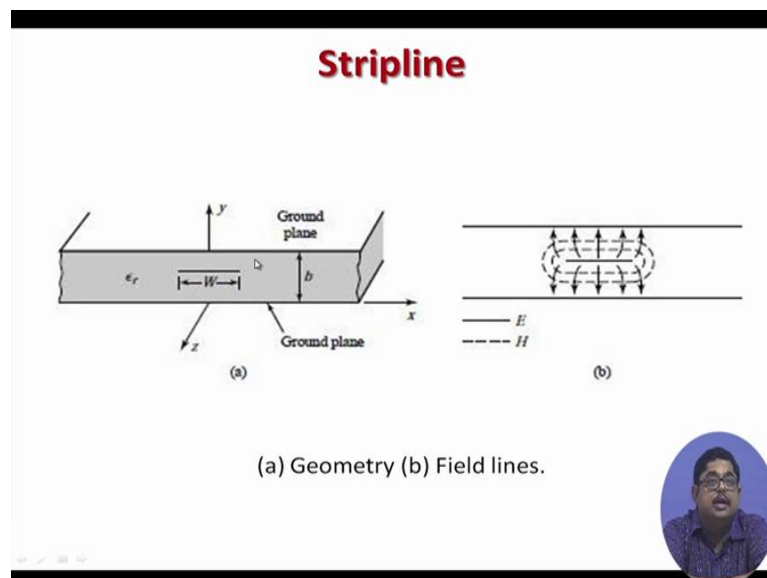


Basic Building Blocks of Microwave Engineering
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Lecture – 09
Planar Transmission Line

So, in this lecture we will now see a new thing that instead of the three-dimensional structure we will see planar transmission line. Actually, nowadays there is a need to have conformal structure planar structures so that the antennae or the transmission line they are on the surface and they are also on a, suppose on a source and load and power transmission circuit all are on a single plane, so that you can reduce the size of the object. So, that is why there is a need for planar transmission line. We will see two very popular planar transmission lines. There are number of planar transmission lines, we will only see the two which are easy to analyze and visualize.

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The first we will see is called a strip line. Now strip line is the geometry shown here, you see that there is a ground plane on the top, there is a ground plane on the bottom and between the ground planes there is (Refer Time: 02:06). But there is another thing sandwiched in the inside this (Refer Time: 02:16), that is a metal. So, there is a, this metal which is infinite in another transverse direction; that means, if we call it y sorry

that is y. So, z direction in z direction in this, there is a strip of this of width W metal. So, this metal sandwiched or embedded inside this dielectric, top is ground bottom is ground.

So, this is a side view of the structure that which is there is a metal, this is top is a metal bottom is a metal in between all are dielectric. So, what will happen generally you can which is ground plane is grounded. So, if potentially zero, this ground plane is grounded and this has got some potential. So, field lines will start from here and will end up here. Similarly, field lines will end up here, the whole thing is this, and this length is quite long. So, that the this strip, line this this central conductor that sees that throughout they are dielectric

Now, the E field is like this that the H field will be something like this. Now this is a structure of strip line, it is a bit difficult to fabricate because inside this you will have to plate this. So, generally it is that the half portion of this is fabricated and then on that another top portion is put.

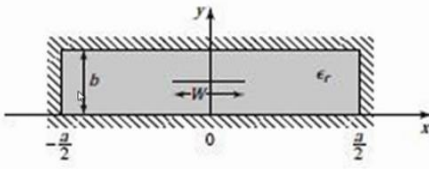
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Stripline

- A planar transmission line
- photolithographic fabrication
- planar and integrated circuit
- A thin conducting strip connected between two ground planes, the remaining entire region filled with dielectric.
- Practically, constructed by etching centre conductor grounded substrate of thickness $b/2$, then covering with grounded substrate of same thickness.


Now, it is a planar transmission line; obviously, you see that the whole structure is plane or not that like that 3D things like wave guide and (Refer Time: 4:35) etcetera by photolithographic fabrication it is made. As I already said that since it is planar and it is amenable to integrated circuits so microwave integrated circuit we used this strip lines as the transmission lines.

