Fundamentals of MIMO Wireless Communication
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Lecture – 11
Small Scale Propagation Received Signal Correlation

Welcome to the course on Fundamental of MIMO Wireless Communication. We are currently discussing this small scale propagation model, we have seen multi path interference model, we have seen the distribution of the envelope of received signal, we have also looked at the distribution of the square envelope of the received signal and we will look into the received signal correlation in today's lecture now.

Before we proceed with the discussion on correlation of the received signal I would like to remind you few things like when we have considered Rayleigh distribution for the envelope that is alpha which is equal to mod of h of t.

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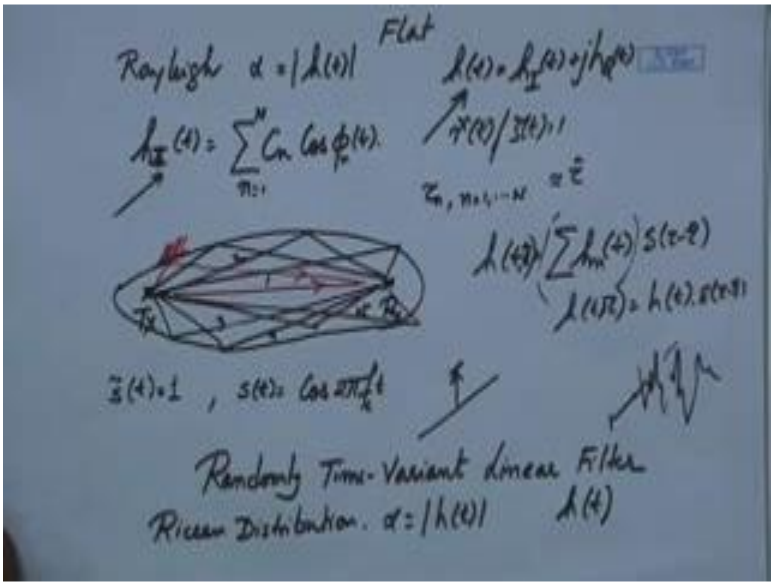

We would like to remind you few things like we have assumed flat fading and when we said h of h I of t that is we said that h of t is made up of h I of t plus j h Q of t, if we look at h I of t that we have explained before it is sum of n components including cos phi n of t. So, when we arrived at Rayleigh we assumed that this is Gaussian distributed following central limit theorem and that was true when the number of paths that are arrived were pretty large and we had assumed or we had considered flat fading when this paths which are different all arrived all most at the same time.

So, what effectively mean is suppose there is a transmitter which be indicated by t x and there is a receiver which is indicated by r x. So, there are n number of paths between the transmitter and the receiver, this we should also remember. There are n number of paths however, the propagation delay for this paths are almost the same and that would happen if these are at the two focal points of an ellipse where the reflectors or scatter error of the object on which the signal impinges and gets dispersed the lie on the ellipse whose two focal points are the transmitter and the receiver.

If we have this then we can say that there are n number of paths, very large number of paths and they are all having this tau n for n equals to 1 to capital n almost equal to tau cap and this let to be the conclusion that, we could have flat fading because our expression of h t we had sum over h n of t delta tau minus tau cap and then we could add up all of this to represent h t sorry h t comma tau. So, h t comma tau was h t times delta tau minus tau cap this led to flat fading. However, there are very large number of components which gets added up because of which we could make the Gaussian approximation and for which we have got the Rayleigh distribution, this one of the things which we would like to remind you.

The second important thing which we would like to remind you is that when we derived these expressions we said that let s tilde t is equal to 1. So, when we said let s tilde t be equal to 1 what we meant was s of t was cos 2 pie f c t for the carrier tone itself which means the (Refer Time: 04:34) a constant amplitude tone in the frequency axis however, whenever it is received across the band of interest it was flat. But, this amplitude that is received fluctuating, it is fluctuating with time and therefore, we have h of t h I of t and h of t or we have h of t time varying and what we have said this it is time variant, what we have also seen is that this variation can well characterized as it is a random variation and we have also seen that this received signal could be represented as this was r tilde t when s tilde t is equal to 1. When this is Gaussian distributed we have seen that. So, this random distribution is Gaussian distributed for this special case and we have also seen that this received signal can be represented as the outcome of the linear filter. So, what we have effectively a channel as randomly time variant linear filter.

