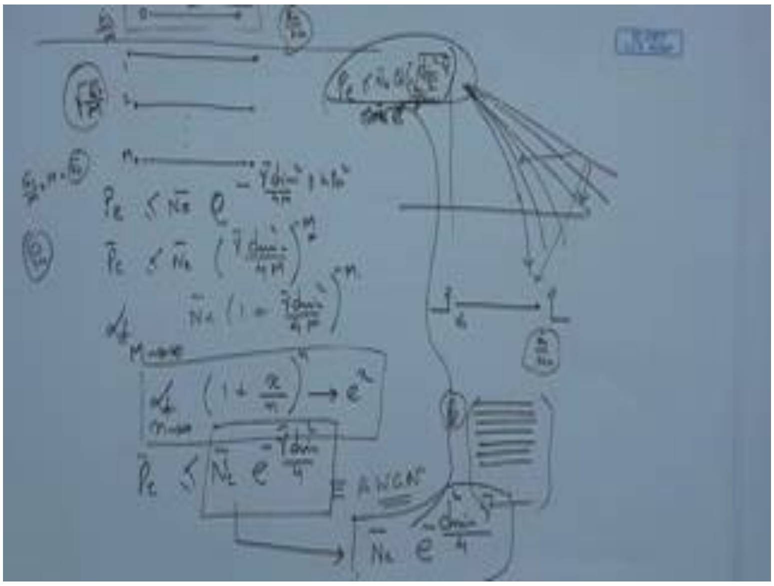
Fundamentals of MIMO Wireless Communication
Prof. Suvra Sekhar Das
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 29
MIMO Transmit Diversity – 1

Welcome to the lectures on Fundamentals of MIMO Wireless communications. We are currently exploring diversity gain. We have discussed several received diversity, we have also discussed several transmit diversity. We have also explained to you the general notion of diversity. So, what I will begin in with is a bit revision of the general notion of diversity and sees the implications that could present. So, some minor or some fine conclusions that I have that can be drawn here from that and once we discussed that we will move on to the transmit diversity mode that are still remaining for us to explore.

(Refer Slide Time: 00:55)



So, when we look at the typical explanation of the diversity scheme where we said we are having several parallel transmissions and these could be a time frequency or a space and where the signal energy was divided equally amongst all the transmitted sequences.

What we had found out was the probability of error expression was less than typically as usually used we used N bar e to the power of minus gamma bar d mean squared by 4 m and h, h square this is what we used this m is coming because of this m over here and when we carried on to calculate P e bar we had found this to be n bar using the n g f method or whatever method you would like to use gamma bar d means squared by 4 m and last time explained this could be d time by 2 m to the power of minus m right and here we explained the about the order of diversity.

Now, so as I mean increases the diversity gain increases we have also said at if this is the performance of no diversity then this is the performance of 1 order of diversity 2 order of diversity and so on. So, the slope increases as m increases that is what we have explained. So, now, in this to the beauty of this particular description that we have is the total transmit power is remaining the same because of this because transmit power per branch is e s by m and if I add up all of them like 1, 2 up to m, I want to get e s that is e s by m multiplied by m is equal to e s. So, that is the total power.

We can compare this with a KSC so that means, a single input single output system we are also it is e s. So, the SNR over here is e s by n naught here also the effective SNR is e s by n naught. So, the total average SNR is the same in both the cases that means, once I take the CISO case or we take the multiple inputs the parallel transmission where we have diversity. So, here is a nice case to compare. So, when we move to other cases where it was not your where it was not e s by m scenario, we did not have this m in the denominator because of the m here for transmit MRC this m got compensated for STBC this m remained and there was like eight in the denominator as you might remember from the other expressions.

So, any way back to this expression this is pure diversity there is no eigen expressed in this that is fundamentals of this no eigen in this expression there is only diversity gain which is here. So, as e s increases as number of branches increases per branch now decreases. So, only I have added only diversity gain is expressed in this. Now, in this we would like to study the limit m tends to infinity. Now, this is a very, very important condition that we were saying that suppose I increase the number of transmissions to infinity, while keeping the same constant on the transmit power what we mean is that suppose I have a single antenna to transmit single antenna to receive.

