Fundamentals of MIMO Wireless Communication
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Lecture – 10
Small Scale Propagation Envelope Distribution

Welcome to the lecture on Fundamentals of MIMO Wireless Communication. Currently we are discussing Small Scale Propagation Model. After discussing multipath interference followed by flat fading we are now going to discuss the envelope distribution for small scale fading.

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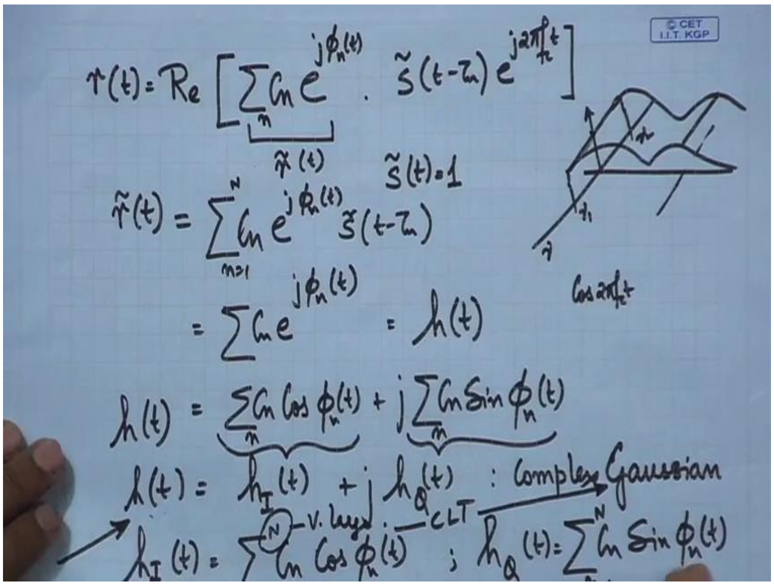

The received signal that we have seen till now is represented as real part of sum over c n e to the power of j phi n of t times s tilde t minus tau n e to the power of j 2 phi f c t. And we have identified this part as r tilde t. Now in order to study flat fading, what we have seen flat fading is 1 where in the entire range of frequency the gain is constant although the gain changes with time with the same gain remaining across the entire frequency. Since, the gain is flat across the frequency range of interest we can study it by sending a single tone in the whole band of frequencies. That means, if we reduce cos 2 pi f c t or in other words we set s tilde t is equal to 1.

So, if we set s tilde t equals to 1 what will get is the s of t is real part of e to the power of j 2 pi f c t which is caused to pi f c t or a continuous wave transmission. If you are sending a continuous wave transmission whatever happens to the signal envelope over here we will expect it to be the same across the entire band of frequencies, because we are encounter a flat fading channel. Now if we have this we basically have r tilde t is equal to sum over c n e to the power of j phi n of t s tilde t minus tau n, n is equal to 1 to n. In other words what we have s tilde t equals to 1, so we will be having sum over c n e to the power of j phi n of t, and which is also equal to h of t as we have seen in the previous lecture.

So, if we look at this part we could write this as sum over c n cos phi n of t plus j sum over c n phi sin of phi n of t. And this is instead of writing as r tilde because we have taken s tilde is equal to 1 we can write this as h of t. So, what we have as the channel impulse response which is basically an impulse in our case instead of a train of impulses which gives raise to this flat fading as sum of sinusoids waved by some coefficients c n in the both and real in the generic part.

Now in order to study this we will take a look at these individual real parts and the imaginary part. So, let us say h of t is equal to h I of t plus j h Q of t, where h I of t is this expression and h Q of t is this particular expression. Where, h I of t is equal to sum of c n cos phi n of t and h Q of t is equal to sum of c n phi n of t. If we look at these expressions this c n’s are basically contributions from the different surfaces on which the impinging waves hit up on get reflected, deflected, scattered or any other process and there is also this phase component.

