Fundamentals of MIMO Wireless Communication

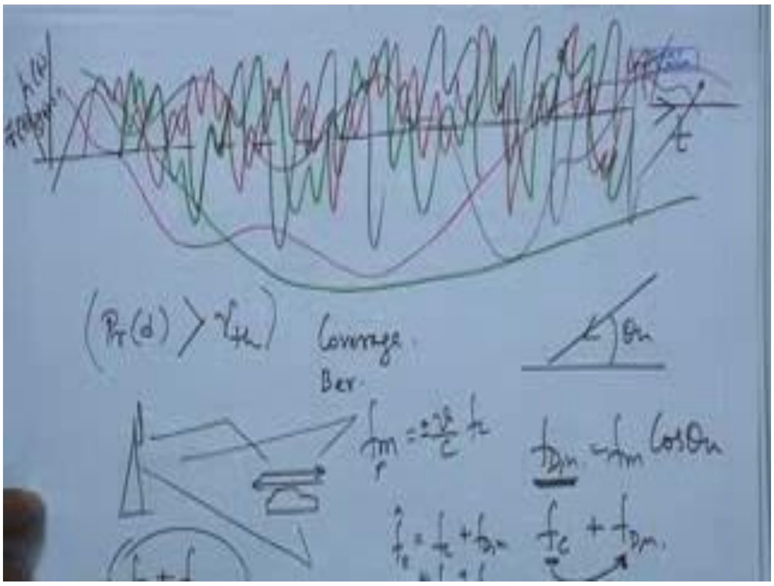
Prof. Suvra Sekhar Das
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 14
Doppler Spectrum

Welcome to the lecture on Fundamentals of MIMO Wireless Communication. In the previous lecture we have discussed the Rayleigh of fading condition which gives raise to the autocorrelation function, which gives a Bessel function; the first kind which 0th order. And we have also discussed what the meaning of coherence time is, we have also discussed the slow fading and fast fading conditions and how they would arise in different operating conditions as well as for different communication systems.

In the last lecture we would just trying to compare the different situations of fasting slow fading, and we have also said that if there is fast fading the single would fluctuate faster than the channel condition that would required better receiver to be built or even better transmitted signal design. At the same time should also try understand or fair idea about what is the problems when there slow fading. The advantage of slow fading of course we have mentioned that when there is slow fading that means signal rate is faster than the rate at which the channel is fluctuating or the signal strength is more or less constant during the symbol duration. That is one view of the whole thing.

(Refer Slide Time: 01:39)



The other view of the thing is the channel is fluctuating at a very very slow rate. If I would draw another channel which fluctuates at even a slower rate, channel which would fluctuate at even a slower rate. So, what I try to draw is I of try to draw the other side of the coin means I could have drawn the signals going up, but what I means to say by this particular picture is which slow fading as the signal rate remains constant during the symbol duration, on the other hand the signal actually means quite constant during the symbol duration.

What does it mean that ones the signal strength falls below a certain threshold, suppose I have a certain threshold of operating point; if the signal strength falls below this particular threshold for it to come back about the threshold it takes quite lot of time. Bow why this threshold is important, because we would required the received signal strength we received at distance d we had calculated to be greater than certain threshold. Only once we are able to this calculation, means this calculation is important for coverage probability. This is important for bit error probability that means to provide the signal covers certain level that means is beyond the certain level for a certain percentage of time the error probability is good. Otherwise you signal is below certain threshold of you would have outage.

So, which slow fading conditions once the signal goes in outage it will remain in outage for a longer time, whereas in a fast fading condition the signal fluctuates so it goes below the threshold as well as it comes above the threshold. So that is the other advantage of fast fading or the disadvantage of slow fading condition. With this quick description would like to move ahead, we will take this particular picture. So, when we say that the signal is fluctuating as a shown in this particular diagram this particular picture shows signal fluctuating, so this is the time access and this is let us say h of t or r tilde of t when s tilde t is equal to 1. That means, when there is continuous if transmission.

