**Fundamentals of MIMO Wireless Communication**
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**Lecture – 26**
**Problems of Error in MRC**

Welcome to the lecture on Fundamentals of MIMO Wireless Communication. We have started to discuss diversity performance .We see selection combining; we have also seen maximal ratio combining. We have seen 1 way of calculating error probability. I love to like to show you 2 different ways for arriving at the same error probability of expression. So, that would be 1 of the most important goals of this particular lecture.

So, before we start on with the error probability expression for maximal ratio combining, i would like to draw equivalence between the 2 kind of description for typical error probability expression. So, those things are less confusing in all future things that we do. So, to begin with,

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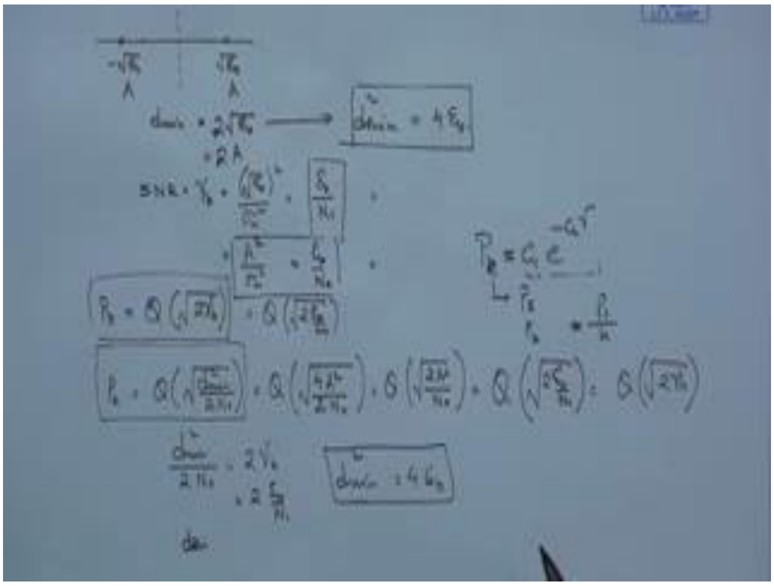

Let us consider b p s k system where we have 1 symbol at minus root e b and another symbol at root e b and this could also be written as A. So, for this situation we can always write that the d min that is the minimum distance between the conciliation points for this simple case is equal to 2 root over p b which is also 2 times A, and for this case if you are to write the expression of S N R signal to noise ratio for e p s k it is gamma b that is S N R for bit this is equal to square root of P b that is this value squared divided by sigma squared n which is the noise variance and this is basically E b by n naught this is expression which we are more use to.

Now, this is a equal to A square because root P b is A by sigma squared n which is equal E b by N naught now given these things that we have definitely we could write E b by N naught is equal S n by sigma squared N or(Refer Time: 02:31) c b.

Generally, we have error probability expression for a w g n as Q function of square root of 2 gamma b which is Q function square root of 2 E b by n naught. In some cases you might find error probability expression is also given as Q root over d min square by 2 N naught. This expression is also used in many places. So, if we expand this expression, we going to get root over d min squared is basically d minimum is 2 A then is four A squared divided by 2 times N naught which is Q of square root of 2 A squared by N naught and this probability expression would be the same. So; that means, the argument of this must be the same and also we find it clearly if the same you take out 2 and 2 E b by N naught should be equal to A squared by N naught and we would have here also E b by N naught as A squared by N naught. So, if you look at these 2 expressions this is equal to E b by N naught gamma B is also equal to E b by N naught.

So, this you could also write it as Q of square root of 2 E b N naught which is equal to Q of square root of 2 gamma b in other words these expression is the same as this expression. So, we need not be confused and just to draw the equivalence for quick reference later on d min squared by 2 N naught is equal to 2 gamma b. So, this means this is equal to 2 E b by N naught or if you cancel out N naught from both sides you have d min squared is equal to four E b. For this is again we will established from here if you see these d min squared if I take form here, I would get d min squared is equal to four E b the same expression whole. So, basically under the circumstance everything holds true. So, we are just using this show that, either this expression or this expression all of them are correct and they will be use to as and when applicable.

