Fundamentals of MIMO Wireless Communication

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Lecture – 17
FSF -Coherence Bandwidth, Delay Doppler Characteristics

Welcome to the lecture on fundamentals of MIMO Wireless Communications, we have currently discussing frequency selective fading we have already see n how frequency selective fading Fourier transform or the transfer function has to be found out. We will now look at the frequency characteristics of the signal and that will describe or give us a full description of frequency selectivity, and we will also try to define coherence bandwidth which is again another important parameter called such a Channel Propagation Model. With that we will come to a close of this small scale propagation models for a single input, single output system and a after that we need to proceed towards the MIMO channels.

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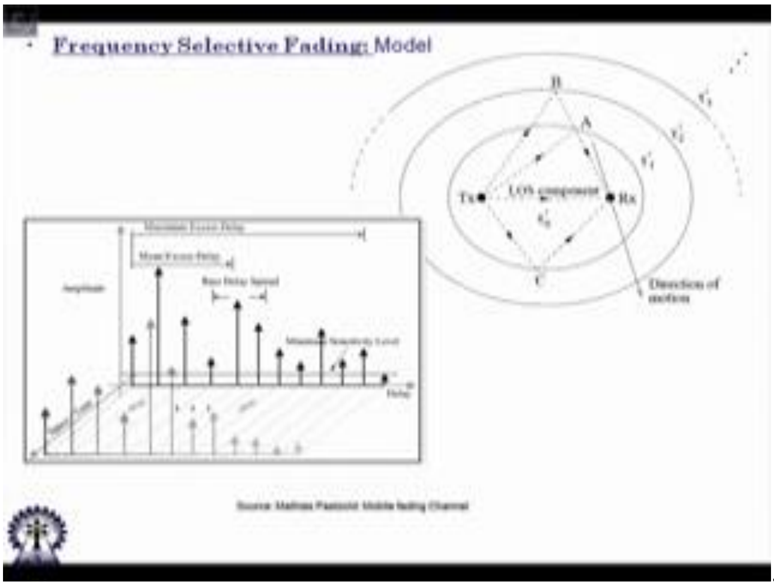

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So, this is the model what we have started looking at, we have described this particular model with the previous lecture.

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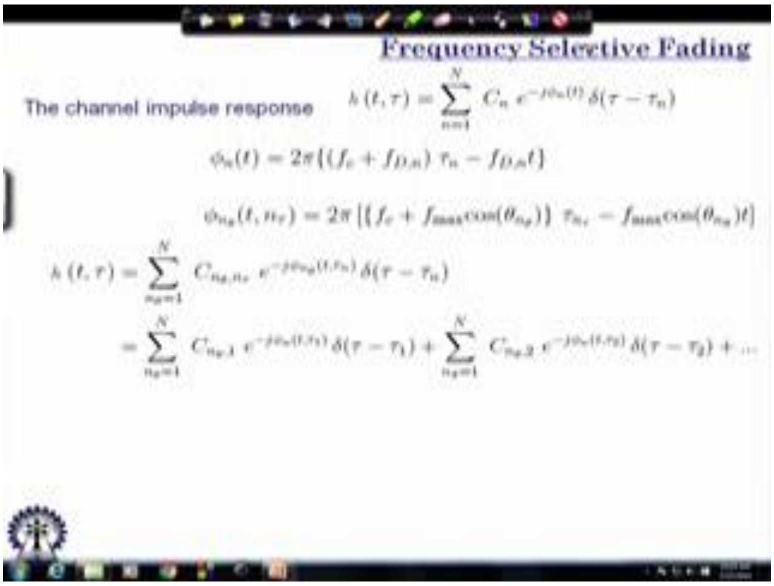

We have also seen how the channel impulse response looks like this is our standard classical description of channel impulse response, we have seen the phase what we have added to the description is now that description is a little more details, where we have said that well the first delay; first delay as given by this is also a some all the delays of all the paths that come at the first delay plus all the paths coming at the second delay and these 2 are getting separated because of delta function.