So, when we are having this the signal is fluctuating. And we have also seen one kind of wave describing this. When the signal fluctuates the rate of fluctuation is defined. The maximum rate of fluctuation we have defined it has f m which is equal to v by c times f c and this could be plus or minus depending upon whether the vehicle is approaching the signal or it is moving away from the signal. So, if there is it tower and there is the vehicle if it is going away it is minus f m if it is going towards us is plus f m, so there are span of plus minus f m that is present in this present situation.

All other cases when there is the signal arriving at an angel theta n we have said f Doppler at a circle angel is equal to theta n. So that lies between minus theta minus f m and plus f m. So this f d what we have seen in our earlier derivation gets added to f c. What is it mean? The signal is transmitted carry of frequency f c, but when it is coming to the receiver the receiver perceives it to be coming at a frequency of f c plus f d n this f d n could be plus f m could be minus f m or could be anything in between. If there is only 1 transmitter and there is nothing in the space, absolutely nothing and this is either moving away in the direction or going towards it they will be only one value that is plus f m or minus f m.

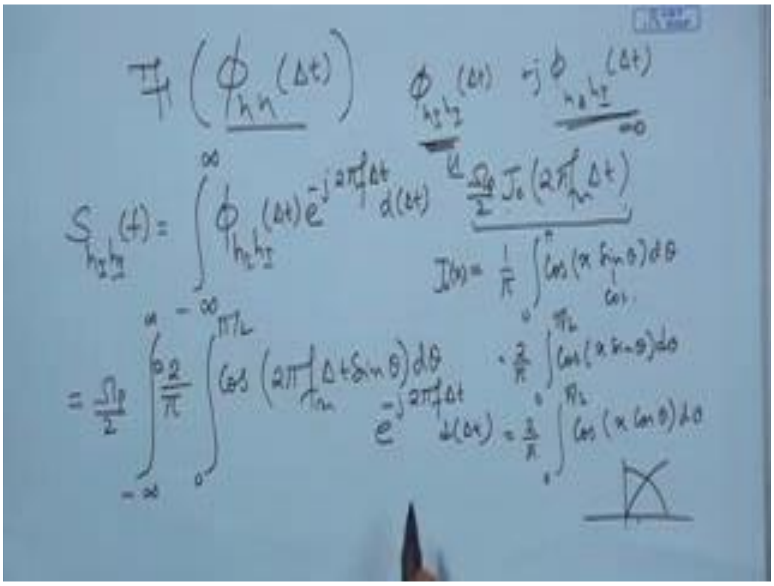
So, there means f c that is transmitted is going to be shifted to f c plus f d n. That means, f c plus f m or minus f m nothing else there is only one single frequency. That is also not a problematic case because the receiver can tune to that particular frequency, that means the receiver considers f c cap as f c plus f d n which is equal to f c plus or minus f m either of it. In this case receiver can tune or to this particular frequency and there is no problem. Whereas, when there are multiple paths because of theta what the receiver says is multitude of frequencies, this is what we already seen. That means, receiver is basically getting f c plus f d n it is basically getting of whole set of frequencies; this is one view.

The other thing which we should considered is when a signal is fluctuating as I was just mentioning there is create of fluctuation. This fluctuation is mentioned or is described over here by means of f m. And what we are noting here is there is a plus f m there is a minus f m and there is a whole range of frequencies in between. What would like to find out is what the weights of these frequencies are or what percentage of which frequencies present in this system. That means we are trying to look at this spectrum of the fluctuation. If there is a fluctuation that is happening what is this frequency content of this fluctuation that is what we are interested in.

Now, since this is a random process or we cannot take a Fourier transform and study it, because a Fourier transform would be of a particular snap short of a signal. That means we are reading particular section of the signal and we are taking the Fourier transform of it. And as time changes this particular snap short is going to change. So, the next snap short for the same process could be this and naturally they are Fourier transform would be different. And in the third case; the third time instant I take the series it could be different and the Fourier transform would be different.

So, what is rather studied is the power spectral density, so that means you take the Fourier transforms or you take this spectrum of snap shorts and then you look at it in average and you find out what is the average content of signal unit.