So, here total transmit power is e s and average SNR is e s by n naught, compare to this I consider a situation, whereas I still have the e s as a transmit power yet I send several of them what is going to happen I am not going to change the average transmit power neither my changing the average received power. So, average SNR of this link is remaining the same there is no arrogant. So, still what happens effectively I am enjoying an average e s by n naught, for this what we need to what we need to see is that if we take limit n tends to infinity, 1 plus x upon n to the power of n, this is the thing. This tends to e to the power of x we use this identity, using this identity if you say limit this tends to infinity that means, ideally the expression that we have is N e bar 1 plus gamma bar d mean squared by 4 m raise to the power of minus m.

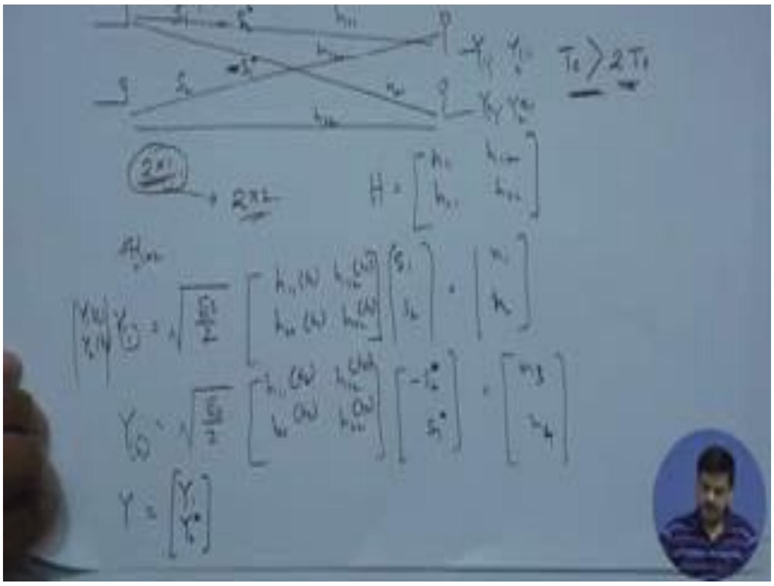
So, basically we have minus m over here. So, you say m tends to infinity this expression would boil down to P e bar is less than or equal to N e bar to the power of minus gamma bar d mean squared by 4. Now, if you would see this expression this expression is independent of m and this is the same as expression of error probability for AWGN, if you look at it any bar e to the power of minus gamma bar d mean squared by 4. So, this is the expression for AWGN what does it mean effectively is that if I keep on increasing the order of diversity the slop increases and finally, at a certain point where at a certain point when it tends to infinity it meets the error probability for AWGN and we can we can clearly see this because we made the; we started off with P e is less than or equal to N e bar q of root over d mean squared gamma bar by 2.

This was the expression that we started with. So, this if you use Chernoff bound this turns out to be N e bar e to the power of minus gamma bar. I will write this expression I can bring it here let say N e bar e to the power of minus d mean squared gamma bar by 2 times 2 is 4. So, this is the same expression as this one. So, once we get from this q function and as limit m tends infinity. So, what we have said is as the diversity order increases without a regain that means, we do not require a regain still we would meet the performance of AWGN system that is very, very important. Remember that as we increase the order of diversity we are actually going towards the performance of AWGN.

However, 1 word of caution that if we have that means, if this m is not there in the denominator, if you have gain in that case along with this increase in slope this curve would also shift to the left and in that case we would beat the performance of AWGN. It would be better than that, but however, again there is we around that if somebody might say that well I want to compare the composite SNR means after combining whatever is the SNR AWGN if a fatigue the same amount of SNR AWGN and try to look at the average or the combined SNR in kind in case of combined scheme in that case AWGN is going to beat that that of the combining scheme.

