Fundamentals of MIMO Wireless Communication
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Lecture - 24
Selection Combining

Welcome to the lectures on fundamentals of MIMO Wireless Communications. We have seen in the previous lecture how do we get a spatial diversity in summary, what we have seen is that by adding an extra antenna at the receiver we can get another copy of the same signal which is transmitted and using this particular signal. We can use combining techniques by which we could at least improve the probability that the signal falls below certain threshold there are many, many techniques by which you could do it. So, we begin with one of the very simple techniques and very effective ones which is known as selection combining. So, let us look into selection combining.

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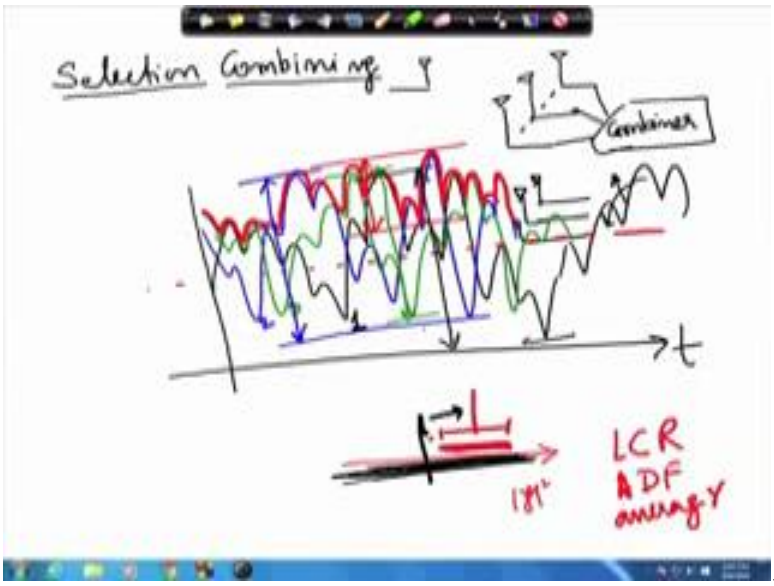

So, we will start our study of selection combining. So, as the receiver suggest as this particular name such as this is a made of toward selection and then combining. So, the straight forward in this case we have one transmit antenna and we have more than one receive antenna. So, we will initially spent time on single output multiple output and these different antenna branches they send their signal to some combiner. So, the signals from the different branches are combined and the combining technique is selection; that means, they select the best out of it.

So, there is a switch you can imagine there is a switch which goes through all the input branches. So, it goes through all these input branches and it come, it selects one of them and the method is basically the one that we have discussed in the previous lecture. So, suppose with time this is the time axis and this is the received signal strength axis. So, this is the signal strength of one of the antennas and if the strength of the other antenna is something like this and the strength of another antenna would of course, be another color.

Of course, it could be something like this and so on, and the selection combining is precisely what we have discussed in the motivation or this kind of work is the one that selects the best. So; that means, here onwards half at the output of the combiner the result would be the highest signal strength; that means, you going to select the branch which having the highest signal strength, if I take black as branch one; that means, if I take this black as branch one and then let say the blue as branch 2 blue as branch 2 and the green as branch 3 then, we can say that we are basically now going to branch 2 then it is branch 3 that is branch one which is the highest then it is branch 2 then again it is branch one branch 3 2 and so on.

So, when we use the selection combining is basically what we have done before what we have seen is that the level crossing rate changes we have discussed level crossing rate. So, if this is a level crossing rate if has changed significantly in this particular diagram the average duration of fade has seen significantly in other words, what we can also see as if there is the change in average of the signal we could also note.

So, there is change in l c r level crossing rate there is change in average duration of fade are the 2 very important things what we can also see is that there is change in average received signal strength and the next thing which is not very straight forward appearing from this, but if one looks very, very carefully one can find it out if we follow let us say the blue one in let us let us follow the blue curve. if we look at the blue curve the range of fluctuation is from here sorry for this range of fluctuation is from this point to this point right. So, this is the dynamic range of the blue signal if, we look at the green signal the dynamic range is from here till here almost similar; if we look at the black signal it is also very, very similar at least in this window it starts from here and go still here. So, black window is there instead of these if we compared the dynamic range of the red signal, what we see is the max value is? Somewhere here and the mean value in this particular picture is somewhere there.

