Fundamentals of MIMO Wireless Communication

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Lecture – 19
Expression of MIMO Channel

Welcome to the lectures on fundamental of MIMO Wireless Communications. We have in the last lecture look at wide sense stationarity uncorrelated scattering and homogeneous channels, with this we carry forward and discuss the single input, multiple output channel, the multiple input single output channel, multiple input multiple output channel and also the narrow band antenna assumption. So, we are going to discuss this, but very shortly I would like to explain 1 particular characteristics of the channels that what we are looking at, so I would like to just describe the propagation characteristics which we have been discussing over the last few lectures very, very important.

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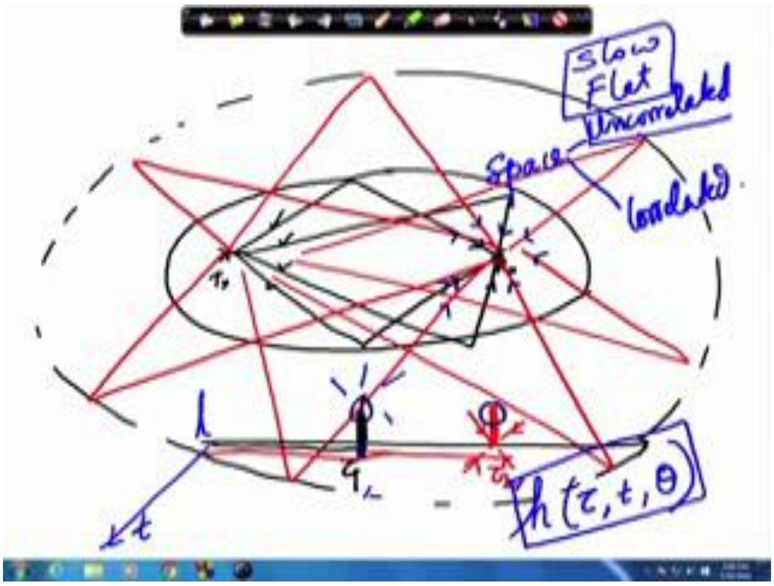

So, if we have a transmitter at this point and we have a receiver at this point, we have been saying that let there be an ellipse and all paths that come from this ellipse are having the same delay which give raise to flat fading this is what we have described. And if there is an ellipse, I will draw another arbitrary ellipse; that means, a representing all scatters which are in this particular ellipse you would have paths coming like this, which are having the delay of let us say tau 2, let us say let us take a very, very simple example right. So, we have these conditions and when we are trying to draw the impulse response that is receiving let say, we have launched an impulse at the transmitter.

So, because of these; that means, instance of colour because of this once what do we get is after a certain duration time that is the delay corresponding to tau 1 we are getting echoes. Echoes which are very close to each other, this is the result of the first circle and if we look at the delays of tau 2; that means, if we proceed further and this total delay is tau 2. So, we are going to get echoes which are all almost on top of each other at delay tau 2.

Now if we concentrate on either of these 2, if we concentrate on either of these 2 let us take these particular set, what we are going to have is there are n number of such multipath coming in and they are coming from different directions 1 could be from this direction, this direction, this direction and direction, which would define the angular spread and similarly for this 1 also they are coming from different directions that would define the angular spread for each of this delays so; that means, if we are able to expand the view here in the theta dimension; that means, we could say that there are rays which are coming in this direction, in this direction, this direction, this direction and hence we are in other words trying to describe h that is the channel coefficient as a function of delay; that means, you already have tau 1 and tau 2 and also theta; that means, the angle at tau, in this angle, in this angle, in this angle, in this angle at tau 2 again this angle and of course, it is a function of time. So, this access is the time access. So, that gives us the delays access the time access and the theta.

So, this is what is actually happening in the system is what we should visualise and keep in our mind, when we are referring to the channel conditions and we although this these things are going on of course, there are many more details also this one kind of abstraction we would always use even further abstractions without which certain results which are very, very fundamental and which are quite insightful are not possible. So, please be careful at every point of time when we are moving ahead and making a assumptions with using this kind of channels. And also I would like to summarise that will be using a slow fading will be using flat fading unless otherwise specified this are the 2 conditions that we will be using and we will also be considering uncorrelated channels this will of course, describe most of the time will consider uncorrelated. So, in this space dimension, when we are in the space dimension and we are going to also take care of correlated, right.

