Fundamentals of MIMO Wireless Communication
Prof. Suvra Sekhar Das
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 28
Transmit Diversity without Channel known at Tx

Welcome to the lecture in fundamentals of MIMO Wireless Communications. We are discussing special diversity. In the previous lectures we have talk about receive diversity as well as transmit diversity, for the transmit diversity case we have discuss the condition when channel is known at the transmitter.

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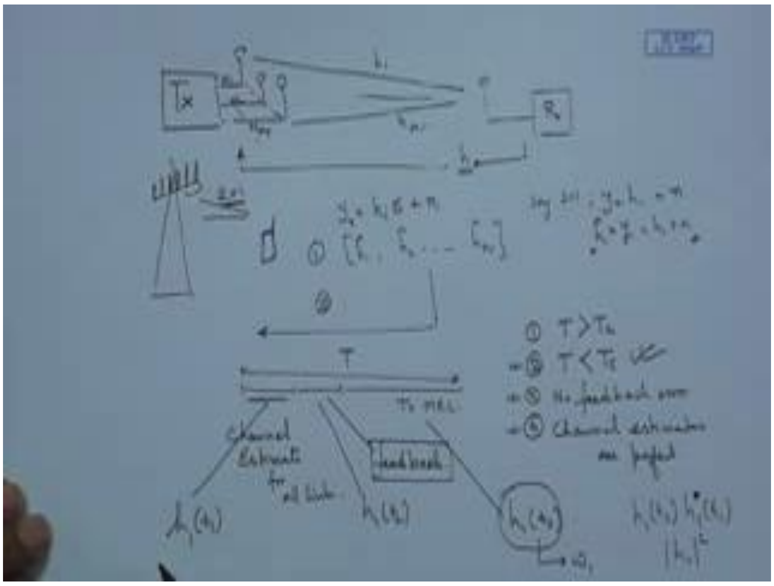

Now, when the channel has to be known at the transmitter we should remember that there is this transmitter with several antennas and there is, let us say the simple case when there is 1 receive antenna will be most simple case. So, at this case there was w 1 w 2 up to w m t that was used at the transmitter; that means, we had rate factors and these rate factors were proportional to the channel gains.

So, if this particular link was h1 and last 1 was h m t, we had all this links, this entire channel vector h had to be fade back to the transmitters now this is significant that. So, for this to happen one must understand that the channel has to be estimated at the receiver. So, channel estimation if you typically take y is equal to h I times s plus n. We need to estimate all of this channels; one way of doing it could be that we sent a signal from here let us say s is equal to 1, from this in that case would be an estimate of h1 and then we would said s equals to 1 from channel 2 from antenna 2 and so on, up to all the antennas and then will be getting h1 cap as the estimate h2 cap as the estimate h n t cap as the estimate. So, all these channel coefficients have to be estimated in step number 1, in step number 2 these channel estimates have to fade back to the transmitter and the implicit it is assumption that we make this at these channels that are estimated then their fade back and which are used is empty during subsequent transmission.

So that means, it is first period then you channel estimate the channel, channel estimate for all links this particular amount of time then there is feedback followed by the subsequent section when you do transmit MRC. So, this is the total interval of time and in the previous case we have assumed that there is perfect channel estimate; that means, channel is completely known at the transmitter. So, when channel is completely known at the transmitter 1 has to be very careful regarding what is known about the channel. So, if this duration of time let us say this duration is t, let us take the case 1, when t is greater than coherence time. What is going to happen? In that case channel that is estimated in this part; that means, h of t 1 here let us say this is h1 of t 2 and here it is h1 of t 3 these are three different times. If t 3 minus t 1 is greater than coherence time this coefficient is very different from the coefficient that was measured whereas, this is used in w 1 the rate.

So, if this is used in w1 in expression that we have written it is said that h1 at time t 3 we did not use the time t 3 in that expression times h1 at times t 1. We were the assumption that these 2 are equal and we said it is equal to mod h1 square, and that is not going to happen. So, one of the fundamental conditions that should be valid for it to work properly is that it is total time must be less than the coherence time, that is number 1, number 2 is that the this is the valid condition for it to work this feedback has to happen in ideal way; that means, do you feedback there is no error and so, no feedback error and point number four that the channel estimates are perfect although it is never true. So, what we discuss is the ideal condition; that means, when you are estimating the channel let us take this example. So, say s is equal to one. So, what do we get is y is equal to h one; that means, I am transmit through only antenna one and not transmit anything through this antennas h plus n right.

