Fundamentals of MIMO Wireless Communication
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Lecture - 27
Diversity Gain and Transmit MRA

Welcome to the lectures on Fundamentals of MIMO Wireless Communications. We were in the previous lecture looking at the difference ways of calculating error probability and we have also taken the general criteria of diversity where we said there are n number of parallel transmissions, this parallel transmissions could be in time domain or could be in frequency domain or could be in space domain and we have derived at the general explanation of error probability where we had shown that the error expression a could be in general given in this form.

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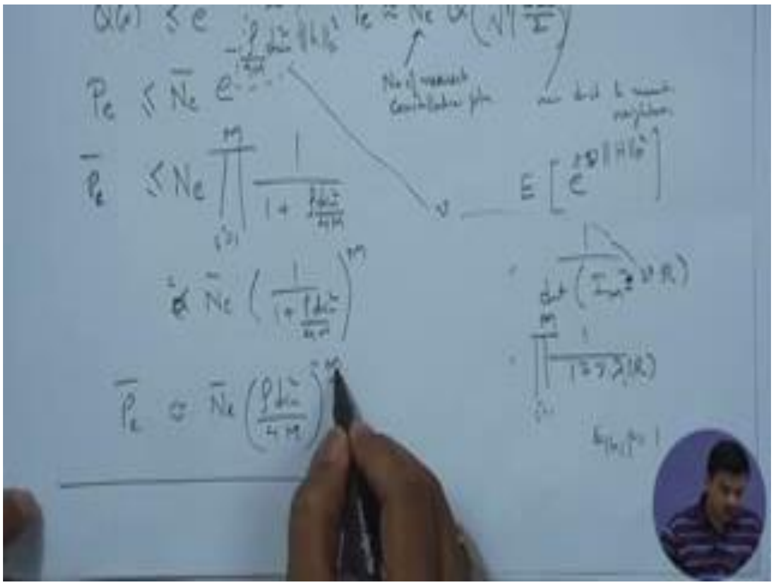

As it is given over here, where the m in the denominator came because there was m parallel branches and the transmit power was divided among the m parallel branches and we also said this a exponent raise to the power of m because typically in the error probability inversely propositional to SNR, but here it is in feast to the order of m which is bringing in the diversity gain.

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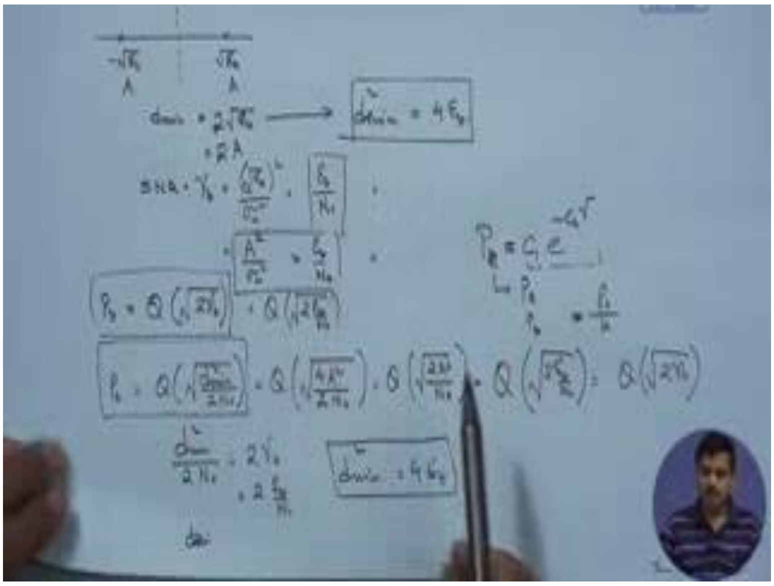

In this expression we have discussed the effect, we have discus the expressions which has d mean squared and in the beginning we had equated this to gamma b here, we said the d mean squared is equal to 4 e b.

