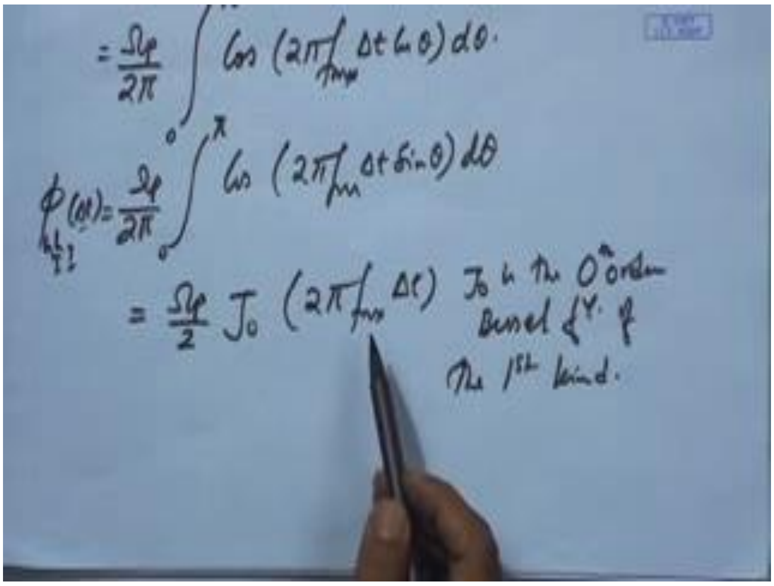
Fundamentals of MIMO Wireless Communication
Prof. Suvra Sekhar Das
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 13
Coherence Time

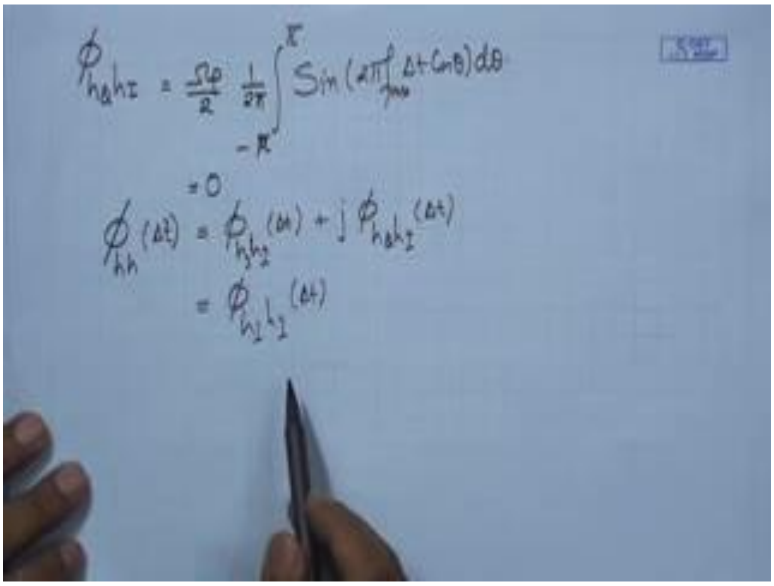
Welcome to the course on fundamental of MIMO wireless communications. We are currently studying correlation properties of the received signal, when continuous wave is transmitted and what is received what we have seen randomly fluctuating amplitude we have seen is distribution, and we have also derived the expression for the correlation of I component.

(Refer Slide Time: 00:48)



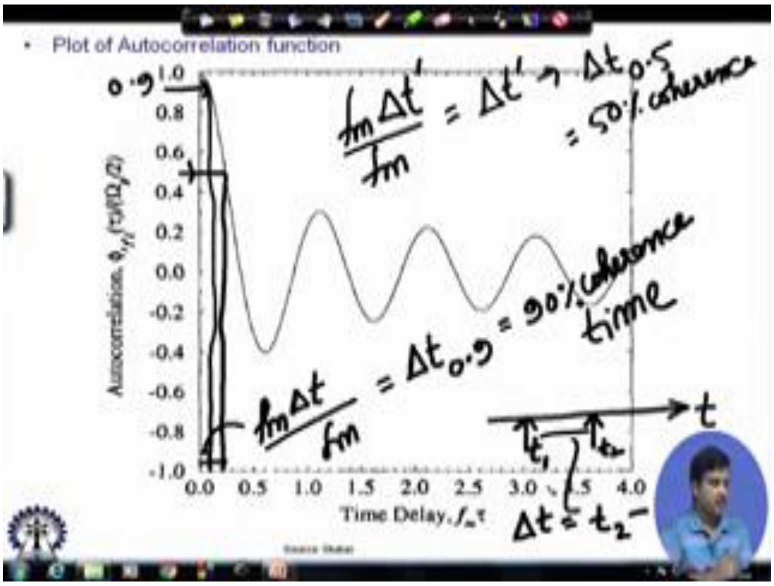
So, what we have last derived is the auto correlation h i, h I with a lag delta t is omega p by 2 pi 0 to pi cos 2 pi f m delta t sin theta d theta, and this turns out to the Bessel function of the first kind of 0th order. So, this is where we had stopped our last discussion and to complete the description.