So, what is happened the dynamic range of the combined signal has also reduced? Now since the dynamic range is reduced. So, what we are saying is if this axis is the mod of the received signal square let say; that means, for the blue one or the black one or the or the green one what we have is the signal is fluctuating a lot in this range right? Whereas the red color signal that is after combining what we see is that the fluctuation range is less fluctuating less in this range write the peak is similar, but lower range has reduced. So, what we can say the variability of the combined signal is less where is a variability of the other signal is more the average has shifted from the previous value. So, average was here average is shifted variability has also reduced and we will study what is the impact of these changes on the performance of a typical digital communications system.

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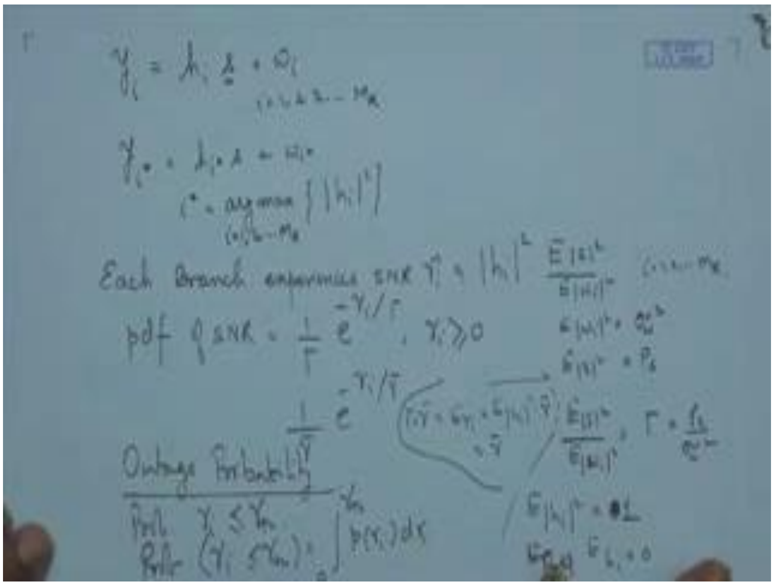

So, here what we do is you will start writing or expression. So, let us say that y of I is a received signal is basically h I of h I times s plus w I s is the transmitted signal h is the channel coefficient and w is white Gaussian noise additive white Gaussian noise y I is the received signal. So, I indicating the ith branch and I is equal to one to 2 to 3 up to m r number of received branches h is flat it is low and it is basically a stationary process we have already said we will assume wide sense stationarity uncorrelated scattering at homogeneous channel for this and when, we do the selection combining we would simply rewrite the equation as y I and put a star is equal to h I star times s plus w I star where I star is equal to arg max of mode of h I square over I is equal to one to up to m r what this essentially means is that this is the received signal in the ith branch. So, whether I is one or m r what we see is at the same received signal is being captured, but it is by a different complex coefficient and there is something else that is added.

As a result of selection combining; that means, when we are doing this particular combining here as we are doing it here what we see is that the best of them best in the sense the highest branch is taken during combining. So, the process of combining can be written down in in this form as we have written arg max means the argument which maximizes this expression and the argument is over i. So, will select that particular branch which has the maximum value of the channel at that instant of time, So, I star is the solution of this case.

So, clearly in one case it was branch one branch 2 or branch 3 at each instant of time this is what we get and the SNR; that means, each of the branch each branch experiences in SNR represented by gamma I which is equal to mod of h I squared times e if s squared divided by e of w I squared e s I squared is the is the signal power e w I squared is the noise power I equals to one 2 up to m r e of w I squared would be equal to sigma omega squared and e of s squared would be equal to p s this is the convention that we used and e s squared over e w I squared would be represented as gamma which would be p s pi omega squared. And we also have the assumption that e of h I squared is equal to sorry is equal to one and e of h e of h I is equal to 0. So, this is for Rayleigh fading, but this is always true is one of the assumptions that we make in this work.