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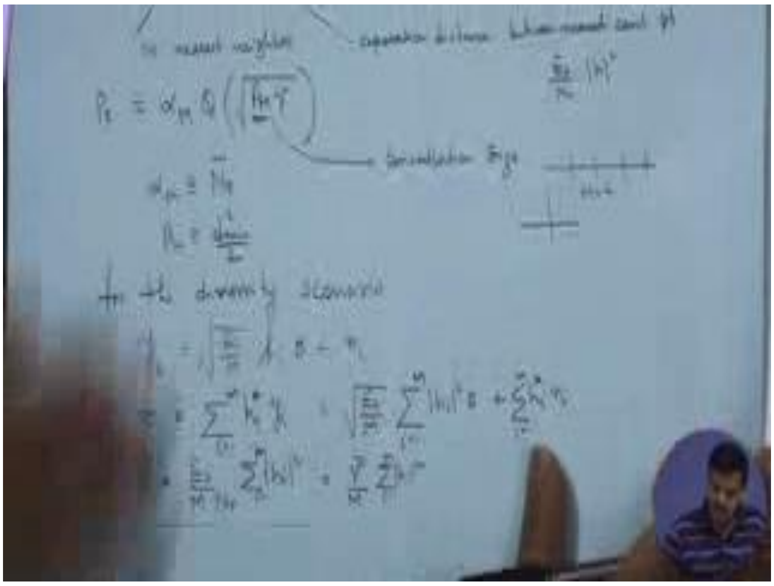

So, in this way we could do the translation at this point. It is also important to note that in some cases we might have another way of writing the expression that means, in some places in some references you might find that probability of error is given as approximation of P e N e bar that I will explain all the things q function of square route of d mean squared by 2 and rather more accurately times eta, where eta or we could also write this is gamma where gamma is the SNR.

If you look at this expression this is the number of nearest neighbors and d mean is the separation distance between nearest consolation point and gamma is the SNR, similar to noise ratio in case of AWGN. This SNR would be e s by n naught in some cases e s is taken to be 1, in that case you have d mean squared by 2 in case of feeding channel. So, this is AWGN in case of feeding channel it should be e s by n naught times h f squared which is norm of x square, it is the channel square a in case of this is simply e s. It is simply e s by n naught times h square.

This is what it is and in some cases in some references you will find P e error probability is given as alpha m q function of square route of beta m times gamma, this gamma is same as SNR and beta m is a co efficient, alpha m is co efficient, here we have been using this expressions in earlier cases, but this m this particular m is the constellation order constellation size that means, if it is pulse amplitude modulation and there are 4 levels. So, m is equal to 4 in this case if it is q m. So, for this case m is equal to 4 for this case m is equal to 16.

So, like this m is dependent upon the constellation size n naught this special m and we might be confused with these expressions. So, there is equivalence there is of course, equivalence in these expressions and clearly alpha m is same as N e bar, so that is how these coefficients are usually derived and this beta m naturally is same as d mean squared by 2. So, if you use these translations then these expressions turn out to be the same and the error probability expressions work out as the same.

So, there is no more problems with them you might find these in different places and for details on these we could refer to many different books 1 of them would be a wireless communication by and the expressions that will be following mainly could be with this. So, 1 and half will be giving the translation with respect alpha m and beta m also show the equivalence. So, of now I will carry on and show you how they are the same exactly when we take just pure diversity case.

For the diversity scenario we said that the received signal in the ith branch or the ith institute of time or ith frequency or ith received signal is root over e s by m assuming there are m transmissions and it is done in such a way that the totally energy elements are same plus this the channel co efficient for the ith copy of the signal s is the signal n i, we have described n i in the previous discussions. So, and we wanted to combine the signal in such a way that the signal to noise ratio at the receiver is maximize. So, we will use maximize ratio combining where we says z is equal to some over i equals to m mind it this m that will be using here is different than this alpha m beta m. So, this alpha m and beta m would be modulation for this m is different. So, wherever affix of alpha and beta is there it is of the constellation and not to the special dimension.