(Refer Slide Time: 01:18)



We also need to write the expression for phi h q h I now, looking at the expression for h I h q we can more or less guess what will be the h q h I that is the correlation of the cross components minus pi to pi instead of cos what we have is sin 2 pi f max delta t cos theta d theta and for cos we got this j 0, but whereas, for sin since it is a odd function this will turn up to 0 and hence the phi h h of delta t would turn out to be phi h I h I delta t plus phi h q h I delta t and this is 0 and then this is equal to phi h I h I delta t or in other words the correlation of the received signal is completely characterized by the correlation of the I components of the signal.

(Refer Slide Time: 02:38)



Now, this Bessel function as we have derived in this would like the one that is present in this particular picture. So, this is this is the picture and we have also briefly shown you this particular picture, and it is clear now that what we have this picture I was telling you in last time it should be little bit modified, because the picture is taken from book (Refer Time: 03:02) directly. So, this this axis this x axis is going to be f m delta t and this axis is as you are seeing is phi h I h I delta t normalized by omega p by 2 because omega p by 2 is the power that is normalizing this particular term. So, this correlation function as we are seen is decreasing from a peak value of towards 0 and because of the assumptions that we have made we are getting oscillation a damp oscillation about 0. So, there might be negative values, but what is of interest I mean of course, it is a model. So, there are problems which problem with a particular model. So, what would be interested fundamentally is in this range where the correlation the coefficient drops to very small values, and since this axis is plotted with f m delta t we can use this particular figure for any values for instance if you change f m, because f m is equal to v by c times f c.

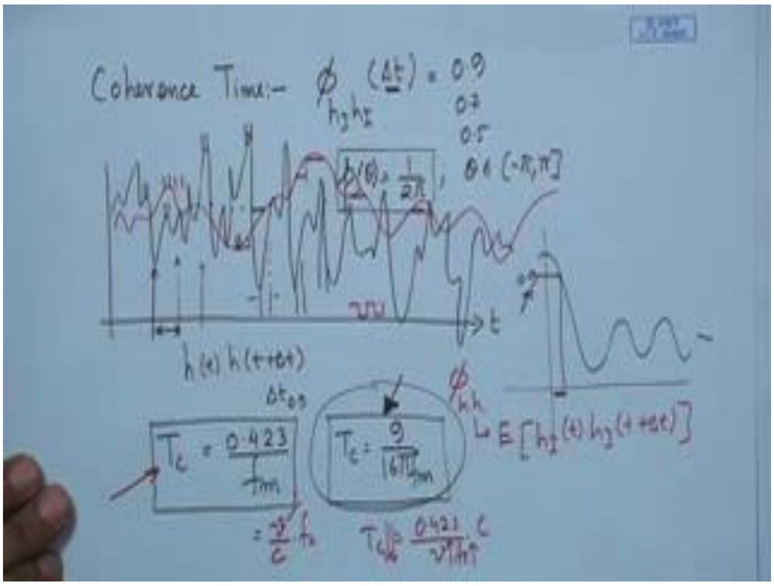
So, as we keep changing f c or as we keep changing v we will get different values of f m. So, what we see over is as f m increases. So, what we are basically getting is this is becoming smaller and smaller. So, a particular value of coherence, this value of coherence appears at much-much smaller duration of f m delta t or in other words if f m is large; that means, if you look at this product f m delta t. So, let us read 1 point if you take this line straight and we reach this point, which is let us say 70 percent. So, what we can read this as at this value of f m delta t which is let us say x, x is equal to f m delta t the correlation is 0.7 or we could also state that phi h I h i, let us x is equal to 0.7. So, since x is equal to f m delta t, as f m increases suppose we increase f m where delta t must decrease in order to maintain the same value of x, as to get the same value of correlation and vice-versa.