So, even though we have send a constant amplitude the received signal amplitude keeps on fluctuating dynamically and this amplitude fluctuation can be described in one instant by a Rayleigh distribution. In another instant we said that it could be Ricean if there is a strong line of site component present or if there is one strong a specular component suppose there is excellent reflector which converges energies, they go there. So, when there is a specular component instead of Rayleigh what we get is the Ricean distribution if there is a specular component. So, alpha which is mode of h t is Ricean distributed by that what we have seen.

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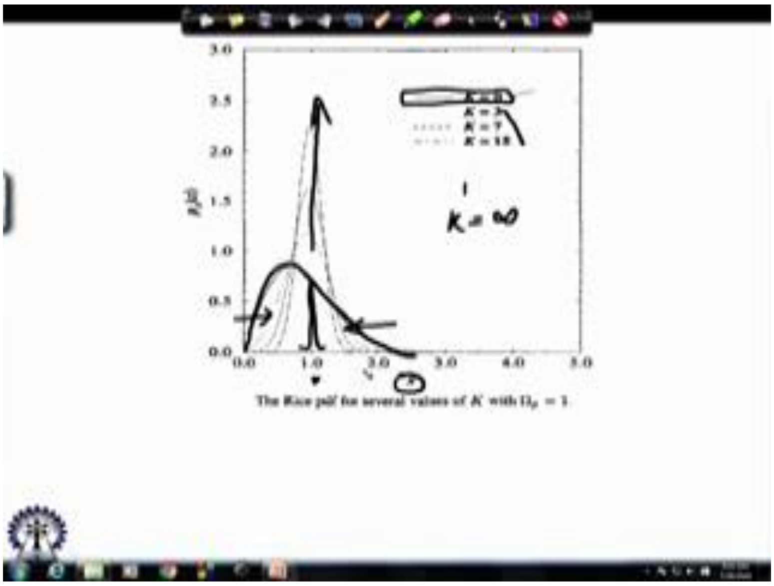

And we have also described that the distribution would look like one we have here and for the values of k, values of k as in this particular figure when k is equal to 0; that means, the weight of the Ricean component is 0 there is nothing in this particular specular component then we are left with all reflected paths and what we end up with a Rayleigh.

However for non zero values operation coefficients for zero we have distribution following Rayleigh, for non-zero values distribution as the coefficients increases slowly becomes narrower and narrower and it peaks. So, when k tends to infinity this will become very very narrow almost consoled in one place and rise in various peaks which would mean that there is hardly any variation of the signal which is denoted by x. So, when there is a very very strong specular component compared to others or we can say that between the transmitter and the receiver there is a very strong specular component.

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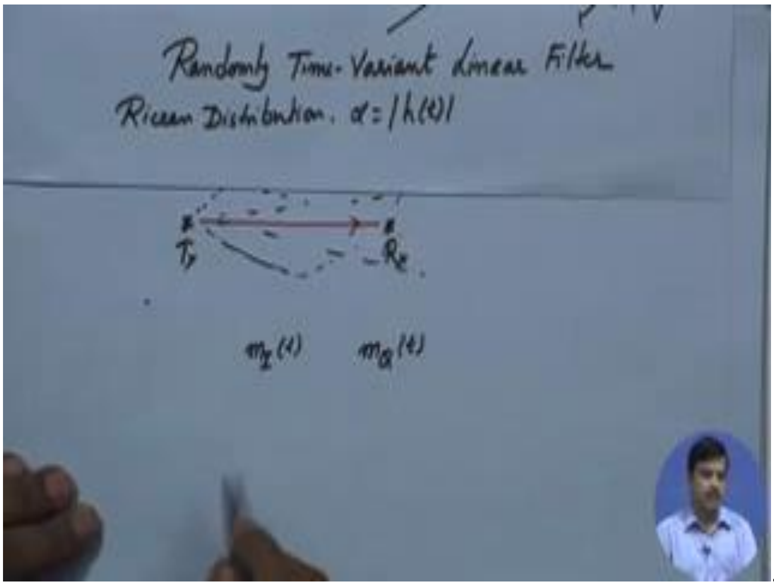

