Fundamentals of MIMO Wireless Communication

Prof. Suvra Sekhar Das
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 16
Frequency Selective Fading – II

Welcome to the course on Fundamentals of MIMO Wireless Communications, we are currently discussing small scale propagation model. We have covered flat fading and now we are discussing multipath propagation and frequency selective fading. We have given you a brief overview about frequency selective fading in the previous lecture, we will today go into the details of it and see what give raise to frequency selectivity.

(Refer Slide Time: 00:43)

So, the typical model for frequency selectivity that we have taken is as in this particular image and we have explained in the previous discussion that these concentric ellipses, these different ellipses that we have they basically provide the distance at which reflectors are presented. So, if we take the first ellipse; this conforms to a certain delay let us say tau 1 and the second ellipse conforms to a second set of delays; that is tau 2, which are resolvable; that means, they are separate and the third ellipse conforms to a set of delay which tau 3 as also indicated by these. You have also explained that suppose, you have an impulse in the beginning and then because of the impulse what you get is echo's. So, there is an echo coming at the first delay tau 1, then there is a echo at tau 1 and so on and these are echo's due to the delays, so tau 1 delay what we explained is all the multipaths that come over here.

So, in other words what we have is this one; that is at the tau 1, one that comes at tau 1 is basically all the multipaths that a come at this particular delay. So, basically there are actually several multipaths, there are coming at tau 1, there are multipaths which are coming at tau 2. These are coming from different directions and the one that is here is because of a larger propagation and they are all due to the same propagation delay and so on and so forth.

So, this is what we have explained in the previous class and we have also drawn a diagram, where we try to explain that suppose this is my t axis; this is the t axis that we have drawn and if we take one of these; that mean, let us take this particular multipath. What we have studied in the previous lectures is that, this h of t comma tau fluctuates with time in a random fashion as in this. The second one would also fluctuate randomly in time, so if this is the delay axis; so at delay tau 1, there would be similar fluctuation; that means, if we were sending a continuous wave, this was the fluctuation we would get at a particular delay, this is sorry; this is at tau 2, this is at tau 1 and instead what we had also done is instead of drawing it this way, we could draw it in a different; that means, if we have time as this axis and this as the delay axis; this tau axis and at the first delay there would be multipaths coming, which would fluctuate with time whose envelope would fluctuate in this fashion; at the second delay that is means this is a tau 1, tau 2; this is going to fluctuate.

We have actually studied this particular thing that is what we explained now we have studied with Doppler characteristics, we have studied the distribution; we have studied the correlation properties. So, we move a step beyond that and what we have is these paths are now resolvable as we have already said. So, instead of sending continuous wave at this particular stage, we send an impulse, so basically there is an impulse. So, if there is an impulse then we are going to get these echos and if we repeat the experiment in time then this in time, if I launch another impulse in time and this is the delay axis here; that means, at the same delay we are going to get a certain echo, at the same delay will going to get another echo, at this delay will get another echo and so on and so forth. So, if we repeat the experiment at another time, at this time suppose we have launched an impulse and we are studying it across the delay axis. We are going to get an echo for tau 1, we are going to get an echo for tau 2, and we going to echo get an echo tau 3.

Now, what we have is these values; that means, this particular one, this is h of t 1 comma tau 1, this corresponds to h of t 2 comma tau 2, this sorry corresponds to t 1 comma tau 2; sorry I made a mistake, not this line; this line is supposed to be for this one. For this one it is h of t 1 comma tau 3, now this t 1 is the instant of the time when this echo is launched, so this is the experiment type at that instant of time we going to get these and we are kind of assuming that this time has a higher resolution than this time.

So, this is little bit confusing, so it is kind of; this you can say that is the time when impulse is launched. So, definitely when things have propagated, so if I clean this up better image, so we have launched an impulse at t 1, the echo must have come at t 1 plus tau 1, this echo will come at t 1 plus tau 2, this echo is going to come at t 1 plus tau 3. So, you are splitting these two variables and we are saying that this echo is because of the impulse launched at the time t 1; this echo because of the impulse launched at the time t 1, so that is a separation that we are doing.

