Fundamentals of MIMO Wireless Communication

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Lecture – 15
Frequency Selective Fading

Welcome to the lecture on Fundamentals of MIMO Wireless Communications. We are currently studying the propagation model to small scale fading. What we have seen is the characteristics of flat fading, and what we have found is that the received signal when continuous wave is sent is random. The amplitude of the signal fluctuates over time and the fluctuation is a random process.

To describe it we have seen the distribution of the signal, we have seen the distribution of the envelop distribution of envelop squared. And also we have studied the correlation properties where we have seen the correlation of received signal. We have seen the correlation of envelop and we have also seen the correlation of envelop squared.

And summarily for the Rayleigh fading condition that means, where arrival angles are uniformly distributed, we have seen that the correlation is proportional to Bessel function of the 0th order of the first kind. And that of envelop and angular squared both are proportional to that of J 0 squared instead of J 0. With that we have summarily covered the random signal.

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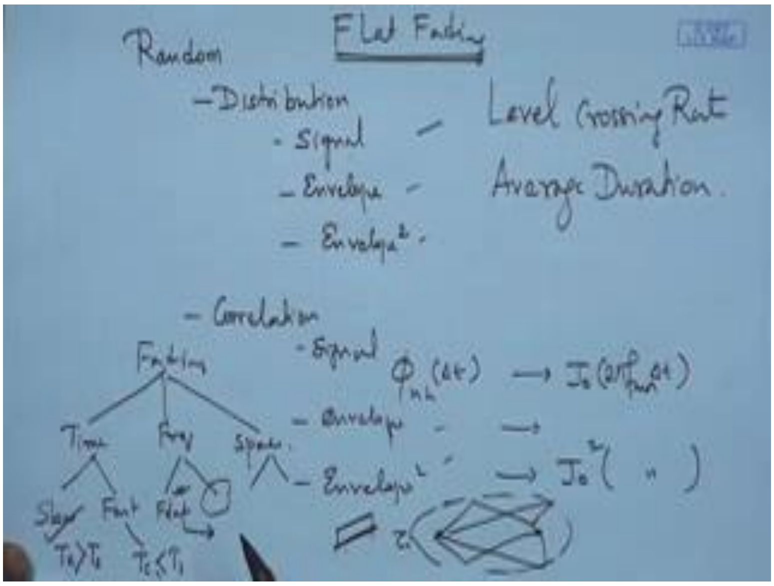

So, to characterize it we have look at the distribution of the signal of envelop, of envelop squared. We have also looked at the correlation of the signal through phi hh delta t. We have looked at that of envelop for one case we have found it to be J 0 2 pi fm delta t. For envelop and for envelop square with founded to be J 0 square of the same term. And why this envelop and envelop square are important because this will be the amplitude and this will be the power of the received signal. The signal is in in important because when we will be doing base band processing this is what will be handling.

So therefore, these are all important. Now along with these to characterize the signals or to design better communication systems we also look at two important parameters; one is the level crossing rate and the other is the average duration of fade. With this characterization we would be fairly well characterizing the received signal for the particular case that we are considering.

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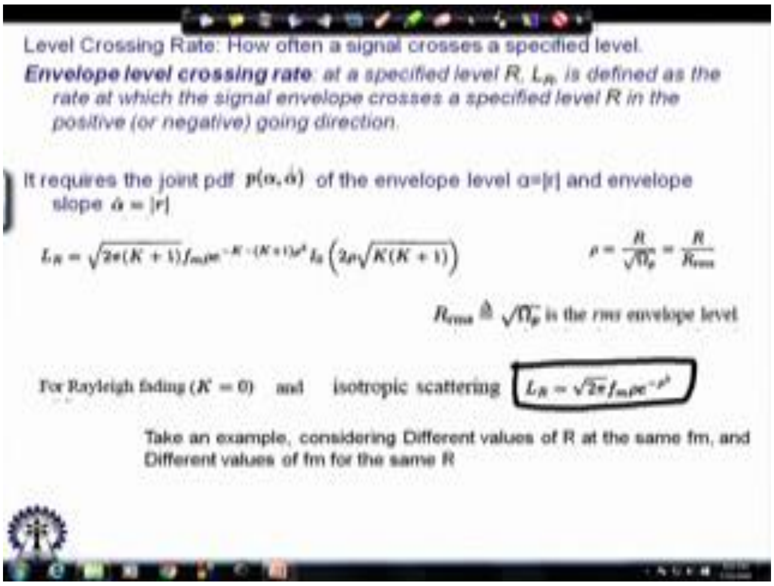

The level crossing rate is often defined as the rate at which the single crosses particular threshold.

