Fundamentals of MIMO Wireless Communication
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Lecture – 18
Spatial Channel Characteristics – I

Welcome to the course on fundamentals of MIMO wireless communications. We have been looking closely at this small scale propagation models right. Now we have covered the flat fading and frequency selective fading. So, we have covered the time and frequency dimensions.

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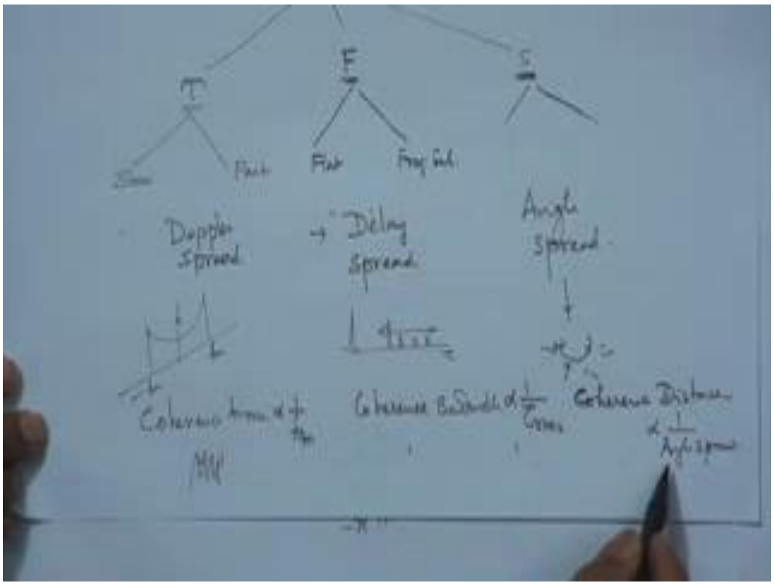

As you have noticed that we have specified the channel the wireless channel in the time dimension in the frequency dimension in the space dimension, last time we have said that in the time dimension we have classified it as slow fading and fast fading. We have described the details in frequency, we have also described there is flat fading and there is frequency selective fading. Now, we reach the special dimension which we are going to describe which would be porous catering or it is catering. So, we will see how this thing happens. So, to begin with we will base out expression on whatever we have studied over here. Based on that this will be helpful to describe, that is why these will become foundations. So, that is why we have spend quite lot of time on those topics.

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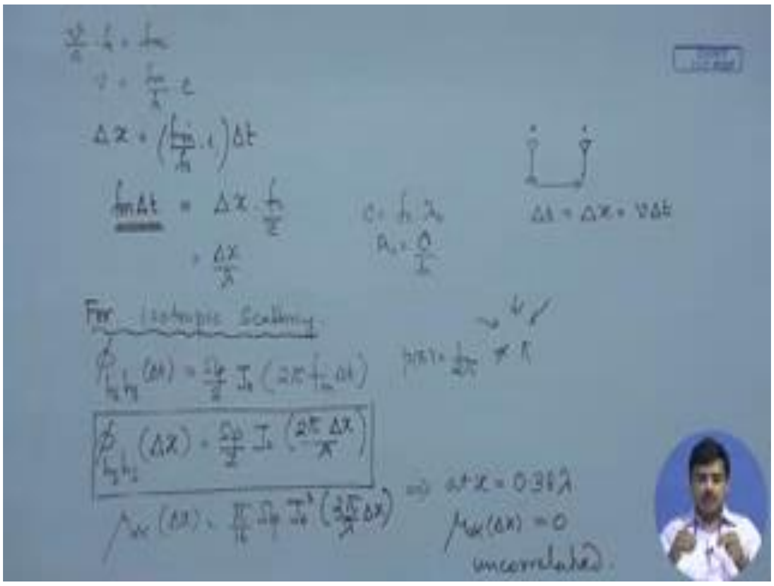

So, if we consider that the mobile is moving and let us say there is some measurement at this point, let us consider that these are receive antenna and that the mobile is moving in this direction and over certain time that is delta t. If we consider a certain time delta t in this duration of time the mobile would cross a distance let us say delta x which is very, very small in this unit of time. So, delta x is equal to v that is a speed of the vehicle times delta t. So, in this distance we receive antenna may have been at the location which is separated by delta x or the vehicle could have move toward there. So, since we know that v by c times f c equals to f m right therefore, we could also write v is equal to f m by f c times c and we could write delta x as f m by f c times t times delta t taking that delta x is equals to v times delta t. Therefore, delta x is v times delta t or we could also have f m delta t; that means, this times delta t is equal to delta x times f c divided by c and since c is equal to f c times lambda or lambda c.

