for these systems to operate the receiver needs to send to the transmitter not the full channel information, but it needs to send 1 the rank estimate of the channel usually through rank indicator. So, the receiver has to calculate the rank and this is not a problem because h is estimated at the receiver this has to be done under any circumstance second thing what the receiver should do is these code books are predefined I will shortly show you the code books.

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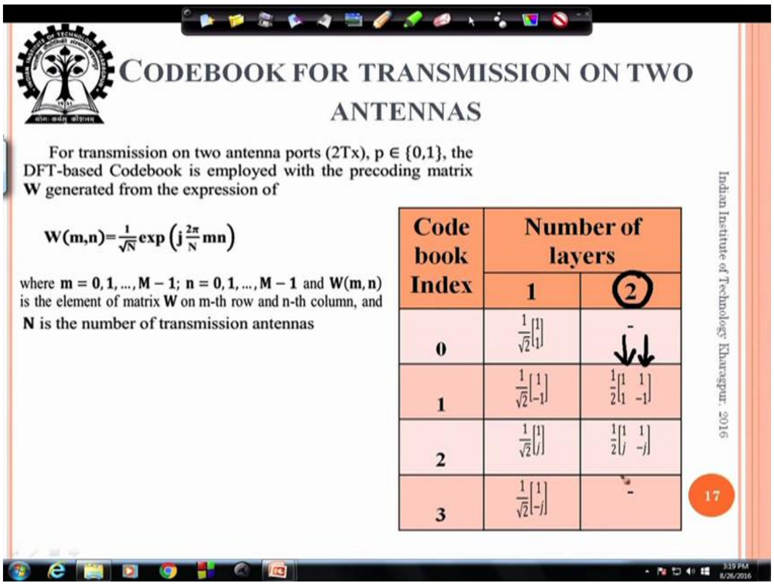

So, this particular example if we take in this particular example we have 2 layers so that means 2 spatial streams or 2 data streams. So, here we can see it has 1 column and a second column right. So, these this predefined code books are used. So, basically what we are trying to say is in this particular expression this are predefined. So, the job of the of the transmitter in the receiver is to know choose the best w if it does not have to use a v it is not require to use a v matrix over here.

So, in case y we had y equals to u sorry in s v d we had s estimate u hermitian times h times v times s. So, this was at the transmitters side this was at the receiver side whereas, in and this is the function of h. So, it is connected to h u is a function of h right. So, this is a not a problem because receiver knows h. So, it can estimate where this is the problem because this has to be fed to the transmitter now whereas, if I use w and w is a whole set of matrixes. So, what we have over here is 1 particular situation.

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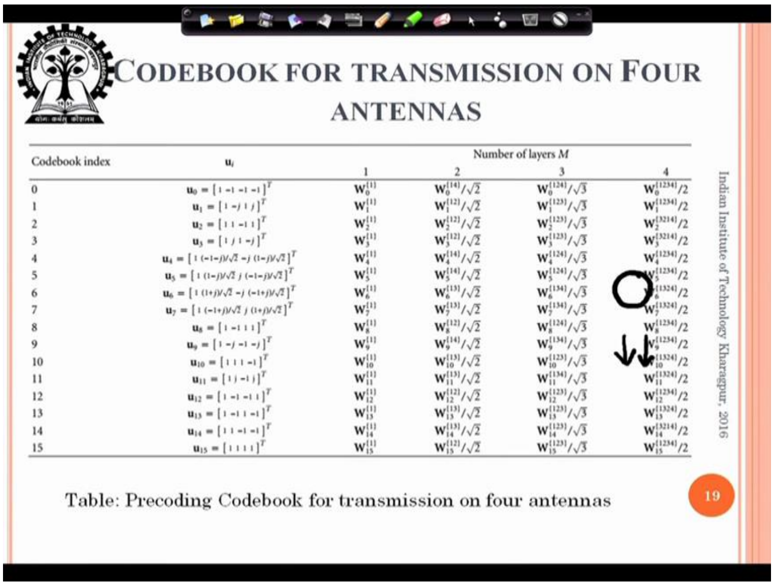