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Whereas the reflected paths are very very small, they are hardly signal strength available in the reflected path. In that case what we have is the Ricean distribution, in that case when this are almost negligible; that means, when this is infinite in ratio with respective to the power in this there is hardly in fading. So, we said last time we could almost imagine this to be (Refer Time: 08:47) condition if there is only additive (Refer Time: 08:50) noise or near to ideal condition with a signal strength and under this conditions earlier we said that h I is 0 mean h Q is 0 mean, but in this case h I is not zero mean, but there is a certain m I that is a mean associative with I component and the mean associative with Q component.

So, what we see is there are two different kinds of very prominent distributions that usually would appear. So, to have uniform model what was done is the Nakagami distribution was proposed

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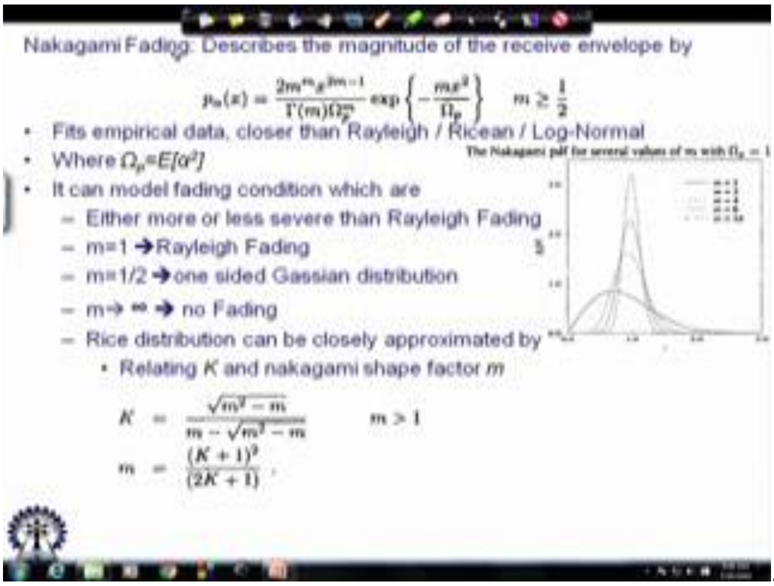

So, the Nakagami distribution is the one which is repeated by the expression as shown in this particular one. So, p of alpha, alpha is mode of h. So, that is the envelope is 2 m to the power of m, x is basically the variable random variable 2 m minus 1 by gamma function or gamma of m is the gamma function omega p is the power of the received signal e to the power of minus m x square by omega p for m greater than half. So, with this one could describe different distribution whether it is Rayleigh or Ricean by choosing appropriate values of m and that is one important reason why Nakagami m is very very important.

So, in this module if m is chosen to be one if you take this particular case this distribution would resemble Rayleigh distribution you can clearly see m to be equal to 1, this would become equal to 1, they would be x e to the power of x square by omega p and whole expression like x divided by omega p, m is equal to 1, e to the power of m x square e to the power minus x square by omega p. So, that would be a Rayleigh distribution for m equals to half it would like a Gaussian distribution again you could clearly see m equals to half means minus x square by 2 omega p and rest of it would turn out. So, this would be a one sided Gaussian distribution for m equals to infinity they will be no fading just like the Ricean distribution and the Ricean distribution can be closely approximated by Nakagami m relating k that means, the rice factor to the Nakagami shape factor which is the m in all this expression.

For example if you would take k equals to this particular expressions square root of m square minus m divided by m minus square root of m square minus m for m greater than 1, you could map a value of m corresponding to a certain value of k. So, if I would chose m equals to 1 I would get a Rayleigh distribution and if I would chose m equals 2 this particular expression as given in one below I can get Ricean distribution corresponding to that particular value of the rice factor. So, using a single expression we could get the distribution of envelope for either Rayleigh or Ricean.

The advantages is, why we are studying all this kind of models the importance of all this kind of this models is that we would like to design transmitter receiver as same. So, design the transmitter and receivers you would need important performance matrix and to get this performance matrix we would often require this two be analytical expression or close form expressions. So, for Rayleigh we have seen the expression is quite easy quite simple and (Refer Time: 12:27) for Ricean it is not.