(Refer Slide Time: 08:32)

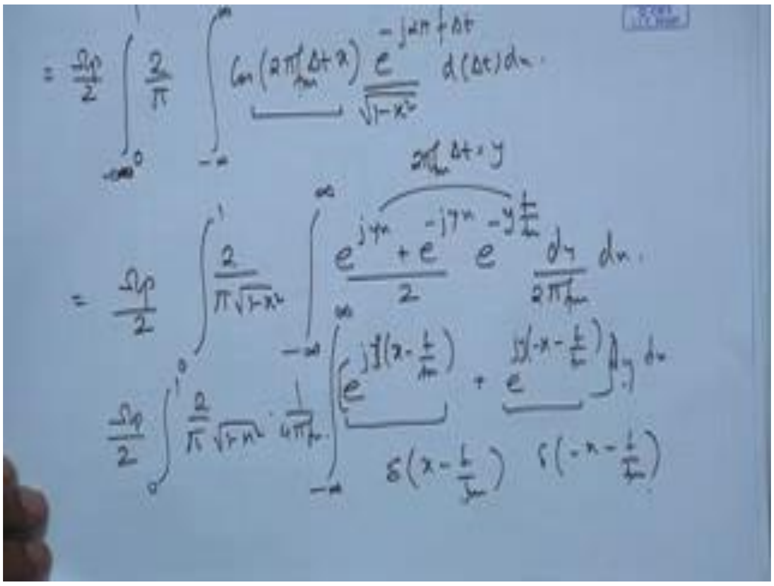


Now, that could be one way of studying it, but there is also in other more effective way of studying it is you take the Fourier transform of the auto core relation function. If you take the Fourier transform of the autocorrelation function then also you can arrive at the power spectral density of the fluctuations, because that would give us what is the content of different frequencies that represent in the fluctuations. Now for us phi h h delta t is basically phi h I h I delta t plus j phi h Q h I delta t, where we have seen this component goes to 0 and this component remains us non-0. And the result for this component is omega p by 2 J 0 2 pi f m delta t.

Now, in order to get the Fourier transform of this so that we get the power spectral density would be getting s indicating the power spectral density h I h I of f is equal to integral of minus infinity to infinity phi h I h I delta t it is the function of delta t e to the power of minus j 2 pi f delta t d of delta t. With this we have to expand this particular expression. Now for phi for J 0 this expression is J 0 of x could be written as 1 by pi 0 to pi cos of x sin theta d theta or this could be replaced by a cos or you could also have 2 by pi 0 2 pi by 2 cos of x sin theta d theta. This could also be 2 by pi 0 2 pi by 2 cos of x plus theta d theta. Because the integral between 0 2 pi by 2 of sin theta and that of course theta would be same is clearly one is going from 1 to 0 the other is going from 0 to 1.

So, the area under the curve would not be different they would be the same in this cases. So, basically these are the different expressions that we can used, so we would write these expression has omega p by 2 minus infinity to infinity 2 by pi 0 to pi by 2 cos of 2 pi f m delta t sin theta d theta e to the power of minus j 2 pi f delta t d f delta t. With this expression we have to carry forward.

(Refer Slide Time: 11:45)