So; that means, as the Doppler frequency increases; that means, as the velocity increases, the time at which there is 70 percent correlation would be smaller and smaller. So, what does it mean this effectively means that as we increase our velocity, the time over which the channel is correlated to itself is becoming less. So, if we read this curve in a other way, any point over here let us say this is the 0.9, let us say this point is 0.9 and we start going down this line come here. So, this much this duration this much duration of f m delta t corresponds to that argument which would lead to 0.9 correlations. Now if I divide this by f m what I am left with is delta t. So, delta t at which correlation coefficient is 0.9 is known as the 90 percent coherence time.

Similarly, if I take this point and go down let us say point here, in this point as a value over here. So, we would have f m delta t prime which is another value of delta t here because the same f m divided by f m would be t delta t prime and for this case we would call it delta t 0 0.5; that means, it is the separation in time which would lead to 50 percent coherence between single components. What it means effectively is that, if we take the time axis this t axis and if we measure it in a 2 instants t 1 and t 2 and in this gap between in this time is t 2 minus t 1 which is delta t right.

So, if this delta t is equal to as delta t 0.5; that means, we are having we are at this point the signal would be 50 percent correlated and what we have made assumption over here it is wide sense stationary; that means, it is not dependent on t1 and t2, but it is simply dependent upon the lag time between t1 and t2. So, this is one of the very, very important things in the propagation characteristics which one should understand very nicely. So, I would ask you to spend some more time in this, may be spend time with the references that we have already mentioned relook at it and try to understand it yourself again because, based on this the design of violence of communication system is done significantly.

(Refer Slide Time: 09:33)



For instance it is, we would say that coherence time, is the time for which the phi h I h I in our case which is h h I delta t. So, basically we are talking about this delta t, is equal to a certain value let us say 0.9 or 0.7 or 0.5 and so on and so forth. So, these are very, very important and remember the result we have got is because we have made one fundamental assumption p of theta is equal to 1 by 2 pi, where theta lies in a range of minus pi to pi. If you check these assumptions the results which we have obtained would be different. So, this particular derivation that we made there was g theta p theta in this where p theta was made 1 by 2 pi because of which we had got this, 1 by 2 pi over here and we made g theta equals to 1 for which we have arrived this expression and then we said if we would change those assumptions this results would be different. So, whatever we are discussing is with respect to now these particular assumptions any other assumptions would lead to different result.

So, what we are essentially trying to say is coherence time is the time over which signal is correlated with itself if this is the time axis this is the h of t signal what received would be fluctuating right like this. So, that is what we have seen and we have been trying to get h of t h of t plus delta t and this is what being try to capture. So; that means, we are taking reading at this point and we are taking reading at some delta t some increase delta t and so on. And we have arrived at this particular expression. So, when we say delta t 0.9 so; that means we are talking about separation in time over which the signals are 90 percent correlated.

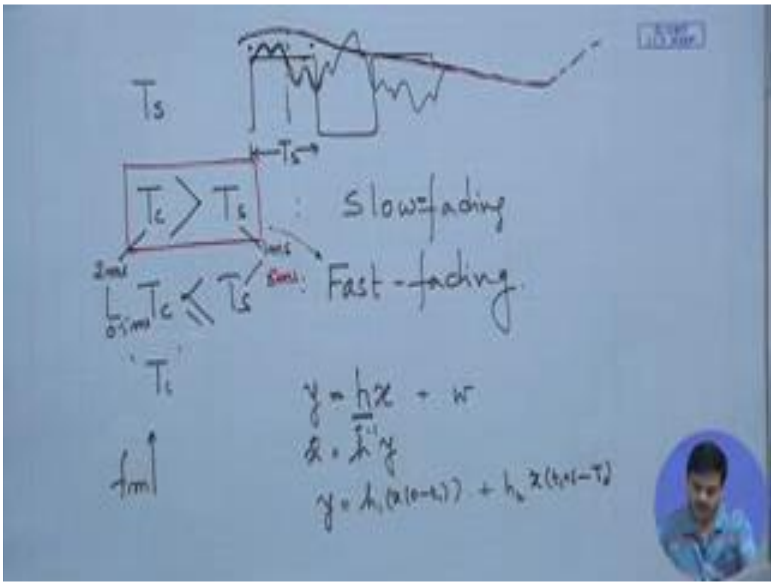
So, in this figure we would probably have this separation which is small as this which is a probably as small as this which is probably as small as this where the signals are correlated with itself, right. If I have point here and a point here is definitely the signals are not correlated because we do not know at this gap there is uncorrelation. So, this duration of gap at which signals become uncorrelated would be a loose definition of coherence time or the time separation at which signal becomes uncorrelated. So, there is rule of thumb for coherence time which is mention as 0.423 by f m. So, if you if you use this particular formula you would get a rough calculation of the coherence time for a particular signal. Little bit stricted definition of coherence time would be 9 by 16 pi f m this is again a rule of thumb.