So, whereas if you look at this particular expression again this could lead to close form expressions, if we use this for calculation of let us say (Refer Time: 12:38) expressions and using one result we could choose different values of m and that would give us the performance for Rayleigh case or appropriate Ricean case or even no fading case. This figure here this diagram shows the different distributions for the different values of m and clearly you can see for m equals to 1, this one looks like most the Rayleigh distributions and as the value of m increases this density function become more and more constraint within the small range of x as well as speaking and if we would look at the previous PDF Ricean distribution we see there almost similar. So, that is the advantage of nagakami m distribution and it is often used in study of violence communication systems.

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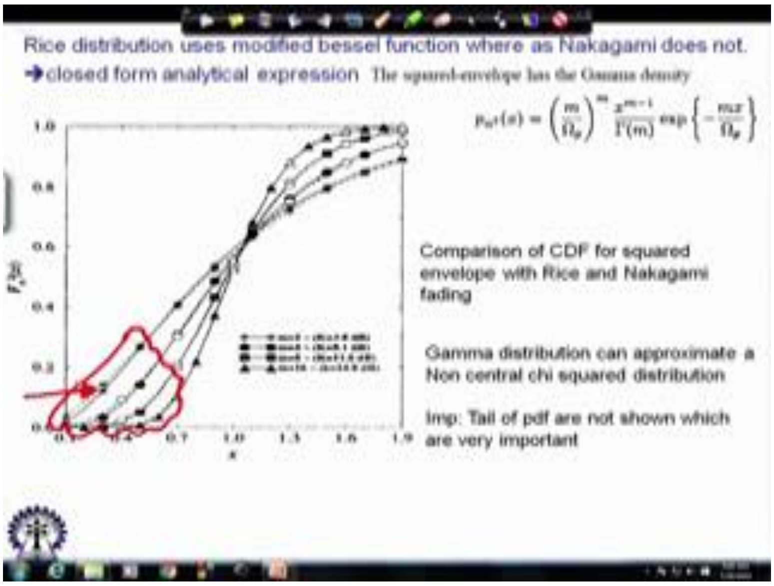

Now, last note regarding this particular part k equals to root over m squared minus m so on and so forth with matches maps k to m and m to k we have to little bit careful. So, this particular figure shows the distribution of the square envelope of gamma density function and on the x axis the variable x whereas we have two different kinds of lines one is the dash line, one is the solid line, but there is the same legend that mean the star, there is a circle and the square and the triangle using dash in the dotted line.

Dash in the dotted line correspond one for the nakagami m distribution the other for the Ricean factor. So, even though we use the corresponding mapping as given over here; that means, even if we use m k match to each other when we look at the corresponding distribution what we find is that if we look at the actual Ricean distribution and if we look at the Nakagami m distribution. So, there is a gap between the two distributions especially towards the cell h. So, you clearly identify the gap and this particular result is for m equals to 2, but as we increase the rice factor this gap decreases. So, when it is closer to Rayleigh the gap is larger, as we seen in the upper portion of the distribution also.

Now, amongst these what is more important is the tail probability distribution for communication system because we are often interested in calculating the outage probability or probability that signal will be below certain threshold. So, when we do that what we should remember is if we are approximating Ricean distribution by gamma distribution or Nakagami m distribution what would we find is that there is a gap towards the tail which could lead to certain (Refer Time: 15:31) values of the outage probability calculations. The error is sometimes very very significant because we are looking at the tail probability. So, we have to keep in this mind we are using Nakagami m in substitute for Ricean distribution, right.

So, with that we move ahead and we take our discussion to the single correlation, now why do we look single correlation? Because, we already said that what we have is a randomly time variant linear filter whose coefficients are given by h of t, so h of t is random although there is a constant amplitude signal that is been sent. So, study random variable or stochastic process we need to study it in multiple dimensions. So, one of the ways to look at it is the distribution function, we have already look at the distribution function we are said that under circumstances when there is no line of sight - only non line of sight is present what we generally get is Rayleigh distribution. However, if there is a strong specular component what we get is the Ricean distribution. So, the non line of sight there is Rayleigh in the line of sight it is Ricean and this are two most common distributions that we encounter and we have also discussed the Nakagami m distribution which can be used to analyze both the Rayleigh case as well as Ricean case by choosing appropriate value of the shape factor that is m.