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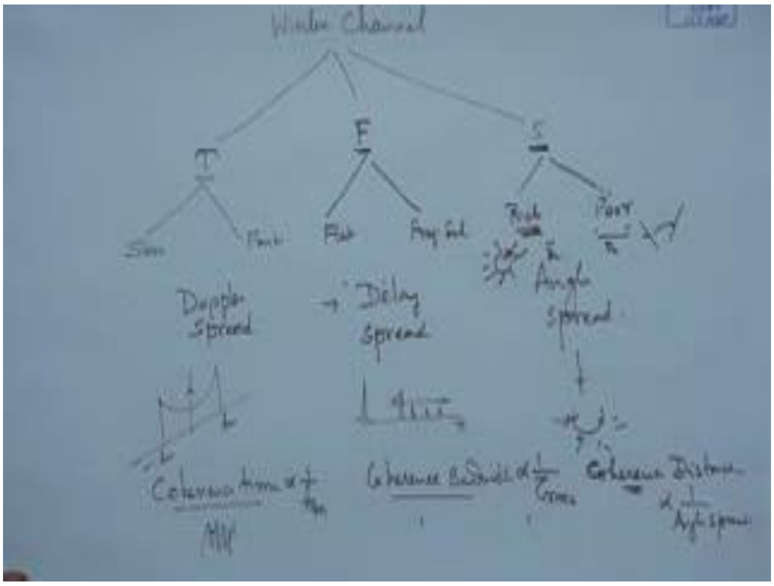

So, just quickly before we go ahead here in this particular picture we said slow and fast flat and frequency selective in the space access we have the rich and poor scattering. So, clearly a rich scattering scenario is one where signals arrive from all directions and poor scattering is 1 where signals arrive over a narrow angle, here signals arrive from all directions. So, if we go by the discussion in the previous lecture we said rich scattering results in smaller coherence distance poor scattering results in larger coherence distance, right. So that means, rich scattering environment would mean that is lot of reflections. So, that in the special dimension there is lot of reflectors around. Whereas, a poor scattering means the rays are coming only from a particular directions and there is not too many reflections around. So, poor scattering is a situation where the coherence distance is larger rich scattering is the situation where the coherence distance is smaller.

So, if we take a 2 antennas lambda by 2, we can assume that the signals are uncorrelated. So, that is how would we define rich scattering environment and a poor scattering environment and that would bring us to a kind of description in the time frequency and space dimension of the wireless channels with which we started quite some time back.

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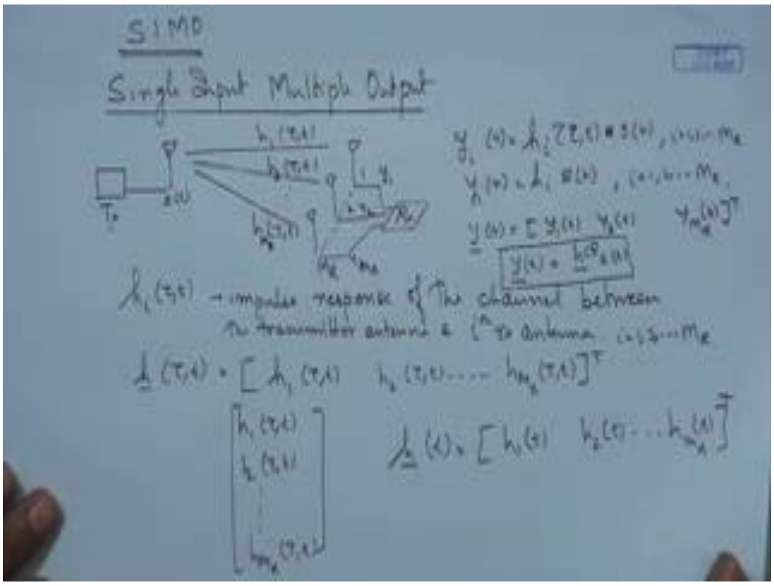

So, now we move forwards and try to look at the SIMO channel the SIMO channel as the names says it is single input multiple output single input, multiple output; that means, we are looking at a situation where the transmitter has only on antenna and the receiver has multiple antennas. So, this is the receiver, we are trying to characterise or write down how would the channels be here, how would we write down the channels under in this particular case, these this is a typical situation which we are going to encounter in the studies. So, it is important that we take a look at this and the number of antennas here we would write as 1, 2 up to m r m r suffix indicating receiver number of antennas and let us have hi of tau comma t. So, if we separate I it is h of tau comma t; that means, the delay and time is the impulse response of the channels. So, that is the impulse response of the channel between the transmitter antenna and the ith receiver antenna. So, we will use this t x for transmitter r x for receiver.