So, basically this y I am saying that h1 cap is equal to y. So, this is equal to h1 plus noise; that means, there is some noise in it in a very trivial sense if I have sufficient large number of such recordings I can reduce the impact of noise, but I cannot the eliminate it. So, under true conditions true operating conditions this estimate is never perfect there will be some noise, we have this made perfect estimate and that is the result that we have given. So, when we do transmit MRC we have lot of difficulties to overcome; that means, we need this coherence time to be pretty large, we need ideal feedback; that means, almost instantaneous feedback without any errors and we required a perfect channel estimates. But in reality this is not easy to achieve, because even during feedback you required channel estimates and so on so forth everything to be going on. So, this this not a very much desired operating condition, we would laugh to use it, but because of constrain it is very difficult to use. So, under nearly static conditions this is fine, but under dynamic conditions there is lot of difficulty on this.

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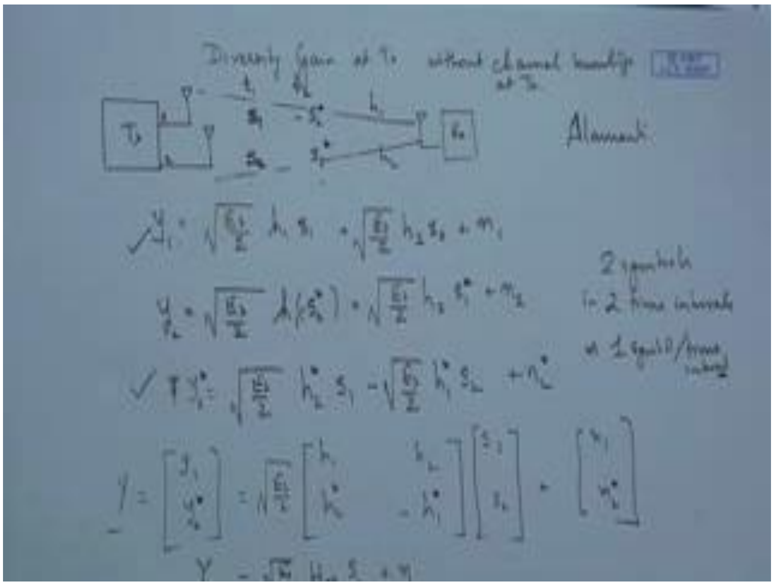

So, to overcome this there are mechanisms by which, you would you can obtain diversity gain at the transmitter without channel knowledge at t x now this is this is very, very important. So, we are not assuming any channel knowledge so; that means, we are avoiding this whole process of feeding back channel information using which1 would descending the communication signals. So, assuming it is not there, something better has to be done at the transmitter when h is not used. So, that is the domain that we are going to look at. So, we will take a look at a very, very simple situation, and we will assume that let there be 2 antennas at the transmitter this the minimum number of antennas that you can have in order to obtain special diversity, and let the receiver have only 1 antenna.

So, this this scheme is a very, very special scheme and this was proposed by Alamouti and this scheme is known as the Alamouti scheme that is what we going to discuss in this particular section. So, under this particular case what is done is we will consider 2 time instance t 1 and t 2 and we will say this is the antenna 1 and this the antenna 2. So, in the first symbol duration we will be sending the signal s 1 and s 2 from these 2 antennas. So, we have to 2 signals s 1 and s 2 to be send in the first time instant we going to send a the signal s 1 from the first antenna in the second time in the second antenna we are going to send the symbol s 2, in the in the second time instant we could send the signal minus s 2 conjugate and we can also send s 1 conjugate in the in the second time instant. So, when we receive the signal at the receiver. So, what we are going to see is that y 1 that is the received in the first instant is root over s by 2 because we have 2 antennas were dividing the power equally amongst then through h1 times s 1 plus root over s by 2 h2 times s 2 plus m one this the noise in this time instant 1 and y 2 is equal to root over s by 2 h1 times s 2 conjugate with the minus sign that is what we are going to send plus root over s by 2 h of 2 s 1 conjugate plus n 2.

