Fundamentals of MIMO Wireless Communication
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Lecture – 23
Spatial Diversity

Welcome to the lectures on Fundamentals of MIMO Wireless Communications. Now since we have studied the properties of the channel that is the flat fading channel, then the frequency selective fading channel, then the space selective channel. We have seen the statistical properties of h which is the matrix relating the multiple input to the multiple outputs. We have seen how it can be factored into its eigen values and singular values, we have also seen that there is distribution of the joint distribution of the eigen values; we have also seen there is the distribution for the minimum eigen value. So, these will be handy when we carry forward with the discussion. We have also seen the movement generating function for (Refer Time: 01:00) of h and we have always said that these are random variables. So, again the movement generating function would be useful.

So, now we are at a time where we can start a journey into understanding the different communication techniques which take advantage of this spatial mode of the channel.

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So, usually the study is can be looked at from the point of view as special diversity. So, we have special diversity to study and the other thing we have is special multiplexing. These are the two very important things in MIMO communications of course, there are being forming and other things, but these are primary paste on which many things involve. So, we will start our discussion with diversity and then once we have complete with this we will move on to multiplexing and then we will have a discussion on diversity multiplexing in trade off; that means, given a setup number of multiple antennas, how can we get diversity gain and multiplexing gain or maybe part of one part of the other or simultaneously both, those are some of the things which are important. So, in some cases people do study a spatial multiplexing to begin with; however, I will take diversity because that is more straight forward that comes directly from your understanding of wireless communications and then little bit more advance topic is special multiplexing. So, that is how we will proceed.

So, when we look at diversity, diversity is this is little basically spatial diversity. So, spatial diversity is we have the input side and we have the output. So, when I say input; that means the transmitter. We have the transmitter side enabled with multiple antennas; we have the receiver side which is enabled with multiple antennas. So, that is the transmitter and the receiver. So, as we have seen that we have many channels the first kind of channel is SISO are single input single output which is studied in classical wireless communications.

Then you have a single input multiple output we have seen how to write these kind of channels, then we have multiple input, single output, we have also seen how write these expressions and then finally, we have multiple input, multiple output. So, our aim would be to understand how to get the diversity again out of all these different modes of operation. SISO mode of operation is what we will not see very explicitly, but that can be covered anywhere. So, we will basically look at how to you extract diversity in this direction. So, when we study special diversity you have the option of doing when I say single input multiple output we mean to say that you can obtain diversity at the receiver side, multiple inputs single output means you can do diversity at the transmitter side and my movements you will be able to do it on both the sides.

So, when you do at the receiver side things are relatively easier, but the movement you have to do anything at the transmitter side there are two possibilities of doing them. So, when you do processing at the transmitter one is you may need a feedback of channel information; that means h, information related to h has to be feedback. There could be another mode of operation where no feedback and this is very very interesting because when there is no feedback the interesting part is you can keep on transmitting without waiting to get the channel information at the transmitter. Our aim would be to cover these different aspects of a communication in this.

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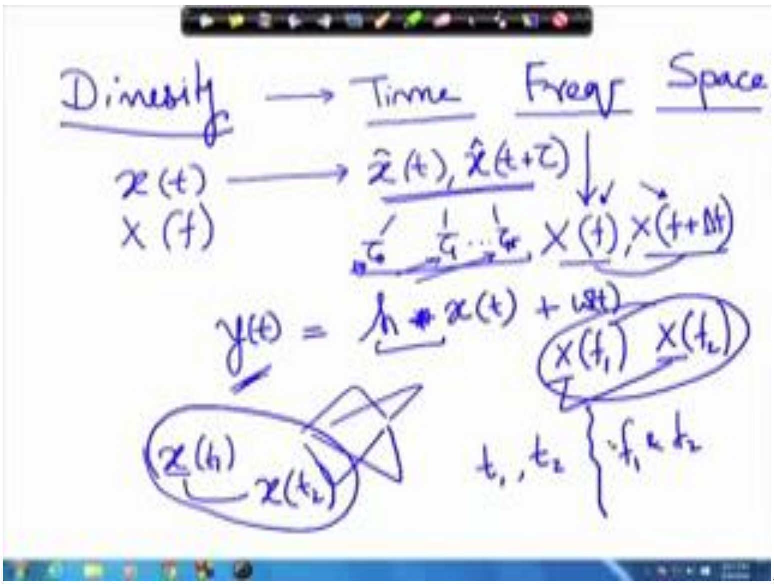