But what is ideally to be done to calculate the coherence time is we should use this particular picture that is the coherence time relationship and as we over here delta t f m and we should say that suppose I want 90 percent coherence time. So, I would go to 90 percent I would read this particular point and I would try to come to the x axis, once I have read this off I know this is x let us say. So, I would take x divided by f m to give me delta t. So, this delta t would corresponding to this value would give me delta t 0.9 for any other distribution; that means, instead of taking the p theta equals to 1 by 2 pi as we have this particular expression, as we have in this expression in this particular expression over here instead of p theta equals to 1 by 2 pi, if we do something else in that case the curve would be different in that case we would not get the Bessel function in that case we would not get the Bessel function it would be something else and if you are looking at 0.9coherence we have to read up this 0.9 read this x axis from this x axis we have to find what is the coherence time corresponding to the amount of coherence that we require.

This amount of coherence that we require could be 90 percent could be 70 percent could be 50 percent whatever is the requirement. Effectively what do you mean by coherence time? Is the separation in time for which the signals are correlated to itself. So, beyond this coherence time the signal would not be correlated to itself. So, if we take 2 samples for instance I take a sample here and I take a sample here they are not correlated. If I take a sample here they are not correlated, if I take a sample at this point and I take a sample at this point they are not correlated. Whereas when I keep the time interval small they are correlated now on the same figure, if I would draw the situation where f m is very small; that means, slowly fluctuating.

So, in this slowly fluctuating case probably the channel would vary in this fashion. In that case the coherence time would be large; that means 90 percent coherence time would be achieved in this separation. So, here it could be 95 percent coherence so; that means, if you look at very small separation distance or it could be 95 percent correlated to itself. Whereas if you look at even larger distance we may find 90 percent coherence we may find 90 percent coherence in a larger distance. So, this again an average value this is again an average value. Because this axis that we are looking at phi h h which is expected value of basically in our case it is h I of t h I of t plus delta t. So, what should we remember is, at every instance of separation of this delta t we would may not get 90 percent correlation, but on an average if the separation is this t c then you are going to get similarity between the values.

(Refer Slide Time: 16:15)



So, with this we go into a very important definition of our study. If suppose we have t s as the symbol duration. Symbol duration means we talk about digital communication system. So, in a digital communication system supposes we are sending symbols which are like this and so on. So, let be this be the symbol duration. So, if the coherence time as we measured in multiple ways is larger than the symbol duration. So, what will happen?

The channel at this point and this point would be correlated. Because coherence time is larger than symbol duration; that means, across this time this time is less than coherence time so; that means, channel is nearly flat nearly equal to this. So, the basically could be fluctuating in this fashion or in this picture, if you look corresponding to the red line the symbol duration could be as small as this symbol duration could be as small as this. So, if we have symbol going like this then that is the situation that we are talking about, if you have this situation then what we have is known as slow fading. So, we had identified the channel to be time selected in time selective if this is the condition then we says it is the slow fading condition.

Where as if coherence time is smaller than symbol duration; that means, the signal fluctuates even before the symbol is over; that means, the channel is fluctuating like this, in that case it is known as a fast fading situation all right. So; that means, once we calculate our coherence time, once again let me remind you we could calculate coherence time using this formula which is little bit relax formula or this particular formula which is little bit stricter. I would recommend you to always use this formula if nothing is mentioned this is very, very important if nothing is mentioned. If we use this you are having better margin and your system design would be better. if with this basically the distance in time approach with signal is uncorrelated.