Generally, speaking we would use probability of error expression as is the approximate expression C 1 e to the power minus C 2 gamma, gamma equation. So, i would say probability of error because this is sometimes will be less probability of symbol error and sometimes probability of bit error depending upon what we are interested in. In all cases these kind of expression is typically used. This is an approximate expression coming from chernoff norm. So, and this is very very useful whenever there is an exp1ntial functions. Usually, the relationship between P s and P b for break coding is P s divided by the number of bit is equal to P b this is again an approximate expression.

So, these are some of the things, we should keep in mind when we will be using the error probability calculations. Now moving forward with this, is the error probability for m i c we have already d1 1 and again, we will use the chernoff bound.

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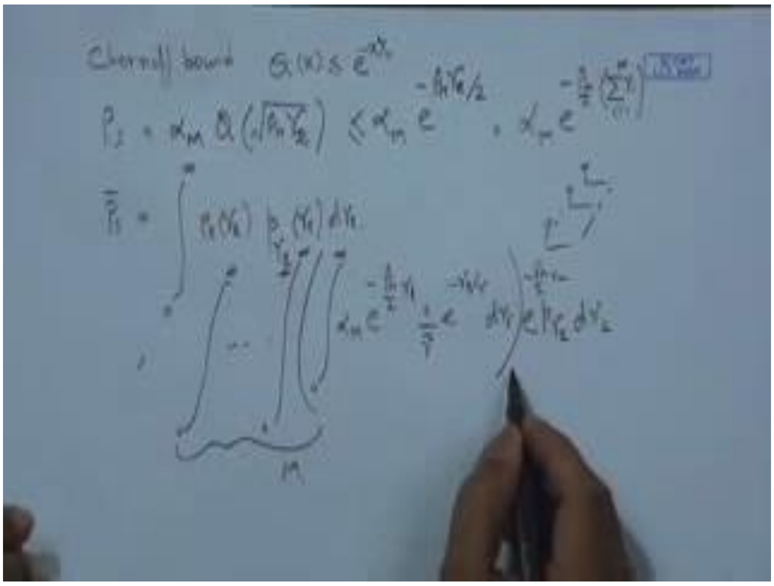

So, the chernoff bound tells fact that Q of x is less than or equal to e to the power minus x square by 2 this basically lead to the earlier expression, that we had that error probability expression looks in this form. So we will take a look at that. So, probability of error symbol error is given as alpha m Q of this is what was explaining root over beta n and gamma i would put summation indicating that of m r c. So, this is less than or equal to alpha n e to the power of minus beta n gamma combination by 2.

So, if we look at this expression that we had written here. This is similar in from over here P 1 is equal to alpha n and c 2 is equal to beta n by 2. So, this is a typical form that is very very useful and in our case this is equal to n e to the power of minus beta m by 2 and this gamma sigma that we have written is basically i equals 1 to m gamma i where, gamma i is the s n r experienced by each branch and as usual we are going to we are going to calculate the P s that means, average probability or error for m r c to do that it is 0 to infinity the probability of error symbol errors as a function of gamma sigma, right this is the function of gamma sigma, you can really see that times p that is the probability of density function of gamma sigma times gamma sigma d gamma sigma.

Now, for simplicity that we would take off this gamma sigma and if you look at this this particular expression is p d f probability of density function of the joint full density function of the gamma that is summation of gamma i what we ideally need is the joint distribution of all these gammas. Now if we take independent branch; that means, all these branches are independent. That is as we have said earlier that we will make that assumption for getting inside into the result then the joint distribution is basically product of distribution and that is what we are going to exploit in both the cases.

So, what we are going to have if I look at this expression it is alpha m e to the power of minus beta m by 2 and the summation that I have I could write it or I could say gamma i and their is a product of it or so, I have taken one of them integrate 0 to infinity now i take this the particular 1 corresponding to gamma i 1 by gamma bar the average s n r in that particular link which is again the same for that link gamma i by gamma and d gamma i. So this is for one of them and p gamma p gamma sigma is basically product of all the gamma, we will have again p gamma if this is gamma 1 gamma 1 is gamma 1 p gamma 2 d gamma 2 again integrate 0 to infinity and. So, on all of them and d will be m in number and for each 1 see here we have this e to the power of minus beta m by 2 gamma 1.

So, here we are going to get or the second 1 we are going to get p to the power minus beta m by 2 and. So, on and. So for every of the integral we want to get that. So, to get to the error probability we need to solve one of the even integrals one of those integrals if you solve them we are going to get alpha m.