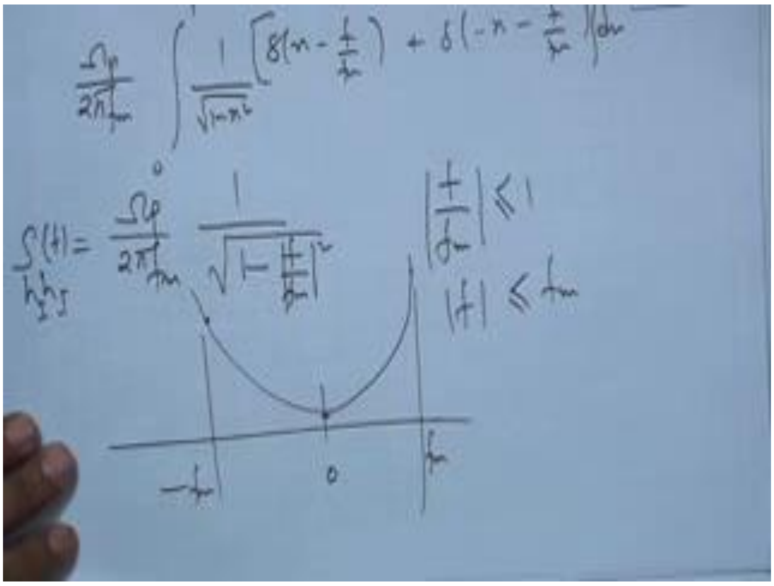


So, this would lead us to omega p by 2, of course we have to make certain change of variables we will make sin theta is equal to x and that would lead d theta equals to d x by square route of 1 minus square sin theta would be cos theta d theta equals to d x and d theta is d x by cos theta, cos theta is root over 1 minus x squared. Because of this and the limit of the integral from 0 to pi by 2 would naturally change to 0 to 1 and what we would have is minus. And we would change the integrals here so we could write 0 to 1 2 by pi minus infinity to infinity cos of 2 pi f m delta t of x e to the power of minus j 2 pi f delta t by square route of 1 minus x square d of delta t d x.

And we could continue from that point and we could do a translation possibly that 2 pi f m delta t is to y, you could you could do it this way also and after one more step it would lead to 0 to 2 pi I am keeping all the constants as it is so that we can take care of them later and instead of cos we could have e to the power of j y x plus e to the power of minus j y x divided by 2 e to the power of minus y f by f m d y by 2 pi f m with this translation times d x.

And if we continue with this you will have to collect these two terms, so you will be getting e to the power of j times y x minus f by f m and we will also be getting e to the power of j times y minus x minus f by f m and integrated from minus infinity to plus infinity 2 by pi root over 1 minus x square 1 by 4 pi f m omega p by 2 0 to 1 d y d x. Now if you look at this integral with d y and this integral with d y even to get a delta function of x minus f by f m and here you would get a delta function of minus x minus f by f m, so will be getting two delta functions and denominator is this.

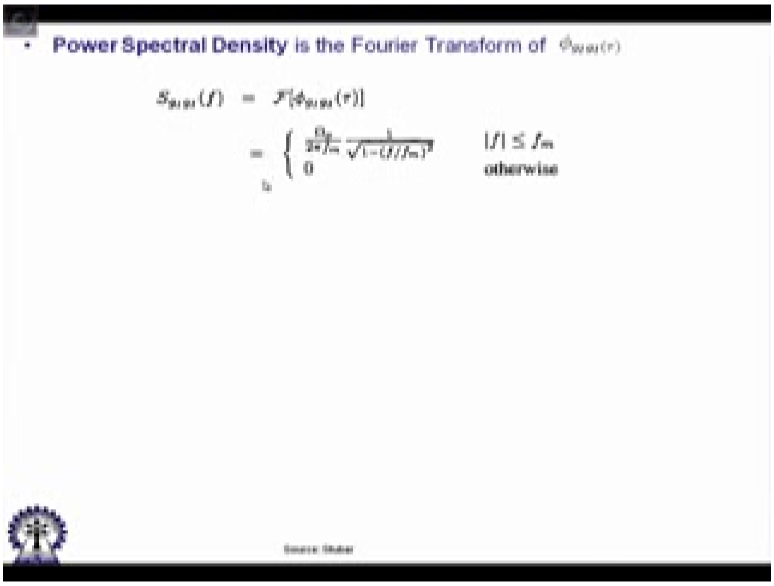
(Refer Slide Time: 14:47)



So, your end expression what will be left with is omega p by 2 pi f m integral 0 to 1 1 by route over minus x square delta x minus f by f m plus delta minus x minus f by f m d x. These are valid only for x equals to minus f by f m and x equals to f by f m, so what you would conclude is this is equal to 1 by 2 pi f m 1 by square route of 1 minus f by f m mod square. Subject to the condition that f by f m is less than or equal to 1 or rather mod of f is less than or equal to f m.