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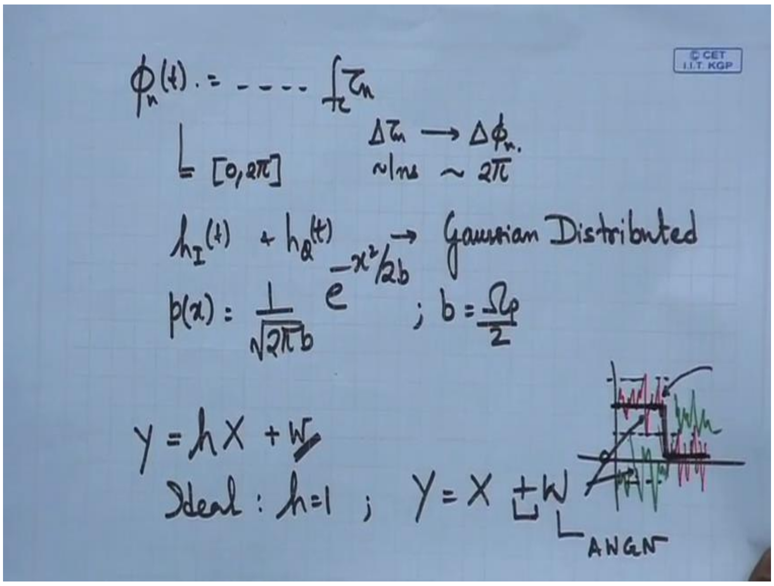

What we have seen for the phase component is that, this phi n of t has a component amongst others f c times tau n. And we have seen that even a small change in tau n gives rise to a large change in phi n. That means, we have seen in the order of 1 nano seconds if there is change in tau n this changes almost by 2 pi.

So, because of this it is reasonable to assume that phi is uniformly distributed in the range of 0 to 2 pi, so this is again an assumption. And this has also been a validated, so this is a reasonably good assumption. This is one of the important things we take. Regarding c n's that we encounter in this expression these as we have seen it is from the propagation environment so there is no fix number for c n these numbers would also be random. And using this method one could generate these channel coefficients directly by the first principle. Although there are many different ways of doing it so this itself captures a way of generating this coefficients h I and h Q together which gives us h which can be used in evaluating performance communication systems.

Now if we will take a closer look into this what we have is phi n's are uniformly distributed between 0 to 2 phi and c n's are random. And one simple assumption on c n could be that c n's are all one that could that is also possible. With that what we have is a large number of additions 1 to n, if n is significantly large one rule of thumb is n be at least 6 or if it is rather more than 6 we can invoke the central limit theorem for this and for this both. And with that we can assume that h I of t and h Q of t, if we take this values they can be assumed to be Gaussian distributed.

So, what we have found is through our realisation that means through our model when we studied flat fading, if we take a continuous wave transmission through which we wanted to studied the signal behaviour across this whole band of frequencies what we could get at the end is this channel coefficient or the received signal corresponding to a tone transmission or the complex envelope of this signal as a real part and an imaginary part. Real part and the imaginary part are made of sum of a large number of variables and we are made the assumption that phi n is uniformly distributed in the range of 0 to 2 pi.

So therefore, this is a uniformly random distribution and cos of phi n accordingly would be distributed in the range of plus 1 to minus 1, same with sin and along with which there is another random component in the amplitude which is because of the surface on which the impinging wave hits upon and then arise at the receiver. So, even if we make the where simplistic assumptions of c n to be 1 we would still end up in a situation where h I and h Q using the central limit theorem we could say that they could be Gaussian distributed. And we could write down the probability density function of h I or h Q as 1 by root 2 pi b e to the power of minus x square by 2 b, where b is equal to omega p by 2, where omega p is the total received a signal power.

So, with this going back here what we have us h t which is again I repeat the received signal when s tilde t equals to 1 as can be seen from this expression or which represents the channel coefficients for a flat fading condition is basically complex Gaussian random distributed; it is basically complex Gaussian. So, what we see is that h which is the channel coefficient has is randomly distributed. Now this is one of the important things which we have said in the beginning that we want to understand the channel, because understanding the channel will help us in designing transmitted sequences as well as devising the receiver structure.

So, one important thing that we come across at this point at this point is this channel coefficients are random. So, if you look at the flat fading condition here we have values which appear random, it is complex so it has real part and imaginary part and it is the distribution is complex Gaussian. If we take a typical digital communication system where let us say x is the symbol that is getting transmitted it is a flat fading channel. So, there is multiplication with h and let us say there is noise which is getting added. And finally, we are getting the received signal y. If there was no channel or if this was ideal condition, that means if h was equal to 1; that means, under ideal condition h is equal to 1 in that case the received signal would be y equals to x plus w. Where, w is the noise component and what we have is additive noise because of the plus. And if you take this as Gaussian and white we have additive white Gaussian noise.