So, you could write lambda c equals to c by f c or which is delta x upon lambda. So, this particular variable or the product which you have written over here is something which you have encountered before. So, you already know that for isotropic scattering we have phi h I h I delta t is equal to omega e by 2, where omega is the total received power j 0, j 0 is the Bessel function of the first kind of 0th order 2 pi f m delta t. This expression we had derived before for the correlation. So, basically when, we studied this flat fading condition and we were looking at the Doppler’s spectrum that is how it would go ahead. So, we studied the correlation function based on which we looked at the Fourier transform of that and we have Doppler's spectrum. So, this is a correlation function. So, now, we could instead of making the function of delta t we could write phi h I using a same expression a function of delta x instead of making the function of delta t by using this translation and which would become omega p by 2 which is 0 2 pi f m delta t is delta x upon lambda.

So, this basically tells us that if we are spaced apart by delta x then, the correlation of the signal at this point and this point could be expressed by this function provided, we have isotropic scattering; that means, provided there is isotropic scattering remember this was derived when p theta was taken to be equal to 1 by 2 pi; that means, there is equal probability of rays arriving from all directions. So, there would be equal probability of rays arriving from all directions under such condition.

We could use the already derived expression to arrive at the correlation of the received signal which are separated by a certain distance delta x as a function of lambda this is very, very important and if you would look at the correlation of alpha, alpha delta x alpha is the mod of h of t. So, that would be pi by 16 this as to normalising factor omega p j 0 square 2 pi by lambda delta x. So, this is also derived directly from the expression of correlation of alpha and similarly it will be for alpha squared. So, if you would look at this particular expression what you can get is that at delta x equals to at delta x equals to nearly 0.38 lambda nu of alpha alpha delta x is equal to 0.

So, from this is what you get so; that means, under such a conditions; that means, under the condition where there is isotropic scattering; that means, rays arriving from all directions with equal probability. If we have the spacing separated around nearly around lambda by 2 this is roughly lambda by 2 then, you get uncorrelated signals. So, if I space them at lambda by 2 this is very, very important results that we have. So, this we can remember that when we have 2 antennas and we are looking at these 2 antennas. So, when these 2 antennas are separated by lambda by 2 the signal in this antennas and the signal in this antennas would be uncorrelated the as we bring them closer and closer the signals are more and more correlated with each other. So, this is one fundamental result and we have derived this for the case where there is equal probability for arrival from all directions.

So, this with this we basically move. So, now, if we try to categorise it in terms of rich and poor scattering we could get situations where, we can have in the case that is rays arriving from all directions with equal probability.

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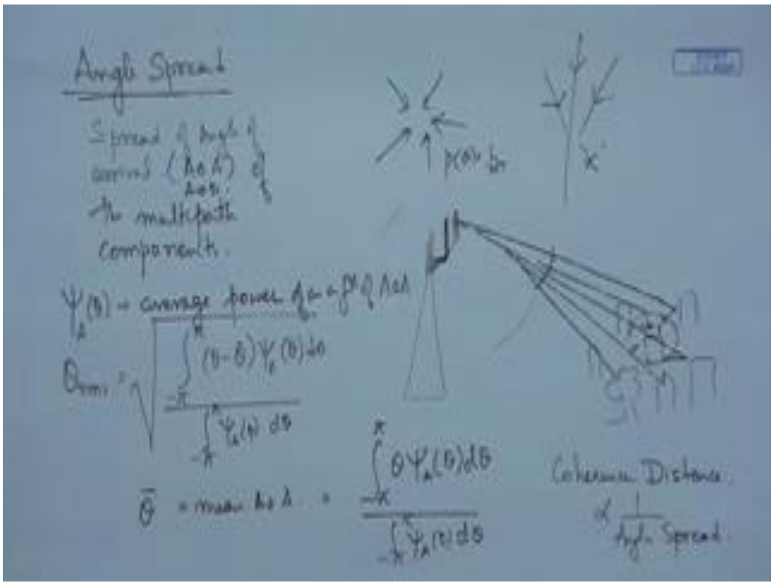