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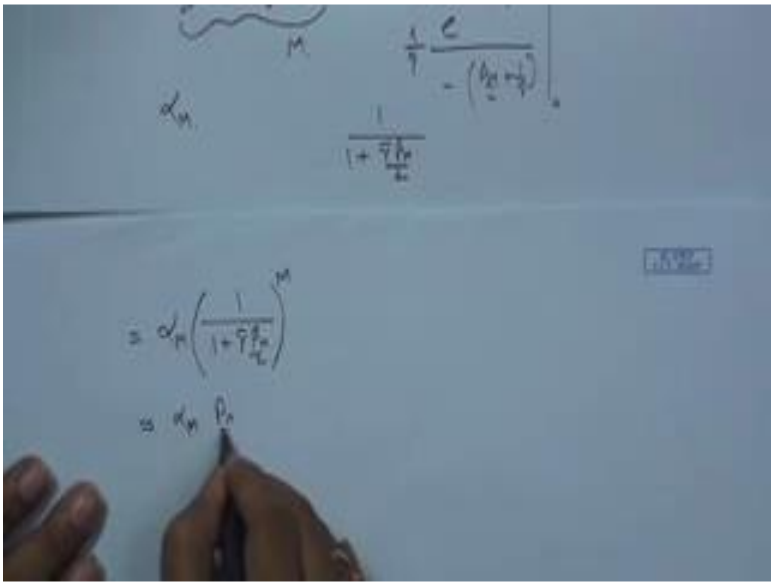

Of course it comes out it is 1 by gamma bar e to the power of minus beta m by 2. So, 1 by gamma bar divided by minus of beta m by 2 plus 1 by gamma bar limits 0 to infinity. So, if you work this out we are going to get this as 1 upon 1 plus gamma bar beta m by 2. If i take out gamma bar this gamma bar in the gamma bar take out over here (Refer Time: 10:45) each other. So, this is a particular expression that we are left with and then what we would be having is 1 by 1 plus gamma bar theta m by 2 which is the result of 1 of the integral and since all of them are identically distributed all this e gammas are identically distributed will be getting again those factors multiplied with each other, will be getting those factors multiplied with each other.

So, effectively what will be getting is raise to the power of m because there are m such integrals alpha m is outside and if you if you make the approximation that gamma bar is much greater than 1 that means, average s n r is pretty high. This could lead to alpha m and beta m by 2 gamma bar raise to the power of minus n. So, this is again the same expression that we have got this expression as well as these expressions are the same expression. So, all we kind to say is by the earlier method or by this method you are able to get to the same result. What we will now do is we show you another way of doing the same calculations. So, that you are well convergent with these techniques which we can apply them other situations which is not been covered in this particular case.

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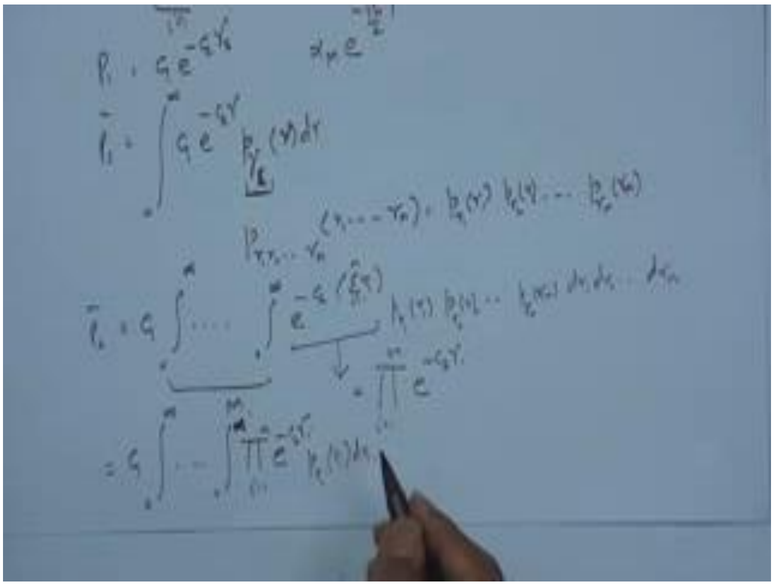

So, again let us take a look at gamma sigma, gamma sigma is basically i equals to 1 to m gamma i that is summation over gamma i and the P s expression of course, we take the same P s expression c 1 e to the power of minus e to gamma s. So, or gamma sigma whatever it is and yeah. So, so what we have taken is basically alpha m e to the power of minus beta n by 2 gamma this is the expression you have taken and again P s bar is equal to 0 to infinity you are going to apply the same thing e to the power of minus gamma p gamma sigma gamma gamma. This will be distribution if for the combined s n r and if we assume independent; that means, if we assume that the joint distribution of all these s n r's are they are independent.