Now, after looking at the distribution for this particular random process, but we also need to look at is the correlation properties because its time varying. So, if it is time varying we need to look at what is the characteristics of time variations. So, to do that we need to look at correlation properties with that we will be able to define the behavior h of t which is the objective of the next part of this particular discussion.

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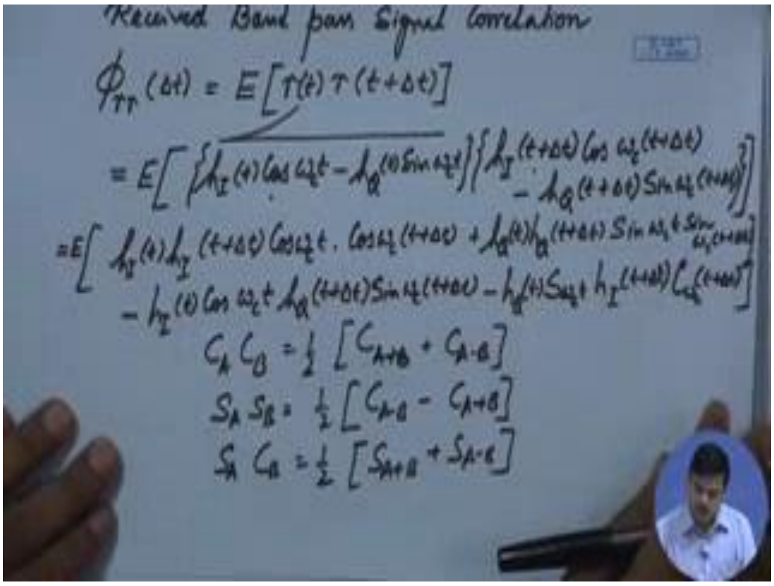

So, if we look at the signal. So, if we look at the received band pass signal correlation we could write phi of rr of delta t is equal to expected value of r of t times r of t plus delta t. So, this is the basic expression for the correlation. What will go through in the next few minutes go through the steps from which we could find this which will lead to very important result that is the (Refer Time: 18:31) time of this kind of channels. So, if we expand this I would put this equal to sign here because that would help us have a lot of space over here we would expand r of t, r of t would be h I of t cos instead of writing 2 phi of c t we will write omega c t just. So, that it helps us reduce the space omega c t minus h Q of t times sin omega c t, that is the first component.

The second component is h I of t plus delta t cos omega c t plus delta t minus h Q of t plus delta t into sin omega c of t plus delta t. So, that is we have r of t which gets expanded in this part r t plus delta t is basically r t, but this t has been replaced by t plus delta t. So, what you see in this two expressions is there exactly the same h I t, instead of h I of t - h I t plus delta t; cos omega c t cos omega c t plus delta t h Q of t h Q of t plus delta t sin omega c t plus delta t.

So, in the next few steps we have to expand this expression do all this multiplications collect the terms, so that we are able to apply e operator and to get our appropriate result. So, if you expand this expression you still e operator, e is the expectation operator which we keep it outside - we have h I of t times h I of t plus delta t cos of omega c t times cos omega c t plus delta t; that means, we have taken this two products plus, we have this two products h Q of t, h Q of t plus delta t. So, which you can identify we are basically multiplying this two terms and we have the sin and sin term, sin omega c t times sin of omega c t plus delta t and of course, there are other terms.

Next we have h I, h Q terms h I of t cos of omega c t, h Q of t plus delta t sin of omega c t plus delta t this product is done. Now we will be doing h Q, h I this term. So, again there is a minus sign h Q of t sin omega c t, I am writing this in short h I of t plus delta t into cosine of omega c t plus delta t. So, now, we have all the expanded forms.