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So, that is if we are taking the signal fluctuation that means is if this is the time axis and as we have said the signal fluctuates randomly, of course we are assuming some inherent fm value in this. So if we set some level, let this level be level which is corresponding to the receiver performance. If this is the level for receiver performance the rate at which it crosses this threshold in one direction that means, either from above to below, so here it is crossing, there it is no crossing, here it is crossing, there it is crossing, there it is crossing, there it is crossing, it is crossing in one direction, or here there it is crossing there it is crossing.

Or we take the rate at which it crosses in the upward direction either the first one or the second one we do not take both. So, either the first one or the second one and we define it as the number of crossings per unit time that means per second, so rate at which it crosses a particular threshold. We could have multiple thresholds, so this is basic definition now. Why this is important? This is important because we would like to know at what rate the signal goes below the threshold that means, what is the rate of outage. So, usually outrage can be calculated from this.

And the average duration of fade is basically if this is the desired level and this is the period for which the signal has remained below the threshold that mean, it is gone down and then it is come up. So, what we are interested in calculating, what is the average period for which the signal goes below the threshold? Now both would indicate our calculations on outage because once it goes below the threshold it will remain below threshold for a long duration or the duration corresponding to the average duration of fade. Here again the discussion that we are having that if we consider another situation where the signal is fluctuating very very slowly this we had said earlier.

As we can see that the slow fading gives us advantage that the rate of change of signal is slow compared to the symbol duration, suppose this is the symbol duration it is slow compared to symbol duration, but once it goes below threshold it remains below threshold for a long duration of time that is the disadvantage whereas, if you look at the other one the rate of crossing is faster, so the two have their own advantage and disadvantage. In this case the signal might be fluctuating within the symbol duration, but since it is fluctuating faster it will not remain below a threshold for long time.

On the other hand it goes below the threshold several times. So, if you can understand this process or if you can know what are the details of this then we could design communication systems accordingly. And this influences the error connection code design more often the forward error connection codes the inter leaver design. These are some of the important things which inter leaver design which affects the communication systems for these particular modules. So, we will just briefly look at how this matrix are defined and then will proceed with it.

So, level crossing rate is basically talking about how often a signal crosses particular threshold, is as defined for the level crossing rate. And as we have just said the envelope level crossing rate is what is important. So, at a specified level is which we have drawn in the paper R which we have drawn in the paper. L R is defined as the rate at which the signal envelope crosses the level R in positive or negative going direction this is what we described. That means, if this is the fluctuation, so either you count the rate at which crosses this direction or you count the direction or you count the other direction.

And the actual calculation is something which requires the joint distribution of the envelope as well as that of the rate of change of envelope. So, basically we have alpha of t and alpha dot of t that is derivative of alpha with respect to t. And that is required because it gives us the rate at which the signal is changing, so the rate is calculated by alpha dot of t. We are not going to do the details of the derivation of this particular expression we will be interested in the in the expression it for this particular course. So, the level is defined with respect to the R ms value. So, if R is our level of interest we defined rho which is with respect to square root of the total receipt power so that is root over omega p.

That is the definition of R and for Rayleigh fading condition which is for interest there is isotropic scattering to the isotropic scattering model, the derivation would give our result which is the rated which crosses the threshold is equal to square root 2 pi so this is the constant what is important is fm times rho in to the power of minus rho square. So if we concentrate on some of the important terms, the most important term as of now would be fm. So, what is clearly from this particular formula is L R the rate at which it crosses the threshold is proportional to fm. And fm is equal to v by c times fc.

So, clearly again as Doppler frequency increases which is because of either velocity or of carrier frequency the rate at which it crosses the threshold increases and we have seen this through the drawings in the paper. So that is what is the level crossing rate, this is one of the important measures of the channel behavior accordingly things we have designed.