So, we are going to get gamma 1 of gamma 1 multiplied by p gamma 2 of gamma 2 and. So, on up to p gamma n of gamma n this is what we exploited in the earlier method. So, again carrying on to the same thing will get c 1 integral 0 to infinity m of ten M n number of them e to the power of minus e 2 sum over gamma i i equal to 1 2 m p gamma 1 gamma 1 times p gamma 2 gamma 2 up to p gamma n gamma n d gamma 1 d gamma 2 up to d gamma n this is what we have done still now in the in the previous case.

So, at this point again the same thing that we will use this particular expression you could write it as equal to product of pi equals to i to 1 to m, e to the power of minus c 2 gamma i. So, these 2 expressions are equivalent. So, hence we could use this expression that is what we have to currently d1, but here we will use a slight modification compare to the last expression the last method that we did. So, we have a and write down the integral again m number of integrals here product of r equals to 1 to n e to the power of minus c to gamma i, if this is the product p gamma i gamma i d gamma. So, so this is the more concise way of writing the expression and if we now focus on 1 of the integral let us see how does it look like this product sign i can bring it outs side and we have d1 before c 1 is of course, outside i equals 1 to m integral 0 to infinity this is precisely what we have d1 in the previous. So, still now the steps are remaining the same there is no modification.

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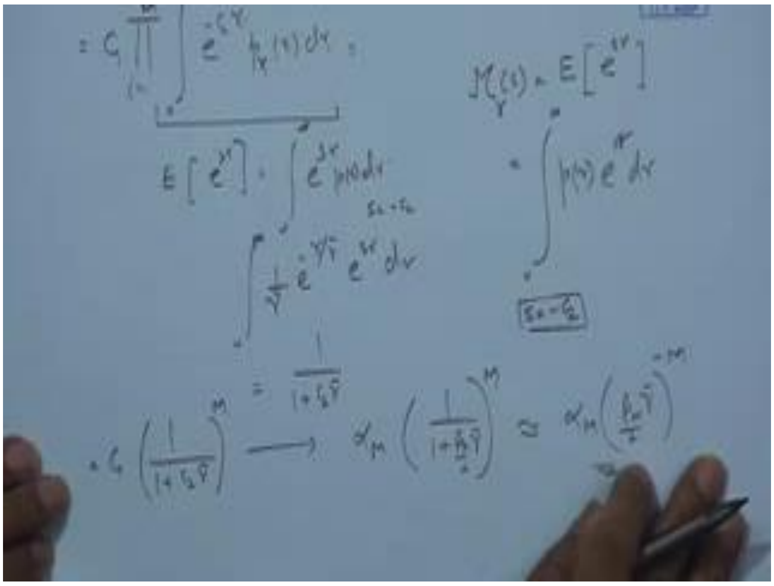

This particular expression, now if we if you observe this is where we make the slight variation, If the result is going to be the same, but still the way we could do it is different. If we look at the this expression it remind us of moment generating function where we say the moment generating function s of gamma let say is equal to expected value of e to the power of s gamma; that means, the expectation of e to the power of s gamma this is the function of s and we are taking the random variable gamma. So, this is 0 to infinity that is the range of gamma. So, expectation means p of gamma e to the power of s gamma d gamma this is the expression. If we look at this expression now it precisely the moment generating function where s is equal to minus c 2 of if we have this result already derived; that means, if we have the moment generating function and we can see that we are using a moment generating function here then we could exploit that expression and we could get it. So, for our case we have e to the power of s gamma is basically 0 to infinity e to the power of s gamma e gamma d gamma and s is equal to minus c 2. So, this is what we have already said if you use this p gamma as 1 by gamma bar e to the power of minus gamma by gamma bar e to the power of s gamma d gamma integral 0 to infinity then we have the same thing the result is 1 by 1 plus c 2 gamma bar. This is the expression that we have. So, this expression now we can substitute over here.