So, let us getting to the study of spatial diversity, when we talk about diversity when we use a term diversity this is not very new to communications when you do wireless communications that different domains that your access are time domain, frequency domain and finally, it is space domain, right.

We have seen the channel can be described channel has its variability in the time where it is slow or fast getting it is variability in frequency, flat of frequency is selective it is variability in the space there is rich or poor scattering. So, we are not referring to that currently we are of course, that is very very critical, but currently we were talking of obtaining diversity from time domain frequency domain and space domain and what I mean by this is that diversity can be obtained in time domain, it can be obtained from frequency domain and finally, it can be obtained in the space domain.

Before we launch into the space domain let us briefly take a look at what is the meaning of diversity. By diversity what I mean is that when I have sent a signal into the air I would like to receive multiple copies of the signal and the receiver. So, when we receive multiple copies we could receive the copies in time domain, we could received them in frequency domain and we could also received them in the space domain. So, before we go to space domain when we talk about time domain when I would like to receive them in time domain, suppose I would sent x of t if I would sent x of t I would right to receive x of t which I would write it as x cap let us say or a time t and I would also like to receive x cap at a time t plus tau. So, once I have multiple copies of the signal then I can do something.

Now will see what we can do when I talk of frequency domain suppose I get write capital x as the frequency domain information of t. So, if I would send it at a certain frequency f, I would like to get it at f as well as at x of f plus some spacing let say delta f right will describe the space domain separately. So, in time domain things can be one can imagine that I would receive the first delay version of the signal tau 0, I would receive the replica of the signal again at tau on and so on and so forth up to tau n; that means, the maximum delay and if these signals are uncorrelated; that means, x at delay t 0 x at delay tau 1 x at delay tau n if they are uncorrelated then we can get some benefit out of it. Simply because, if you remember with the way we write if I say y of t is the received signal and what I get is basically h convolution with x t plus noise.

So, I would be receiving multiple copies because of this convolution and if this copies an uncorrelated, uncorrelated scattering then I could take advantage of it and do certain thing there are methods of doing it. If it is flat fading; that means, sorry if flat fading means all these are at the same delay then still you have multiple paths signal does come from multiple path, but since the delays are the same they would appear at the same time and we cannot separate them. They are appearing at the same instant of time. So, in order to get multiple copies I would have to send x at t 1 and I would have to send the same x again at a time x t 2, that means, I am using two instance of time to receive two copies.

If I look at the frequency domain, again if there is the spread in the Doppler frequency I would receive the signal at f and I would again received the signal f plus delta f. Whereas s n and if there is frequency selectivity then also the signal the copy of the signal I received at f and the copy of the signal I received at f plus delta f or again uncorrelated, but in the frequency domain rather I have to send a signal at a frequency x f 1 and a frequency at x f 2. That means, I will be sending the same signal at two different frequencies and I will be receiving them.

So, what we see from here; that means, I am sending at two different times and I am sending at two different frequencies that mean I am using time t 1 and time t 2 or I will be using frequency f 1 and frequency f 2 to get two copies. So, in wireless communication we all know that the time and frequency resources are very very costly and that is the spectrum basically. So, I would like to make maximum use of it and there are different ways of achieving diversity again. So, if I am going to use this particular method, that means use replicated transmission in the time domain or replicated transmission in the frequency domain I am using more than one resource of time or frequency to send the same information or they could be smarter wise of doing it. So, this could be one of the possibilities, but may not be always recommend. So, this is valid, but whether this one is to be used or not is a secondary question. You may be aware of a forward error correction codes, in short it is written as FEC that is forward error correction code.