and they could be rays arriving at particular direction here p theta equals to 1 by 2 pi here p theta is not 1 by 2 pi and we have seen certain cases in one case we have seen there is a specular component that is there is k factor the other cases we have seen the Doppler spectrum to be Gaussian and not a Jakes spectrum. So, there are certain such situations which we have already seen for Doppler conditions. So, what we will now look at is the to describe this particular situation is described by means of angle spread see here, when we looked at dimension of time we had said there is when it is slow or fast fading we had a described in terms of the Doppler spread here, we had the term delay spread and similarly here we would have angle spread in this case we had transmitted a single tone, but that was received as a jakes spectrum when they was 2d isotropic scattering or some other ways here when, a delta function was transmitted what was received was echoes of the delta function in the delay access.

So, a delta function was spread in time and we had in the delay access we had the delay spread excess delay and here a even though there is transmission from one direction because of scattering from multiple directions you get an angular spread of the received signal you get an angular spread. So, these are the description in the 3 dimensions and here we will see what the effects of it are and how it is described. So, typically if you look at mobile station usually the base station is at height in most cases the base station is at height and the mobile is usually surrounded by buildings and lot of scatters that is usually the scenario; that means, the raise that come they get reflected from different scatters and so on. So, usually at the mobile location we are going to get signals arriving from all directions with equals to this is more or less true, but if you look at the base station I means suppose there are multiple antennas at the base station it is not true that rays are coming from all directions with equal probability so; that means, the angular spread at the mobile is not the same as the angular spread at the base station and this is very, very fundamental this is also very, very important in deriving spectral results.

So, we describe angular spread as the spread of angle of arrival as a spread of angle of arrival or A o A of the multipath components multipath components or the angle of departure. If it is the receiver it is angle of arrival if it is transmitter it is angle of departure. So, if psi a of theta is the average power of average power as a function of angle of arrival then, we could define theta root means square as square root of integrate from minus phi to phi theta minus theta bar the mean angle of arrival psi a; that means, the power at the receiver coming at different angles d the theta divided by the normalisation factor which is psi a of theta d theta and theta bar is the mean angle of arrival which could be again defined as minus phi to phi theta psi d theta again divided by the normalising factor psi a of theta d theta. So, this is this is basically the average power as a function of angle of arrival. So, as a function of theta you are normalising it by at the denominator and this particular thing would cause space selecting fading and usually you would define coherence distance.

You would define the term coherence distance see again, if we go here we had coherence time in the time analysis we had coherence bandwidth in the frequency analysis and here we have coherence distance in this space analysis. So, in coherence time we said that coherence time is inversely proportional to the Doppler's frequency, the max Doppler frequency coherence bandwidth is inversely proportional to r m s delay spread or coherence time is inversely proportional to the Doppler's spread coherence bandwidth is inversely proportional to the delay spread similarly coherence distance would be inversely proportional to the angle spread. This means as the Doppler frequency increases the coherence time decreases; that means, the faster heat fluctuates the faster heat fluctuates the distance of time for which it is coherent is less similarly over here. The more is the spread less is the coherence bandwidth here also it is talking about the same thing more is the distribution of the angle of arrival lesser is the coherence distance and less is the spread larger is the coherence distance.

So; that means, what we can say from this coherence distance is inversely proportional to angle to spread let us write this so; that means, we can say that at the receiver; that means, in this point where signals are arriving from all directions the coherence distance is smaller because angular spread is larger and here coherence distance is larger much larger because angular spread is less. So, at the mobile it is advantages for us that we can place 2 antennas which are close to each other. So, we can place 2 antennas which are close to each other and they are uncorrelated. So, when they are close to each other they would be uncorrelated and where as at the base station they need to be spaced far apart to get uncorrelated signal.

Now just look at the description that we give at this point is probably we required a larger coherence time we would like to have probably larger coherence time of course, there are merit's and demerit's again we may like to have a larger coherence bandwidth again with merit's and demerit's with coherence distance same does apply. That means, here in one case when, we are discussing equalization we said that if the signals expressing slow fading; that means, over the signal bandwidth there is no over the signal over the symbol duration there is no fluctuation and over the signal bandwidth there is no fluctuation; that means, if we would draw a signal time frequency grid in a way that let this be the frequency access.