So, carrying forward we would like to combine it as h i squared y i. So, this would lead to a route over e s over m times of course, there is a summation i equals 1 to m i again this is the special dimension mod h i squared s plus h i conjugate sum i equals to 1 m n i, this is the received component and when we calculate the signal to noise ratio. So, the gamma in this particular case would be the square of this term. So, we going to get e s over m and when I take this square at the power of this term will be getting e of n i n i conjugate and we have already described previously how n i would be this will be 0 mean circular symmetric complex Gaussian.

So, and they are independent of each other that we have already mentioned in in our previous discussions. So, we are going to get m naught in the denominator because of this and will be get in the numerator sum over mod h i square i equals to 1 to m this will happen because square of this divided by square of this will be equal to be left with this particular term and this you could write it as if I take e s by m naught as gamma bar gamma bar is e s by m naught divided by m because there are m transmissions each having 1 by m of the power times mod h i squared i equals 1 to m. So, this is the same as we had before and then as we deviate or as we start comparing that 2 expressions.

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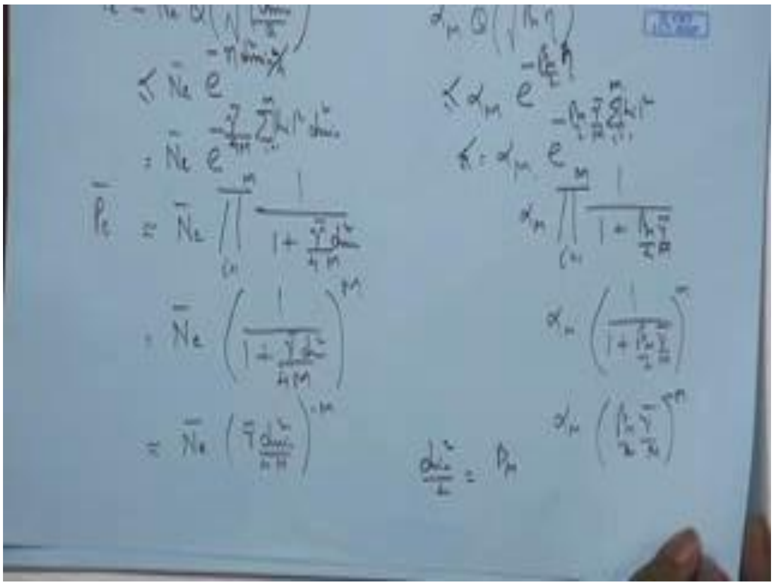

So, clearly P e is by 1 N e bar q of square route of eta d mean squared by 2. So, this is what we discussed in previous lecture and the other 1 is alpha m q of square route of beta m times eta or gamma. We would use eta for this particular expressions its gamma over here. So, that is using the Chernoff bound this is less than or equal to N e bar e to the power of minus will eta d mean squared by 4 2 and another 2 because of Chernoff bound i thing here is less than or equal to alpha m e power of minus beta m eta by 2 and this is equal to N e bar e to the power of minus if you see this expression is gamma bar by 4 m gamma bar I want to take on this times mod h i squared i is equal to m and this 1 is equal to alpha m e to the power of minus beta m that is also the same basically gamma bar sum over mode h i square i equals to m.

The next step would be to calculate P e bar that means, expected value of probability of error. So, using the technique which we done before this would turn out to be N e bar times product of 1 by i equals 1 to m 1 plus gamma bar. So, I get this a d mean squared a d mean squared we had missed there for m d mean squared and here also what you get is alpha m pi i equals to 1 to m this is the special dimension 1 by 1 plus beta m by 2 gamma bar of course, gamma bar beta m by m right.