If noise is not present we receive the perfect signal. If there is noise we get distorted signal because of noise. In that case suppose we have sent a signal which is like this what we would receive is because of noise addition, but now what has happened is there is h getting multiplied. Suppose h takes a values even if you take it take the real part of it something it is multiplied and that multiplication would lead to the level being shifted to any point. If it is opposite in sin it could go below 0, in this case or it could go above 0. So, the received signal could be anywhere, it could have been here. And in this case the received signal could have been here.

On top of it we do not know the exact level and this scaling factor can be different for different symbols. The job of the receiver design and the transmitted design would be such that when this is the signal level those are transmitted, whereas this or this is what is received we have to reconstruct this signal back we need to understand the behaviour of this. From the current discussion what we could get is that these h coefficients are Gaussian distributed, so that means these amplitude it could go to any level although with different probability values and accordingly we have start working on the receiver design.

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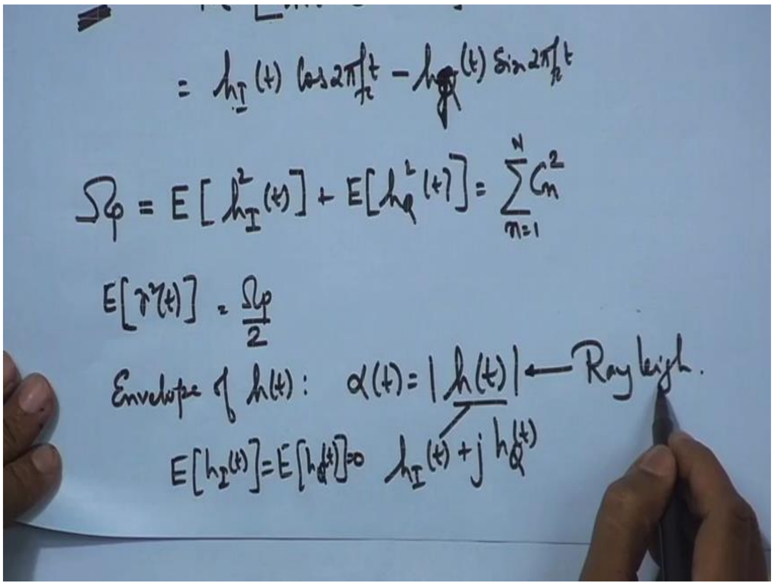

So, based on this we could write the received signals has the real part of h of t e to the power j 2 pi f c t, and which is equal to h I of t cos 2 pi f c t minus h Q of t sin 2 pi f c t. And once again when s tilde t is equal to 1 the received signal would be that corresponding to a continuous wave transmission, so this would be the pass band channel coefficient which can be expressed in terms of the band pass in terms of the base band or the complex envelope of the channel coefficients translated to the appropriate carrier frequency. With these we also make the assumption that or put the symbol that omega p is defined as the total received power that is expected value of h I squared plus expected value of h Q squared of t which is equal to sum over n equals to 1 to n c n square and expected value of r square of t which is the a pass band signal would omega p by 2 because this in the power in the pass band.

Now, once we have seen that h I and h Q are Gaussian distributed and h is complex Gaussian distributed what we are interested in as is the title of today lecture is the envelope of h of t which we define alpha t is equal to mod of h of t. So, h t is complex Gaussian, h I of t plus j h Q of t. So, this is Rayleigh distributed provided expected value of h I and expected value of h Q goes to 0. Now if this is random and this is uniformly distributed between 0 and 2 pi and c n is independent of phi n we can easily get that expected value of h I is equal to 0 and expected value of h Q is 0.

In that case we would have e of h I of t is equal to e of h Q of t is equal to 0 and the envelope of h t which is defined as alpha t that is mod of h t would be a Rayleigh distributed.