However, if we keep the average SNR per branch as the basis for comparison if there is gain definitely it will better than AWGN whereas in the other case when there is no gain what we just saw it the combining scheme would reach the performance limit which is the same as that of AWGN, this is very, very important for us to remember.

(Refer Slide Time: 08:52)



So, this is very nice. With this we would like to move on to transmit diversity schemes. So, we have already described 1 transmit diversity scheme that is the Alamothi scheme. In the Alamothi scheme, we have already said there are 2 transmit antennas and we are going to be transmit s 1 and s 2 and s 2 conjugate minus s 1 conjugate. So, this is the scheme that that what you are transmitting in case of transmit Alamothi and when we transmitted this particular scheme, we actually send minus s 2 conjugate and s 1 conjugate we had only 1 received antenna. So, now, instead of having just only 1 received antenna could be have a scheme where we have 2 received antennas so that means, we have h 1 1 this could be h 2 1 this going to be h 1 2 this could be h 2 2. So, this could be little bit modification of the earlier scheme. In earlier scheme, it is Alamothi with 2 plus 1 combination 2 transmit 1 is the antenna and then we move on to 2 cross 2 system.

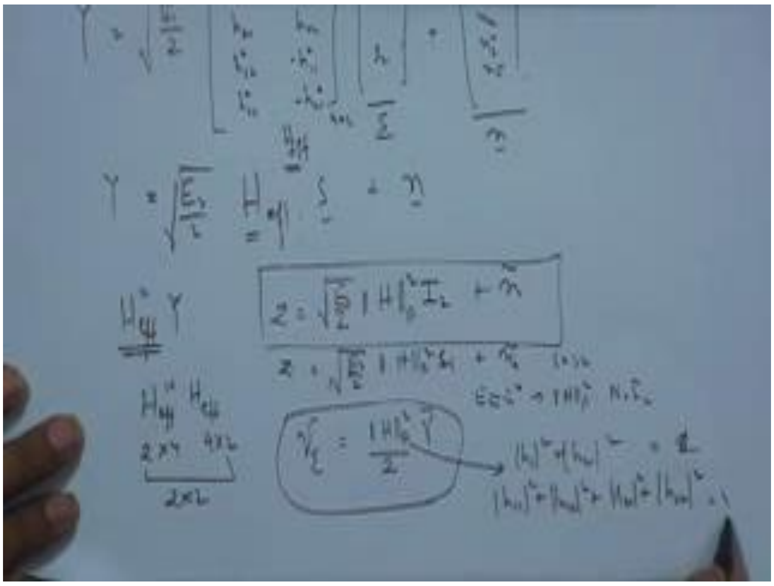
So, although we specified it is a 2 cross 1 the h matrix is actually 1 cross 2 and in this case it is 2 cross 2. So, that we should be clear when it is a 2 cross 1 we are actually meaning 2 transmit 1 is your antenna when we are mentioning the h matrix the number of rows is indicating number of received antennas. So, we should be careful when we are noting this. So, if we have the Alamothi scheme with 2 received antennas then what is going to be happen in this case the h channel is basically h 1 1, h 1 2, h 2 1, h 2 2 this is the h channel and the y 1 signal that we are going to get we would write it as root over e s by same e s by 2 we will transmit the same symbol sequences no change in the symbol sequence.

We are going to have h 1 1, h 1 2, h 2 1, h 2 2 and the symbol s 1 and s 2 and we are going to receive them in 2 antennas n 1 and n 2. Now, while we discuss Alamothi scheme we should always remember that we are sending the symbols in 2 time instance t 1 and t 2 and we have used a coefficients h 1 and h 2 in that case now we are using h 1 one and h 1 2 h 2 two it is very important to remember we make the assumption that the coherence time is greater than 2 symbol durations if this is symbol deviation 1 this is symbol deviation 2 it should be greater than that. So, that the channels remain constant across the 2 duration 2 time 2 symbol durations that is very, very important for everything to hold.