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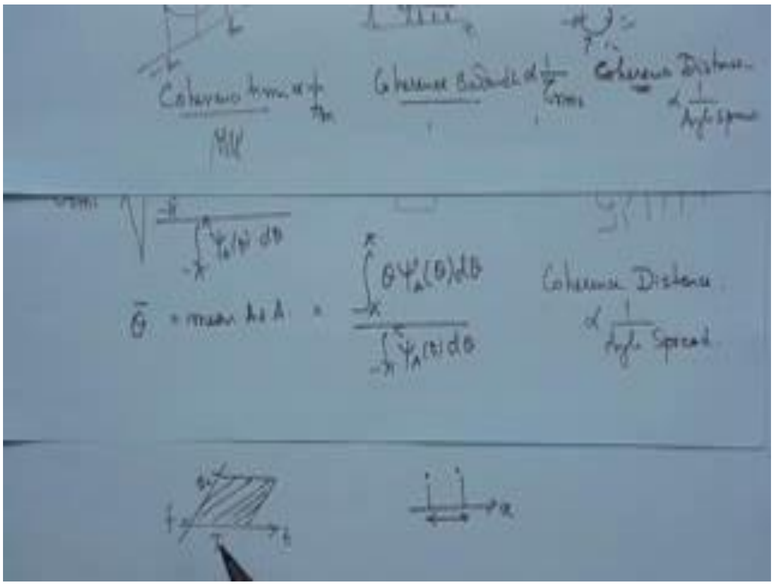

Let this be the time access. So, if this is the time frequency space a signal occupies and the channel is nearly flat over the space equalization would be easy. Whereas when we talk about the special dimension we have the x axis let us say. So, we are kind of finding out over what distance would the signals be uncorrelated is defined as the coherence distance. So, now, in this case what is of interest was is how close is it? The closer it is the better it is for us compare to the coherence time or coherence bandwidth situation here we would loved to have a longer coherence time because and a longer coherence bandwidth because we could do with easy equalization.

Of course, there are problems because that would mean once the signal is in a certain state at the because of the channel the signal or the channel would not come out of that state very fast if the coherence time is longer, if the coherence bandwidth is very large whereas, here the situation is not a function of a time of frequency it is with space. So, if coherence distance is small we could place 2 antennas close together. So, if we have a mobile which is let us say 6 inches or 5.5 inches wide and if we take the frequencies of operation let us say 24 100 mega hertz. So, we could easy place 2 antennas on that phone. Whereas at the base station we need to place them far apart, but again the distance would be manageable.

Now, this we would this is this is this is 1 of the very, very interesting things that we have at hand. Now if we look at this particular expression what we see is that it is also a function of lambda; that means, longer lambdas would mean smaller values over here. So, that would affect the coherence distance. So, there again what we get is a smaller very, very small values of lambda would be again the coherence distance is short compared to larger values of lambda. So, the frequencies also effecting the operation over in this particular dimension as well. So, usually we would find that at the base station the coherence distance at the base station; that means, this particular picture at the base station coherence distance is roughly let us say 5 lambda where is here it would be nearly 0.5 lambda roughly I mean that is how that is how it is. So, if you would take 2 d isotropic scattering at lambda by 2 it is uncorrelated at 0.25 this correlation becomes seventy percent or so.

So, we would like to have it would like to have lambda spacing sorry antennas spacing lambda by 2 in order to be uncorrelated where as at base station this distance becomes around 5 lambda or so, because angle of arrival is nearly around twenty degrees to thirty degrees in that range. So, this is 1 of the very important things that we are moving into when we look at the special channels. So, now, we would like to look at one particular characteristics of the channel and that is known as wide sense stationary uncorrelated scattering, because that we would require describing what is called homogeneous channel, and this is 1 of the important assumptions that is made in the study of wireless channels. So, if we look at the expression for the channel coefficient; that means, you usually write it has h t comma tau and we take h conjugate at t plus delta t comma tau.