So, in case of exponential distribution what we can write is p d f of the SNR; that means, when this is Rayleigh this is high squared or exponential distribution is given by one by capital gamma e to the power minus gamma I by capital gamma. Where gamma I is greater than or equal to 0. We could also write it as 1 by gamma bar e to the power minus I by gamma bar and of course, this. So, basically meaning this the average received signal strength now if we take the expectation of gamma I which is equal to gamma bar or capital gamma expectation of gamma I it is expectation of this times this that is e of h I squared times expectation of this which is that itself.

So, basically gamma bar e of h I squared is one that is basically gamma bar. So, this is what you have. So, this is gamma bar is the notation which you could write for this case and we could calculate the outage probability one could calculate the outage probability; that means, probability that the gamma I is less than or equal to some gamma threshold let us say it is gamma t h or gamma 0, but also at this gamma 0 is basically probability that gamma I is less than or equal to gamma t h which is equal to integrate from 0 to gamma threshold p of gamma I d of gamma i.

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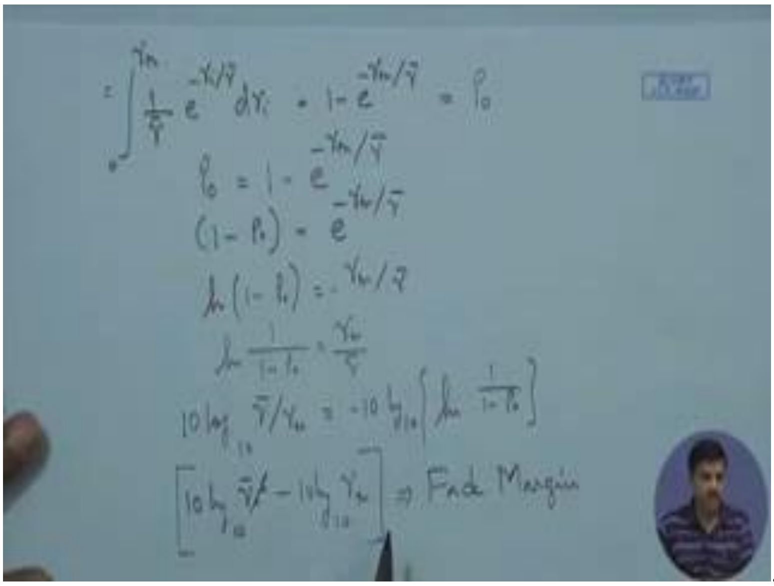

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So, if you continue with this you are going to get that is equal to integrate from 0 to gamma threshold what is p gamma I is one by gamma bar e to the power minus gamma I by gamma bar d gamma I and this if you would solve it turned to be one minus gamma t h by gamma bar.

So, this is the probability that signal falls below the threshold now when we do this particular calculation we have done for any one of the branches. So, for anyone branches this is the probability signal falls below the thresholds. So, if we go to this particular figure that we have drawn here if this is my gamma t h suppose now I change this axis to SNR axis these the gamma threshold. So, the area under this would be if it on the curve if we take average times things below this line compared to the total signal strength over the entire time what we are going to get is this particular probability; that means, the percentage of time signal remains below the threshold.

So, yes, this is the particular probability that the signal falls below threshold for any one of the link this is very, very important. So, we would we can write this as pout or p 0 is the probability that signal falls below the threshold or you could also write that p 0 is equal to one minus e to the power minus gamma t h by gamma bar and then you could say that gamma 0 or one minus p 0. If I bring one minus p 0 on this side it will be equal to e to the power of minus gamma t h by gamma bar and then you could write l n one minus p 0 is equal to minus gamma t h over gamma bar, if I move the minus side on this side you are going to get l n one minus p 0 because minus is here minus goes on top is equal to gamma t h by gamma bar and if I would take this as the has the ratio and take the log based 10 on both the sides then, what we have is 10 log base 10 of gamma bar by gamma t h is equal to we could write minus log 10 of l n one minus p 0. This could be the expression.

So, if we look at these what we have over here is 10 log base 10 gamma bar we could say it is SNR in d b we could say it minus 10 log 10 of gamma t h. So, this difference that we are seen is basically the fade margin it is basically the fade margin because this is the average SNR in in decimals and this is the particular threshold that we are talking about. So, if we want the signal to have a certain outage probability then the difference between the average SNR and threshold should be given by this particular expression.

So, this is with these we can find how much of fade margin that that we required.