So, that is also, but it is not that type design, where is an exact design would be using the derivation of correlation function for the corresponding p theta and from that you plot the curve and you rid of the 90 percent coherence point go to the x axis from which you calculate in the time difference. So, from these three different ways you could calculate the coherence time then you compare with the symbol duration once you find coherence time is larger than symbol duration, you know it is slow fading condition. if it is smaller than symbol duration it is fast fading condition.

Now, why it is important? Because a clearly if the coherence time is larger than symbol duration; that means, we have this particular scenario, where the channel is fluctuating slowly. So, the channel is fluctuating slowly could be because of 2 possibilities; one the symbol duration is really, really small in the other is the mobility is very, very slow in either of these 2 cases we would get slow fading condition. Because we are doing relative measurement, we are doing it in comparison we are comparing the symbol duration with respect to coherence time.

So, suppose symbol duration is 1 millisecond let say let us say symbol duration is 1 millisecond and coherence time is 2 milliseconds. So, if, 2 milliseconds are greater than 1 millisecond it is slow fading right now you consider that this one millisecond is remaining same. So, suppose this is one millisecond and this is 2 milliseconds. Now using the same 1 millisecond, if there is increase in mobility; that means things start to move faster if things move to faster the f m goes up. So, if f m goes up; that means, you are shrinking the x axis, if that happens then the coherence time reduces. If coherence time reduces and as we increase the mobility, if coherence time becomes comparable or less than the symbol duration you end up the fast fading situation.

Now, when you reach fast fading situation; that means, let us say this has become 0.5 millisecond. Now when we when you are in the fast fading condition; that means, signal amplitude fluctuates faster than the symbol duration what is going to happen. Suppose I have sent x there is of course, noise at the receiver and what we said is fast fading condition h gets multiplied. So, in a slow fading condition h would remain same throughout the symbol duration. So, I would receive the symbol y if I could estimate h then I could equalize the effect of h for instance x cap is equal to h inverse times y and I have able to recover x; that means, I would divide y by h and I would getting back an estimate of x where as if the channel is fluctuating like this; that means, if I have y is equal to h one times x for the interval let us say 0 to t 1 and then we have h 2 for the interval let say t 1 plus delta to capital T. so; that means, I am assuming 2 different kinds of 2 different kinds of notable value of h 1 over the period of 0 to t 1 the other of the period of t one to t capital t that is the symbol duration or t s in that case we have to an advance equalizer, sometimes known as fraction space equalizer which would sample at a higher rate and would try to do fractional equalization or even more complicated procedures.

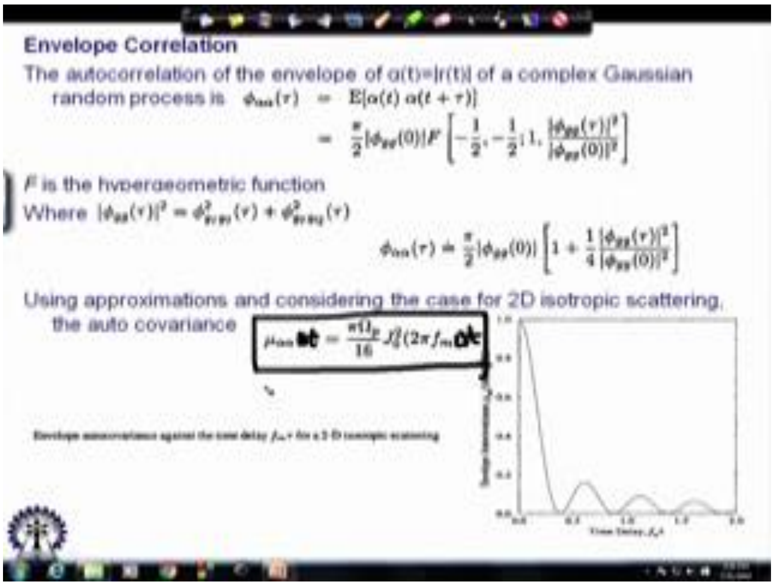
So, typically we would like to design systems which would experience a slow fading condition rather than a fast fading condition. If you cannot help creating a situation of slow fading you have to design advance receivers which would able to handle the fast fluctuation of signal strength.