Since tau 1 is not equal to tau 2, we cannot add them up. So, here also that delta function separation was available. So, this is what we have described in the previous lecture.

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We want to see the frequency domain representations of we want to take the Fourier transform in a delay domain. And what we have is the discrete Fourier transform is what we are taking in this case I have also describe we could take the continue Fourier transform also, and this is how it would look like and the expression of the Fourier transform is the one that we have here. So, this is the expression that we have tine correction here would be tau 1 and tau 2, this is this is the right over here this is this is k over here

So, what we have here is basically the Fourier transform of the echo's coming at the first delay plus Fourier transform of the once coming at the second delay. So, what we last discuss is if we write in the in the discrete domain let us look at in the discrete domain itself this is equal to the Fourier transform of at time t of k because of tau 1 plus h of t k because of tau 2 and. So, on up to Fourier transform of in the k th frequency because of tau n plus dot, dot, dot plus Fourier transform of at time t k tau n max; that means, a maximum resolvable delays. So, we just remember there are 2 things one is n, which is ideally to be n tau n. So, basically the n we have used in this is ideally to be n sorry it is suppose to be n suffix tau n; that means, the number of multi paths at the n th delay; that means, the first delay would have 6 to 10 multi paths let say the second delay could have different number 1, 2, 3, 4, 5, 6, 7 this is second ellipse the third one could have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

So, basically this is the third delay. So, what we can say is n tau 1 let say in this particular example is 10 n tau 2 could be, let say 20 n tau 3 would be let say n 15 and. So, this could be variation. So, we should keep in mind and this is the number of resolvable delay. So, we could go up to n tau 6 we could. So, this is basically indicated by n tau max and this is indicated by n ideally speaking there should tau n in the suffix, but we are omitting it for simplicity. So, we should keep this in mind.

So, if we proceed what we just said is if there is a single multi path we have already seen it what would happen that would result in flat fading, but now we have addition of several such once. So, what I will show you now is the frequency response picture of a sample realization. So, remember this particular one that we have over here, yeah this particular one is a sample realization mind it at a function of time. So, if I say h of t prime where t prime is not equal to t comma k; that means, k th frequency we will get h of t prime comma let say tau 1 for the first path and this is not equal to h t prime, because there are different time instance. So, that we understand. So, same would have to happen with the frequency selective fading. So, that is why we say we are looking at a single snapshot proceed with this of course, we are given this description.

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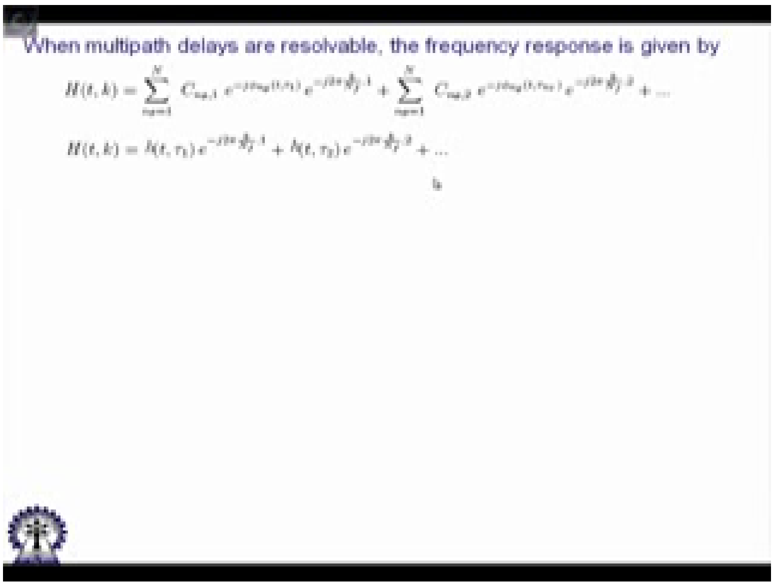