In other words; you can say that you are taking away t 1 from one of this that is also another way of looking at it. So, with this let us proceed with our discussion, so we cleaned it up and so let us look at another instant of time, which is at time t 2, at time t 2; if we launch the experiment, we are going to get an echo here which would be h of impulse launched at t 2, this is the impulse; which is launched t time t 2 and the echo received at a delay of tau 1. So, we will look at this and we will look at this both have the delay tau 1, see both have the delay tau 1 it is basically the same channel coefficient, which is being studied at two different time instance, that is t 1 and t 2.

If we look at this, we are basically talking of h t 1, tau 1 changing over to h t 2, tau 1. This we have already studied under flat fading, so everything that we have studied under flat fading would apply to this. So, same would happen for this delay, same would happen for this delay, this delay and all the delays right. So, when this happens the impulse response, if I clean it up the impulse response that we are going to get at another time would be an echo here and echo here, an echo there and echo there and echo there and so on and so forth.

So, this value and this value will differ by a certain amount, this value and this value will differ by certain amount, this and this will differ by certain. The difference would be because of this delay in time which is t 2 minus t 1, what you can easily guess is if this time is less than the coherence time which we have discussed then these two values would almost be same as each other. If this is less then coherence time then this value and this value would be almost equal to each other that is h of t 1 comma tau n; I write in general is almost equal to h of t 2 comma tau n. In other words or rather be very specific if we write it if it is less and lesser whereas, if this distance in time is much much greater than coherence time, these values will be different. So, if there was a continuous measurement, you would get fluctuations and if the variability is very fast this is the kind of picture with the variability is slow, the channel is going to fluctuate slowly.

So, this we have to clearly understand if we understand this, we have understood one of the most important things of this multiple propagation in time and delay axis. So, will go ahead with this description further and will say that this dark colored line; the dark colored images are basically response because of an impulse launched at this particular instant of time therefore, these ones are basically the channel impulse response. So, to read it; it is the response of the channel, so the channel is there you launched at impulse of the transmitter and this is what you have received. So, it is the response of the channel due to an impulse launched at the transmitter, this is a channel impulse response; what we had seen before is that, this is basically you have launched an impulse, what we have seen before; I will draw with the different color. You had launched an impulse all the echos have arrived at the same delay, they have all arrived at the same delay and that was tau cap. Now here the delays are resolvable, that is a fundamental difference between what we have studied before and what we have studying now.

So, there also it was channel impulse response, but the impulse response was not spread in time, it was located at one location of time because all the reflectors where on one of the ellipse, with this we go ahead further this are a very important description. So, another important thing I will just quickly mention here is that when we study such things along with the impulse response, so now we look at impulse response term; response due to an impulse. Typically, if you studied filters, you would talk about filters impulse response; that means, you launch an impulse into the filter and what is the response you get. So, which looks similar and what you can see is, it is almost looking like an fir filter response, finite impulse response filter and mostly that is why we can model this propagation channels thus finite impulse response filters. We have drawn such a transversal filter diagram earlier when we are giving the description above the flat fading contribution and we will take a look at that once again.

So, as if this is an impulse response, so when you study a filter along with the impulse response, we also study the transfer function. What is the transfer function? Transfer function is basically the Fourier transform of the impulse response that you have already studied in other course in signal processing. So, in this case look at this what we is h of t comma tau is the impulse response, we will describe it; how to describe it, we will look at it and you take the Fourier transform of it, so when you take the Fourier transform; you do not take it across the t, you take it across this delay axis because in filters what you have studied is you have studied filter impulse response of h of tau and there was no t in it because it was a time in variant response, here what you see is if I would launch the impulse here; the response that I am going to get at different delays would be different than the situation when impulse was launched at an earlier time.

So, if I look at the impulse response at time t 1, impulse response at time t 2 they would be different and hence we are going time varying response. So, therefore, the transfer function which relates the input and the output in the form that the let us say; s of f; that is the output f is equal to the transfer function, it is let us say h of f times the input or let us say x of x f, so that is how you would relate the transfer function, so the same thing applies; only thing is that here there is a time factor which should get associated.