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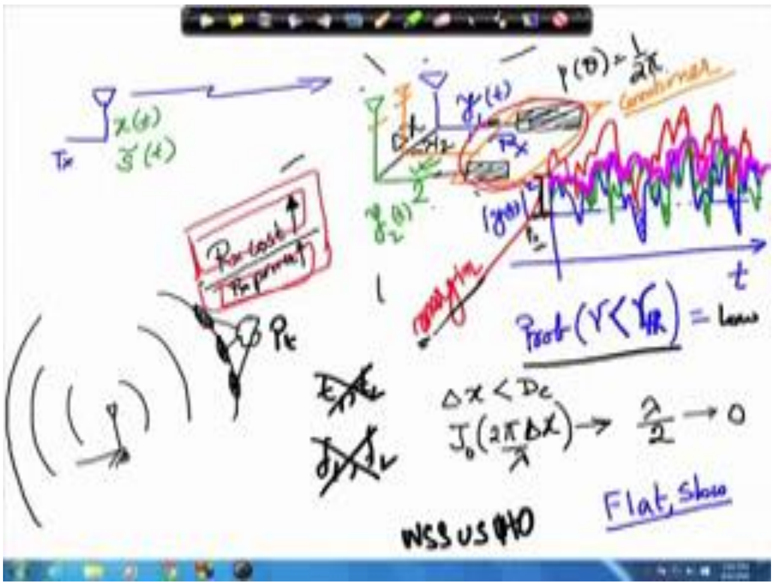

So, if we go to the previous page yes. So, here we are talked about the fade margin here we have talked about this margin; that means, this gap this gap is what we have talked about in this particular figure is basically the margin. So, it is the difference between the average SNR when margin is used and this threshold. So, if I would maintain this extra amount of SNR then I could obtain this certain particular probability of p out which I desire. So, and this is the relationship that has to be used in order to get this particular calculation.

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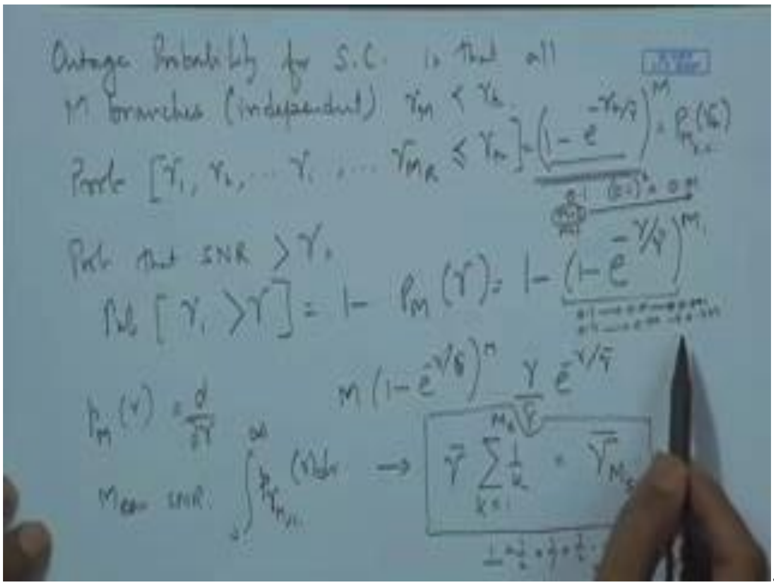

Now, as we move ahead with this selection combining what we see is that the outage probability for selection combining now this is different we have outage probability for selection combining I will write in short s c is that all m branches. Here we assumed independent; that means, we are assuming the separation between the antenna elements is sufficient; that means, is more than the coherence distance or it is more than lambda by 2 in case of p theta equals to one by 2 pi; that means, they are uncorrelated in case of Gaussian they are independent. So, we will be assuming independent branches for independent branches when received signals are received simultaneously amongst all the branches specifically outage probability for selection combining is that all m branches provide combined together have SNR; that means, gamma combined is less than a certain threshold this is what we want.