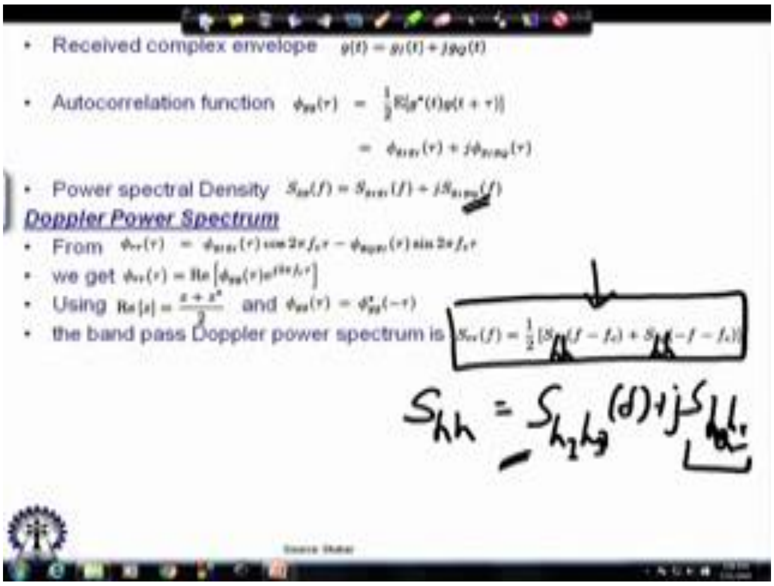
So, this is the spectrum of s h h of f; that means, the spectrum of the base band signal by if you try plot particular signal over here what we see is that when f is equal to f m or very close to f m this value almost tends to infinity. So this is 0, this is f m, this minus f m, there is very high value and when f is equal to 0. That means, this is 1, so you have some minimal value over here and all values would lie in between. So, what you would have spectrum is looks like this that is known as the Jakes spectrum and we will take a look at it.

(Refer Slide Time: 16:23)



So, the spectrum looks has an expression of this quantity which we have just derived.

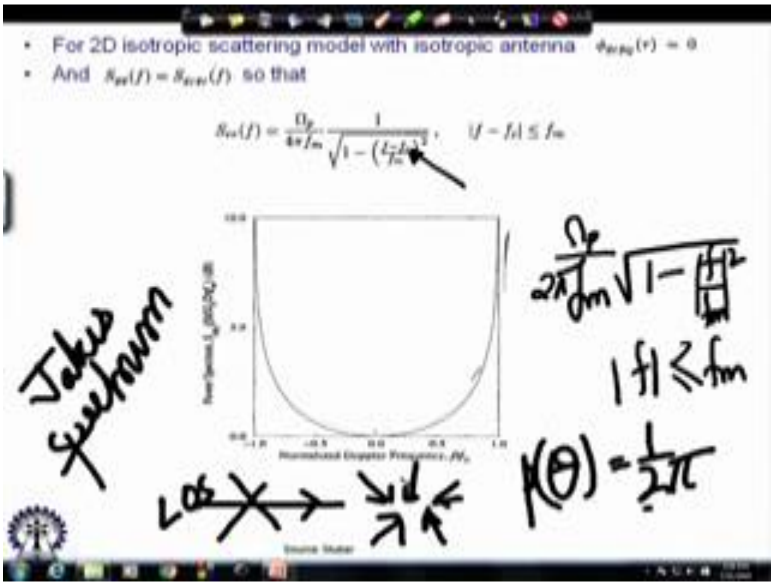
(Refer Slide Time: 16:32)



And if you would look at the pass band spectrum; so the pass band spectrum can be calculated from g is basically h in our case we have actually used h in our used g. So, s h h is equal to this particular expression, that means s h I h I of f plus j s h Q h i, but now this term would be 0 because this is the Fourier transform of the cross co relation coefficient which is 0.

So, basically this is effectively giving us the entire thing. If you would use the previous expression which you derived into this what we would get is this spectrum in the pass band.