So, this is what is going to happen rather we could say that this is time instant 1 this is time instant 2. So, this link is basically h1 this link is basically h2. So, we device this particular mechanism where we are sending 2 symbols in 2 instants of time and in encoded in such a way that in 2 time durations we are sending total of 2 symbols that is s 1 and s 2 because this symbols are modifications of s 1 and s 2. This is s 1 conjugate this minus s 2 conjugate. So, basically over all we are sending 2 symbols s 1 and s 2 in to time instants so; that means, we are not reducing the rate we are keeping the rate constant. So, if you see this signal we can form a vector y in terms of the signals. So, to do it better we would rather use y 2 conjugate. So, when we use y 2 conjugate would get root over s by 2 and I would bring this expression here first. So, I would have h2 conjugate s 1 and minus root over s by 2 h1 conjugate s 2 plus n 2.

So, if I would focus on y 1 and y 2 conjugate and the received signal received vector y I could format as y 1 y 2 conjugate which I could write it as root over s by 2 and I could send signal s1 and s2 from this 2 antennas I could I could effectively write the expression in this way that root over s h1 s 1 plus h2 s 2 and then y 2 conjugate is equal to h2 conjugate times s 1 minus h1 conjugate times s 2 plus n 1 n 2 conjugate. So, when I write the expression in this way we effectively are writing y is equal to root over s by 2 h effective this is effective channel times s this is the matrix plus some noise vector. So, we have a linear equation. So, we could use matrix algebra in in what you on this.

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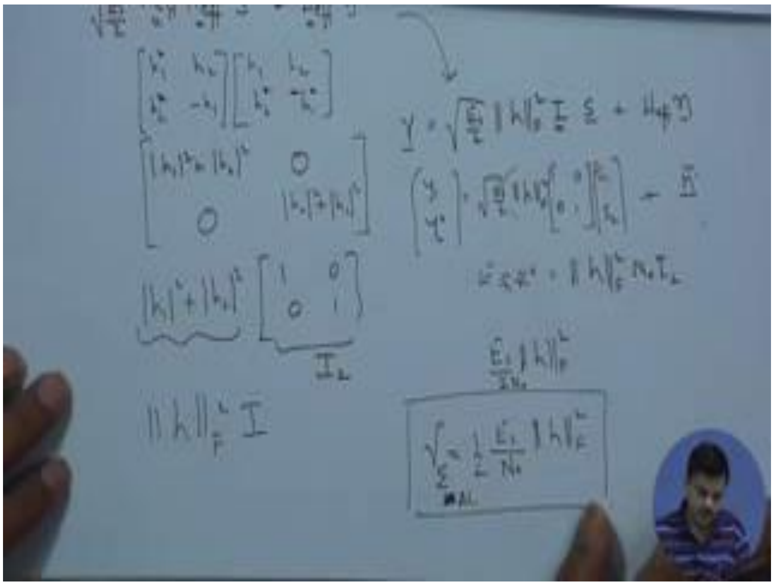

Now, to work to get a solution one of the easiest ways would be to have equalizer where the equalizer is h effective permission multiplied by the receive signal. So, if you multiplied this with the receive signal what we would get is root over s by 2 h effective times h e f times s plus h e f f that is h effective permission times noise. So, we are multiplying this particular co efficient, if you if you expand this coefficient what we would get is let us first write down this h1, h2, h2 conjugate h1 conjugate with the minus sign the permission of this would be h1 h2 this row becomes this column this row becomes the next column there is a conjugate, conjugate transpose. So, h1 conjugate h2 conjugate h2 will become h2 and minus h1 right times s and so on. So, will focus on this particular product what we see is that we are getting h1 mod squared plus h2 mod squared the second term would be h1 conjugate times h2 minus h2 times h1 conjugate.

So, they cancel out each other we get 0 there. So, when we take at this, this row times is column h2 conjugate h1 minus h1 h2 conjugate. So, again we can get a 0 and we have h2 mod squared plus h1 mod squared. So, what we get effectively is h1 mod squared plus h2 mod squared times 1 0 0 1, which is an identity matrix of size 2. So, this is basically h f squared times i, we can say that h effective that got created here this is a orthogonal matrix that could have be seen before also and that is what we would seen here as well. So, in this way if you proceed we could write this expression here as root over es up on 2 times h f squared times I times s plus h effective n bar.