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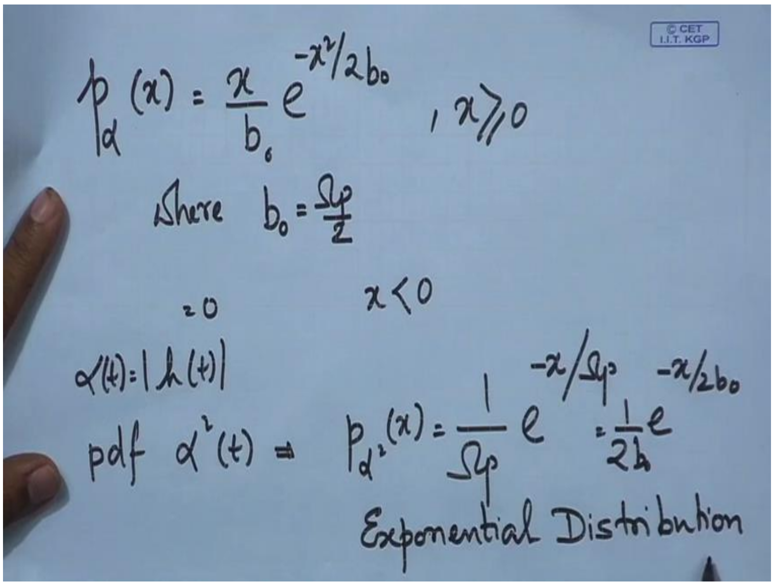

And the Rayleigh distribution would be given by p alpha of x is equal to x by p 0 e to the power of minus x square by 2 b 0 for x greater than or equal to 0. Where b 0 is equal to omega p by 2 and it is equal to 0 for x less than 0, because alpha is mod of h t and the lowest value that alpha can get is 0 anything below 0 it is not defined and hence the probability density function is 0.

The interest in alpha t which is mod of h t lies because mod of h t gives us an indication of the signal power. So, that is why this is a very very important quantity and this Rayleigh distribution is also one of the most important distribution that you would encounter in the study of wireless communications. Of course, there are many other distribution, but this is one of the most popular once which is widely used in understanding simplest communication systems and gives us a simple expressions and whether we try to understand the behaviour of a system when it goes through wireless channel.

Now we would also be interested in the probability density function of alpha square of t. Alpha square of t is clearly the square envelope and squared envelope is going to give us the signal strength directly, and we could write this as p of alpha square of x is given as one by omega p e to the power of minus x divided by omega p which is equal to 1 by 2 b 0 going by the expression above e to the power of minus x by 2 b 0.0 And this kind of expression is the exponential distribution.

So, what we can say is that the power of the received signal over a wireless fading channel for the case that we are considering is exponentially distributed. Now just to remember some of the important things that we have come across while doing this derivation is, we made the assumption that phi n is uniformly distributed between 0 to 2 pi; is one of the important assumptions that we have made. Second important assumption we have made is this n is very large so that using the CLT; that means the central limit theorem we can make this Gaussian approximation.

So, with that we could arrive at the distribution of the alpha that is the envelope which is Rayleigh distribution and the distribution of the envelope square which is giving us an indication of the signal strength is exponential distribution. So, the signal strength is exponentially distributed at the receiver.

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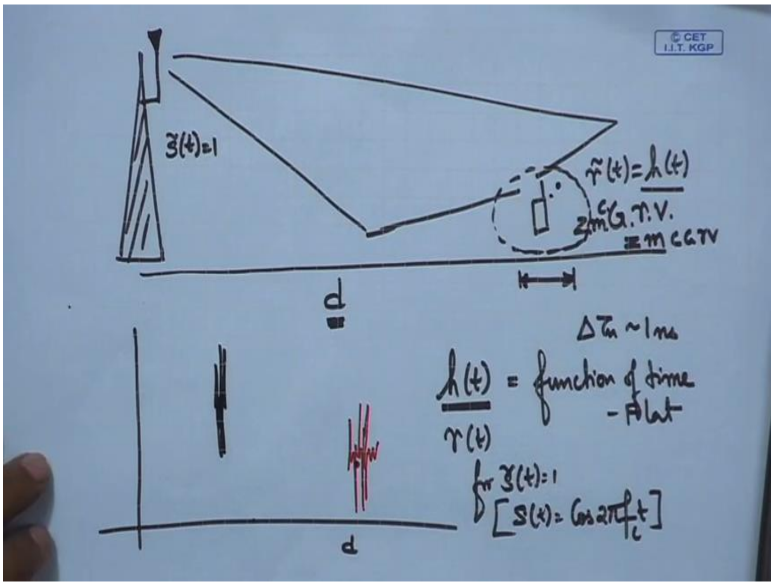

Now, just to explain what it means is, suppose we have transmitter with an antenna this is the separation distance and let that be a mobile. In all this expressions in the last two lectures and this lecture we have never used d, we have never said that the mobile has moved from it is location. However, what we have said is there are paths which are reflected and the reflection could be from moving surfaces or this could be moving, but there is no notable change in the distance within the consideration. Or what we mean is this mobile is within a region where the average received signal strength is remaining the same, because we have always been talking about tau delta t n in the order of 1 nanosecond. We have taken those examples and it is almost practically static mobile condition.