(Refer Slide Time: 17:23)

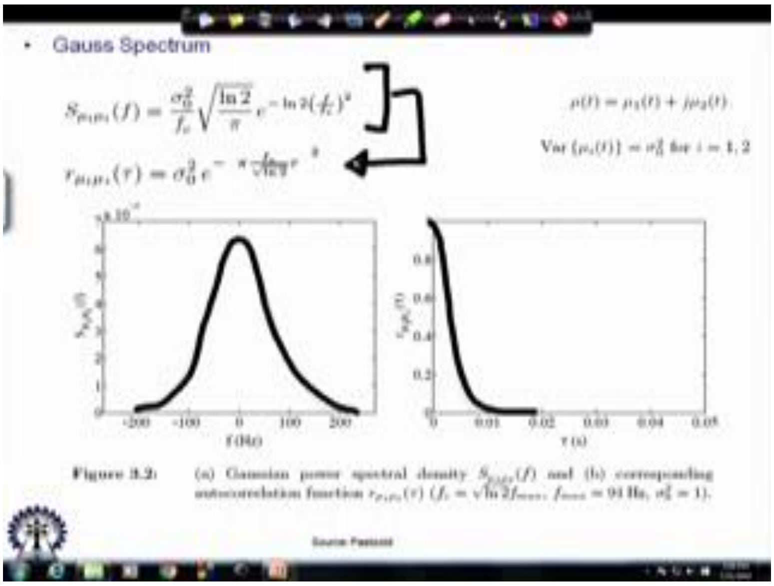


In the pass band this spectrum would look like this. In the base band just to remind you, the base band spectrum would like omega p by 2 pi f m square route of 1 minus f by f m mod square mod f is less than or equal to f m, that was the criteria that was said. So, here that gets modified to f minus f c, so that means f c was said equals to 0 you could also imagine it in that when it is a base band this f c is equal to 0, whereas it in pass band f c is not equal to 0 it is present in this particular expression and this spectrum looks like this.

So, spectrum looks like the shape that has been drawn over here. This is the famous Jakes Spectrum; this is also known Jakes Spectrum. Again I would like to remind you all this derivation is because p of theta was set equal to 1 by 2 pi. If p theta is not set equals to 1 by 2 pi the expression would look different. P theta beings said equals to 1 by 2 pi means raise arriving from all directions with equal probability, so that inherently means that there is no line of sight component present in the situation. Because there is equal probability of raise arriving from all directions that means this strength of wave from scattered components are all equal. In that case if there was a line of sight there will be a specular component which is not present in this particular scenario.

So, this is known as the famous Doppler spectrum or Jakes spectrum. We call it the Doppler spectrum because this spectrum is due to Doppler Effect due to mobility effect and hence it is known as the Doppler spectrum. And this particular shape of the spectrum know as the Jakes spectrum which appears for p theta equals to 1 by 2 pi which is again pretty famous. This is observed in reality all though we had derived it analytically, this is observed in reality and there is a match between observation and there derivation. And that is why this particular model is a very, very famous model and is the basis for several results in the domain of fall is communications.

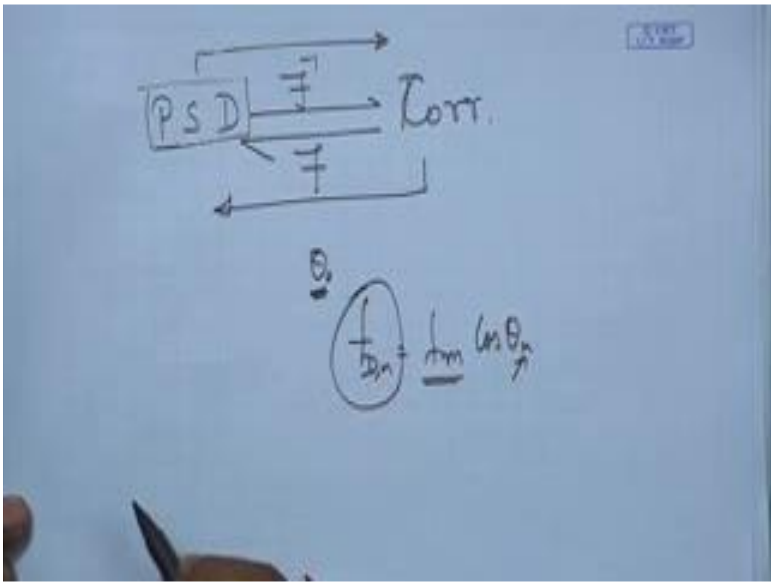
(Refer Slide Time: 19:51)



We have another spectrum here which is the Gauss spectrum. The Gauss spectrum is here, the corresponding auto correlation function is here, this again this particular result is taking from mobile fading channels by (Refer Time: 20:10) and these this particular graphs are there. So, for the Gauss spectrum the spectrum would look like this.