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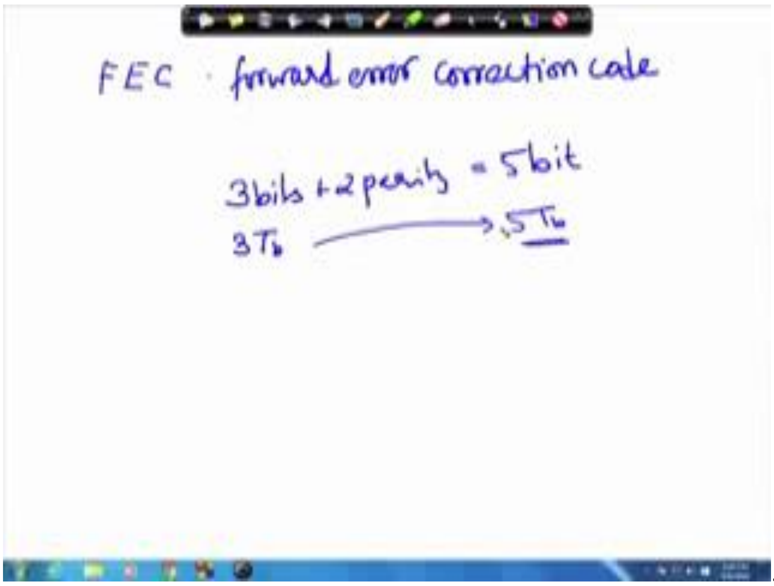

So, in forward error correction code usually parity bits are added; that means, we are having a sequence of bits and there are new codeword is formed. So, if I have lets a 3 bits and 2 bits of parity are added I generate a new codeword which is 5 bits. So, when we have these 5 bits of information. So, instead of sending 3 I am sending 5 bits if it is a very plain and simple system I will be using 5 bit durations. So, previously I was using 3 bit duration; that means, let us say I have 3 bits to send I had 2 parity bits then I get 5 bits in total.

So, previously I was using 3 time each bit duration indicated as T b, now I have to use 5 times the bit duration. So, I am actually using extra time and I am spending some of my time resources, it is costly, but this is of course, necessary and the other advantages we need not necessarily send this in time domain they could be many, many different ways of doing it. If you are using high order consolation then the probably all this bits could be easily captured into one symbol and we can send them out, without losing time and we could also distribute them our frequency and we could also gain benefit out of it and lastly if you are using very very long codeword's then your efficiency is very very large and in that case the loss due to the parity would be significantly less. So, this is another mode of gaining getting some gain, but it is somewhat related to this diversity again that we will be discussing.

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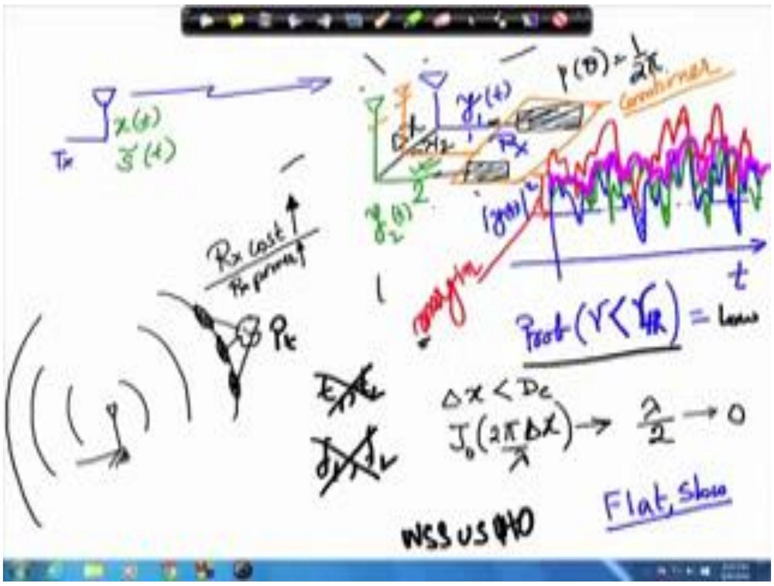