So, what we saw in the example is that even though the symbol duration is remain the same as mobility as increase coherence time as become smaller and then the system from slow fading as now is experiencing fast fading. On the contrary if we say the earlier situation where coherence time is 2 millisecond, I have a system which as band which as a symbol duration of 1 millisecond, now suppose I use a different communication system I use a communication system where symbol duration is much-much larger; that means, the symbol duration is instead now let us say the 5 millisecond. So, when symbol duration is 5 millisecond all though mobility is giving 2 millisecond of coherence time it is experiencing fast fading.

So, what we what we see is that definition of slow fading or fast fading is comparison of coherence time with respect to symbol bandwidth that means, if I am transmitting from some information let us take the example g s n and l t e, this 2 systems different symbol durations. So, these 2 systems will experience a different channel condition; one could experience slow fading other could experience fast fading for the same mobility condition. The coherence time as you have seen here in this particular expression is affected by f m which is the Doppler frequency this f m is a function of velocity as well as it is a function of carrier frequency as well as f c. So, basically f m is equal to v by c times f c; that means, as I increase v, f m increases as f m increase, t c decreases right because clearly t c is equal to 0.423 by v times f c times c. As I increase velocity coherence time decreases as I increase carrier frequency coherence time decreases right. So, as we increase mobility coherence time would decrease.

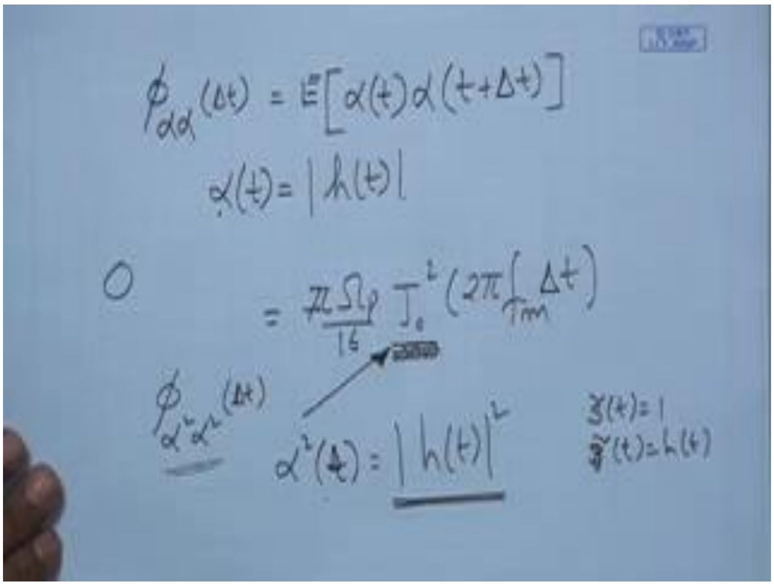
On the other hand if I change by symbol duration again although the coherence time could remain same we would experience different kind of fading condition.

(Refer Slide Time: 25:25)



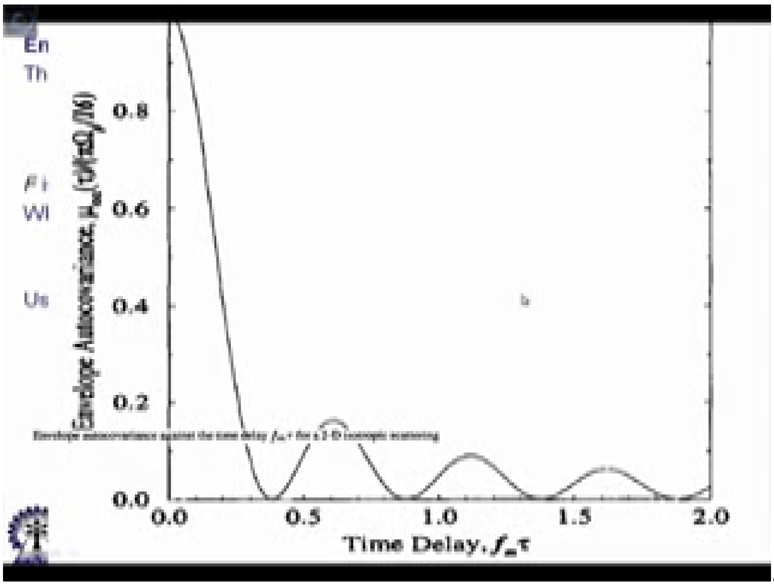
So, with this we move on to see what happens to other properties of the signal. So, what we would lastly like to see is the envelope correlation. Envelope correlation is basically the correlation of alpha alpha t basically we talk of phi of alpha alpha delta t which is equal to the expected value of alpha of t times alpha of t plus delta t where, alpha of t is equal to mode of h t.