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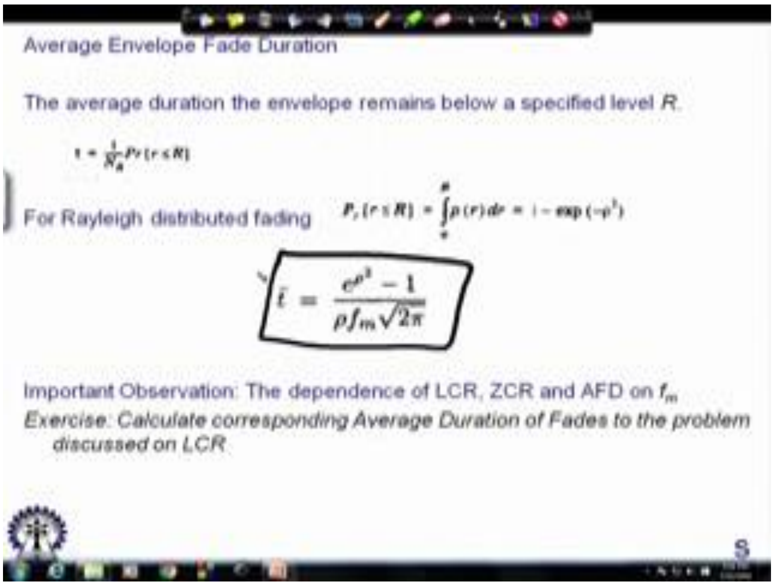

Moving ahead further, the average duration of fade is another important quantity which is often used. So, the average duration of fade is defined as the average duration, this is very important it is the average duration not the exact duration. The envelope remains below specific level. This is what I have already described in this figure, that what is the time duration for which it remains below this threshold? t is talking about this particular duration that remains below the threshold in this case and this cases. So, what we are interested is in the average duration that it remains below the threshold.

Now to do this calculation what is done is we first calculate the probability of the envelope remaining below R, if you look at it that is probability that the envelope is remaining below some level R divided by the rate at which it crosses that level in one particular direction; let us say in one direction. So, that will give us on an average how long it is going to remain below the threshold. Now probability that the envelope is below a threshold is given by this expression if you take Rayleigh distribution and you integrate this you want to get 1 minus e to power of minus rho square. And the average duration of fade would be this probability, this is expression divided by the rate at which it process. So, if you would take the LCR calculated in the previous page and divided it by this you are going to end up in the expression of which is the average duration fade. So, together using these two quantities is often some of the important modules of digital communication system are designed.

At this point we will stop our discussion on small scale fading and flat fading. So, what we have essentially discussed is all this things as is (Refer Time: 11:48) for flat fading. We have also discussed time selective fading that means, slow fading or fast fading. So, when we have fading we had said there is time selective there is frequency selectivity, there is space selectivity. For time selective there could be slow or fast, they could be for frequency it could be flat or frequency selective for space also.

So, we have studied the slow fading when we said that coherence time is much much larger than the symbol duration and fast fading when coherence time is smaller than the symbol duration. Flat fading we have already seen when the signal is flat across the band width of interest and that happens when all the taus the delays because of multi path, these multi path effects that we have drawn they are all on the ellipse or which the two focal points or the transmitter and the receiver.

That means, they are having almost the same delay so all the reflected paths arrive almost at the same delay. So, the impulse response is basically and impulse in that case the response is flat across frequency, so across the frequency access the response is flat. We have seen that and what will now proceed to see is frequencies lead to fading.

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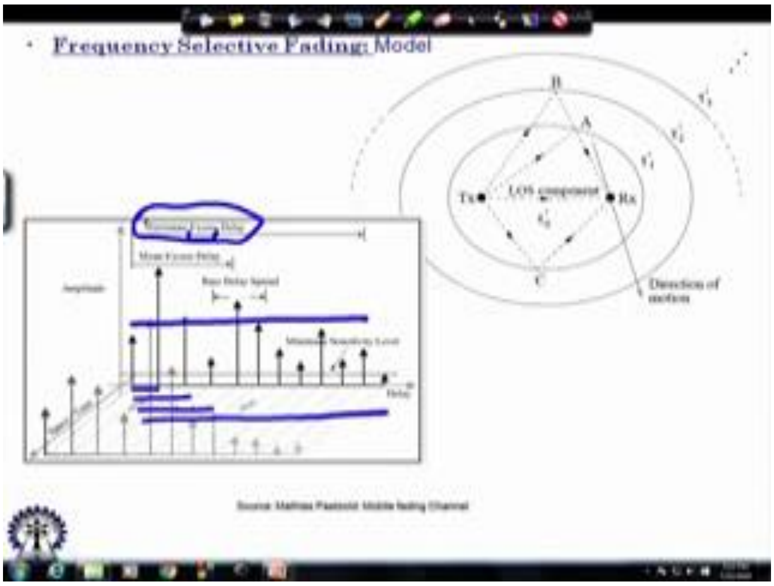