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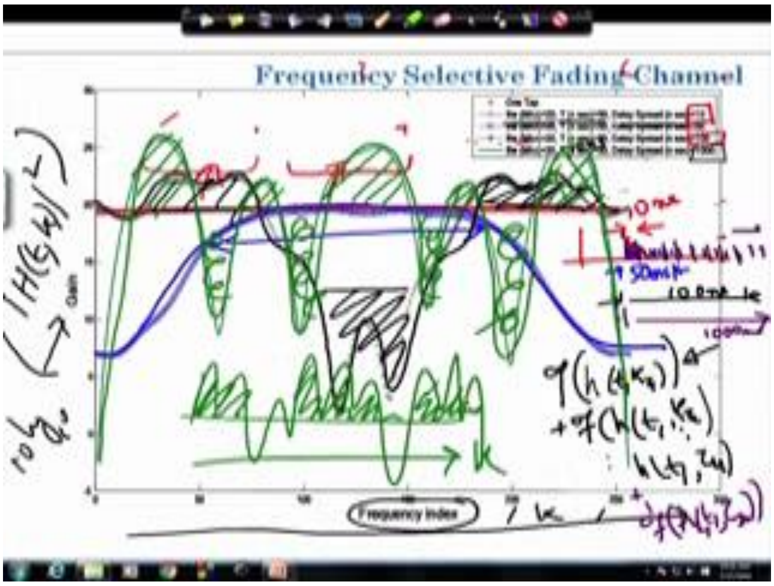

So, now suppose I have taken one particular situation and this situation is described by certain parameters which is not very much necessary at this point of time we can look at them later through an example. So, when there is only one path or one resolvable path we have seen it is flat across frequency. So, what you are going to see is this particular one that is in red color across the frequency index. So, this is the frequency index clearly the x axis with k axis frequency say and is the gain; that means, you are basically talking of h of t comma k squared and this is in dB. So, basically 10 log 10 of this function, this that is the x axis this is the x axis. Whereas, if you describe it shortly we have taken bandwidth of 20 mega hertz r m s sorry this 50 nano second as the spacing and r m s delay speed of 10 nanosecond we will describe more of this. So, this is what we get if we move further what we get is what we get is the second situation where we say the delay spread has increased if we look at this the delay spread has increased in this particular case.

So, I go back a little bit allow me. So, in the in the second case what we have is delay spread from 10 nanosecond has increased to 50 nanosecond, what does that mean? Previously suppose this is an echo if this is impulse that I have sent the delay spread is very, very small delay and this width is 10 nanoseconds. Where is in the second case where it is 50 this number is 50. So, if I clear it up if you look at this number here I take a different color if you can if this is readable 10 and 50 the second one is 50 in that case let the first echo come over here and then there are echo's.

So, this width is now 50 nanoseconds. So, how can this be 50 nanosecond this can be 50 nanosecond, if the reflectors or scatterers are spread in such a way that if this is the transmitted this is the receiver, if this is an ellipse in the first case everything is almost-almost here in the first case. In the second case reflector are slightly oriented in way there are multi paths which are having longer delays. So, this is 50 nanosecond right so; that means, the spread is more, if the spread is more we are going to add up more resolvable paths; that means, if we look at the previous picture; that means, we are getting more resolvable paths, we are getting more resolvable paths as we are seeing this plus this, so more, more and more.

So, this gets added if delay is less if delay is less you have only one you have flat if there are more number of paths; that means, delays are more. More number of resolvable delays if tau 1 tau 2 and so on. If this increases this spread increase. So, as this spread increases what we get is the situation represented by this particular figure, by this plot that is the frequency response that is happening. If we move on further and we say that we have a possibility where I am erasing this for visibility and yes now we have seen – for, 100 nanosecond which I will try to explain with different color; if it is 100 nanoseconds it means that in the response is due to an impulse, impulse is launched at here is echo's are coming in 100 nanoseconds; that means, echo's are coming for 100 nanosecond.