So, this we should remember and will use it later on, so with this let us progress further in our discussion. So, what we have described in the previous lecture is the maximum excess delay this particular term we defined it very very clearly in the previous lectures; I would recommend that you take a look at that and now will describe certain other things. So, if we look at this impulse response, it is then needs a certain way of describing this impulse response and there it is, there is a method which does like power delay profile; which we look at power delay profile is a method of describing this impulse response; that means, what is the power or the average power delay profile is the better term what is the power at a certain delay the average power.

So, if this is the instead of amplitude if I would write amplitude square, I am going to get the power access. So, basically what we are talking about is h of t comma tau mod squared and the expected value of it. So, if we take the expected value expectation about time because this stochastic process, so this thing goes away; so what we are left with e of h of tau square; that means, on an average what is the power of the signal at certain tau t 1, at a certain tau t 2, at a certain tau t 3 and so on.

This is one way explaining it; we will look at certain power delay profiles. Along with this and this is not a very easy description because you need to describe so many things, you need to describe this versus; the time. So, basically you will be giving this tau axis, tau table and e of h tau mod square, so you will be giving these values through which will be describing the power delay profile. Now beyond that we should be able to describe it much much easier fashion and it is described in easier fashion by using certain other single variables. So, one of them is the r m s delay spread, so we have already seen that this is having an excess delay of 0 because this is where the first time this signal is appearing; this distance is the first axis delay. So, basically this is t 2, tau 2 minus tau 1; if we proceed we are going to go to the last point which is tau n minus tau 1.

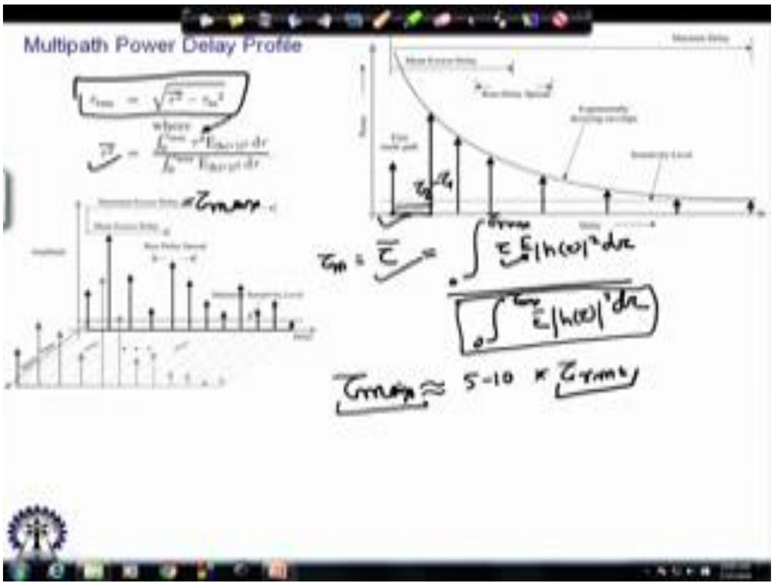
So, when we look at this; what we see is that the impulse which was sent, how is an impulse; impulse is almost occupying negligible amount of time; very very small amount of time and it is going almost theoretically to infinity; however, the area under the curve is equal to 1, so finite energy (Refer Time: 17:11). So, the impulse when it is sent, what we receive is not an impulse anymore; what we see is the impulse is spread from this location of time to this location of time, it has become wide, it has become spread.

So, to measure the spread; we would use the term, I would like to measure the delay which was almost 0 in an impulse. Delay has now becomes spread out, that is why delay spread, what is the root mean squared of the delay is what we need to measure that is one of the important ways of describing such a profile. By a single parameter we can say that on an average what is root mean square of the spread that an impulse is getting, this is one way of describing it. I will draw quick reference to our earlier discussion on flat fading, in flat fading we had studied the flat fade using s tilde t is equal to 1; that means, we had s of t is equal to cos; 2 pi f c that was the single tone. If you recall what we have studied is, we have studied the correlation properties that is pi, h, h delta t is what we have studied; from this we have taken the Fourier transform and we found s h h f that is the Fourier transform of the correlation function which is gives us the Doppler spectrum. So that means, this f c got modified to f c plus f d n and this n range from n equals to 1 to capital N so; that means, there was a whole bunch of frequencies which is now getting received.