Codebook index	u_i	Number of layers M: 1	2	3	4
0	$u_0 = [1\ -1\ -1\ -1]^T$	$W_0^{\{1\}}$	$W_0^{\{14\}}/\sqrt{2}$	$W_0^{\{124\}}/\sqrt{3}$	$W_0^{\{1234\}}/2$
1	$u_1 = [1\ -j\ 1\ j]^T$	$W_1^{\{1\}}$	$W_1^{\{12\}}/\sqrt{2}$	$W_1^{\{123\}}/\sqrt{3}$	$W_1^{\{1234\}}/2$
2	$u_2 = [1\ 1\ -1\ 1]^T$	$W_2^{\{1\}}$	$W_2^{\{12\}}/\sqrt{2}$	$W_2^{\{123\}}/\sqrt{3}$	$W_2^{\{3214\}}/2$
3	$u_3 = [1\ j\ 1\ -j]^T$	$W_3^{\{1\}}$	$W_3^{\{12\}}/\sqrt{2}$	$W_3^{\{123\}}/\sqrt{3}$	$W_3^{\{3214\}}/2$
4	$u_4 = [1\ (-1-j)/\sqrt{2}\ -j\ (1-j)/\sqrt{2}]^T$	$W_4^{\{1\}}$	$W_4^{\{14\}}/\sqrt{2}$	$W_4^{\{124\}}/\sqrt{3}$	$W_4^{\{1234\}}/2$
5	$u_5 = [1\ (1-j)/\sqrt{2}\ j\ (-1-j)/\sqrt{2}]^T$	$W_5^{\{1\}}$	$W_5^{\{14\}}/\sqrt{2}$	$W_5^{\{124\}}/\sqrt{3}$	$W_5^{\{1234\}}/2$
6	$u_6 = [1\ (1+j)/\sqrt{2}\ -j\ (-1+j)/\sqrt{2}]^T$	$W_6^{\{1\}}$	$W_6^{\{13\}}/\sqrt{2}$	$W_6^{\{134\}}/\sqrt{3}$	$W_6^{\{1324\}}/2$
7	$u_7 = [1\ (-1+j)/\sqrt{2}\ j\ (1+j)/\sqrt{2}]^T$	$W_7^{\{1\}}$	$W_7^{\{13\}}/\sqrt{2}$	$W_7^{\{134\}}/\sqrt{3}$	$W_7^{\{1324\}}/2$
8	$u_8 = [1\ -1\ 1\ 1]^T$	$W_8^{\{1\}}$	$W_8^{\{12\}}/\sqrt{2}$	$W_8^{\{124\}}/\sqrt{3}$	$W_8^{\{1234\}}/2$
9	$u_9 = [1\ -j\ -1\ -j]^T$	$W_9^{\{1\}}$	$W_9^{\{14\}}/\sqrt{2}$	$W_9^{\{134\}}/\sqrt{3}$	$W_9^{\{1234\}}/2$
10	$u_{10} = [1\ 1\ 1\ -1]^T$	$W_{10}^{\{1\}}$	$W_{10}^{\{13\}}/\sqrt{2}$	$W_{10}^{\{123\}}/\sqrt{3}$	$W_{10}^{\{1324\}}/2$
11	$u_{11} = [1\ j\ -1\ j]^T$	$W_{11}^{\{1\}}$	$W_{11}^{\{13\}}/\sqrt{2}$	$W_{11}^{\{134\}}/\sqrt{3}$	$W_{11}^{\{1324\}}/2$
12	$u_{12} = [1\ -1\ -1\ 1]^T$	$W_{12}^{\{1\}}$	$W_{12}^{\{12\}}/\sqrt{2}$	$W_{12}^{\{123\}}/\sqrt{3}$	$W_{12}^{\{1234\}}/2$
13	$u_{13} = [1\ -1\ 1\ -1]^T$	$W_{13}^{\{1\}}$	$W_{13}^{\{13\}}/\sqrt{2}$	$W_{13}^{\{123\}}/\sqrt{3}$	$W_{13}^{\{1324\}}/2$
14	$u_{14} = [1\ 1\ -1\ -1]^T$	$W_{14}^{\{1\}}$	$W_{14}^{\{13\}}/\sqrt{2}$	$W_{14}^{\{123\}}/\sqrt{3}$	$W_{14}^{\{3214\}}/2$
15	$u_{15} = [1\ 1\ 1\ 1]^T$	$W_{15}^{\{1\}}$	$W_{15}^{\{12\}}/\sqrt{2}$	$W_{15}^{\{123\}}/\sqrt{3}$	$W_{15}^{\{1234\}}/2$

Whereas this particular tells us that there are whole lot of W s available. So, basically if we try to do it very quickly.