So, there is straight forward this expressions for it could be written as 1 by 1 plus gamma bar by 4 m d mean squared raise to the power of m and here alpha m 1 plus beta m by 2 gamma bar by m raise to the power of m and for gamma bar greater than greater than 1 we had N e bar gamma bar d mean square by 4 m raise to the power of minus m and here also alpha m beta m by 2 gamma bar by m to the power of minus m. So, again everything is the same.

There equivalence here also it is d mean square by 2 is equal to beta m. So, with this equivalence everything was true. So, we will be using either of these expressions where as and when applicable. So, there is no reason for confusion amongst this expressions they are all equivalent in their own places. So, with this we would like to carry 1 with our discussion on diversity. So, as we move forward with our expression of diversity I would like to explain something about of the diversity gain. So, the order of the diversity gain is typically represented by this exponent of error probability this is very, very important.

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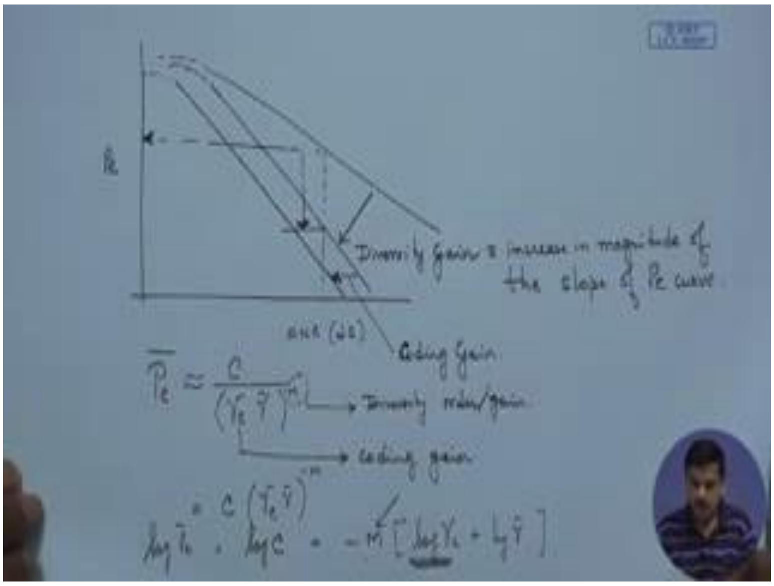

So, if we would like to compare let us say this is the probability of error axis and this is the SNR axis signal to noise ratio in d b, lets say this is the SNR axis in d b and this is the error probability.

So, if without any diversity let us say this is the error probability curve with diversity what happens as we clearly see in this expressions here that the error probability is now raise to the power of m so that means, clearly there is the slope of this is order of m. So, when there is diversity gain this particular curve becomes like this. So, there is an increase in slope in the error probability expression this is the diversity gain. So, what it leads to is if I take a particular SNR value at that SNR the error probability was some point here now because of diversity the error probability has come down to this point.

So, there is significant gain in diversity in error probability because of this diversity gain and this kind of gain because of which there is increase of slope of the error, probability curve is usually described as diversity gain is basically increase in magnitude of this slope of this can be described in terms of increase in magnitude of the slope of the probability of error curve along the this particular point. It is also important that we described another kind of gain which is associated with these is known as the coding gain in coding gain what we would experience is that the error probability curve shifts laterally to the left direction is the laterally shift in the error probability curve and this is usually noted as the coding gain a sometimes array gains also reflected in this kind of in these kinds of performance.

So, this coding gain is usually comes from smart ways of encoding transmitted coding signals which could be error correction codes or any other kinds of codes. So, there is the left shift in the error correction in the probability of error curve. So, the 2 distinct kinds of gains can be observed in the error probability expressions what is SNR and we can identify whether there is a diversity gain or whether there is a coding gain. So, if we would look at the probability of error expressions, the average probability of expression in such an error probability of error expression we could say that the expression looks like as if there is some constant and there is gamma c a coefficient times. Let us say gamma bar which is the average error expressions average SNR raise to the power of m now this constant we cannot do anything, but this particular exponent is basically giving us the diversity order or the diversity gain that is available.