So, instead of receiving only f c; you are now receiving f c plus f d max or f m and f c, this is the upper range and f c minus f m. So, even if a single tone was sent, we were receiving a whole range of frequencies around f c; that means, a frequency was getting spread and that is why we had called this the Doppler's spread so; that means, a single frequency was getting spread in frequency domain, what we have over here is an impulse is getting spread in the time domain. So, we are getting spreading in this case in the delay access, in the earlier case we had spreading in the frequency access. So, moving ahead further; we would describe this r m s delay spread and there is also a mean axis delay that is important in the description.

So quickly let us take a look at how we proceed with the description briefly I will describe this particular thing and then will go back.

(Refer Slide Time: 20:06)



So, this multipath profile we have already described and the r m s delay spread as we are trying to say, we describe in terms of square root of tau squared bar minus tau m squared. I will describe what is tau m; tau m is basically tau bar, tau bar would be integration of 0 to tau max, tau e of h tau squared; d tau divided by this normalizing factor tau max e of each tau squared d tau. If we define tau bar, if you look at the expression it is tau that is axis delay; so this is having an axis delay of 0, this is having an axis delay of tau 2 minus tau 1. So, that is what we are talking about here h tau square that is the average power at that particular delay, so basically we are weighing the delay with the average power and of course, its starts with 0, this is having a delay of 0. So, this is not having any weight, this is having a weight, this is having the value which is relative to the starting point and denominator is the energy in the impulse response of the channel.

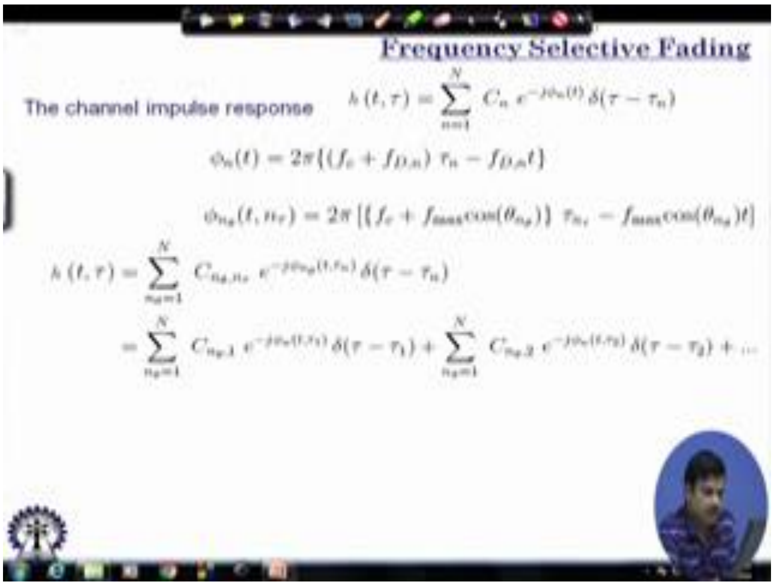
So, this is how we would be measure tau bar, this is how would measure tau squared bar, so the difference is here you have tau squared and the root means squared can be calculated in this method, so this is how you would carry calculate the r m s delay spread. So, once you calculate the r m s delay spread; you can characterize the propagation channel, so this one you would get a single number which will tell you if an impulse is send how much is the spread of the impulse.

This one of the very important characteristics of such a channel and there is also rule of thumb, which would connect and say the tau max that is the maximum axis delay this is the value which is tau max, rule of thumb you could say approximately 5 to 10 times, I am giving a very very large range but we cannot help it, this is how it is varying from indoor to outdoor conditions; times tau r m s.

So, if I give you the root mean squared value the delay squared, I am basically characterizing the channel, we can calculate how much is the maximum spread of the channel through this rule of thumb and of course this is specific to propagation environment, some propagation environments may have axis delay of around 5 times the r m s delay spread, sometimes it is 10 times; the r m s delay spread. There are many more details to it, which are not very relevant for us in this particular subject, but it is very important in general for a wireless communications, where I would just briefly touch upon that and you can find out more information. This value r m s delay spread is not a constant value, it is also probabilistic, and it changes for the particular environment from location to location.