So, rather it should be black in color which is indicated by this black line. So, if I would draw it with corresponding colors, we could have a better picture could have the better picture in the sense that the first one, I would say echo impulse launched the echo's are here. So, this gap is 10 nanosecond the second case I would choose a different color and say that echo's are now spread to 50 nanosecond, right in the third case which is indicated by the black one I would have echo's which are coming from faraway places; that means, there are reflectors or scatterer's which are located far away. So, this distance is 100 nanosecond this means that, we have more things to add up; that means, we will have what we have is h of the Fourier transform of h of t at the frequency at delay tau 1 plus Fourier transform of h at time t 1 h k at tau 2 and this for the red case is only limited to one for the blue case it is possibly limited to 2 for the black case it is possibly limited to 3 or 4; that means, h t one comma tau 4, let say and so on. And as we increase this length this line is now having tau the delay spread as 1000 so; that means, the impulse if we could draw with let say yellow color.

Now, yellow is probably not nicely visible with this violet color let say. Echo starts here echo skip coming for and it goes on for a duration when, this distance goes beyond this particular screen is 1000 nanosecond so; that means, you will have several such things getting added plus dot, dot, dot Fourier transform of h 1, h t 1 comma tau let say 20; that means, there are 20 resolvable multi path for example. So, what we observe is as we increase this delay, what we are getting is a for the first case when there was no single there was a single resolvable delay this is a flat fading the second case when there are let say 2 or little bit close 2 we have somewhat variability in the frequency domain; that means, some white portion of the frequency is getting selectively pass through while the others are not in the next case what we have is this particular contour indicating that there is some portion of the frequency which is getting signals not attenuated much where as there is another portion where the signal is attenuated heavily.

The third case that we are going to see is or the forth case is when the signal is fluctuating according to the line that I am facing screen has become very, very dirty. So, there is much more fluctuation in the frequency domain, there is selectivity these shaded bands are the ones which are having less attenuation, they are passing the signal properly where as this region is having attenuation let say relative attenuation. So, what we see as we increase the path. So, as we increase the delay the selectivity in the frequency range in the frequency increases; that means, there is more crossing, there is more number of crossing if we have a thresh holding the frequency axis there is more number of crossing and some portions of the frequency axis this is the frequency axis are selectively allowing the signals to pass through and hence this is known as frequency selective fading.

In this same tone, I would like to measure I would like to mention that the time selective fading there it was instead of p instead of k that was the time axis and things looked very similar and we had a time selective fading, in time selective fading what we remember we had discussed certain portion of the time signals are passing through less attenuation certain portion there was more attenuation and we describe level crossing rate and average duration of fade. So, here also similar thing applies in the frequency domain and we move quickly to the description.

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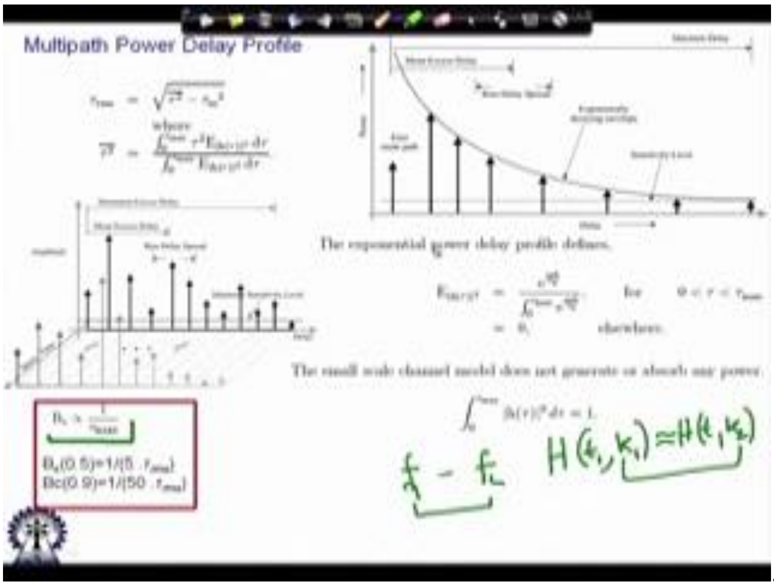