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The diagram shows a rectangular waveguide structure in the x - y plane. The x -axis is horizontal, with the origin 0 at the center. The y -axis is vertical. The structure consists of a central conductor of width b centered at $x=0$, extending from $x = -b/2$ to $x = b/2$. Below this conductor is a ground plane at $y=0$. The region between the central conductor and the ground plane is filled with a homogeneous dielectric of relative permittivity ϵ_r . The top boundary of the structure is at $y=b$. The region above $y=b$ is also filled with the same dielectric. The diagram is labeled with x , y , 0 , $-b/2$, $b/2$, b , and ϵ_r .

- Two conductors and homogeneous dielectric \rightarrow TEM wave
- Like parallel plate waveguide, it can also support TE & TM
- higher order modes can be suppressed by shorting screws between ground plane and choosing $b < \lambda/4$.
- flattened out 'coax'
 - \rightarrow both have center conductor completely enclosed by outer
 - \rightarrow uniformly filled by dielectric.
- no simple analysis like coax or waveguide
 - \rightarrow conformal mapping is standard procedure for solution of Laplace's equation.



Now, as I said this, so now if we look carefully that, sorry, if you look carefully this is a planar version of coax. I can say because coax is also like this there is a central conductor and covering that circularly throughout coax actually is the outer conductor. Here instead of that coax actual thing, it is a rectangular structure but the central conductor is all the sides it is seeing the all the side means that at least at the top and bottom it is seeing the outer conductor.

So, that is why you see the field lines starting from the central conductor are all going to the either up ground plane or lower ground plane. So, it is something like that. But there is a difference that in a coax the field lines are all radial to the structure, here it is a rectangular structure but the field lines are not I will say they are not either in horizontal or vertical direction, it is a mixture. So, that is why the analysis of this strip line is not as straightforward as the coaxial line. In coaxial line because of the circular symmetry the (Refer Time: 06:38) equation will take a nice form. Here also you can apply Laplace's equation to analyze that, but one thing is since it is a quite large distance here and theoretically a strip line is extending infinitely in you can say z direction so and it has finite thing here.

So, you will have to apply the Laplace's thing here. Also, the structure and the coordinate system they are not properly confirming. So, that is why you need to have some special function to analyze that.

So, Laplace's equation is used but special permissions are needed to handle that and also so that is why people try that with the another way of attack is people try by having some conformal transformation so that you get a better conformal structure by transforming it in some another plane and their you can get a good.

But one thing we will say that we need to as an engineer, we need to this transmission line means if we know the it is characteristic impedance, particularly these lines as the thing suggest that since we have a something like (Refer Time: 08:25) line. So, and here the whole, the field lines are existing between the two conductors so TEM wave is supported here. So, since TEM wave is supported so the thing is we can have characteristic impedance defined for this structure and that if we know, basically engineers engineering design requires that knowledge of the characteristic impedance of this transmission line in cases where TM things propagate.

Now that can be done. So, people have come out with some approximate method of analysis of strip line and also people have come out with some empirical formulas for this strip lines so that we will see one by one. One is two conductors and homogenous dielectric, so as I said TEM wave should be there also like coax or parallel plate waveguide higher order modes TE and TM also should be there.

Now, higher order modes can be suppressed by shorting screws at the points where their maximum takes place so that people have time and variable to suppress the thing. Now as I said that we can consider it as a planar version of flattened out coax, both have center conductor completely enclosed by outer conductor uniformly filled by dielectric board. But as I said the reason that due to the non-conformal, nonconformance between the structure of the transmission structure and the coordinate system and the field things we are forced to do conformal mapping is standard procedure for solution of Laplace's equation in case of strip line.


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Analysis

- Phase velocity of TEM mode

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{c_0}{\sqrt{\epsilon_r}}$$
$$\beta = \frac{\omega}{v_p} = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} = \sqrt{\epsilon_r} k_0$$
$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{LC}}{C} = \frac{1}{v_p C}$$

[L & C are per unit length parameter $v_p = \frac{1}{\sqrt{LC}}$
So, if we know C, Z_0 can be obtained.
 Z_0 is enough for engineering performance.
Solve Laplace's equation but involves special function.



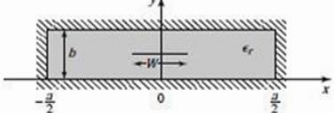
Now, in any TEM mode we know the phase velocity is given by the velocity of light by epsilon r provided we are assuming and that is true for strip line that there is no magnetic material. So, mu not, mu you can always take to be mu not. So, in that case v_p will be simply c by epsilon r, beta we know it is same as k but in case of dielectric it will root over epsilon r k and Z_0 if we take that 1 by c , L and C are the per unit length induct and capacitance then it is 1 by v_p ; that means, phase velocity into c .