So, what we get is c 1 m times 1 by 1 plus c 2 gamma bar. So, m times the same thing is this again we if remember what is c 1 we have alpha m 1 by 1 plus beta m by 2 is c 2 is beta m 2 that's what we have identified here, which is c 2 is basically beta m by 2. So, that is what will be using. So, we have it rise to the power of m again the same expression for large gamma bar approximation, this will become alpha n beta m gamma bar by 2 s to the power minus m. So, what we have essentially done is done it in different ways and still we arrive at the same expression of probability error. So, the main idea of doing this particular thing is that you are able to use this technique wherever which 1 you find more applicable and you can calculate the probability error for the appropriate scheme that is under consideration. With this we would quickly start or take a look at the general diversity that is that is what the general order of diversity or general way of doing diversity that is if we look at if we try to analyze a typical diversity system you will say that.

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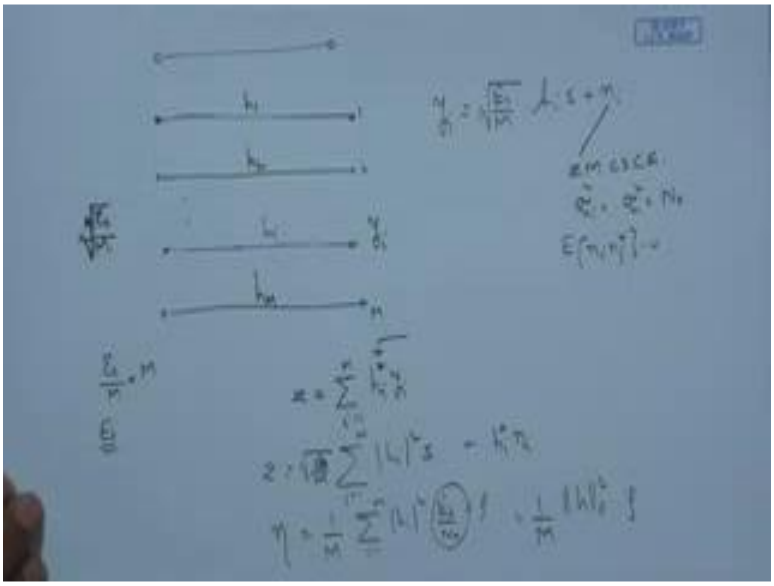

There are n number of link between transmitter and a receiver. N number of links between a transmitter and a receiver and the first link has the channel coefficient h 1 Secondly, (Refer Time: 18:35) channel coefficient h 2 like that the last link has channel coefficient h m and we have 1 2 up to m receiver basically y i is the i'th received signal and from the transmitter we are sending root over E s upon m as the signal energy for the signal power for transmit branch.

So, when we are doing transmit side diversity; that means, you can consider time diversity frequency diversity or even special diversity when we are doing diversity from the transmitter side general diversity that is what we consider we have this particular thing at hand. So that means, the power is done in such a way that the total power at the transmitter remains as, if you take square of this we are going get E s 1 n power any 1 of the branches multiplied by n number of branches, basically the total power e s that that indicating that the total power of this system is not the distance from the case where i have a single input and a single output case.

So, this is primary assumption which we will make. So, the receive signal Y i is equal to root over E s upon m times h i s plus n i. So, this is if it is straight forward s i is the channel coefficient s is the signal and n is the noise is 0 mean circular symmetric complex (Refer Time: 20:13) this is what we have already discussed before and sigma square n i is equal to sigma square n typically what we use for noise and it is n naught and of course, we Expectation of n i n j conjugate is equal 0.So; that means, they are uncorrelated there independent they are (Refer Time: 20:31) these typical assumptions as we are made earlier also holds true, and at the receiver we would like to maximize the s n r. So, we would say that i would like to have combiner at that they receive signal Z is formed by i equals 1 to n h i conjugate Y i, this means i have knowledge about the i'th channel h i I have knowledge about the i'th channel which is h i and i use it i I take the conjugate multiplied by y i and get it.

So, if you if you expand this expression you are going to get i equals 1 to m y i of course, we will have root over e x upon m mod h i squared s plus h i conjugate n i this is the expression you will get. So, we are adding up all of all of these terms if we write eta as the expression of s n r then we are going to get the square of this term divided by the square this term given h. So, will be getting 1 by M square of this and some over i equals 1 to m mod h i squared E s by n naught E of s by N naught and this is where we (Refer Time: 21:59) write it as rho.

So, you could write it as 1 upon M and if you would look at whatever is this, this basically the (Refer Time: 22:06) norm of h times rho this is the s n r expression that we have at the receiver for this particular scheme where you are dividing the power equally among all the transmit branches the total power image as this.