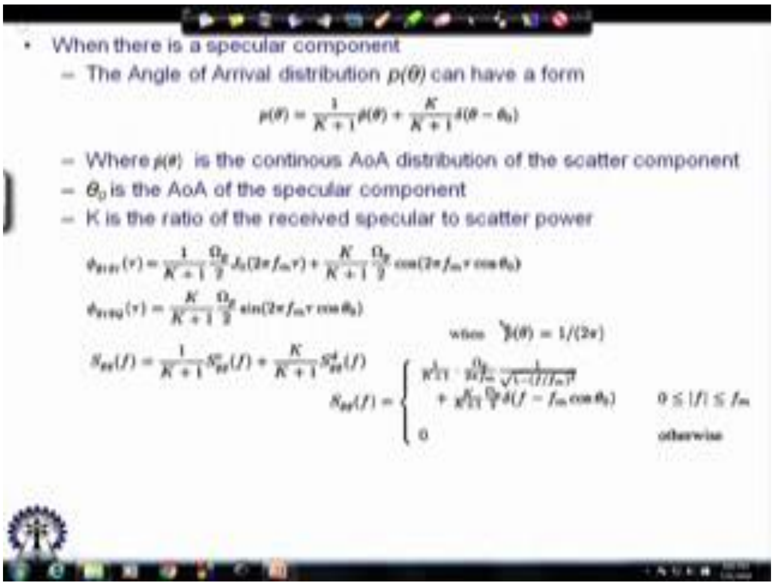
If we do the inverse Fourier transform you would get the auto co relation function. So, when we arrived at this spectrum we did the Fourier transform of the autocorrelation function. Similarly, there is also another way of arriving at the autocorrelation function instead of the wave we had done, is you arrive at the power spectral density. From power spectral density you take the universe Fourier transform and you arrive at the autocorrelation function.

(Refer Slide Time: 20:41)



So, basically power spectral density takes the inverse Fourier transform you get correlation function. You did the Fourier transform you going to get the power spectral density. So we have traveled this path, you can travel this path also. But in that case arriving at the power spectral density is not very rigorous it is kind of through explanations but that is pretty satisfying it is used in many, many cases. That is little bit easy and compared to the particular process that we have followed in this particular method.

(Refer Slide Time: 21:23)



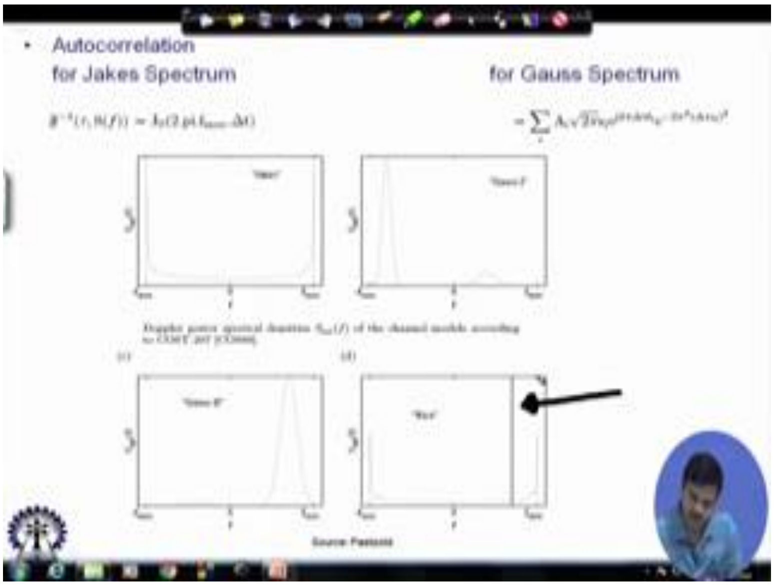
So, we continue with this. Now it is important, so instead of p theta being 1 that is whatever we have said so if there is a specular component; and I remember specular component we have discussed and we said whenever p theta is not equal to 1 that means we have line of sight, we talk about line of sight, one component we stronger than the other, k is the weight of the line of sight or the specular component. For the distribution we had talked about ration distribution. In this case we will talk about the particular spectrum. So, p theta is the continuous angle of arrival distribution of this scatter component and theta 0 is the angle of arrival of the specular component.