So, going beyond this if we look into the spatial dimension, if you look at the spatial dimension suppose I have one transmit antenna at the transmitter side I am drawing this has the transmitter and in a typical single input single output system I will call this as the receiver. So, when the signal goes out to the receiver let us say x of t it is easily received as y of t in at the receiver and if I would draw the x axis, let say this is time and this is the gain of the received signals strength let us say y of t mod square let us say we do this and what do we expect and let us assume on, all cases I will be assuming flat fading and will be also assuming slow fading; unless otherwise mentioned and these will be the prime assumptions.

So, with time I am going to get fluctuation of signal strength which is random we have already discussed this, if it is rarely distributed then this will be exponential distribution the envelope squared would exponential distribution. We have already discussed about the here the variability being captured by Doppler, the coherence time and all kinds of things we have already discussed related to this. Now we have also discuss that there is level crossing rate if there is a certain threshold, rate at which it crosses the threshold we have also discussed there is a average duration of fad one signal goes below the threshold. So, we have actually talked about this indicating that these are performance matrix since if it goes below threshold there is (Refer Time: 15:12) and what we usually defined of course, other than probability that the s n r, received s n r as a function of time or received s n r. Again received s n r is less than certain threshold. So, if s n r is less than a certain threshold what is this probability? So, I would like to have this probability as low as possible that is one of the desired things.

So, if we want this probability to be low what usually do is instead of sending the transmitted signal by the wave we have drawn it, we would like to increase the level of the transmit power. So, that the received signal strength something like what we are drawing here that means, if I push transmit power the they receive power increases. So, instead of the blue one whatever is in new line that we have drawn is what we have going to get and if this is my threshold that is gamma t h then what we will see is that since I have increase the average transmit power there is hardly any probability that the signal goes below this threshold. So, we are reducing the probability. That means, we are able to in; we are actually giving a margin - margin is by means of transmitting extra power that we are able to achieve this condition.

Instead let us look at the situation that we have another received antenna, this is antenna 1 and this is antenna 2. So, we can use a different color for this lets say this one it may not be clear, but this green in color. So, and the same signal will now we received in antenna 2 as well, I have not send any multiple signal we have send x of t or according to our earlier restriction s tilde of t in the base band equivalent or the low pass equivalent signal whatever was received here I would receive another copy of the signal. Like, this is very very cleared because there are reflectors around and we would be getting scattered components, we will use the narrow band antenna is assumption; that means, the signal here remains the same (Refer Time: 18:00).

So, basically with this whatever signal I received here let us say in the first antenna we received the signal y of t or y 1 of t whatever we are going to receive in the second antenna we could write it as y 2 of t and if this distance of separation if this distance of separation is let say some delta x let us say this distance is delta x. Now what we have studied when we studied the channel in the spatial domain we said that if delta x is less than the coherent distance then the signal received in both these two antennas are correlated, whereas if this distance is greater than the coherence distance they are uncorrelated.

We have also seen that when signals arrived from all directions with equal probability; that means, when p of theta is equal to 1 by 2 pi we have studied this case, we seen that the correlation is a function of j 0 2 pi delta x by lambda. And this leads to then if I look at the mode squared of the signal what we have find is that at separation of lambda by two approximately then it is found it to the 0.38 lambda the correlation goes to 0; that means, decorrelation distance is lambda by 2. So, if this separation is more than lambda by 2, let us say if it is comparable to lambda by 2 the signal received here y 1 and y 2 would be uncorrelated. In that case we can tentatively draw or received signal I will take that blue one is the transmitted signal, we could take uncorrelated signal coming in at that place. So, what we can see is statistically the blue and the green signals; that means, received by antenna 1 and antenna 2 they appear similar, there is the same threshold, signals go below the threshold write the average is more or less same. But now if we take both the signals simultaneously and for instants all though this picture is getting a bit clattered will use one pink color.