So, when we look at frequency selective fading we will concentrate first on the; so we will take a look at this particular picture. We will not take a look at this picture as of now. So let us assume it transmitter is located here, the receiver is located at this point and there are rays which will go from the transmitter to the receiver is a rays which are going from the transmitter to receiver. For this two rays that we have drawn since they are get on the same ellipse there getting reflected from the same ellipse their path delays would be the same.

So, if there where rays coming from this and this they would be multi path as we have studied before, but since they are getting reflected from the same ellipse or reflect as on the same ellipse what do would be have is on the delay access the echo's would appear at the same point the wont be different. So, we will be getting flat fading, because if you take the Fourier transform across the delay axis h of t comma tau you want to get a flat response across the frequency, this is what be able to.

Now instead, we relax the assumption and we say that the signal propagates gets reflected this also gets reflected. However, there are rays or waves which get reflected from reflectors or scatter us which are further away that means, the propagation delay is not the same. While deriving the flat fading we had made the assumption that tau 1 is almost equal to tau 2 is almost equal to tau n and that finally getting equal to tau capital N. So, this is the assumption that we made. And this was said equal to tau cap and then we had h of t comma tau is equal to h of t times delta tau minus tau cap this is what we have had. So, now in this case we do not make these assumptions we said they are not equal to tau; they are all different they are distinguishable. So, that is what is represented by this particular picture. That means, this ellipse is the one where the delay is tau 1 the second ellipse is the one which represents the delay of tau 2 that means, wave propagating from here getting reflected coming over here would have a delay of tau 2 and the one getting reflected on the third and coming back would have a delay of tau 3 and so on.

So, these are basically resolvable delays, the delays which the receiver can distinguish. For instance, if the receiver has a symbol duration which is 1 millisecond it will not be able to distinguish between the paths which are coming within 1 millisecond. So, this will be relative to the particular receiver. Of course, we will we will take better look at.

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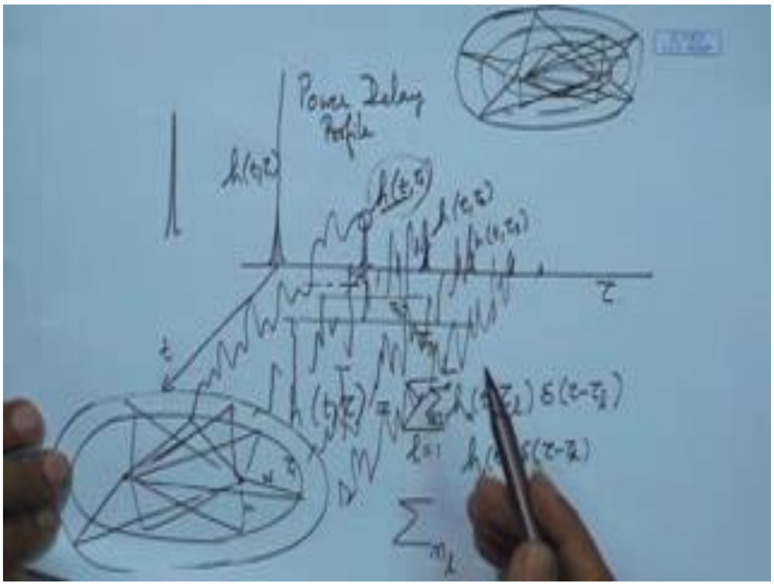

So, now we will try to we will try to write it here, suppose I launch an impulse and that impulse propagates through the air and it comes to the receiver after multiple reflections. So, if I draw the delay access. So, after some proportion delay which is tau 0 I have launched an impulse I want to get the echo after that is certain delay what we have seen. In case of flat fading the other echo would also be on top of this, and there will be n number of such echo, this is going to happen in this case. Now when there is second set of delays is coming from a reflector which is on the second ellipse that means, it is a transmitter receiver this is on the first ellipse there is a whole set of reflectors on the second ellipse let us say they will come with the delay of tau 2, so if this is tau 1 they will come from the reflectors at tau 2; one path, second path, third path and so on. So, they are all going to come in be on top of each other, this delay is tau 2.