This particular coefficient is indicating of the coding gain. So, this how these 2 gains can we understood in if when we looking at the probability of error expressions and if will if we take a look at this particular curve a we would be evident that this is also equals to c gamma c gamma bar raise to the power of minus m. So, if I take the log of probability of error because this is in the log axis right. So, we could take it in the log domain this would be some logarithm of c it is a constant term plus minus m times log of gamma c plus log of gamma bar. So, clearly what we can see is that there is this left shift because of this and this is causing the additional increase in probability of error expression.

So, clearly you can also understand it in these terms. So, essential diversity gain as we have seen as SNR increases as SNR increases the gain due to diversity is significantly higher where as the array gain is almost the same in all range of SNR. So, the curve essentially from here just simply shifts to the left. So, it translates whereas, here the curve changes the slope and as SNR increases the gain that is achieved is much more than compare to in the condition where the SNR is less and that is also evident here because the SNR is rise to the power of m.

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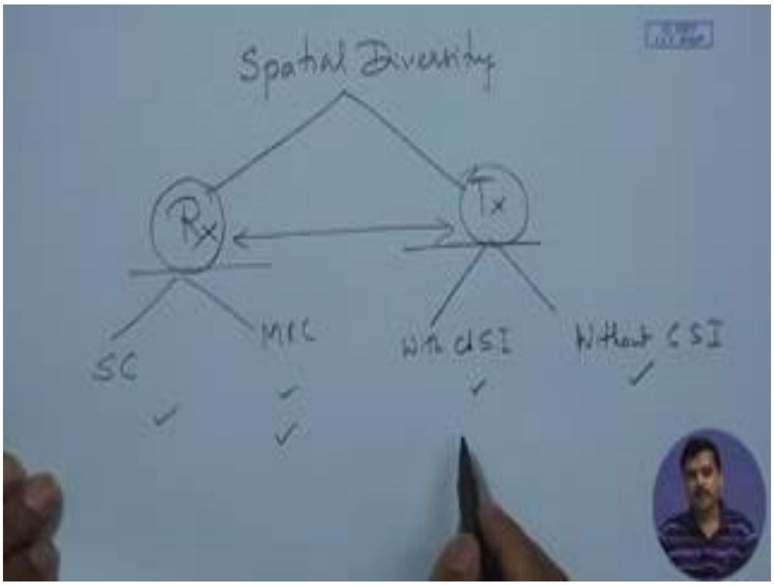

So, higher SNR this higher order and gains are even more this is something important which we would like to remember when we understand gain or coding gain. So, after this in the we can see that when we look at special diversity when we look at special diversity we said you could obtain it at the receiver or you could obtain it at the transmitter or you could obtain it on the both side on the receiver side, we have seen there is selection combining, there is maximal ratio combining.

Now, we have seen the error the PDF of SNR in both the cases we have seen how to calculate error probability in case of MRC of oscillation combining we have given in the expressions to get the exact numbers 1 has to numeric technique. So, now, it is time that since we have looked at the how diversity can be a taken advantage of at the receiver we would like to see how diversity can be taken advantage at the transmitter. So, when we go to the transmitter again there are 2 techniques 1 is with channel state information at the at the transmitter and another is without channel state information at the transmitter will take look at with channel state information as well as without channel state information and finally, you could obtain diversity gains by using both transmitter and receiver at the same time we will also have a look that slowly in the few lectures to come.