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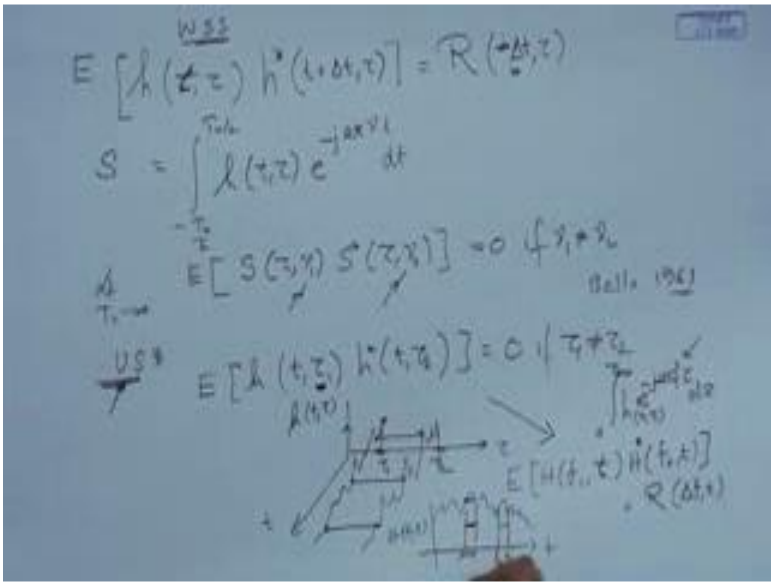

And we take the expectation of it what we are talking about is the correlation in time. So, this is t right and we said that this is r tau r r delta t comma tau; that means, this is not dependent on t it is dependent only on the lagged time between the 2 channel coefficients and if you take the Fourier transform and we would write the scatter function as s equal to minus t u by 2 to t u by 2 h of t comma tau e to the power of minus j 2 pi and let us put this frequency nu t d t then is how taken the Fourier transforms in the in the time domain and in the limit that t u tends to infinite we going to get and we take expectation of s tau common I am just changing the locations of the variables.

So, that it is easier s conjugate tau comma we have nu 1 and nu 2 at 2 different frequencies this would be equal to 0. If nu 1 is not equal to nu 2 this is 1 of the results that we are using this comes from wide sense stationary. If you would remember we applied wide sense stationary to arrive at this particular condition. Now again if you would use wide sense stationary on this and you look at the Fourier transform of this; that means, you looking at the scatter function take the expectation of it you are going to get this result. When nu 1 is not equal to nu 2 what does it mean is that 2 different a Doppler frequencies they are uncorrelated I mean; that means, the channel at 2 frequencies Doppler frequencies are uncorrelated and this is because of famous people by Bello 1963 those you are interested they can look at it and along with that there is also this uncorrelated scattering u s uncorrelated scattering which says that expectation of h of t comma tau 1 times h of t comma tau 2 is equal to 0 if tau 1 is not equal to tau 2. Now what it means is that if we look at this diagram which we are draw before if this is the tau axis and this is the t axis and this is h of t comma tau.

So, at tau 1 we have an impulse; that means we have send an impulse we have received an echo and this is fluctuating with time then at another tau 2 this also we have seen that is another echo which is coming. So, what it says that every instant of time at every instant of time, if we try to take the product and do then averaging the result is 0; that means, this and this are not correlated this 2 are uncorrelated. So, that is that is the result of uncorrelated scattering now this would give raise to the lag frequency correlation or expectation of h f 1 tau h 1 t times h conjugate of f t 2 t this is equal to the correlation function of delta f at t. This we have already seen what it means is that if we take the Fourier transform; that means, now we are taking the Fourier transform across this tau e to the power of minus j 2 pi f tau d theta integral 0 to tau max of h t comma tau. So, this what, we have to take. So, you are taking at 2 different frequencies. So, if we are getting it as a function of frequency we get the transform function the channel transform function.

So, if we make the assumptions that at 2 different delays the channels are uncorrelated the result we get is the frequency correlation function is not dependent on the frequency, but only on the lag; that means, if I take the Fourier transform of this access I am going to get the transform function; that means, h of f at a particular instant of time this is f. So, the correlation between this point in frequency and this point in frequency is dependent only on the gap and not on f 1 and f 2. So, again if I would take these 2 points in frequency f f 3 and f 4, if this gap and this gap is the same the correlation would be the same. So, that a lag frequency. So, this is the famous wide sense stationarity uncorrelated scattering channel model. So, with this we would move on to describe the homogeneous channel for special condition. So, basically here what we have talked about is second order stationarity in the time and frequency domain. So, that is what is described by wide sense stationarity uncorrelated scattering.