So, what we are calculating is the average value; if you carry out experiments is one small location do it in another small location, we will be giving it one average value for a specific location, but if you are designing a communication system which is really advanced and which you takes advantage of this variations, then we should keep this in mind that this although we are specifying it as an average for one area, it can be varying within a larger area and there are detail measurements on this which you can find out when you study more about the propagation channels.

(Refer Slide Time: 23:56)

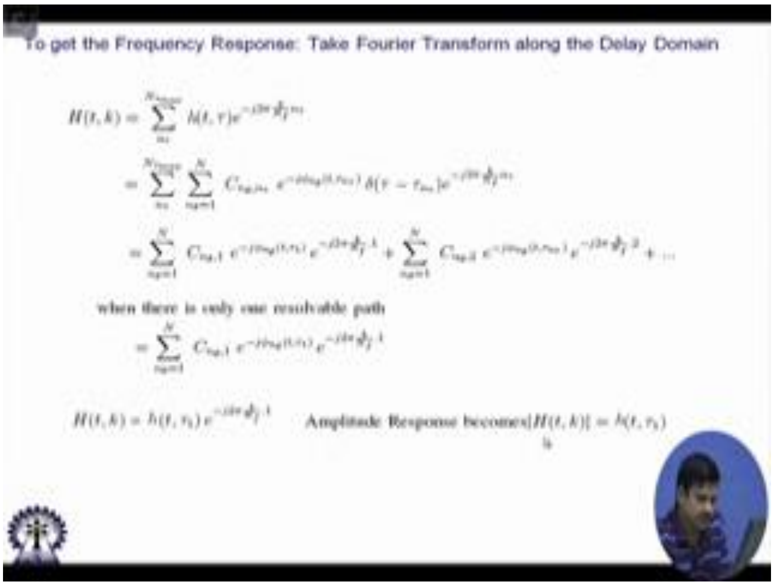


So, with this we go back to our discussion on the channel and if we look at the impulse response now, what we have is the channel impulse response by this particular expression, what we had seen before is present here in front of you; I do not need to explain this much except that this h of t, let me see about here h of t comma tau we had this expression, this is not new for us; we have seen this expression where c n are the coefficients, pi n are the phases and this n is having this delay of tau n and this is the expression of phi; we have used it in our earlier derivations. Now we modify this a little bit and we say that let phi n be phi n theta and this f d n is actually this expression f m cos theta n and we have of course, n theta and so this is the phase expression and of course, we have it here why we put n theta because there is along with the nth path, there is a phase associated with that path and so on and so forth.

So, we proceed with this right, so h t of tau can now be written as if you look at this expression phi h t of tau is basically some of n theta equals 1 to capital n. So, now, this is c n theta, n tau that is a variation that we have over here. So, when we expand this expression, now what we write over here is n tau is the first one, this is the second one and this one corresponds to this delay that is tau 1, this two also corresponds to this delay of tau 2. Now this n theta summation and this n theta summation which is adding to c n theta of 1, which is adding to c n theta of 2, are because at the first delay there are several paths coming. So, these are getting added by n theta as to 1 to n number; ideally speaking this should be along with tau n corresponding to the nth delay right.

This one is basically sum of all the coefficients which are getting added because of the second order of the delay and so on. So, basically this expression can be expanded in this way where what you seen over here is, this summation over n there is a delta tau n; ideally speaking we could have written a summation n tau equals to 1, 2 tau n that is maximum delay that we could have written over here we avoided first simplicity. So, we have the expression below where this n which was inside this delta has now been broken out separately as the first delta, the second delta. Whereas for each tau equals to tau one; there is summation over several multipaths; moving ahead further with this expression.