So, this is how it is given by and we can represent h tau comma t. So, of course, I is equal to 1, 2 up to m r you would write it has h 1; that means, first link h 1 tau comma t, h 2 tau comma t and so on, up to h m r tau comma t h 1 tau comma t h 2 tau comma t dot, dot, dot, dot up to h m r tau comma t with the transpose. So, transpose of this vector. So, at the outside let me remind you that we would use linear algebra and most of the studies on MIMO communication are using results from linear algebra. So, 1 would require a basic of linear algebra vector operations and matrix operations to revise that all points of time. So, when we say this what we mean is that, effectively we are talking about a column so; that means, h 1 tau t, h 2 tau t and so on, up to h m r tau t although we are writing it in this ways tau comma t most of the time will be interested in talking about h of t because we will assume flat fading.

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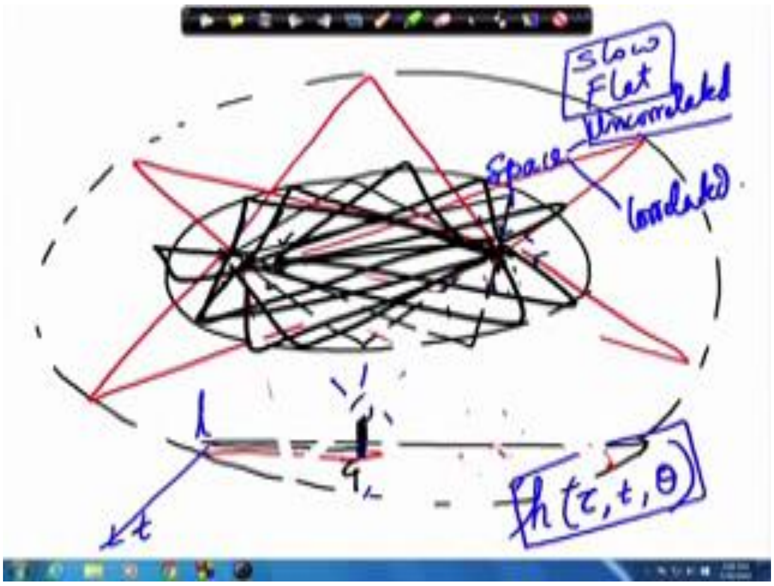

Flat fading would mean that all the multipath are at the same point; that means, in this particular picture the red colour is not present; that means, only the black is present. So, we do not have we do not have the red once so; that means, to make it faster we do not have this red once. So, only tau 1 is present so; that means, all the delays these are not present only the black paths are present. So, if you could imagine that that is the flat fading that is just for the reminder case just for the reminder. So, only the black lines are present. So, only these only these reflectors no others paths. So, all the delays are the same almost the same no red paths only this paths. So, that is when we have this. So, usually we would right h h of t, is equal to h 1 of t h 2 of t up to h m r of t this is mostly what you would encounter transpose, for the single input multiple output channel and the received signal. So, what we have going to get here is y1, y2, y3 this is what we are going to receive.

So, we could write that yi at the ith receiver is a function of time is equal to ideally speaking hi tau comma t convolved with s of t fort I equals to 1 to up to m r, but what we have written over here we could write it as for flat fading scenario h I since it is no delay it is a multiplication s of t for I equals to 1 to up to m i. So, this is what we are going to get and for I equals to 1 to m r you could write it in vectorial form, y of t is equal to y 1 of t y 2 of t up to y m r of t transpose or we could write y of t is equal to h vector times s. So, that is what is in short form you could write s of t h of t, we could write this in the vectorial notation as the received signal, so this clearly tells that I am receiving y1, y2 up to y m r, so m r different copies of the same signal whatever is transmitted over here; that means, if s of t is transmitted. So, we go through the channel we see over here, go through this channels we see over here, go through this channels we see over here. So, there are multiple copies of the same signal which is received at different antennas. So, this is the typical single input, multiple output channels.

Next we move on to the multiple input single outputs so; that means the reverse of this condition. If we take this condition and from this becomes the transmitter this becomes the receiver.