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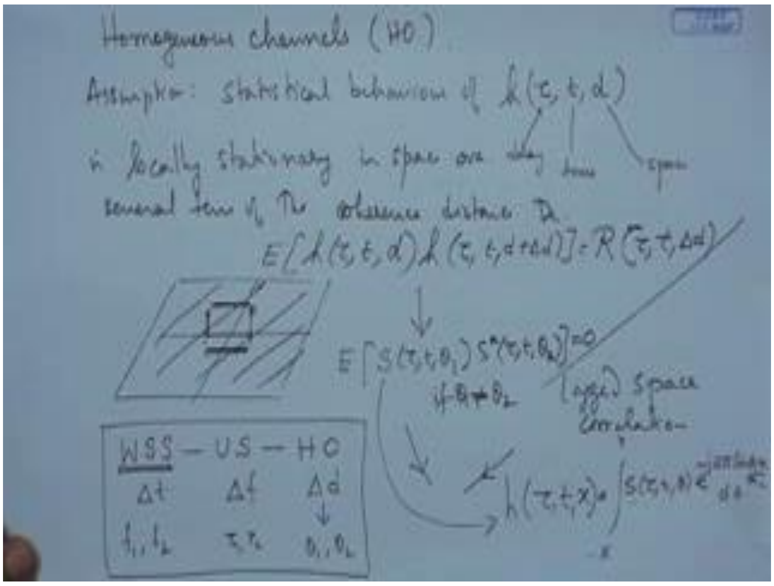

So, similarly there is also the second order stationarity in the space domain and that is that gives raise to the homogeneous channel the homogeneous channels. So, here the assumption that we make is that the statistical behaviour of h tau t d. Now we have the third dimension this is the delay this is the time this is the space is locally stationary in space over several tens of the coherence distance dc. So, previously we have said that the coherence distance is inversely proportional to the angular spread and now, what we are talking about is a homogeneous channel which says that over a certain region which is several tends of coherent distance.

Distance is of course, x axis or the y axis so; that means, we are talking about the y axis or x axis. So, it is the space. So, several tends of coherence distance in this what we have is homogeneous channel that is which is described in the form that e of h tau t d and h of tau t d plus delta d is equal to correlation function which is a function of tau t and delta d and not of d; that means, lag space correlation lagged space correlation; that means, over a certain region in space the correlation between 2 locations the signal at this location and the signal at this location which is separated on certain distance is not dependent on this location or this location, but only upon the separation.

So, that is what is homogeneous channel and this is this is very, very important, so along with wide sense stationarity. if we have wide sense stationarity which gives us that the correlation function is dependent only on the lag time and which gives raise to the two Doppler the channels a 2 Doppler frequencies are uncorrelated along with we have uncorrelated scattering which gives us that the channel coefficients are 2 delays are uncorrelated and that would bring raise to that would give us that the channels at 2 different frequencies are correlated only by the separation in frequencies and that is extended to homogeneous channels; that means, the 2 signals at 2 different points in space are correlated as a function of only the separation distance and nothing else.

So, here it is giving a raise to separation as delta t here it is separation in delta f and here it is separation of delta d. So, this again gives raise to the result that signals coming again s is the scatter function at 2 different theta; theta 1 and if I take this scatter function at tau 2 theta 2 this would be 0. If theta 1 is not equal to theta 2 so; that means, a signals coming at 2 different directions are uncorrelated that is what this gives results in so; that means, 2 different theta 1 signals at this 2 are uncorrelated signals at tau 1 and tau 2 are uncorrelated signals at f 1 2 different Doppler frequencies are uncorrelated.

So, that is what dual description of wide sense stationarity uncorrelated scattering homogeneous channel gives raise to. So, this particular channel s is described as h of tau t x instead of d we are writing x is equal to minus pi to pi s of tau t theta e to the power of minus j 2 pi sin theta times x by lambda c d theta. So, this is what we have the expression the relation between h and s. So, the Fourier transforms relationship between h and s. So, this is also known as a scatter function. So, this is the arena in which we will be discussing the rest of the course.

So, this is very, very important assumption that we will use in whatever analysis that we are going to do in this work.

Thank you.