Suppose I will use a thicker line suppose we use device a method by which out of these two signals it is y 1 and y 2 we select the best of the two that means, in this part of the time we select the blue one, in the next part of the time we select the green one, next one blue is high, then green, then blue, then green is high we are writing the (Refer Time: 20:53) blue is high, then green is high I am just following whichever is highest the blue is high green, blue, green, green, green, blue, green, blue, green and so on so forth.

So, this is the signal that I am going to get if I select the best out of that two, so that means, if I carefully study the situation we are a receiving signal at antenna 1 and also receiving at antenna 2 we have not use the extra power; that means, we have not done what was done during the red color. So, we have kept the same average part as earlier, but whatever is received here the same average received signal will be here this is because of stationarity; that means, within the coherence distance within several tens of coherence distance - spatial distance, which is within a certain region at the homogeneous channel assumption basically we had made wide sense stationarity uncorrelated scattering and homogeneous this is the assumption that we made.

The average received signal strength is the same in a certain area then what we see is that even though this individual links, the blue link and this green link green link they go below the threshold whereas, this pink color which is riding the (Refer Time: 22:21) of them, which is taking the best of the green and the blue is hardly going below this threshold. They might be vocations, but in this particular diagram we do not have it. So, what we see is that by virtue of getting independent signals in these two branches there is a possibility that the probability received signal strength goes below the threshold becomes very very low, we can make it very very low.

So, this is in short what you can say the advantage due to diversity and in this case we are not using extra time, we are not using t 1 and t 2, we are not using f 1 and f 2, we are not using extra power. The transmitter is sending at the same power p t, we have not increase the power simply because we are used another received antenna this is at no cost, no cost to the spectrum no cost to the power at the transmitter we have been able to improve the performance. The cost of course, nothing comes for free. So, there has to be a cost that is paid. So, the cost that is paid over here is receiver complexity because each of these branches these branches is blue and this green branches these are all separate branches. So, in these two branches are separate branches they have their own receiver circuitry, they will have their own circuitry. So, your receiver cost would go up.

That is the penalty that your paying for this, otherwise nothing else and receiver cost is going to go up and along with that because the complexity is up what you can also guess is receiver power consumption would go up, simply because your processing at more than one branch this is called a branch, is one of the received branch is another received branch. So, you have to do processing with as many branches if there are m r antennas, your signal processing cost goes up at least in the r x section of base band section complexity would be different. So, at this cost at least you are able to get the signal out of the deep fade conditions where there was single antenna that is used. So, this is the prime motivation that we go for diversity.

Now, in the case that we replace the antennas which are not separated by coherence distance; that means, suppose we placed an antenna which is somewhere in between and which is not less than the coherence distance. In that case the signal would be correlated, but we have to find certain combined technique they would be combining techniques at the receiver. So, we have to use certain combiner technique and still you could get something extra.

The fundamental reason why this is happening if you have to understand intuitively the movement I add an extra antenna, movement I am adding extra antennas what I am doing is effectively capturing more energy this is one intuitive explanation. That means, suppose I have a signal, suppose I have a radiator it is radiating. A received antenna is having a certain size if I put one more antenna its capturing a little bit more of the surface area if I put another capture, another antenna it is capturing little bit more of the surface area. So, I am capturing more energy. Now when I do this then I have to combine the signals in such a way that I can take advantage of these at intuitive graph explanation of what is happening, so for this there is no change in the transmitter configuration only the receiver configuration changes and we can get benefit out of it. So, if this is the basic motivation for using received diversity.

We can do a similar technique at the transmit side and that will see of course, at a later time. So, with this we start our journey into finding the (Refer Time: 26:39) that we can get out of such signal processing techniques using spatial diversity.

Thank you.