In the second interval, we received this second vector that is again e s by 2 we have same channel matrix h 1 1, h 1 2, h 2 1, h 2 2. Remember, in the previous case we had h 1 1 and h 1 2 we did not have this, in this case. We are having this ideally speaking it should be at time t 1 and this should be at time t 2 ideally speaking this should be the case, but if we assume this even at t 1 and t 2 and they are the same that the assumption and here the signal is minus s 2 conjugate and s 1 conjugate. So, this is the special construction that is done for the scheme Alamothi rather, this is n 3 and n four or rather it is n 1 at time t 2 n 2 at time t 2.

So, this is the overall signal space that we have and the single that we need to construct is y 1 vector followed by y 2 conjugate. So, if we construct the signal in this way we are we will be able to de decode the signal that is been transmitted. So, this is what we did in the previous case it will be in the same situations. So, this is indicating time instant 2 and this is indicating time instant one, but the 2 entries of that means, y 1 and y 2 y 1 of t 1 y 2 of t 2 are basically the input at a 2 antennas y 1 y 2 at time 1 and again y 1 at time 2 y 2 at time two. So, that is how the overall expression is. So, this is want we need to construct.

(Refer Slide Time: 13:21)



So, if we construct that expression the t 1 that will be left to is y big y is equal to root over e s by 2 then we have h 1 1, h 1 2, h 2 1, h 2 2 and then h 1 2 conjugate h 1 1 conjugate with a minus h 2 two conjugate and h 2 1 conjugate the minus just like we had in the previous time s 1 and s 2. So, we are taking that conjugate of y 2 if you are taking conjugate of y 2 this will become s 1 and s 2 the minus sign would we absorbed. So, we taken the minus sign inside it the expression would appear in this form plus n 1, n 2, n 3 conjugate n 4 conjugate right, n. So, it is basically again y equals to e s by 2 square root h effective times is a matrix time is a vector plus n is this vector this is the s vector this is the h effective matrix and again the way we decoded is we simply multiply h effective harmation with y we can guess what is going to happen is that this harmation means this is going to come in the row. So, basically this is going to multiplied by itself. So, it would be h 1 1, square h 2 1, square h 1 2 square plus h 2 2 square which is the norm of the channel matrix h effective and if you take the cross elements I mean this multiplied by this we are going to be get the terms which we could cancel out each other. So, h 1 1 conjugate times h 1 2 and they would be again minus h 1 1 conjugate times h 1 2 over here.

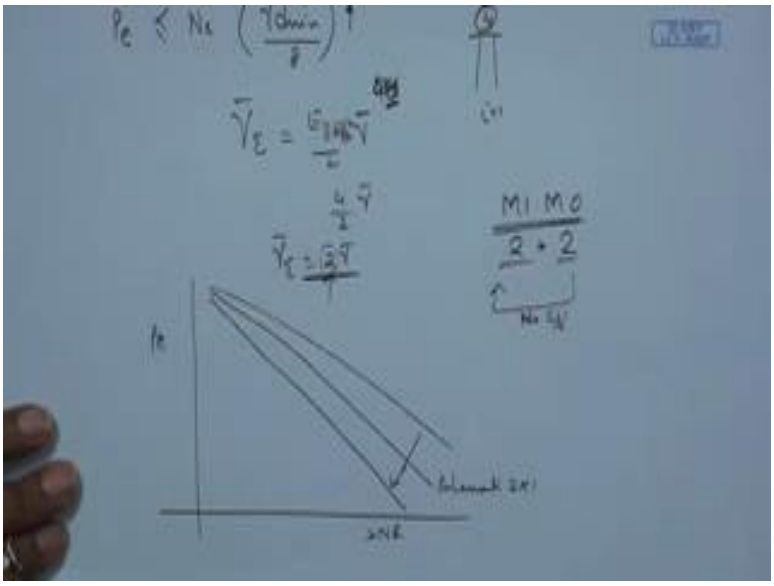
So, they are going to cancel out with each other and the effective expression that we are going to get out of it is said is equal to root over e s by 2 mod h f squared if 2 because this is a 4 cross 2 matrix. So, if we do this, we are basically doing h effective harmation times h effective. So, this is a four cross 2 and this is a harmation. So, it is a 2 cross four.