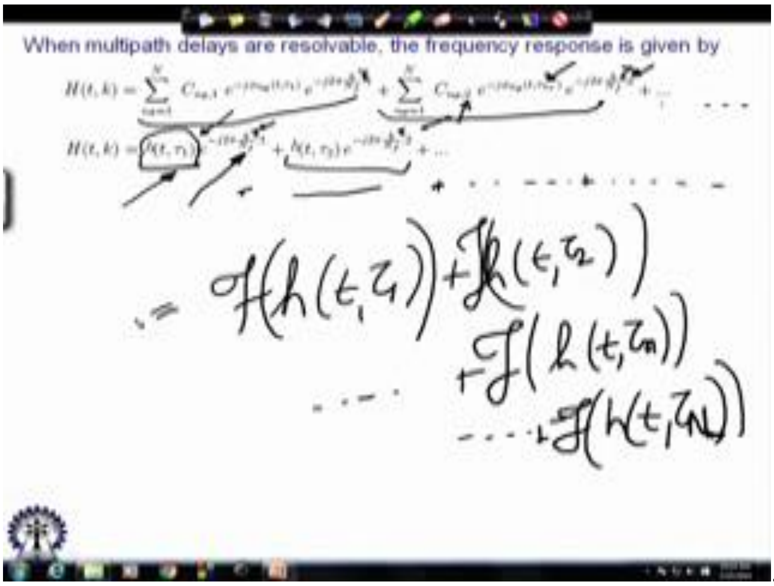
(Refer Slide Time: 27:06)



So, we now need to take the Fourier frequency response that is what I have just briefly explained little while above and in order to take the frequency response, we take Fourier transform along the delay domain, I have explained this little while here ago; so if you take that we write it as h capital h of t comma k look at this, so t has remained t and h t of tau. So, t is remaining as t and this is the tau and summation over n tau that is this index of tau to tau max; that is the maximum number of resolvable delays minus j 2; pi k by n f, so we are taking in the discrete domain, discrete frequency; n tau indicating the particular delay. You could do it also in a way h; t comma tau; e to the power of minus j 2 pi; you could write f tau, d tau integrate 0 to tau max and you could write this as h of t comma f; this way also you could describe it either way is fine, you could do it in any of the ways.

So, that would mean; I would need to expand h of t or need to expand h of t and this is what has happened here; the first, the second last expression of the previous slide. So, this is what is happened and then you expand this expression as you see. Here you can clearly see this second expression; that means, it is getting expanded to the delays, so what we have done at this point is; this is due to the first delay sum overall multipaths, this is sum overall multipaths at this second delay and so on, so this is n this should be tau 2; so n of 2; that means, the second delay and so on for all the resolvable delays. So that means, for or all these things we have broken it up in two parts; when there is only one multipath, which you had seen before only the first coefficient could remain; that means, these will vanish when there is no resolvable multipaths; that means all the delays are very very close to each other there is only one. So, that there is impulse responses basically here impulse that is the first case what you had seen, whereas in the other case these will be the other delay cases.

(Refer Slide Time: 29:51)



.

So, in that case you remember; we take the Fourier transform and what we get is the flat fading situation, which we had studied earlier. In this case, when the resolvable we have the Fourier transform of the different paths separately. So, let us look at this expression if we look at this expression; this is the Fourier transform of the delays that are arriving at the first delay of tau 1; see we have put tau 1 over here. This summation, this particular expression is indicating all the multipaths, similarly here it is the second delay and e to the power minus j 2 pi k by n f multiplied by 2 indicating the second delay and so on and so forth. Sorry this is yeah this particular phase due to the second phase, so what we would have in turn is h tau; h of tau 1, so this will be tau 1, this will be tau 2 and so on.

So, what we have is h t; tau 1 with this expression plus, so basically this whole thing gets a Fourier transform gets converted here, this is due to the second set of delays. So, basically what we have, is Fourier transform of the delays of the multipaths of the first resolvable delay plus that of the second plus that of the third plus that of the forth and so on.

So, basically what we get as the frequency transform function as addition of several impulse response occurring at different delays, where there is an amplitude factor which we have seen before and there is a phase factor. In the flat fading case, all had phase tau cap; what we are writing over here is all the d paths with the delay tau 1 are associated here; here all the paths with delay tau 2 are associated here and so on. So, now, basically you are having h; t; tau 1 plus h; t; tau 2 and the Fourier transform of this and the Fourier transform of this and the Fourier transform dot dot dot dot dot h of t comma tau n; dot dot dot plus the Fourier transform of h of t comma tau n; that is the max delay, tau n max this is what will be the expression that we get in the frequency selective fading and we will see how this gives raise to frequency selectivity in the next lecture.

Thank you.