Now, knowing the dielectric we can find the phase velocity. So, once we know the phase velocity it can be seen that to find impedance I need to find the per unit length capacitance. So, both capacitance and inductance determination is not necessary. So, the standard procedure for all these planar transmission lines including strip lines is find out per unit capacitance. Now how per unit in capacitance can be found out? Now capacitance you know that, if we give some if you can calculate some charge distribution or if we can find some charge distribution and find the total charge and if we can find the potential function then q is equal to $c v$ is the root by which we can find the c . So, that is done for all the planar transmission lines.

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Approximate Analysis


- TEM fields obey Laplace's equation. Stripline extends to $\pm a$ in x direction. Hence difficulty.
- Field lines do not extend very far away from central conductor.



Electric walls at $\pm a/2$.
 $a \gg b$
So, fields around central conductor not perturbed.
So, Laplace's equation in a closed region can be applied

$$\nabla_T^2 \phi(x, y) = 0, \quad |x| \leq a/2, \quad 0 \leq y \leq b$$

B.C.

$$\begin{aligned} \phi(x, y) &= 0, & x = \pm a/2 \\ \phi(x, y) &= 0, & y = 0, b \end{aligned}$$


So, you see that as I said that since it is TEM field so it will obey Laplace's equation. But the problem is another strip line extends to plus minus; that means, infinity in both these directions x and this z direction. So, that is a problem that is called micro Laplace's equation in a large domain if you want to apply if the computational its create problem.

So, what people have done that if you see the field lines; obviously, far away from these strip lines, the fields will die down the fields will gradually become field in fields and after certain distance away their own field. Because of the structure that the strip line is existing only at the central option of this whole structure. So, people have found that for confining the field we will say that at a distance plus a by 2 and minus a by 2, I have 2 more walls. Obviously, there are 1 ground plane then another ground plane here, but I will put sorry 2 more, 2 more ground planes or we call it electrical walls here that conducting walls. So, that the structure is now in confined.

And by doing this when we are putting this thing means this is far away so obviously I am putting in discontinuity here because from a epsilon r I am putting a metal. But then the equation expected from that that will be write down here there is a discontinuity there will be higher even as an modes etcetera, but that will be a, so here it would not change much this is the idea.

So, now we can put Laplace's equation to our use that this is the Laplace's equation this is the potential function. So, we are in this region we have the Laplace's equation

converse laplasion then put boundary condition again that at x is equal to plus minus a by 2; that means, that 2 sides there are electrical walls. So, the potential function will be 0 and also top and bottom ground plane where the potential is 0.

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- Separation of variables
- Central conductor has a surface charge density. So, $\phi(x, y)$ will have a slope discontinuity there.

$\vec{D} = -\epsilon_0 \epsilon_r \nabla \phi$ is discontinuous at $y = b/2$

So, separate solution for $\phi(x, y)$ in

- $0 < y < b/2$
- $b/2 < y < b$

$$\phi(x, y) = \begin{cases} \sum_n A_n \cos \frac{n\pi x}{a} \sinh \frac{n\pi y}{a}, & 0 \leq y < b/2 \\ \sum_n B_n \cos \frac{n\pi x}{a} \sinh \frac{n\pi}{a} (b-y), & b/2 \leq y \leq b \end{cases}$$

So, by this whole boundary condition then put separation of variables again everything will be given in the notes, but the idea is this the central conductor will have a surface charge density because it has a potential non-zero potential. So, there will be a discontinuity of the potential function around that strip line central conductor. And we know that if we have a potential if we take the gradient we get electric field, negative gradient of the electric of the potential function gives us electric field, electric field multiplied by the appropriate permittivity gives us the displacement vector.

Now, displacement vector if I know then from Maxwell's equation we can find out the magnetic field etc. so field analysis becomes straightforward from there. Also, here what we do once we know the displacement vector from that we relate it to the Maxwell's the or process law that del dot b is equal to rho and from that we find out relate the rho and this potential function and ultimately manipulate it to find out that q is equal to c b. So, c we get from that because rho is the charge distribution, rho gives me q and this potential function gives me potential difference and since they are related here. So, we can always find out. So, that is the approximate analysis people do. So, here now it is true that since there is a discontinuity so; that means 2 sides will have 2 different solutions.