So, basically what we are saying is that probability at gamma one SNR of first branch gamma 2 SNR of second branch let us a gamma I, ith branch up to gamma m r. So, basically you have m r number of receivers are all simultaneously less than certain gamma threshold if there all independent this will be 1 minus e to the power minus gamma threshold by gamma bar multiplied by that of the second and third and fourth because each 1 is this would be raised to the power of m. So, this is the outage probability for selection combining given a certain threshold I would put selection combining over here and the probability that this the coverage probability that SNR signal to noise ratio is greater than a certain gamma than for 1 or more branches is basically probability that gamma I is greater than certain SNR is 1 minus p m of gamma, which is 1 minus 1 minus e to the power minus gamma by gamma bar raise to the power of m and if I would this is if I would like to calculate the PDF of gamma these to the density function of this.

We take the d by gamma of the above expression and this would lead to m one minus e to the power of minus gamma by gamma bar raise to the power of m times gamma by gamma bar e to the power of minus gamma by gamma bar if you do the derivation you are going to get this and then mean SNR are can be obtained by integrate 0 to infinity p gamma m selection combining gamma d gamma and this would result in gamma bar sum over k equals 1 to capital m r 1 over k. So, this is the expression for gamma bar m selection combining. So, what we get to see is that there is some increase in the average SNR and the SNR is as if added 1 plus 1 by 2 plus 1 by 3 plus 1 by 4 and so on the gain decreases as there is more and more number of antennas and now this can be understood quite easily. If we I take a look at this or we can draw a new figure.

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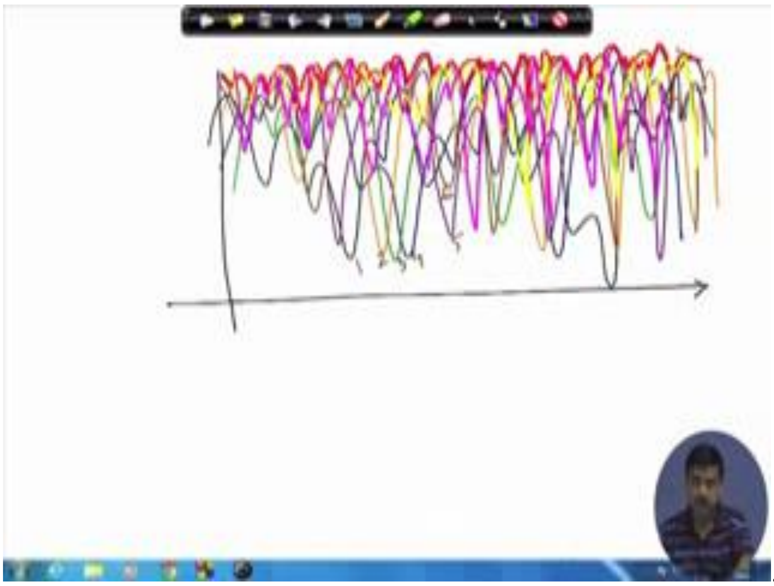.

So, suppose we have one of the branches and let these are the signal to noise ratio of one of the branches and the second branch would have a signal to noise ratio. Which is like this the third branch maybe following this particular one the fourth branch could be this one fifth one I am adding simply trying to add more and more branches. Now compared to the previous pictures and trying to ensure that their average SNR and that is being is a same. So, that later we can do this combining. So, after as many branches we have a 1, we have 2, we have 3, we have 4 and we have the 5, we have 6, we have drawn. So, many different branches now, if we try combining them what do we see pictorially I use the thicker color here.

So, I trace the peak of it. So, this is random imagination again what we see is that if I take more and more number of branches this becomes more and more stables if I would add another branch. Let us see I would add another branch to this whole thing if I would add another branch to these whole thing and curves this is yellow it is a very difficult and I had one more branch in a certain color let us a pink yeah and carry on doing it then what do I get when I modify selection my curve would now become this instead of dipping for that the curve would become this instead of dipping down the curve goes there instead of dipping down the curve goes there. So, what we can clearly see instead of dipping is there it is there the variability even decreases further. So, as it increases the average SNR increases. of course, but the gain from mod number of antennas is less and this gives you of course, some benefit.