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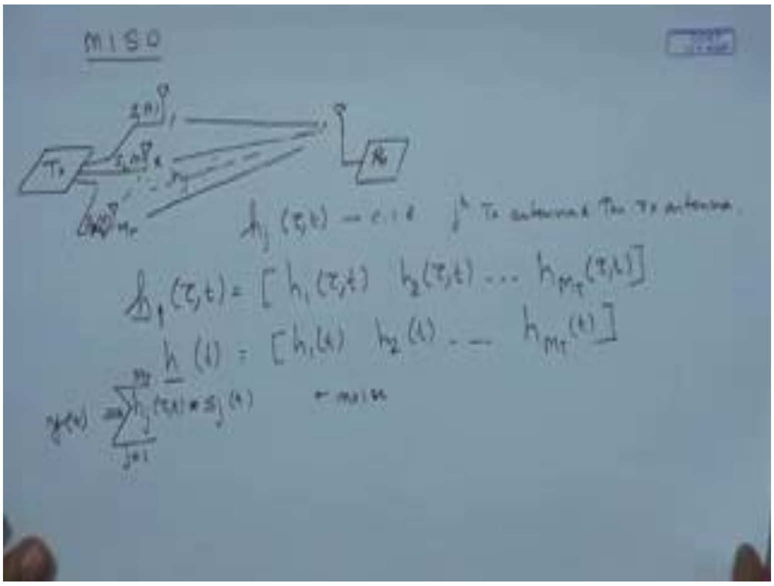

So, that is the case, as if there is 1 transmit antenna and multiple transmit antennas and there is only 1 receive antenna. So, this is the reverse condition, now 1 example of this could be if this is the base station base station is pretty large it can hold multiple antennas this is the mobile which as only 1 antennas. So, this is the uplink direction and here this is the downlink direction; that means this is the base station with multiple antennas this is the receiver with the single antennas. So, this could be a potential situation that we could take it as an example. So, here we would mark them as the first antenna the second antenna up to m t indicating the transmitter antenna and so, here there are m t numbers of SISO links, each of them are SISO; SISO means single input single output.

So, these are the different SISO links and here would identify each channel as h j of tau comma t which would be the channel impulse response between the jth transmit antenna and the receive antenna. So, that is the way to write it h of j tau comma t and what you would have h h of tau comma t the vector is h 1 tau comma t h 2 tau comma t up to h m t tau comma t no transpose any more. So, this basically a row vector whereas, for this case it was a column vector, there is a transpose here that is a difference and again if we consider flat fading which will mostly do. So, in our case we can write it as h 1 of t h 2 of t up to h m t of t. So, this is the standard way of writing it and now there is a slide difference between the transmitted signal and the transmitted signal in this case we will have s 1 of t transmitted from the first antennas, s 2 of t transmitted from the third antennas, s m t of t transmitted from the m tth antennas.

So, what we are going to get is s j of t getting convolved with h j of tau comma t by the jth link any particular link this is the h j and then, at the receiver this signal that is getting transmitted from the first antennas is getting added to the signal which comes to this path is getting added to the signal that comes for this path is getting added to the signal that gets that comes to that path. So, we have a summation of j equals to 1 to m t right and of course, there is some noise. So, this is the received signal y of t. So, if you look at the fundamental difference between the h of t there and the y of t here and the y of t there the difference is here y of t consist of several ys; that means, I have received 1 2 up to m r number of copies here I have received only 1 y, but I have transmitted m t number of copies. So, that is pretty natural and it goes by the name and here as h each of this links would have a noise associated with that. Whereas, here there is only a single noise added to the combined component simple because there is only 1 received antenna.

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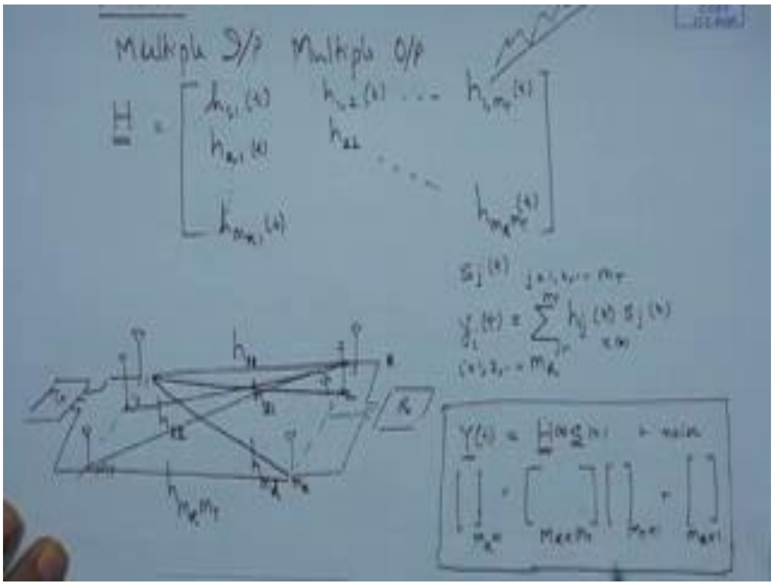