(Refer Slide Time: 25:46)



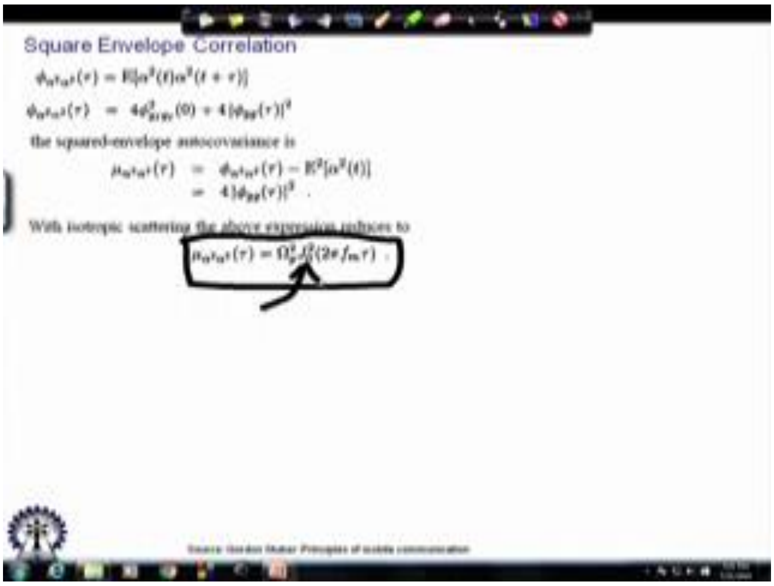
So, result that we have is what we are going to share with you over here, again the derivation is (Refer Time: 26:11). So, we not go through that, if you want you can go through it. So, this particular expression, now this results this things are taken from Stover I have used the description from Stover. So, basically we have is phi of alpha alpha delta t this one would turn out to be pi of sigma p by 16 j 0 squared 2 pi f m delta t. So, what you see the difference between the previous 1 and the current 1 is that it is j 0 squared instead of j squared so; that means, it is decreasing faster, typical example of this would be the figure as shown one over here. So, if you see this particular figure this is the envelope correlation.

(Refer Slide Time: 27:11)



So, this is decreasing faster than the other one.

(Refer Slide Time: 27:23)



And if you look at this square envelope correlation the square envelope correlation also gives us the same thing that this is proportional to j 0 square surprisingly both the results give correlation to j 0 squared.

Now, the importance of alpha phi of alpha squared alpha squared delta t you can clearly guess because alpha squared t is equal to mode of h t squared. Now h for s tilde of t equals to 1 r tilde t equal to h of t. So, basically this giving us signals strength right. So, when we study the correlation of alpha square we are basically studying the correlation of the signal strength. So, what we have over here is that correlation of the signal is proportional to j 0 2 pi f m delta t where as the correlation of the envelope alpha or the correlation of the alpha squared are both proportional to j 0 squared 2 pi f m delta t.

So, this is under line thing should remember in this particular discussion and just to summarize what we have also talk about in today's discussion is the calculation of coherence time we, have explained what is the meaning of a correlation function and how this correlation is effecting and we have also explained rule of thumb for calculating the coherence time for the special case that we are taking the special case is defined by p theta equals to one by 2 pi and finally, we have also described the fast and slow fading conditions, where we have said that the fast fading condition is when the channel fluctuates faster than the symbol duration and slow fading condition is when the channel fluctuates slower than the symbol duration.

So, it is quite easy to remember when the signal fades at a rate which slower than the symbol rate it is a slow fading condition. Whereas when the signal fades at a rate which faster than the symbol rate it is a fast fading condition, and with most of a analysis in this particular course we will be mostly assuming a slow fading condition; that means, within the symbol duration the channel is nearly constant and it is not fluctuating. However, from symbol to symbol there is fluctuation of signal strength with time.

Thank you.