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Determination of Charge and voltage

- From boundary condition (continuity of fields at the central conductor) constants A_n and B_n gets determined
- Total charge on the central conductor can be written from the potential function knowledge by applying Gauss's Law (Maxwell's 3rd Equation)

$$\rho_s = 2 \epsilon_0 \epsilon_r \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} A_n \left(\frac{n\pi}{a} \right) \cos \frac{n\pi x}{a} \cosh \frac{n\pi y}{a}$$

$$Q = \int_{-w/2}^{w/2} \rho_s(x) dx = W \text{ c / m}$$
- From the potential function, voltage between conductor is determined

$$V = - \int_0^{b/2} E_y(x=0, y) dy = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} A_n \sinh \frac{n\pi b}{2a}$$
- Capacitance per length

So, potential function, the potential function, the idea is within this zone; that means, from 0 to $b/2$ there will be one function 1 type of potential distribution and then $b/2$ plus $b/2$ there is another potential function because of the discontinuity of the surface charge density on this. So, that is why it is broken into 2 and then first a/n and b/n the constants are determined then total charge can be written from the potential function knowledge as I said by applying Gauss's law. So, you have ρ_s . So, that ρ_s is related to the d by applying Gauss's law so that means once from the field you know this ρ_s then q will be get that along x direction and get that total charge. And from potential function you get the voltage between conductors by since I know E_y , E_y is known again from the gradient of the voltage potential function. So, calculate V , now you can find capacitance per length.

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Determination of Characteristic Impedance

- Per length capacitance

$$C = \frac{Q}{V} = \frac{W}{\sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{2a \sin(n\pi w/2a)}{(n\pi)^2 \epsilon_0 \epsilon_r \cosh\left(\frac{n\pi b}{2a}\right)}} \text{ Farad / m}$$

- Characteristic Impedance

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{\epsilon_r}}{c C}$$

So, by that method it comes like this that the capacitance depends on the width of the strip line then a is the place where you are how far away you are putting the electrical walls; w is already there the width the width of the thing and b is the strip line width that means, the separation of the 2 ground plane the top and bottom, we have separated a already known.

So, now once you have that then this c is known there you put it c , this is the constant this the velocity of light and epsilon r is the known dielectric constant. So, you can find the characteristic impedance. Once characteristic impedance of the device is known all the engineering per applications can be done like how much power it will take, how to impedance match it etcetera all those things will come.

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Empirical formula for Characteristic Impedance

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b}$$

where $W_e \rightarrow$ effective width of the center conductor.

$$\frac{W_e}{b} = \frac{w}{b} - \begin{cases} 0 & \text{for } w/b > 0.35 \\ (0.35 - w/b)^2 & \text{for } w/b < 0.35 \end{cases}$$

- This formula assumes strip thickness to be zero
- This formula is accurate upto 1%
- Note that Z_0 decreases as strip width W increases



So now people have also come out with empirical formula for characteristic admittance but there they need to have some effective width of the center conductor and that effective width is given by this this formula. So, these are basically if you look at these formulas, no one expects you to remember these formulas, but by refereeing to these you can find approximately the up to accuracy of 1 percent this formula is all correct. But this formula assumes the strip line the central conductor its thickness to be 0. There are other better formulas also people come out with that for a thick strip lines there also some formulas. And one thing is to be noted that from this that if actually you see W_e is related to W and so; that means, if the width increases strip line width it extends width in the Z_0 conduct increases then the characteristic impedance that decreases. So, that is one.

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Design of stripline

- Find W from the given values of Z_0, b, ϵ_r
- S_{10} inverse formula $\frac{W}{b} = \begin{cases} x, & \sqrt{\epsilon_r} z_0 < 120 \\ 0.85 - \sqrt{0.6 - x}, & \sqrt{\epsilon_r} z_0 > 120 \end{cases}$
- TEM line, so, $\alpha_d = \frac{k \tan \delta}{2}$ where $x = \frac{30\pi}{\sqrt{\epsilon_r} z_0} - 0.441$
- $\alpha_c = \begin{cases} \frac{2.7 \times 10^{-3} R_s \epsilon_r z_0}{30\pi(b-t)} A, & \sqrt{\epsilon_r} z_0 < 120 \\ \frac{0.16 R_s}{z_0 b} B, & \sqrt{\epsilon_r} z_0 > 120 \end{cases} \quad Np/m$

$$A = 1 + \frac{2w}{b-t} + \frac{1}{\pi} \frac{b+t}{b-t} \ln \left(\frac{2b-t}{t} \right)$$