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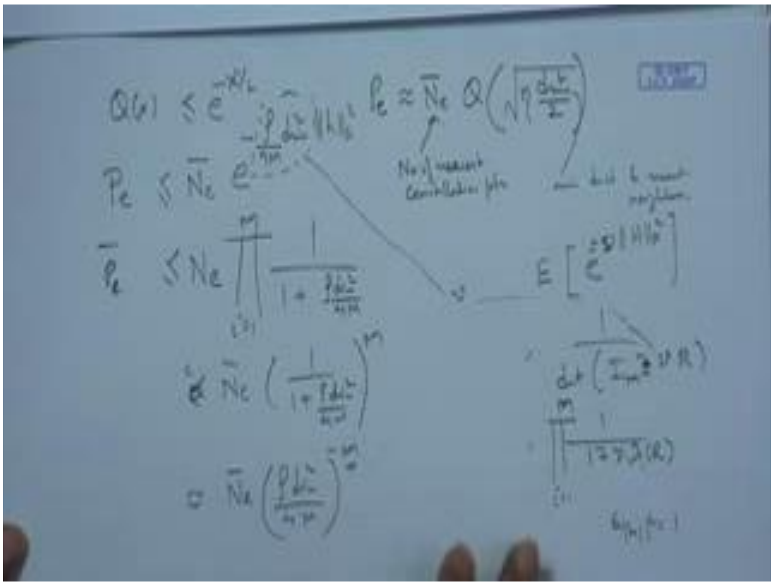

So, with this we can we will be using the chernoff bound mechanism to calculate the error probability. So, the (Refer Time: 22:30) of course, Q of x is less than equal to e to the power of minus square by 2 i am writing it often. So, that we remember this is the one that will be using. So, given this probability of error is less than or equal to N e bar this means this is another approximation for error probability, but if you look at this expression it is similar to what we have N e bar effectively means I would give probability of error approximately equal to N e bar times Q of root over d min squared by 2 times eta. So, N e bar is the nearest minimum distance of separation sorry this is the number of nearest conciliation point number of near conciliation point.

In case of this (Refer Time: 23:29) it is 1 in case of e p s k it is 2 and this is the minimum distance to nearest neighbors right.

So, we have given this kind of an expression before and eta is s n r to basically in case of a w g n channel it is E s by n r and in in our case we have channel coefficient also multiplied with that. So, given this we could write probability of error with this approximation is less than we will use we will expand the full of this rho by over m d min squared and h s square just look at it carefully. So, eta the expression that we have eta is rho by n by n we have rho by m rho by m h x squared that is h squared d min square and 2 multiplied by this 2 is 2 is four basically rho by four m d min squared h square. So, this is this is the error probability expression and now we can use our earlier results. So, basically what you want is P e bar error probability again will be using the moment generate function earlier we had said expectation of equity power of s mu h s square we had negative sign there is equal to 1 by determinant of i the dimension of this in this case it is m plus if i use a plus i use a minus over here if i use a minus i got a plus over here mu times r. So, that would be 1 plus r product of r equals to 1 to m in this particular case 1 plus plus or minus it may upon what we have mu times lambda of r lambda i of r. So, when it is independent when h is independent we are going to get these eigen value to on 1.

So, what will be left with the will be average s n r. So, when you left with the average s n r you could write the average probability of error to be the expectation over this. So, we will have the product i plus 1 to m because this is a size n the maximum size of size n 1 plus rho is basically rho d min squared by four m this is whole term is basically mapping to our v in this expression or s whatever you would like to use and that of h squared is the basically 1 because again remember we have mod of h s square equal to 1 and these are of eigen value. So, this is outside this when we multiplied with all of the eigen values. So, this is 1.

So, we have this expression now this we could further say is equal to or this is equal to N e bar times 1 by 1 rho d min square by four m raise to the power of m and for large rho when s n r is large this is approximately equal to N e bar rho d min squared by 4 m raise to the power of minus m. So, once again you are seeing that it is the diversity the error probability expression is raise to the power m and the (Refer Time: 27:21) decreases because of this diversity branches. So, we will stop this point in this particular lecture with this expression and in the next lecture we are going to see that what is the significance of this expression and what we can understand more of diversity from a typical expression of this and what else this expression help us in future to get expression of error probability there understand things in a better way.

Thank you.