So, with this if we move ahead and the next natural step for us would be to describe the MIMO case. So, that is the first time we are encountering MIMO multiple input, multiple output. So, in this case naturally it is a combination of the previous 2 cases. So, what we have is the h matrix, now we write it in capital H this is usually the notation. So, the way we have come across is the way of started from time frequency now to space and then we have taken Fourier transpose of the channel in the delay access in the term time access inverse Fourier transpose from the Doppler access then, there was the space this angle there was. So, many things are going on you would often get confused with the number of variables that are required. So, in many a cases when you are not studying this space dimension; that means, when you are not studying the multiple antennas dimension usually people usually the or the technique is to represent capital H as the Fourier transpose of small h and; that means, we are talking about the transpose function of the function, but when it is the special dimension the usually notation is to use the capital H.

But there are many places where people would use the c there would use a g they would use a small h. So, we would go by this because we are taking flat fading channels. So, that is not going to confuse us. So, this is t he notation that we are going to follow, but as of now what I write here is not with flat fading this with full fading statistics so; that means, h 1 1 right. So, here we describe that transmitter as m t number of antennas transmitter has m t number of antennas similarly the receiver would also have m r number of antennas. So, this link would be h1 1 this is received an antennas 1 transmitted by antennas 1 the second antenna link, if I draw the second antennas lets for a sake of convenience. So, this link if you look at it goes to the second antenna 1 2 h of received in antenna number 2 transmitted from antenna number 1 similarly this particular 1 is h received in antenna number m r. So, this m r transmitted from antenna number 1 whereas, this link is denoted as h which is received in antenna number 1, transmitted from antenna number 2 and this link is h received in antennas number m r, transmitted from antenna number m t.

So, this is how at the typical link restriction would happen. So, here h 1 one means this particular link it is tau comma t is can simple write h t and then h 1 comma 2 which would mean received in antenna 1 transmitted from antenna 2 of t and so on up to received in antenna number 1 transmitted from antenna number m t function of time just a quick reminder at this point this, this you have taken the flat fading.

So, they are fluctuate in time there is a time access they would fluctuate right then, if you take the Fourier transform if this you are going to get the Doppler's vector or. So, you going to take the auto correlation and take the Fourier transform of that you going to get the Doppler spectrum whatever you have studied would apply for this cases. So, here we have h of 2 comma one; that means, received in antenna 2 transmitted from antenna 1 and dot, dot, dot, dot up to h m r comma 1 of t this would be h 2 2 and so on. Here we are going to get h m r m t as a function of time. Now this is how the full model would be. So, I would like to go back a little bit when I say the multiple input single outputs in this case. So, the vectorial notation we could write it as y of t is equal to h of t times s of t where s of t is given by s of t is given by s 1 of t s 2 of t up to s n t of t in a transpose. So, that would help us in describing the equations.

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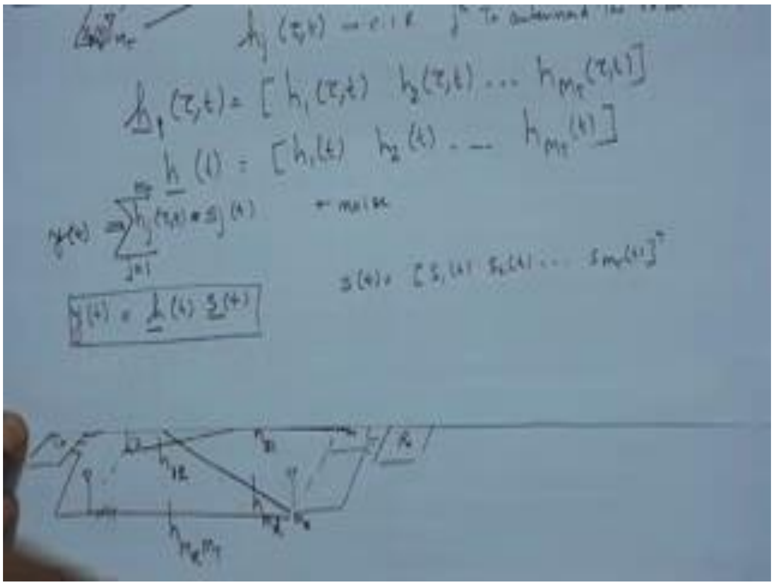