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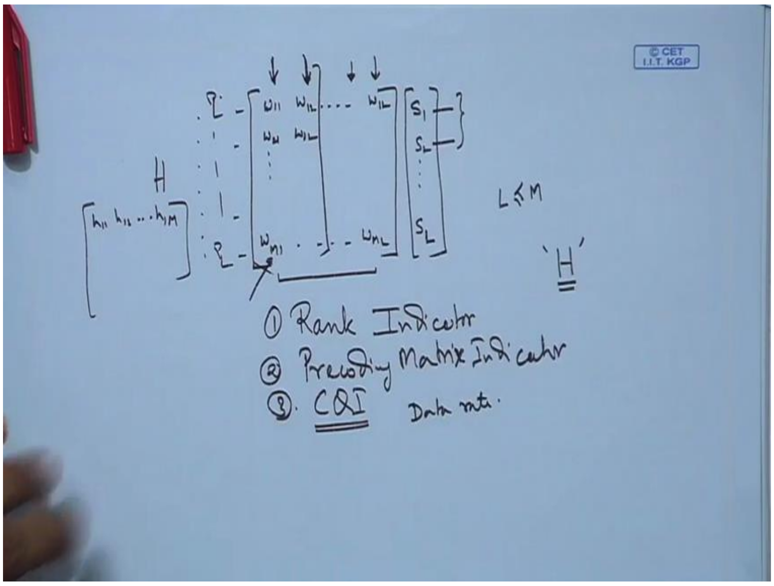

We were saying that there is w 1 which is a matrix there is w 2 which is a matrix w three which is another matrix w four which is another matrix. So, these matrixes are all predefined the all predefined 1 does not have to define w in case of s v d your w is basically v and at the receiver what do you do at the receiver it is not a problem because 1 of the easiest way for a receiver is if i multiply h hermitian and then w hermitian to y this could be 1 of the easiest ways of doing it the other way I could do it is h inverse times y times w in hermitian and I would ensure that w hermitian w is equal to identity.

So, in this way we have predefined matrixes. So, this hints us if we are choosing predefined matrixes the gain that we get is we do not have to calculate v we do not have to send back v. So, if we have let us say a sixteen different choices of w all we need is sixteen log two; that means, four bits of information to select 0, 0, 0 up to 1, 1, 1 to indicate different W s. So, the receiver has to find out which w is the best choice because you can solve this also you can solve this equation and find out which w is the best and you have to select and send it. So, this is known as the PMI indicator or pre coding matrix indicator.

So, we have a rank indicator and pre coding matrix indicator and number three the third thing that we require is a CQI indicator which is a channel coding channel quality indicator or in other words what kind of data rate is to be supported because when we are choosing let say 2 spatial pipes definitely the SNR in 2 spatial pipes are not going to be exactly the same. So, if there is significant difference between them and we have to maintain the same error probability we cannot choose the same data rate we have to choose 2 different data rates the higher SNR link can support higher data rate lower SNR link support a lower data rate. So, when you support to lower data rate you have to choose a particular modulation and coding rate when you support a higher data rate you would either choose a higher modulation or a higher code rate.

So, this combination gives you one particular data rate in LTE there are 31 different possibilities of options which could be signalled by either 15 or even 31 different options. So, in that case you would indicate which particular data rate to be used for which particular channel. It has been reduced to 3 different information instead of sending back the whole v matrix, if you are to send back the whole v matrix you had to have let say m cross n elements that would be an m squared, so m squared elements which would be quantized by let us say 14 bits of information. So, 14 times m square is the overhead that is required to be signal back to the transmitter. So, that is you reduced by use of such code books in LTE.

Now, details of this are available in any standard book on LTE or in a reference material and some things are also included in this particular slide which you can read yourself and try to get a feel of what is going on in the system. So, there is also beyond this something known as cyclic delay diversity which is also used in LTE and beam forming is also allowed in LTE, in beam forming amongst different modes, one can think of that if one has knowledge about the channel at the transmitter then what one could essentially do the SVD of the channel and find the different the vectors of v and if 1 uses the appropriate v vector then what essentially 1 is doing is 1 is conditioning the channel towards the particular eigen mode. So, that is very, very fundamental. So, one could do that and achieve many important out comes.