As you can see there is a delta theta minus theta 0, delta theta minus theta 0 indicating that there is this much amount of power ratio of k there is this much amount of power coming from with theta 0 direction, whereas this is the weight of the power on this scatter components. That means, there are raise coming from all directions and there is a strong specular component which is having this ratio of k is to 1 with respect to all this powers. There is a specular component and that is coming at an angel of theta 0. That is what is mentioned by this particular expression.

If that is the case what you are going to get is a complicated expression, the expression for phi g g that means phi h in our case it is h and not g in our case it is h, so wherever is g you would treated as h in whatever we have done. This particular section matches the result for the non line of sight. This is the additional component of line of sight because of this part. Additionally if there is phi g I g Q component which is not present earlier, because remember in this case we have a mean m I and we have mean m Q which was 0 for the case of p theta equals to 1 by t 2 pi is non-0 for this particular case.

And this is the power spectral density and which would look in this fashion as given over here. This is for the ration case in looks more complicated and that is why Rayleigh is the case which is easier and gives us an insight into performance of systems.

(Refer Slide Time: 24:05)



So, this would result in spectrum as shown in this particular slide. If you have a specular component this is the Jakes spectrum as you can see. So, there is draw on top of it this is the Jakes spectrum. And whereas, the (Refer Time: 24:24) spectrum as because the racial distribution there is the Jakes spectrum on top of which there is this strong specular component. And what we also have over here is the Gauss spectrum, but as of now what is the important for us is comparison of the Jakes spectrum and the (Refer Time: 24:43) spectrum. This pictures I have actually taken from the book by (Refer Time: 24:47) which is on mobile fading channels. That is also have very famous book it is very detailed books so if you are interested in study more about channel model effects the book by (Refer Time: 25:59) which is on mobile radio channels is also very well recommended.

So, what we remember is when we have the ration component. the ration component would mean that there is this string frequency component and this is clearly because we have this theta 0 which is giving raise to particular frequency and we have f d n is equal to f m cos theta n. In this case this is theta 0. So, basically is the very strong component, a very strong component means this particular frequency is strong that particular frequency beings strong means it is appearing on top of the Jakes spectrum. So, the Jakes spectrum is here, on top of the Jakes spectrum this is clearly visible. So that is because of the line of sight present or a specular component being present.

So, we stop at this point and what I would like to tell you is be have covered some of the most important things required to understand wireless communication systems, the rest of the communication systems would build on these things. Although we have some more things to covered, but whatever we have done till now is I would underline that very very important. So, I would urge you to go to these parts again and again so that you understand them thoroughly, because this is not very easy I would say because it requires sudden visualization it is not then they math’s and other things are very difficult. But it is probably difficult to connect because we are doing some mathematics over here, whereas what is actually happening in the nature.

So, go through the references look at the materials which have provided, go through the discussions so that you understand is improved. Because once you understand this then only your understanding of the rest of the course would be meaningful. Means will be not referring to this directly we will assume all the things that we are mentioned over here are well understood. For instance, will assume slow fading will assume fast fading sorry will assume flat fading so that would clearly ring a bell that we are talking about the these spectrum over which the signal is not fluctuating, but talking about the time over which the signal is not fluctuating.

So, that is the very relatively easy condition, but it is also important you understand what happens in other cases because this lays of foundation to your journey to the domain of wireless communication.

Thank you.