If there are reflectors further away a scatter us they we call to get reflected it and come back to this. Then there again they would be reflections and this delay is tau 3. So, what we see is that the delta, here what we have is h being the coefficient which is function of time as we have seen because when this is not considered we have seen this h of t and the delay was tau 1 for all of them. We had basically written h of t comma tau is equal to h t times delta t minus tau cap and that is tau 1. The second one we have h of t comma tau 2 and so on h of t comma tau 3. So, all these are function of time, but they are different delays. So, what we have h t comma tau is basically sum of we have L equals 1 to capital L lets say and h of or we will have to change the index t comma tau L delta tau minus tau L.

This is because h at the delay tau L and that is being specified by delta tau minus tau L and this could also be written as h L t 1 delta tau minus tau L that means, this is at a delay of tau L. What we should remember is there is an additional summation n equals to 1 to n h n that means, at the delay tau L there are n number if multi paths. That means, this there are n number of multi paths, so I draw the figure again. So, if there is the first ellipse path number 1 path number 2, path number 3, path number small n up to path number capital N. In the second ellipse now all of these are at a delay of tau 1 that is what we have. The second ellipse they would again be a large number of them.

So, ideally speaking what we should have is sum over m suffix L indicating the number of paths multi paths at Lth delay. So, that is what should indicate. That means, a number of reflectors on the second delay is not necessarily equal to the number of paths in the first delay and that with the third delay. So, this number n would vary according to the delays. So, what we have is this is one of the echo's; this is the second echo's, and this is the third echo's. And with time if I draw time on this axis if I draw time on this axis and would track this, what you we would find is h of the function of t that we have already seen so this is going to fluctuate with time.

What we have seen is the distribution of this we have seen. The distribution to be Rayleigh distributed, the envelope to be Rayleigh distributed and the received signal is complex Gaussian distributed, the envelope is exponential distributed for p theta equals to 1 by 2 pi. So, what we have studied is basically one of the paths. What we have studied is what could happen in the second delay, what could happen in the third delay. This is what we have already studied.

Now, what we have is whatever we have studied for one delay is getting replicated in the second the third the fourth and so on and they are staggered in the delay axis, so all are fluctuating this is very important to remember. So, all that we have studied till now the properties that means, whatever we have mentioned here the random distribution of the signal envelope, envelope square, the correlation, the Doppler spectrum we have also studied the Doppler spectrum is everything for this path; because this path is made up of several multi paths coming at that delay. This is another group of multi paths coming at a second delay. This is another group of multi paths coming at another delay.

We have studied this axis. In this axis we have studied the correlation in this axis we have studies the Doppler, so we have taken the Fourier transform across the time access and what we have reached is the is the jakes spectrum. And we have also studied specular component in this case where they would be ration. We have talked about the distribution to be Nakagami on this. So, if I cover up everything else whatever we have studied is this particular one.

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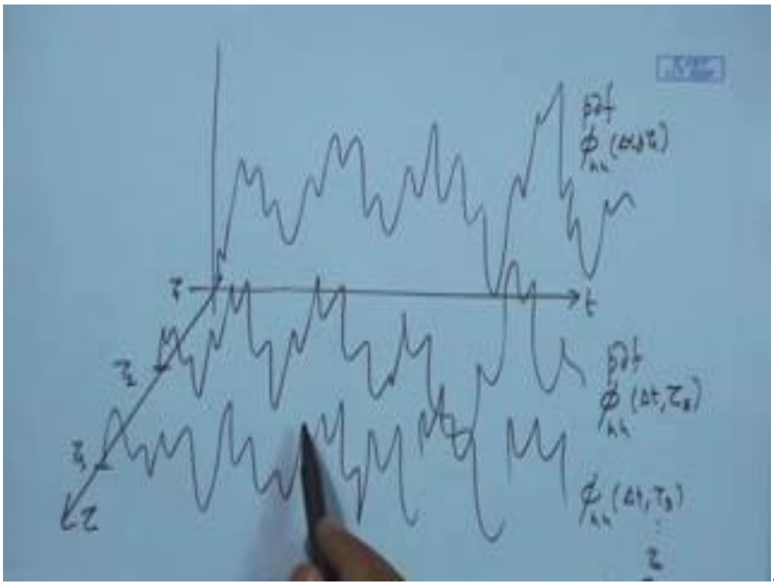