So, this is the vector y that is received. So, if you look at this expression. So, this is s 1 s 2 1 0 0 1 h f squared times square root of s by 2. So, we have the signal s 1 coming out into y 1 and which is not interfered with that of s2 and of course, that is this processed noise. So, we could write this n tilde bar and now we would like to calculate the signal to noise ratio for this particular case. So, we need to also calculate e of n tilde, n tilde hermitian and this would turn out to be from this we can see it could turn out to be h f squared n naught times I 2 so; that means, here also we have this particular coefficient n naught is the variance and I two; that means, it is identity matrix. So, each of the noise coefficients are h f squared times n naught. So, if you have to calculate the signal to noise ratio for each of the branch we will be getting s by 2, this squared divided by this squared. So, it is basically h f squared times and this n naught will turn out in the denominator.

So, we have s by m naught of h f squared this is the effective SNR because of Alamouti scheme that that we get and which will be useful for us in calculating the error prohibit expression. So, if we move ahead further and we look at the probability of error expression.

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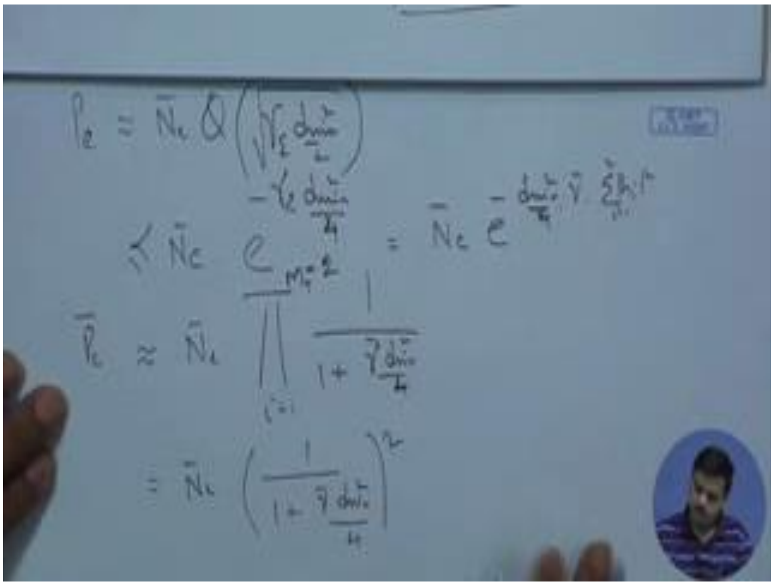

So, pe this sum is the proximately equal to n e bar times q (Refer Time: 17:20) square root of gamma combined d min square by 2 or we would also do it in terms of alpha m and beta n. So, this is less n r equal to n e bar e to be power of minus d min square by 2 times the combined and we would be interested in p e bar. So, p e bar would approximate is to be n e bar and this 1 is clear it is product of I equals 1 2 m t, m t is equal to 2 in this case one by 1 plus this particular expression is. So, we need to expand this n e bar e to the power of minus d min square by 2 and gamma bar times has sum of h I squared I goes 1 to 2. So, we have one to 2 gamma bar d min square by 2 right.

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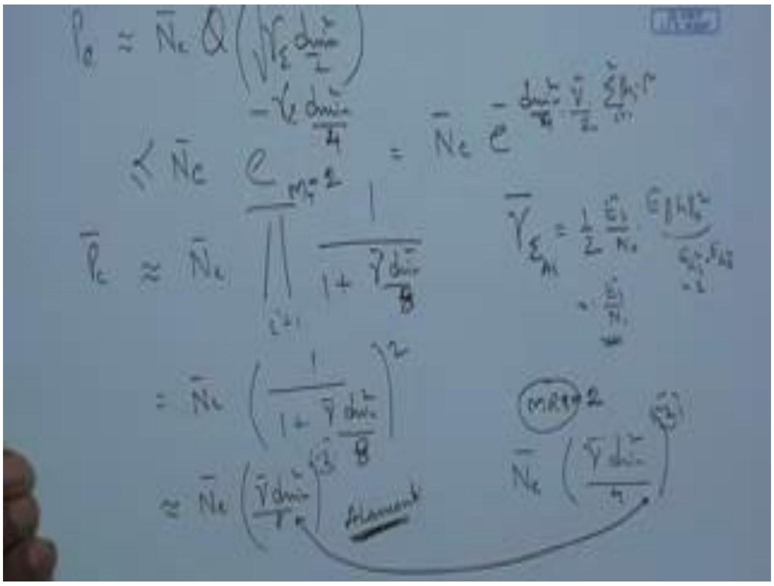