So, the average received signal strength is remains the same. In all cases what we have explain is that h if t which represents the channel strength or r of t for s tilde t equals to 1, that means further case where s of t is equal to cos 2 pi f c t is function of time. And we have also discussed the case where it is flat across the frequency. So, when we studied the different kinds of channels we said in small scale fading there is time selective, there is frequencies selective, there is phase selective.

So, at least we have seen two things and the partially defined one of them; the first that we could note is this channel values are fluctuating with time. That means, here this the signal is not constant. What we have sent is s tilde t is equal to 1. What we have received is corresponding to that r tilde t which we have defined as h t in this case for the case s tilde t equals to 1 is a random variable. For the case where there are large number of paths coming from all directions and where the mean received signal is 0, what we found is this not only a random variable it is Gaussian random variable and it is a 0 mean complex Gaussian variable usually represented as z m c g r v; 0 mean complex Gaussian random variable.

So, even though a constant signal was sent, what we have started receiving. So, even though a constant signal was sent from this point what we kept on receiving is time fluctuations of the received signal strength. And the received signal envelope fluctuates as Rayleigh distributed as given by this distribution and the received signal strength is distributed following exponential distribution and the signal itself is complex Gaussian distributed.

So, I repeat that even though we have sent a signal with constant amplitude what we receive is signal with time varying amplitude it is a function of time, and the amplitude is distribution is random. This is a very significant observation that we could make in our progress of wireless fading channels. And this time domain variation is resulting in the fad in time direction. In the frequency direction it is frequency flat and not frequency selective.

So, what we keep in mind with respect to this figure is we have not generated the expression where receive signal strength changes with distance that is already over in the discussion where we talked about large scale propagation model. In this particular model we are at a location d, now this location could be anywhere either close to the base station or far away from the base station; at that location there is fluctuations of receive signal strength, at that location there is continuous fluctuation of the received signal strength.

So moving ahead further, what we have discussed is the Rayleigh distribution. Now instead of Rayleigh distribution in practical scenarios there could be other distributions also and one of the other important distributions is the Ricean distribution.

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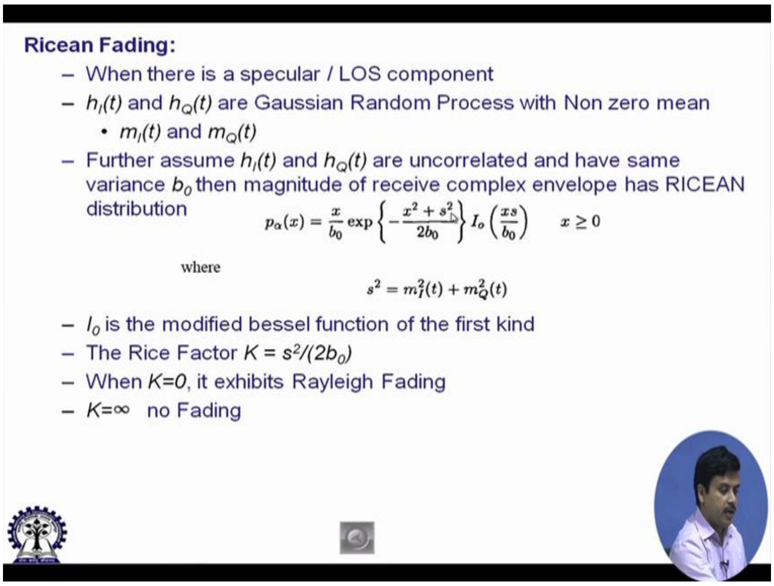

Ricean distribution is present when there are line of sight components. So, what we mean by a line of sight components is or specular component.