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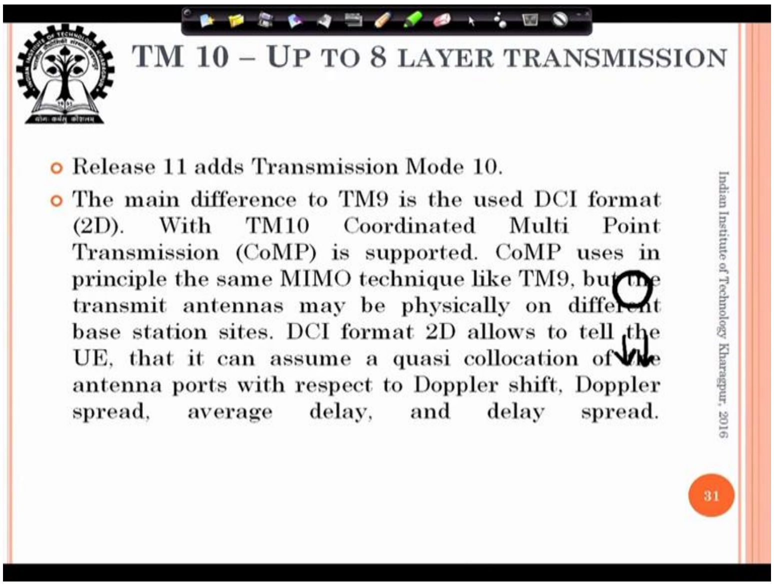

So, we move on and finally, I will give you briefly that whatever we have studied using that one can imagine the situation.

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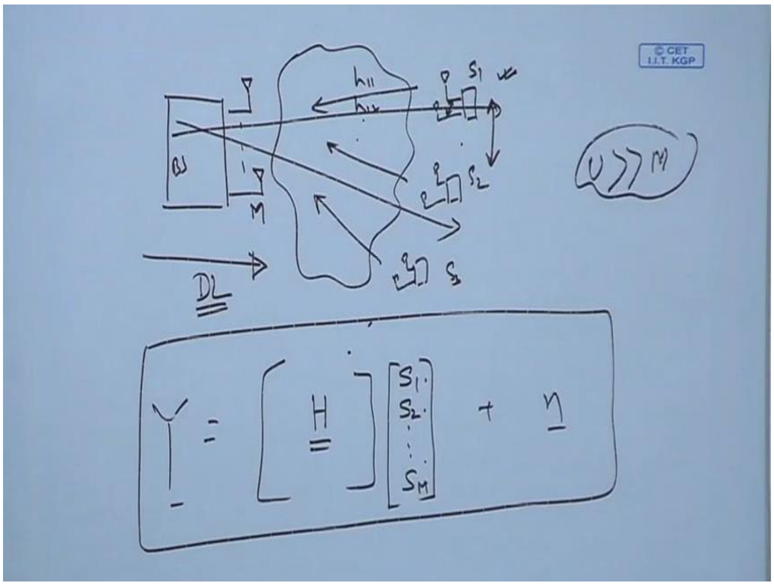

Now, that 1 has multiple antennas at the base station and 1 could have users with single antennas or multiple antennas and so on. So, the other mode which is supported in such MIMO communication system is 1 is multi user MIMO mode which itself is a literature and it would require a good amount of time discussing all this details and whereas, all the things that we have discussed in this course on fundamentals of MIMO communication at the basic tools which enables you to explore a further into this things.

I will very briefly very, very briefly touch upon this particular thing to tell you what it is all about. So, that you can explore further and get in to the details as per your interest. So, in this schemes if you are thinking of uplink in this direction the users can send their own data in this direction. So, if there are m number of transmit receive antennas at the base station and there are a different users. So, up to m different parallel streams can come to the base station 1 can also understand that in this situation the users are usually far apart compare to the separation with the antennas and what we have studied is that as the antenna distance become smaller and smaller correlation increases between the antennas.

Whereas, when the antenna separation increases correlation decreases and we have also studied the effect of correlation that when there is more correlation in the between the antennas 2 things happen 1 is the diversity gain decreases and capacity gain decreases.

So, whereas, when we have this particular situation, this antenna separation is large it is of low probability that will be high correlation.

So, all these links can be of almost independent nature in this case you can almost achieve m parallel links that are getting transmitted. So, we could set up the equation as if there are m different symbols being send by different users of course, there must be synchronisation and there is this large matrix h which gets formed which is basically the composite of all these links. So, basically between this and this there will be certain links between this and this there will be certain links and so on and so forth. So, you could write as h 1 1 then they could be h 1 2 and so on and so forth. From the first antenna similarly from the second antenna and all of this links will form the large matrix each, 1 of them sending separate links and at the receiver you receive this signal.