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So, along with this multi path power delay profile we would describe with something known as the coherence bandwidth this is very, very important. So, coherence bandwidth is described as the portion of bandwidth where the signal remains coherence with itself just like we have defined coherence time now we are defining it in the frequency domain; that means, single is coherent in the frequency domain, if I take to frequencies f one and f 2 over these 2 frequencies if the if the signal is quite same; that means, if h of t one let say it is the time and instead of f let me take k 1; that means, one of the frequency is is almost equal to h of time and k 2 that is what we have used k; that means, this k 1 and k 2 are within the coherence bandwidth. We have a way of defining it as of now we will describe it very briefly there are many detail description to this also. So, what we clearly see in the previous slide in the previous picture, what we have seen is as your number of paths increase or your delay increases your frequency selectivity increases. So, as your delay increases naturally the delay spread is going to increase.

So, as you delay spread is increasing the coherence of the signal across the bandwidth decreases. So, if you take the first case coherence across this entire bandwidth is there. So, coherence bandwidth is very large and delay spread is very, very small because there is hardly spread in the delay. In the second case what we have is in the frequency domain there is some more fluctuation delay is little bit more in the frequency domain the coherence has reduced, if we take the black case then there is more delay; that means, it is become 100 from previous case of 50 and 10.

The coherence in the frequency axis is less now that can be visibly seen over here and as we move to the case of 1000, what we clearly see is that coherence bandwidth visibly it has become less. So, one way of quantifying this is another way of describing the frequency selectivity of the channel 1 is r m s delay spread the other is this particular definition of coherence bandwidth, where coherence bandwidth is defined as inversely propositional to r m s delays spread, and there are 2 coherence bandwidth descriptions the 50 percent coherence bandwidth description and the 90 percent coherence bandwidth is descriptions, these are exactly similar to the coherence time description the philosophically they are the same.

So, the 50 percent coherence bandwidth is given by this expression one by 5 time tau r m s and 90 percent coherence bandwidth is one over 50 times r ms; that means, once we calculate the delay spread one divided by 50 times r m s delay spread is basically the portion of bandwidth where signal is 90 percent correlated with each other. Whereas 1 by 5 tau r m s is the portion of the bandwidth where signal is 50 percent correlated with each other. So, 2 important definitions, if we want the signal to be nearly constant across frequency we would find the bandwidth given by 90 percent coherence bandwidth this 90 percent coherence bandwidth is across which the signal is nearly flat. So, over this bandwidth you have flat fading you can say that is a good approximation over the 50 percent coherence bandwidth there is some similarity, but there is some variation also in the signal.

So, with r m s delay spread and with coherence bandwidth, we can describe the channel characteristics of a frequency selective fading.

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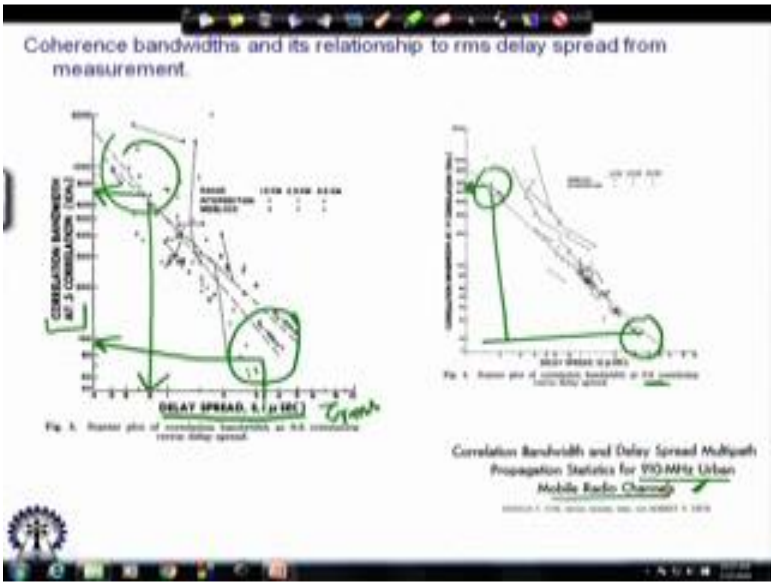