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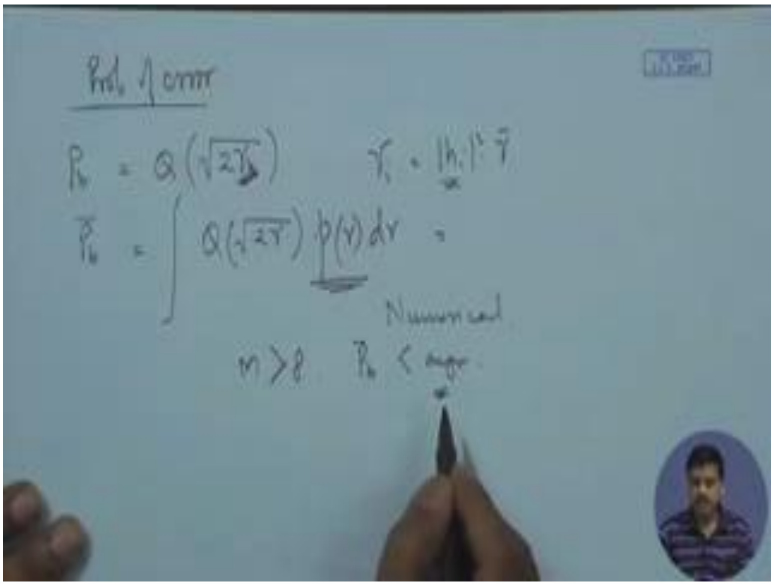

Now if we have to calculate the probability of error for this particular case probability of error calculation the average probability of error now this is very, very important for a wireless communications what we are seen is that the signal strength is fluctuating clearly because there is a channel the signal strength is fluctuating because h is random. So, if h is random SNR is fluctuating as per this p d f then, what is going to happen the probability of error in case of a w g n for q p s k could be q function of root over 2 gamma b, let us say the SNR and in that case in our case this SNR is not constant because SNR. We have seen per branch is equal to h I squared times gamma bar previously this was constant now it is constantly fluctuating.

So, we have to get the average SNR to get the average SNR we have to average the error probability with p of gamma d gamma this is very, very important. So, when you do this the error probability that you get by using this p m is pretty complicated pretty complicated because the expression we have already seen before in the previous expression. And if that particular expression we have to use if you have to use this particular expression it is not very easy to get it through analytical techniques. So, we have to use numerical techniques generally it is numerical techniques and for m is greater than eight we can remember that the average probability of error turns out to be less than that of a w g n.

So, this is quite interesting we will see some of the error probability curves later on. So, even with this very, very simple techniques you could improve the outage probabilities significantly as we have seen over here with this particular expression. So, whatever suppose the outage probability of a single branch is, let us a 10 percent 0.1 and I have a 2 branches; that means, I have m equals to 2 then, the outage probability would become 0.1 raise to the power of 2; that means, 0.01. So, just by virtual of having the second antenna the outage probability from 10 percent has that will dropped to 1 percent.

So, if I add m is equal to 3 this would be 0.1 raise to the power of 3 n that would be a significant reduction in the outage probability and on the other hand, I will be getting the probability of coverage from this particular expression and that that would from this particular expression which would again by 1 minus of the probability given by this. This particular thing, this is what we have calculated single branch from 0.1 it will go to 0.01 the movement it is squared and then it can go to 0.001outage probability and the coverage probability from 0.9 it will become 0.99 to 0.99 nine. So, it is a highly covered. So, if you have 3 branches there is hardly any chance of outage a when, you do selection combining. So, there is a great advantage that multiple receive antennas can provide us and that is why multiple received antennas are highly favored and in this selection combining the advantage that we get is that if we try to look at the previous pages.

So, there we said that cost we said that cost goes up of course, that is true, but in case of selection combining if we are measuring the branch and using only 1 of them then, we could reduce the complexity over here and could have simpler receiver architecture and then the cost rise may not be that high, but the problem here is that you have to constantly a measure the received signal strength that is as given in this particular expression you have to continuously measure it. So, there is a slight variation of techniques that can be used.