So, this expression is well known to us if the channel is orthogonal that means, it may be possible the channels are orthogonal because users are separated by far distance one could usually use mass filter receiver with the mass filter if it is orthogonal one could still identify the received signals, otherwise one could use a channel link version if it is a invertible channel or one could use successive interference cancellation and proceed.

Whereas, in the reverse direction from the based station that means, to the down link direction a things are not that simple and one of the very famous methods of doing it is known as dirty paper coding through which dirty paper coding is kind of pre-coding technique by which the interference that is experienced by different users are pre estimated they are pre cancelled and they are transmitted with appropriate weight. So, that the signal that is received by user 1 is independent or is free from interference of the signal that is intended or transmission to user 2 and it is not a very, very straight forward thing to achieve.

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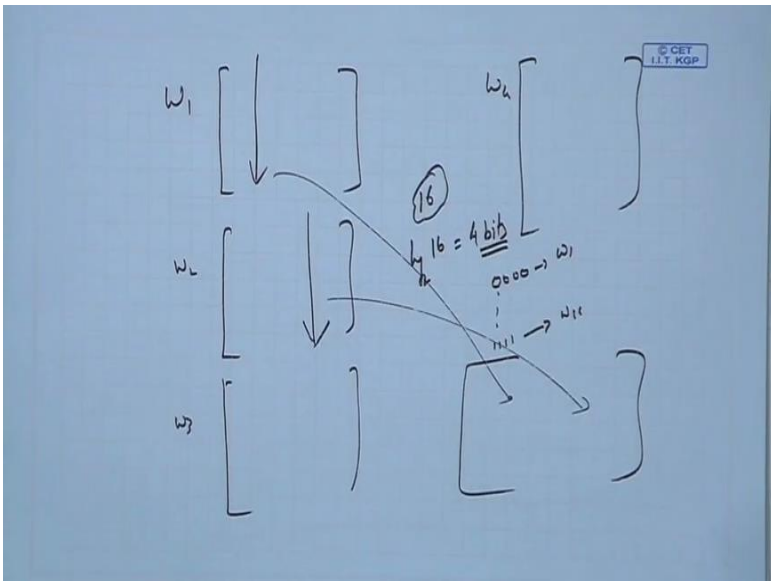

Whereas in LTE by virtue of having code books it is still possible to multiple users. So, that in a fashion, for instance one could choose a different columns of these different matrixes and form 1 new matrix which is using columns from different users in such a way that different users data or these streams they do not interfere with each other and in this wave in the signal is transmitted across to the users would have almost interference free communications.

So, in this case what I would like to point out you can easily guess is it is important that usually the number of users in the system is much, much larger than m; m is the number of transmit antennas. So, if we have to choose at most m users then we have to choose those m users which would result in orthogonal channel because if we can get on orthogonal channel. We have very, very simple receiver to implement otherwise our receiver architecture would be difficult. So, if I have very large number of users in the system and I have to choose a few from them then there is the high possibility that I can select users who would provide me a orthogonal set of channels amongst themselves.

So, this is the big advantage that a multi user MIMO gives us. So, at least in the uplink direction we have to find users who would provide mutually orthogonal channel. So, user selection user association is a very important task in this particular process and followed by scheduling activity; that means, if I choose a group of users orthogonal now then after in the next scheduling interval I have to find and another set of users who would be mutually orthogonal.

Otherwise I would be choosing the same set of orthogonal users continuously and the other users would be deprived of service, whereas in the down link direction the problem is more to find appropriate code words which would yield mutually orthogonal signals to the to the receivers because here it is assume that the receivers are low complex receiver that means, users equipment are low complex, user equipments on the other hand since the base station is much more capable device, it is much more resources to operate it can perform this signal process algorithms which would otherwise not be possible at the user end. Whereas if it is assumed that huge amount of interference cancellation is possible at the receiver then one could do super position coding to achieve capacity in this particular links.

So, I would stop at this point with in this particular course on MIMO communications and I hope we have been able to give you with a foundation or the basic building blocks or the elemental signal processing techniques and signal models which would allow you to explore the domain of MIMO communications further and help you in understanding deeper better systems more complicated systems and able to design further systems as well as come up with improved version of systems which could be potentially effecting new generation communication systems.

Thank you.