So, this is the vectorial equation and; that means, we have this row vector we have the column vector. So, we have a single scalar as the output in this case we move ahead here. So, s of t since we have multiple antennas s of t would be defined as in this case where we have multiple input s of t is defined in this way. So, we for every channel the link the output is this 1 every receiver antenna the output is this 1 and they will be m r number of receiver antennas. So, that that be the expansion as to happen, so we could write that s j of t is the transmitted signal from the jth transmit antenna and y I of t is the signal received in the ith receive antenna I equals to 1 2 up to m r j equals to 1 2 up to m t.

So, y of I is equal to sum over h j of t times s j of t right of course, h I j sum of j equals to 1 to m t is the received signal. If you would represent in terms of tau then of course, there a convolution sign as to happen; that means, if you would say h I j tau comma t then there was a convolved with s j of t. And in vectorial notation this could be written as y of t vector because 1 y I equals to 1 to up to m r is equal to h I have put a double under underline to indicate it is a matrix I have out a single underline to indicates it is a vector.

So, y is a vector because y1, y2 up to m r is equal to h time's s of t. So, this is a function of time this is the function of time. So, if you look at the matrix notation it is a matrix this is a vector and this is a vector the size of this is m r cross 1 the size of this is m r cross m t size of this is m t cross 1 and of course, there is noise and noise is naturally m r cross 1 because there are m r number of received links so; that means, this 1 is y1, y2, y3 up to m r and each received link each signal that is received here is a combination of a transmitted signal from all the transmit antennas, the m rth received signal is also a combination of the signal that is received from all the transmit antennas.

So, with this way you could represent the received signal when, transmitted from multiple input multiple output system and this combination this particular final expression that we are writing over here is giving us the expression to take care of all situations of whether we have SIMO case or MISO case or MIMO case in case of SIMO if you are having SIMO case in this case single input multiple output; that means, m t is equal to 1 m r is equals to of course, m r.

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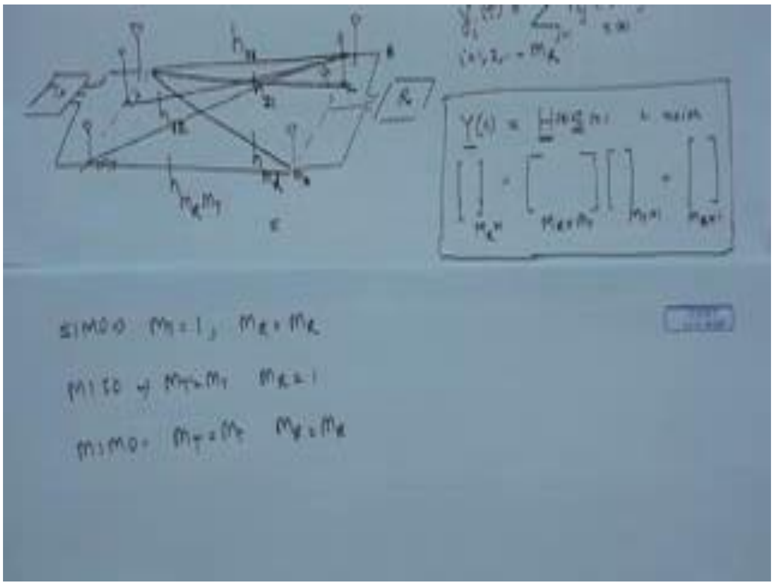

In case of MISO system you have m t equals to m t and m r equals to 1, in case of MIMO system you have m t equals to natural m t and m r equals to m r so; that means you have this particular matrix at hand which is useful. Whereas, in other cases just these numbers would help with this, so, if we take one particular example let say SIMO m t equals to 1; that means, this is only 1 that is transmitted whereas, m r equals to m r. So, we get all these values. If we take a MISO system; that means, m t equals to m t; that means this is row vector this is a column vector. So, this gets multiplied becomes 1 m r equals to 1. So, this is 1 cross one. So, we have a scalar. So, this particular expression or this particular structure takes care of all the three combinations that we have and our study of the MIMO communication system will be based on this particular model that we have just written down.

Thank you.