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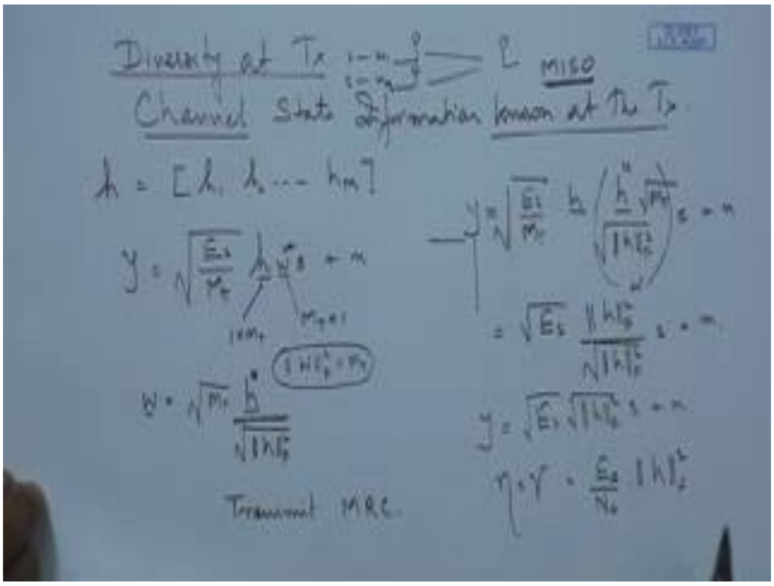

So, when we look at the transmitter diversity. If I am look at diversity at the transmitter and that too we will be focusing at channel state information to known at the t x we will use t x at the transmitter in short we can say the channel known at the transmitter and we will begin with the case. We can always consider the case where there are multiple antennas at the transmitter and there is single antenna at the receiver. So, this is the case which we will consider, this basically the MISO; multiple single output scenario. So, we will remain or with our assumption of the channel. In this case, will assume that the channel vector is given by h 1, h 2, h m t this is the convention that we have used in case of SIMO that is single input multiple output this was column instead of row.

Now, basically we have this particular configuration and will assume that there is some weights associated with each antenna are the weight associated with each antenna same signal is being transmitted. So, the received signal which is combined we have written the expression earlier is given as route over e s by m t. There are m t number of transmit antennas h times w times s as is a single these a vector these a vector and clearly this will be a column vector this is the row vector and this w is basically the weight of the matrix weight of the channel coefficient weight of the antennas that are that are to be used.

So, this is m t cross 1 and this is 1 cross m t and s is of course, a scalar and we have to choose that the Frobenius norms of w is equal to m t. We have already divided this by m t for each branch and will be this together it will ensure that there is e s powers. So, there is no change of total transmit power in this particular case. So, and we will further choose the w vector to be route over m t h hermitian divided by route of Frobenius norms of h. So, what we are essentially doing if you would look at it carefully h hermitian, so that means, h is the row vector h hermitian is the conjugate and it is the column vector.

So, the row vector is h and this is the w getting multiplied with each other, we can predict that almost h 1 1 square plus h 2 2 square plus dot, dot, dot of m t square is going to appear over here and that will be Frobenius norms square and there will be this m t and this m t will be cancel out each other. So, we are going to get something like the receive MRC. So, what we have over here is in other words you can say you have transmit MRC it transmit maximal ratio combining this is done in order to maximize the received SNR.

So, with this thing when we write the expression of the received signal that is y what we get is y is equal to route over e s 1 m t of course, h h hermitian square route of h h square that is w this is w this is the w that we have here plus there is of course, this noise sample that is present. Now, in receive MRC we had channel coefficient getting multiplied here as well, but here it is not the story, but overall performance would turn out to be the same as we would see. So, we are getting the square route of, sorry there is the route this factor we have missed out route over m t.

So, m t m t goes out you are going to have e s this is basically Frobenius norms of h squared time divided by h f square route of times s plus n. So, essentially what we have is route over e s square route of h squared s plus n this is the received signal that clearly from this we can write from the expression of SNR eta or gamma whatever we write it as e s by n naught this square divide by power of this times h squared. So, if you would see this expression it is again the same expression as that of MRC expressions. So, naturally we are going to expect the error probability similar to that of MRC expressions.