Ah just to support whatever we have discussed this particular slide contains some results taken from papers which are from Donald Cocks and others. This particular picture represents this particular picture shows us the x axis as delay spread so; that means, tau r m s and this axis is the coherence bandwidth that 50 percent coherence bandwidth see point 5 correlation is 50 percent coherence bandwidth. So, one is the measured versus some theoretical description of it, what we clearly see is as the delay spread increases the coherence bandwidth decreases.

So, what you see over is as see if you look at this portion delay spread is large coherence bandwidth is small we large delay spread small coherence bandwidth. If we look at this portion of the curve, delay spread is small coherence bandwidth is large this is matching measurements with theoretical values. This is the 90 percent coherence bandwidth description again it is quite similar what you say for large delay spread smaller coherence bandwidth smaller delay spread a larger coherence bandwidth. So, that is as per description and matching, matching with results this is for this 9, 100, 10 mega hertz urban mobile radio channel; that means, in the city area in 9, 100, 10 mega hertz of center frequency.

So, what we see is that these descriptions are supported by measurements also. So, we are actually following proper model, we are not without model finally, towards the end because there are many things to discuss.

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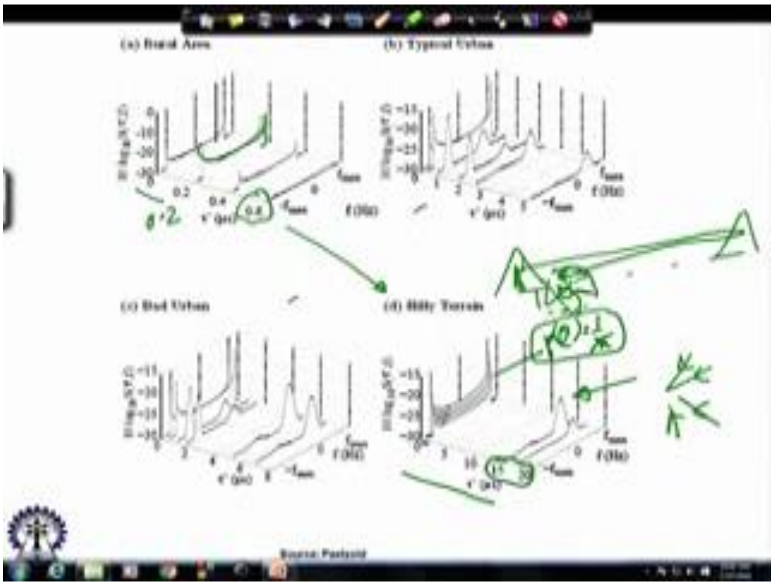

But we will directly go into something known as this scatter plot what we talk about in this particular slide is the total description of the channel in the delay Doppler domain. So; that means, we have seen the delay domain it is the tau axis see these are all tau axis tau is mentioned over here this we have seen, the tau axis. What I had drawn previously was the time axis; that means, I have drawn the time axis and I had said that signal would fluctuate in time this is the tau axis, at another delay signal would fluctuate in time another delay signal would fluctuate in time.

Now, we said that this delay could have 10 multi paths, this delay could have 20 this could have 15 and so on and so forth, and also this delay could have a p theta which is equal to one by 2 phi. Where as this one could be the one which has a specular component this one could have a different p theta not equal to one by 2 pi they could be different. Now p theta affix the Doppler that is what we have seen because well calculating the Doppler we were calculating phi of h h of delta t. While we calculated phi phi of h h of delta t we had e of theta, now while having e of theta we made the assumption that let g of theta; that means, antenna gain equals to one and p theta equals to one by 2 pi that is 2 d isotropic a scattering model and there we said this gives raise j spectrum that is what we have seen before. So, here instead of this time variation which is fluctuating in time, if we look at the Doppler's spectrum which we expect to be the remaining static over a region and over certain duration of time, because it is the second order statistics. So, it is not the instantaneous coefficients.