So, effectively we are getting a 2 cross 2. So, that is why we have identified matrix of size 2 plus n tilde. So, this is the whole expression which we get from this again we have to calculate the SNR and z i any particular symbol. So, this contains 2 symbols this is a 2 cross 2 matrix.

So, s 1 and s 2 estimates that we get e s by 2 h of squared s i plus n i tilde i equals to 1 to a. So, this is what we have and if we come compute the SNR for this we first have to consider e of n tilde n tilde harmation which turn out to be h f squared times n naught times i f 2. So, that is going to be this and basically we have h f squared multiplied by n naught as sigma s power and here also we have that. So, h f square in the denominator this whole squared. So, 1 of them goes off come up. So, gamma combined would turn out be h f squared e s by n naught 2 would definitely come which we could be write it as gamma bar and this 2 would be remain in the denominator now this expression looks same as the earlier case.

However, h of squared has changed in the previous case it was h 1 squared plus h 2 squared that means, in Alamothi 2 plus 1 square in this case it is h 1 1 square plus h 1 2 mod square plus h 1 2 mod squared plus h 2 2 mod squared. So, basically this is a bigger number. So, if you would take the expected value of this in this case it is 2 in this case it is four so; that means, here the size is more and we have have a bigger value to this

(Refer Slide Time: 17:47)



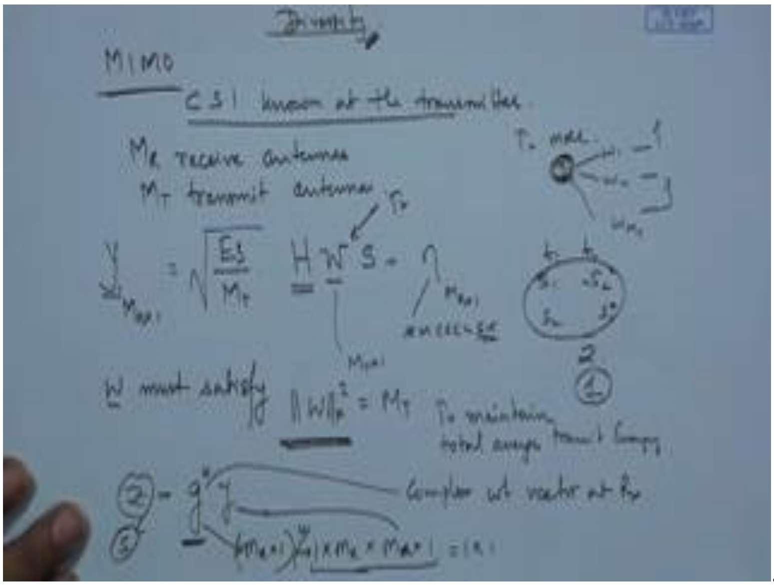
So, moving on to forward with the error probability expression what we want to get it is probability of error average expression of that going by this would be n e bar gamma bar d mean squared by four m if you remember. So, this would give us a 2 to the power of minus four because here h of squared has four entries.

So, there are four entries in it and the phi that we had phi i equals to 1 m r or 1 to m in this case it was 4. So, that gives us this four over here this 2 is a value of m because we have 2 transmit antennas because we have 2 transmit antennas because of which we had e s by two. So, this is the because of the diversity that we are getting and that 2 is leading to this four because we have we had m is a number of transmit antennas in that case. So, that is leading to 8. So, this is the average probability of error expression and if you would look the combined SNR this would be the average value you are going to get it as gamma bar expectation of h s squared by 2.