So, what we see there is a cosine in a cosine and there is a cosine in sin, sin and sin and there is a sin and cosine. So, we need to use our laws of trigonometry. So, which is summarize them we have cos A times cos n. I am writing in short is equal to half of cos of A plus b plus cos of A minus B and we have sign A sin b we will use this results cos of A minus B minus cos of A plus B, and we have sin A cos b is equal to half sin A plus B plus sin of A minus B. So, using this like we have cos A cos B we can apply this expansion, sin A sin B we can apply this so on and so forth will be able to reduce this expression.

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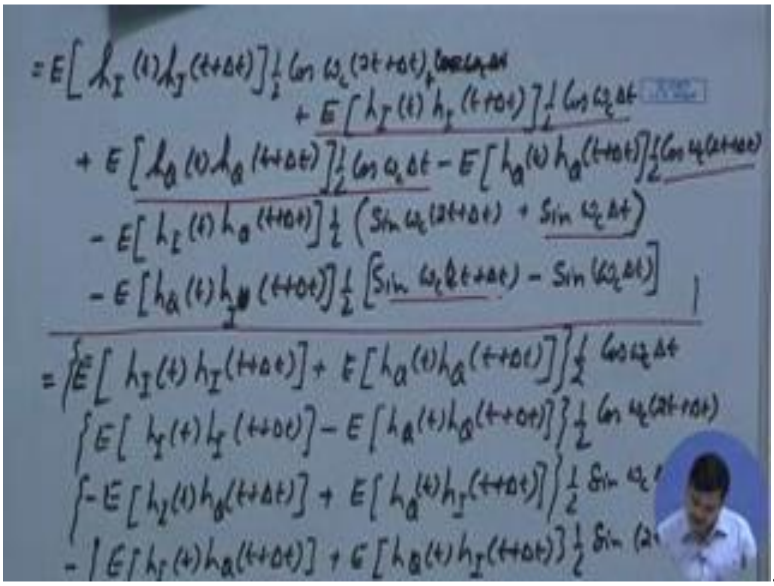.

So, as we go ahead with this expression I would still keep the e operator. So, we have h I of t h I of t plus delta t. So, we have cos A, cos B so we are going to reduce this expressions to half cos A plus b and cos of A minus B. So, we are going to have cos omega c 2 t plus delta t and cos of omega c delta t sorry we going to have a plus E of h I of t h of t plus delta t into half cos omega c delta t plus expected value of h Q of t, h Q of t plus delta t and (Refer Time: 24:38) up, so that you can follow from whatever was in the previous lines into half cos omega c delta t minus E of h Q of t h Q of t plus delta t times half cos omega c 2 t plus delta t and then we continue to have minus sign because of this minus expression.

So, what we have is E of h I of t, h Q of t plus delta t and we will write in short in this occasion that is sin omega c 2 t plus delta t plus sign omega c delta t and the last expression again we will combine h Q of t, h I of t plus delta t half sin omega c 2 t plus delta t minus sin of omega c delta t. So, with this we have been able to capture all the components of the product of this particular line that are there they would open up this and then we could collect all the terms and which are together; that means, we could collect this particular term, we could collect h I h I cos omega c t and we would collect h Q h q cos omega c delta t because there is a cos omega c delta t and there is a h I t h I t plus delta t and we could also collect the terms which are associated with cos 2 t plus delta t.

We would also collect the terms which are associated with sin delta c separately and this separately. So, if we do that the expression what we end up with would be E of h I of t h I of t plus delta t plus E of h Q of t h Q of t plus delta t - this whole thing having the coefficient cos omega c delta t. We also have e of h I t h I t plus delta t minus E of h Q of t h Q of t plus delta t with half cos omega c 2 t plus delta t. And then we have the other term that is e of h I of t h Q of t plus delta t with minus sign in front of it plus E of h Q of t h I of t plus delta t. This having with it sin omega c delta t and then finally, we are left with the last term that is minus of expected value of h I of t h Q of t plus delta t plus e of h Q of t h I of t plus delta t multiplied by half sin 2 t plus delta t.

So, we have arrived to the expression which is interest to us. So, this particular expression we have been able to separate the terms to one where the cosine component is without the t, it is having only the delta t. Then, we have the terms which have the t component and there is also sin which has the t component and we have the sin which is omega c delta t.

So, we stop at this point in this particular lecture and we will continue with this expression to derive the expression of correlation of received envelope using certain properties which we able to describe very soon.

Thank you.