So, another easy way of drawing this matching with what we have done before is if this is our axis of time let us say have reverse the time axis. In the previous we said this was h of tau, so the received signal for continuous wave transmission we saw it to be fluctuating like this with time. If I would draw tau on this access now I have rotated the access, so this could be at some delay tau 1 at another delay tau 2 we would find same thing happening. At another delay tau 3 we would find this happening. So, whatever we have studied for this with pdf is h delta t would apply for these also. And the pdf would apply this would be phi hh, now instead of just putting delta t we would put at tau 2 in this case, this was at tau 1. So, whatever we have studied for the delay tau 1 this would be phi hh delta t for delay tau 3 and so on. This is what we have actually studied, the pdf and all.

So, one question that can a raise in your mind is the correlation property the same across all of this and if the pdf would be same across all of this. Now that is not necessarily true that it is going to be the same for all of them. Some of them would be Rayleigh, some of them could be Ricean, and wherever there is Ricean they would be according distribution of the Doppler and the correlation function; so they could be a mixture model of all of this which could apply in each of the delays. Typically we would refer to have the diagram as in this that means, here we have made this has the time axis this has the delay axis. Usually, we would refer to describe it in terms of power delay profile because there is a certain kind of profiling that happens in this delay axis, this is the delay axis this is h of t comma tau axis.

So, typically the reflectors which are close by, the propagation path is smaller compared to propagation path on the second ellipse or the third ellipse we would except signals from the first (Refer Time: 25:16) reflectors to be stronger than the one from the second and the third because of dissipation. But that is not necessarily true of instantaneous cases on an average that could be true and we would describe these in terms of power delay profile which will describe shortly. So, what we have with us in this particular picture.

Now we concentrate on the second picture which showing us the profile delay profile and there to understand it we could say that we have launched an impulse at some point here. We launched and experimental impulse and because of the impulse we are getting first echo here that means, all multi paths coming at that delay. This is contributing to multi paths from the second group delay. In this picture I have changed from what I have drawn in the paper over here, here we had said that the first delay is having stronger signal then the second and the third and so forth. So, it could be the other way also.

The first one is having lot of observing material where we have reflected from, whereas the second (Refer Time: 26:29) that means, this particular one could be having lot of metals so they could be specular component there will be strong reflections from them. So, that is what we have drawn here and this is the third group of reflector. You can see this is the tau 3, this is the tau 1, this is the tau 2, and this is the tau 1 corresponding to the ellipses here and so on. So, these echos will keep coming till the time we are able to resolve them.

In order to resolve them or in order to record them we usually said in noise floor or the sensitivity level of the receiver. Once the echos fall below the sensitivity of the receiver or we no longer be able to record them and till that point from the first point of occurrence of the signal to the last point of the occurrence of the signal is what is the important for us and we usually refer that as maximum excess delay. This as you can clearly see it is from this point to this point, because this is gone below the threshold. From this point to this point that is called the maximum excess delay. Why it is called excess delay, because what we are interested is in the initial delay is due to propagation and that is common for all. If you look at this section is common for all. So, basically there is a tau 0 and or tau 1 at this point, tau 2, tau 3, so we are interested from the point that it occurs is when then signal comes for the first time. Now if there is no further delay these would not be present, this section would not be present. That is this section is not present, this section is not present, this section is not present. Basically, if there was flat fading only the first one would be present nothing would be present.

Now, beyond that condition we are at a situation where there is an excess amount of delay over here, there is an additional amount of delay. So, these delays are therefore called excess delay and this duration from the first to the last recorded value is the maximum delay and this is the excess with respect to the first one and hence it is called the maximum excess delay.

So, we stop our discussion at this point in the next lecture we will go in to the details of this particular model. And how this particular model helps us in characterizing the received signal and what will be able to understand is what is known as frequency selective fading. That will bring us to the end of understanding the small scale propagation effects which is necessary for this particular course.

Thank you.