So, as we said this value is going to be 4 by 2 times gamma bar that is 2 times gamma bar. Now, you have twice the SNR of the CISO case or per branch SNR. So, if we compare the error probability curves we would have the probability of error curves in this case if this is the curve for probability of error for no combining and if you just have 2 plus 1 diversity, let say Alamothi 2 plus 1. So, this is going to be the 1 if we have this 2 cross 2 it will be some where there, but it is the additional increase in diversity gain because of the order four that we have over here and much better than this thing and additionally there is a additional SNR gain because of 2 because of 2 received antennas. So, we have this four order of diversity and we have this 2, 2 times increase in average SNR. So, we have a gain in this case as well as we have the diversity gain. So, we have exploited all the four branches in diversity gain and we have also used up the 2 received antennas for achieving a 0 gain are compared to the previous case. So, this is another method and you can simply go on and adding and keep on adding antennas at the receiver in order to have more and more a gain in to the system as long as you keep your transmit antenna with 2 the same processing algorithm should help in getting the appropriate diversity gain and average a 2 gain that we are looking at.

So, with this case with this way we can obtain diversity gain even without providing channels information at the transmitter with this. So, basically this mechanism if you would see carefully we have multiple inputs and multiple outputs. So, we have 2 antennas at the transmitter and 2 antennas at the receiver the minimum number required for MIMO communication and yet we have obtained a 2 plus 2 order of diversity in in this case; that means, all the 2 times that is four order of diversity that means, all the antenna branches have been used in this case; however, in this case there is no CSI that is send without 1 without translated information we have obtained this ah.

(Refer Slide Time: 21:47)



.

Now, we would liked to move on to the scheme which uses the channels state information at the transmitter and tries to obtain this performance. So, we are still at a diversity we are still at diversity and in diversity what will be looking at is multiple input multiple output we have already seen such scheme, but in this particular mechanism we will say that the channels state information is known at the transmitter. So, it is known at the transmitter. So, we will consider system, which has let say m r receive antennas and we have m t transmit antennas.

Basically it is a m r cross m t channel coefficient. So, we could write the expression of the received signal as the vector y which is m r cross 1 is equal to root over e s upon m t, so that is e s upon m t times h matrix times weight matrix or weight vector at the transmitter times s which is the signal. So, so remember at this point when we are doing diversity technique and we are doing c s i known at the transmitter we did already 1 thing which is transmit m r c. So, in that case we have send the symbol s across all the antennas, but they were wade by appropriate way to vectors or coefficient and send to the different antennas. So, because we are sending actually 1 data we are actually talking about diversity.

In case of Alamothi, we had s 1 and s 2 and s 2 conjugate minus s 2 conjugate and s 1 conjugate. So, here we could argue that we send 2 different symbols, but overall in 2 different time t 1 and t 2 overall we have send 2 symbols that means, per symbol time we have sent 1 symbol. So, overall we have actually not gone to something known as the multiplexing although it appears to be multiplex it is not multiplex in which we will see later.

So, at this point all i am going to say is when we are doing diversity you are basically sending the same information across all the different branches or the different paths that exist. So, we have this s over here plus noise. So, this is also m r plus 1 and this would be specially white; that means, zero mean or to the symmetric complex Gaussian specially white we could add this thing and this is m t cross 1 we have an m t again to ensure that the total power is maintained so that means, we would put the criteria w must satisfy if Frobenius norm of the w squared is equal to m t. Remember, we said this is the total strength of the particular matrix of the vector that is there and this is to maintain total average transmit energy or power constrain and we would consider or receiver z is equal to g harmation y. So, basically these are the equalize of coefficients and this would be of course, m r cross 1 and this y that we have g is a m r cross one. So, g harmation and when this is harmation it is 1 cross m r right it is 1 cross m r g harmation is 1 cross m r this is m r cross one. So, there is product between them.

So, the end result that you have is basically 1 cross 1. So, this is recovering s in that case. So, it is a going to get back s. So, this is what we have to do and g is the complex weight vector is complex weight vector at the receiver. So, we have w at the transmitter and we have g at the receiver and when we do this MIMO based diversity gain with c s i known at the transmitter or we have to choose w and g in such a way that be maximize the SNR and that is that is our objective and we will see in the next lecture how do we maximize this SNR and what do we get at the end when we maximize the SNR following a special scheme and that will require you to revisit the linear algebra and the important parts of single validity decomposition based factorization of matrices.

So, we will be using some of the resolves from single validity decomposition which will help us in the derivation of the result for this particular transmission scheme.

Thank you.