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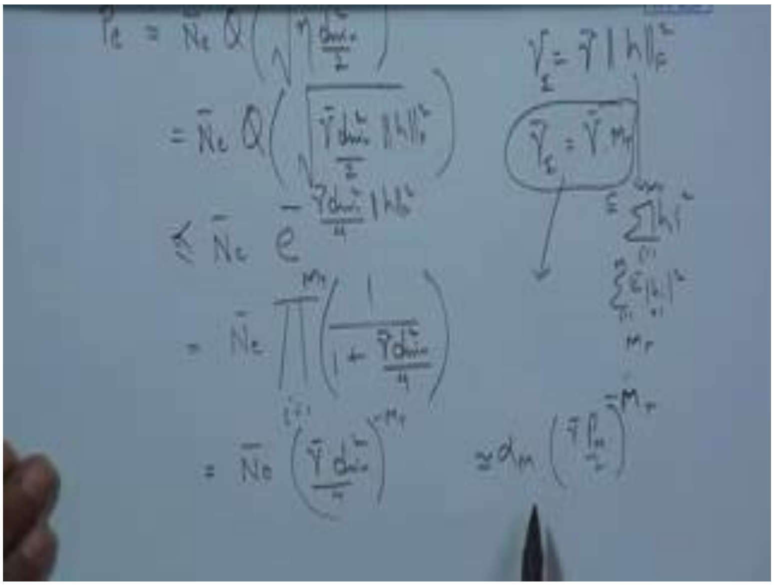

So, when we write the probability of error expression probability of error expression was approximately given as N e bar q function of square route of eta d mean square by two. So, that is N e bar q function of square route of for this we could write as gamma bar h e squared. So, basically we going to have gamma bar d mean square by 2 times h s square rest of it could follow N e bar this is less than or equal to e to the power of minus gamma bar d mean squared by 4 times h f square this 4 because this whole squared divided by 2 the argument of this squared 2 the 2 times is 4.

So, we have this now this would naturally lead to is equal to N e bar pi i equals 1 to here it is m t, m t 1 by 1 plus gamma bar d mean squared by 4. So, what is different than the than the pure diversity case there is m over here because they were like m separate branches, but in this case the m is gone because we are doing this kind of combining its because of the particularly because of w that we have chosen in this case. So, that helps a lot and this would lead to approximately N e bar times when gamma bar is much bigger than 1 gamma bar d mean squared by 4 raise to the power of minus m t.

So, when m t equals to m this is the same as the other 1 when d mean squared by 2 equal to beta m and this is equal to alpha. So, you have alpha m gamma bar beta m by 2 minus of m m or m t. So, this is again the same expression as that of MRC now here what we should note is when we are calculating this expressions eta. So, eta or gamma is given as gamma bar times h f square. So, if I to take to the this I would write as gamma sigma the expected value of gamma bar gamma sigma bar is gamma bar times the expected value of this the expected value of this is a h 1 one square plus h h 1 squared plus h 2 squared up to h m t squared and e of h 1 squared is equal to e of h m h h 2 square and so on.

So, and e of h 1 squared. So, basically if you at look at this is the sum over h i squared i equals 1 to m t and there is an expectation. So, with independent assumption you have e of h i square and a summation and each of these is equal to 1 and the summation is equal to 1 to m t. So, together we are going to have m t. So, basically this times m t. So, what you get is that there is an average increase in SNR as well as there is this diversity gain of order m t even in this scheme.

What we can say in this form of transmit diversity when channel is known at the transmitter we have the transmit MRC and what it boils down to is the performance is same as that of the receive MRC. So, if you are doing transmit MRC with full knowledge of channel at the transmitter you would get the same error probability expression as that of receive MRC and the performance is identical to that of this is the receive MRC we stop this lecture in this point

Thank you.