So, that is equal to n e bar you have a 4 over here 2 times you have a four over here one plus gamma bar d min squared by 4 raise to power of 2. And if you look at the average expression from this Alamouti average is equal to half s by m naught times this; that means, expected value of h f squared this is basically h1 squared e of h1 squared e of h2 squared mod of course, this is equal to 2. So, what we have is s by n naught, what we see in this case there is no average increase in SNR compare to the previous case where there was an m t at the at the at the transmitter.

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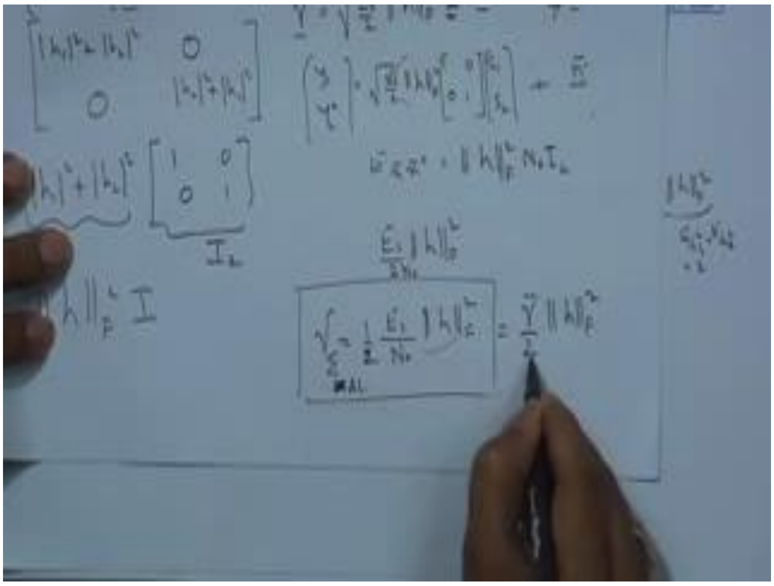

So, with this we have we have y bar half of this of s by n naught. So, we are missed out 2 over here. So, this would be gamma bar by 2 because this particular expression is gamma bar by 2 h f squared this is 2 (Refer Time: 20:02) this 2 comes over here this e s by m naught comes over here.

So, when we take this expression this is fine when we move for gamma n e bar is n e bar e to the power of d min square comes over here 4 comes over here this expands to gamma bar by 2 some I equals 1 to 2 mod h I squared. So, we have an 8 in the denominator 2 times 4 is 8. So, we have 8 in the denominator we have 8 in the denominator. So, this approximately even gamma bar is much-much greater than 1 you have n e bar gamma bar d min squared on 8 to the power of minus 2. So, if you had if you would have transmitted MRC or receive MRC you would have had n e bar gamma bar d min square by 4 raise to the power of minus 2. So, clearly you can see there is the factor of 2 that is different over here and that would be evident if you look at the error probability curves for these things.

So, what we clearly see here is that the average SNR that is the experienced this experience in this 2 cases this is for MRC where, it is transmit MRC or receive MRC and this is for Alamouti scheme where there is no channel information at the transmitter. So, there is a change in the average SNR. So, in MRC you can clearly see there is increase in average SNR compare to transmit MRC were as the order of diversity is the same in both of them because they have this 2 branch antennas.