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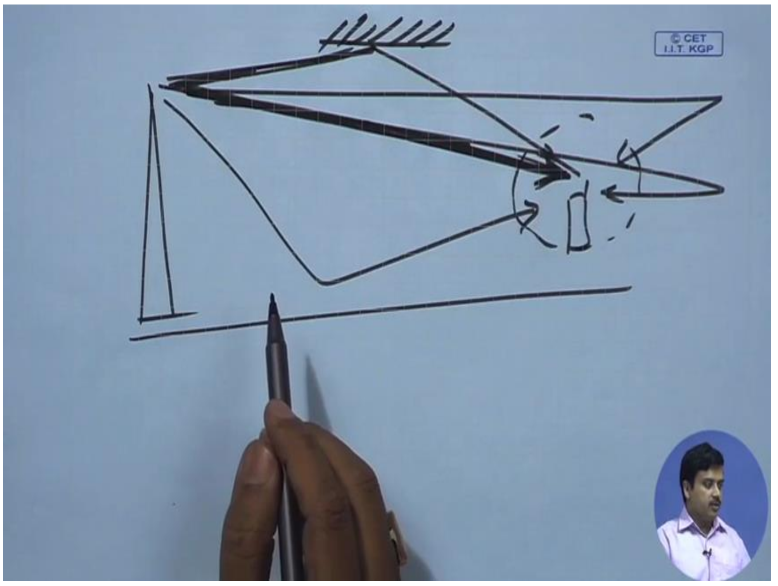

That means, between the transmitter and the receiver there is either a line of sight or there is a very very strong reflector and one of the paths stand out distinctly compared to other reflected paths. So, each signal strength is much-much stronger. Definitely if there is line of sight usually this is one of the strongest paths, if there is a strong reflector that could also be a strong. So, compare to other paths when one such typical path is present what we get raise to is Ricean fading which we are going to describe. In that case we have h I and h Q that we have seen is Gaussian random process, but with the non zero mean as written over here, which means m I corresponding to the real part of the signal and Q corresponding to the imaginary part of the signal.

We will also make the assumption that h I and h Q are uncorrelated the details of such things we will see in the following lecture. In such a case p of alpha, alpha defined earlier is given as x by b 0, b 0 has also been defined exponent of e to the power of minus x square plus x square divide by 2 b 0 and I 0 of x s by b 0 for x greater than 0. Where, s is defined as the sum s square is this square of the mean of I component and that of the Q component and I 0 which is present in this expression is the modified Bessel function of the first kind.

In this expression the Rice Factor K is defined as s square by b 0, so this is the s square corresponding to the rice factor and b 2 b 0 is the denominator term. When k equals to 0 that means, when this k is equal to 0 s is equal to 0 you will find this expression becoming the same as the Rayleigh distribution. This is going to 0 this will be I 0 of 0. Hence, this will be x by b 0 e to the power of minus x square by 2 b 0 which is the Rayleigh distribution. For k equals to infinite there will be no fading because this will be a very very strong component and which will over shadow all the other components.

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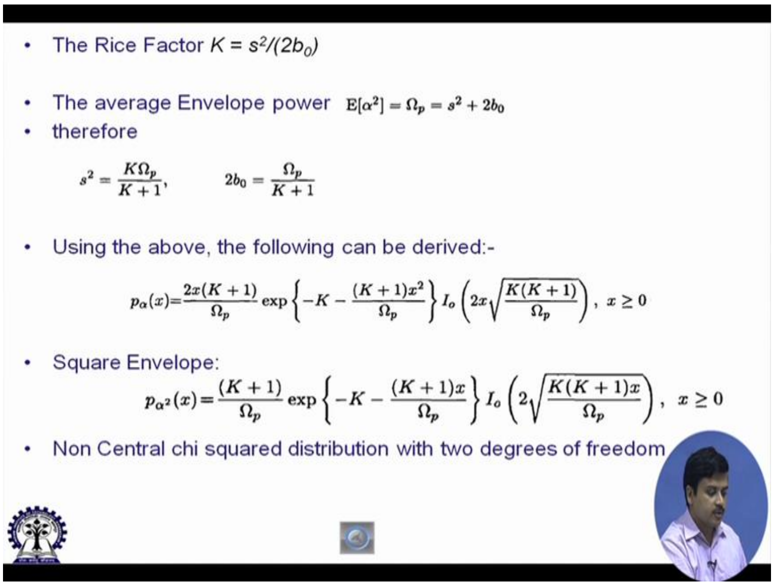

So, for the rice factor k equals to s by 2 b 0 we have defined and the average envelope power which is expected value of alpha square is equal to omega p which we have defined earlier is s square plus 2 b 0. Where, 2 b 0 is that power due to the non Ricean part that means, all other parts and this is power due to the Ricean part. Therefore, s square is equal to k times omega p divided by k plus 1 and 2 b 0 is omega by k plus 1. If you add this two components, this plus this what you will going to get is omega p k by k plus 1 plus 1 by k plus 1 which is equal to nothing but omega p and which is described in this equation. Basically, this gives the ratio of the power of the Ricean component compared to the total envelope power and this gives the fraction of power from the non Ricean component with respect to the total power.