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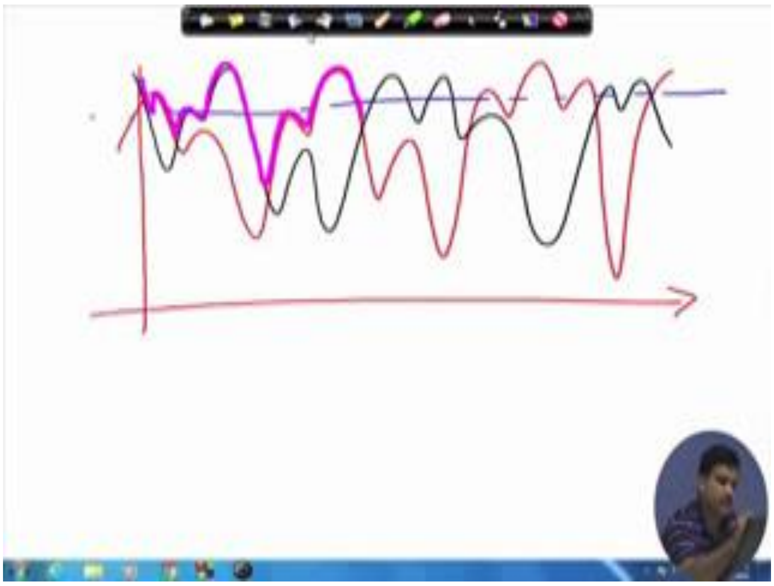

That is from the switched combiner switched being combining in that what is done is if the received signal strength fluctuates. Let us say like this and one of the branches I will take just 2 branch for simplicity and the second branch is let us see like this. So, in in that case one would decide a particular threshold, let us say this is one threshold right and they would not do unnecessary switching and the switching would happen only if, only when the signal falls below threshold. So, if one is following the black it will carry on till the point it goes below the threshold then, it jumps to the red one follows through and now there is no other options it has to go down and then select the black one no other option carry on. Then go to the red now when it when it is there is no other option you simply cannot do it, but the movement I mean you just simply do not switch because if there are I mean this if there is another let us say we redraw this we redraw this.

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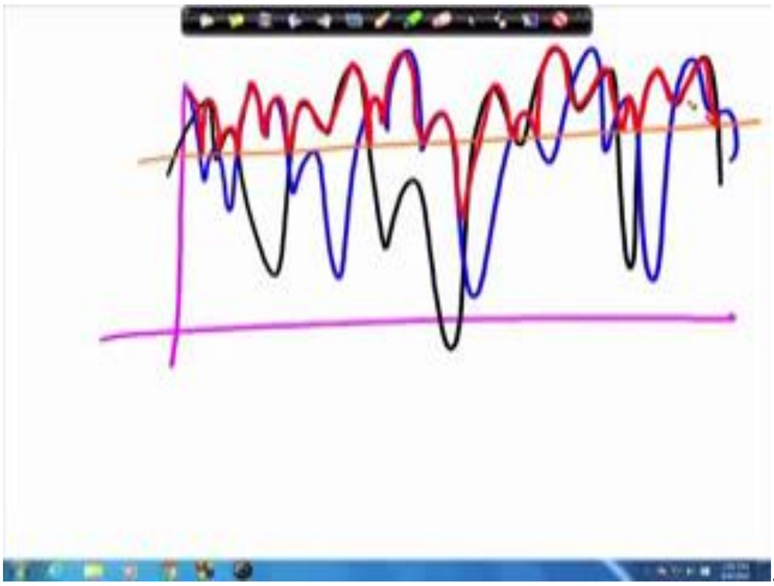
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Suppose one of the branches is having this particular this is the structure of the SNR and the second branch is having an SNR which is like this. So, in this case suppose we have this of threshold which is here which is here yeah in that case the combiner would work like this it is going to select one branch it is going to go with that branch until the signal falls below this threshold. Because the signal is pretty good when it falls below the threshold then it jumps to the next one and it follows through. So, here it is here and then it switches even though the blue one is going better it carries on with the black till it reaches the threshold at that point it switches.

Here you cannot help it here it crosses over and then switches, here it would cross over at that point it would go through the switching it will stick with the black then, it switches it was stick with the black the reason the advantage of this is unnecessary switching is reduced in this particular technique and the complexity can be well handled. So, this is a rather more practical technique which is also used in case of selection combining with this we complete at least one technique of receiver combining and through which we can gets spatial diversity gain, the b r curves are of course, not very straight forward they have to be calculated will see some of the b r curves.

With this we would move like to move on to the next diversity gain technique which is known as maximal ratio combining.

Thank you.