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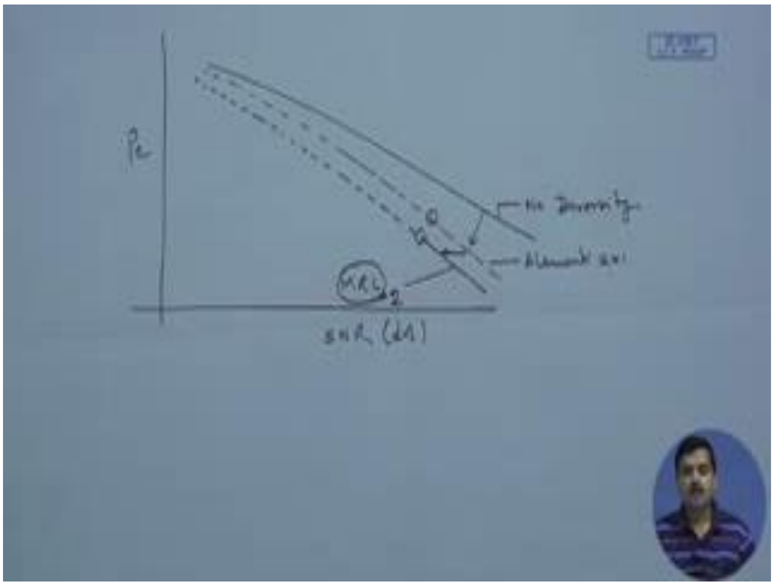

So, we have combining MRC when number of MRCs is equal to 2. Who have with this case, if we would look at the error probability expression for or the error probability curves in this case we will get some performance and this is the SNR in d d a and this is the probability of error expression.

So, when there is no diversity suppose this is the error probability curve if we have the Alamouti scheme we going to get curve which is going like this it is giving increase in slow because we can clearly see over here there is exponent whereas, where when we are getting the MRC what we are having is along with the increases in slope there is an average increase in SNR, is also there, this is the case for MRC this is the case for Alamouti scheme 2 plus 1 and this is no diversity. So, this is the kind of n that we would between Alamouti transmit MRC and receive m r c, sorry between Alamouti scheme and

MRC m r c be transmit MRC or receive MRC when number of branches is equal to 2. Now since the slop is the same between this and this the order of diversity is the same, but here there is an additional gain which is an array gain that is present because of the coherent combining that is available in case of MRC scheme.

So, what we have seen till now are in special diversity modes you could have channels information at the transmitter as well as without channels information at the transmitter you could also gain diversity, but with special scheme that we are consider Alamouti scheme is to be very, very simple scheme. You can get the same order diversity gain, but the array gain is not available which is there in case channels at the information being available at the transmitter or in case of receive MRC just I would like to add at this point when we are considering the transmit MRC case that was we are considering this expression this is particular scenario or we can look at this particular scenario this is very, very useful just if you like to imagine the situation where there is a base station and it is enable with multiple antennas where is user devices enabled with same will antenna and would like to do that because would like to have a mod processing capability at the transmitter to more cost of the transmitter at the at the user end.

We would like to have lower cost. So, we could use 2 antennas and we could use 2 cross 1 transmit MRC or 2 cross 1 Alamouti scheme by which we could gain order of diversity and because we are having this order of diversity gain definitely when compare to see saw scheme one would be able to get better e r performance of over a single input single output transmission mechanism.

So, with this what we have is even without knowing the channel at the transmitter you can improve error probability at the receiver by using special coding scheme where we sent 2 symbols over 2 symbol durations across 2 antennas. So, again I would like to take a look at what we are doing. So, what we are doing is here if you see over all there is s 1 and s 2 in these 2 symbol durations not more symbol. So, basically we have 2 symbols in 2 time durations so; that means we have one symbol per time interval. So, basically we are not losing out in terms of symbol rate at every time we are using the channel we are sending one symbol as in the case of see saw and we are getting the full order of diversity means in this case there are 2 diversity branches we are able to get 2 order of diversity because, in the error expression we can clearly see that there is this 2 in the exponent.

So, what we have achieved is the full rate; that means, a maximum rate that is possible one symbol per symbol duration as well as we have achieved 2 order of diversity that was the maximum order of diversity. So, with this particular special scheme you can achieve full order of diversity as well as full rate an no other scheme can give you, this kind of diversity order and this kind of rates there are there are many other coding scheme when the number of transmit antennas is more than 2 it may not be possible to see it in this particular course.

But they are even now ready through what we have discuss to understand those typical coding schemes by which you can achieve diversity at the transmitter by using more than 2 antennas and you could achieve little bit of array gain if it is possible or in most cases array gain is not achievable, if you had more antennas at the receiver along with this you can potentially get array gain along with the diversity gain using transmit and receive antennas both.

Thank you.