Using these expressions you could expand the earlier expression shown in the previous slide which looks a bit cumbersome, but it straight forward if we replace. We had e to the power of e to the power of x square by omega p and I 0 of x times s, so s can be expressed from this and b 0 can be expressed from this. If you replace these expressions into the previous one you are going to end up with this. And this squared envelope has a distribution has I described by this particular expression it is a bit cumbersome. Because these are a bit cumbersome these are not very popular and not very used for an earlier (Refer Time: 30:37) into the system, whereas you are seen for Rayleigh these becomes exponential and these becomes also quite easy to handle we get very good results.

So, an early inside is easily attended through a Rayleigh distribution, whereas for this we often need to do numerical techniques or even simulations. Usually, numerical technique work out with this it is a little bit more cumbersome then with the Rayleigh distribution. It is a non central chi squared distribution with 2 degrees of freedom.

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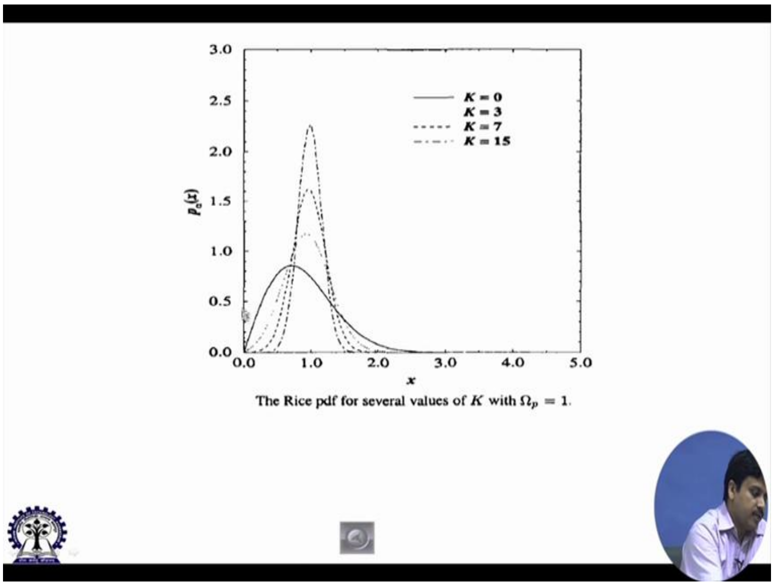

If we look at the distribution of the envelope for a Ricean case what we will find is on the y axis are the PDF x axis are the values for different values of k again this picture is taken from (Refer Time: 31:22), so I am just reusing the figure for sake of for easy explanation. As we see for k equals to 0 there is this particular curve which represents the one which is similar to a Rayleigh distribution. And as you keep on increasing the value of k this becomes sharper and sharper and the spread decreases and slowly as k tends to infinite there is almost an impulse with hardly any fluctuation in the value this becomes narrower and narrower, so as been explained in the previous slide.

With this, what we can see is that the amplitude coefficients which are received are random. Although you are sending a constant amplitude signal, a continuous wave transmission, and the signal is complex Gaussian distributed, the envelope is Rayleigh distributed, when there is equal distribution of power from all directions we will see more details. And when there is a specular component it is instead of Rayleigh distribution there is a Ricean distribution. For Rayleigh distribution the squared envelope which indicates the signal power is exponential distributed, whereas for Ricean case it is more cumbersome.

For Ricean case if you put k equals to 0 you end up in Rayleigh distribution and if you put k equals to infinite that means, very very strong line of side just imagine a satellite link with hardly small portions from multiple paths you can get almost no fading and that would remind you that we will be almost getting AWJN condition. So, if you said Ricean factor to be very very high tending towards infinite you almost have no fading situation. So, all the results that has been used for AWJ and could almost be approximately applied in those cases.

And for the case where k equals to 0 is one of the difficult situations, so analysing for k equals to 0 gives worst case analysis of a system as well as the results are insightful we can conclude things easily and quickly with Rayleigh, although it is not applicable to every situation, but it does apply in many situations.

We will continue with this discussion in the next lecture where we will talk about the (Refer Time: 33:43) distribution, and followed by we look into the signal correlation properties.

Thank you.