So, this is one way of looking at the channel. So, in the tau axis we will take a look at the average power in the in the frequency axis we will take look at this spectrum. So, what we see in this particular picture which is the rural area diagram where its says at the first echo the Doppler spectrum is a js, along with it there is a specular component; that means, it is rise it is the vision. At the second delay which is at 0.2 micro second we have j spectrum and so on and so forth. Whereas if we take look at an opposite one I mean these are also similar this and these are similar. So, we take look at another one there are lot of echo's coming in very short duration and after a very large duration if you at this is around 15 to 20 microsecond where as this is around point 6 microsecond this is because it is a hilly region there is hill across far away there is another hill. So, it takes long time for the signal to get reflected and come back to the receiver.

So, mobile this is the transmitter right. So, in the in the reflectors which are close by because there are let say houses around the mobile for example, things gets scattered almost quite well from all the directions what you have is a j spectrum because in these cases p theta is equal to 1 by 2 pi. Whereas here the one that comes reflected from a far away reflector which could be a hill, it is caused indicating it is not p theta is not equal to one by 2 pi; that means, probably p of rays coming from all direction is not equal the rays are coming from a specific direction and the spectrum is given by the gauss spectrum which we had seen earlier. So, with this kind of a diagram you can almost completely characterize the single input single output channel, and look at how what would be the signal experiencing. There are many details to it, but because of course, our interest we would limit ourselves to this without this description it is difficult to proceed with the other things, but this is just the initial part of it there are many more details which can be studied we would simply like to stop at this point with the very short description that we have seen the small scale fading, where we have seen time selective frequency selective in time.

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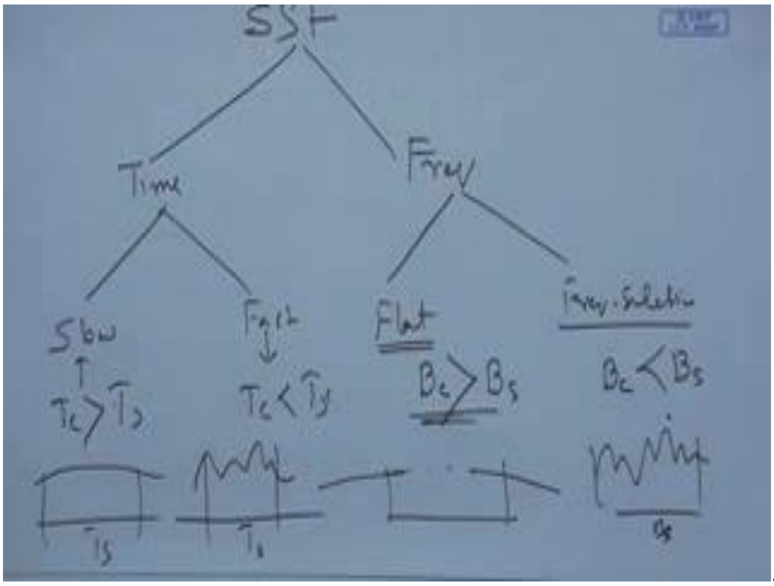

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We have seen slow fading and fast fading; slow fading is one where coherence time is greater than symbol duration fast fading is one, where coherence time is small than smaller than symbol duration. In frequency selective fading we have seen there are 2 cases one is flat one is frequency selective, flat is the case which we could define as the coherence bandwidth is larger than this system bandwidth and frequency selective is the case where coherence bandwidth is smaller than the system bandwidth. Then is within the system bandwidth if this is your system bandwidth, channel fluctuates in frequency and this is the case within the system bandwidth the channel is nearly constant it is fluctuating later. Here within the symbol duration channel is nearly constant here beyond the symbol duration or within the symbol duration the signal is fluctuating.

With this we would bring our discussion on frequency selective and frequency flat fading channels towards an end. There would be a certain numerical which we will try to provide as additional material which would be useful for this particular part.

Thank you.