$$B = 1 + \frac{1}{(0.5w + 0.7t)} \left(0.5 + \frac{0.414t}{w} \right) \ln \left(\frac{4\pi w}{t} \right)$$

t is thickness of the line

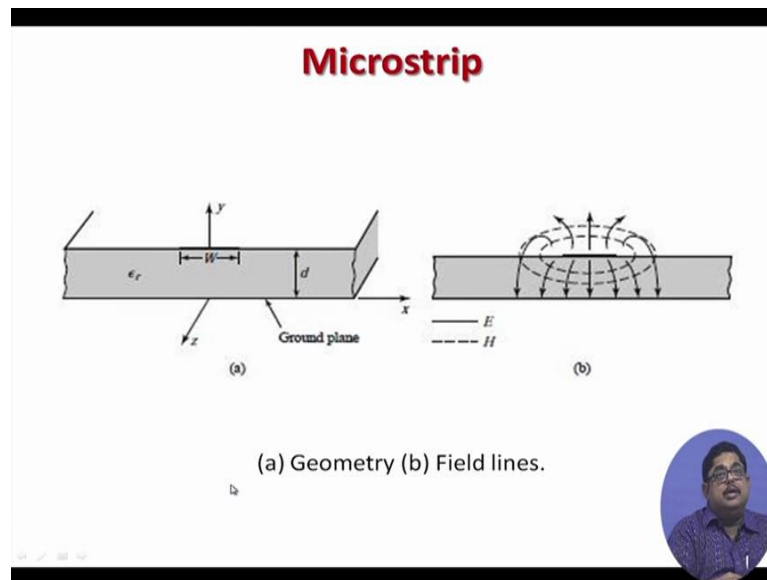


So, now you people have if any design strip line is to be designed then the job is actually to find the width from the given value because generally the characteristic impedance of any transmission line is specified. So, strip line what characteristic impedance is required that is specified and b is the separation of the 2 ground planes and epsilon r these will be given. So, from that by putting into those formulas we will have to find out what is W.

Now, the either by computer programming (Refer Time: 22.06) you can find that or the same formula which we have shown before these formula people have inverted and this is the form that if you are this epsilon r and z not if they are less than this 120 you take this or if it is more than that you take this. So, based on that you can find out what is the value of W.

And if this is the value given for it alpha b that is the attenuation constant due to dielectric plus and this is the attenuation constant due to the conductor loss. Complicated structure but you see that we need to find out what is W and what is the thickness of the line that? Means, thickness of the strip line, if you know that people can find out from these formulas.

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Now, another popular thing as I said that in strip line the problem is it is sandwiched between 2 they are sandwiched or embedded inside the dielectric, its fabrication is a bit difficult. So, people have thought that there is a line of symmetry about the strip line. So, if you remove the top dielectric and the top ground plane then it becomes that you have a ground plane, you have a dielectric on that there is a metal thing. So, you always put that metal, metalize some portion of the dielectric and that is very easy to fabricate. So, that is why that is called micro strip, it is very popular as a transmission line is popular. It also can radiate and behave as an antenna we will see here only the transmission line part. So, this micro strip it looks like this. That means it has the ground plane is the metal and then on the top there is a metallization not extending fully it is partly extended and between that it carries the power and between the 2 metals it is the dielectric that is taking power that is epsilon r is the dielectric constant of the dielectric d is the you can see with our the height and depth of the thing W is the width etcetera etcetera.

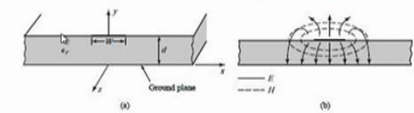
Field lines if you see that definitely this is the just like micro strip lines fields from the center conductor which is at you can think now this is the central conductor this is the outer conductor. But one thing is it is not a coax or strip line that stops the central conductor is not central to the outer conductor, here you can say this can be somewhat like a 2 conductor, but 1 conductor is central another we are calling outer. So, field lines will come from here because, these is that this 2 are not at same potential, generally this is grounded and this is at a potential. So, field lines will come here and this side is here.

So, some field lines will also go to here; but obviously due to dielectric constant of this dielectric by choosing it to be higher most of the field lines can be confined inside this dielectric, but some will definitely go to air.

Now, these are the H lines as you see. So, its fabrication is again photo lithography easily integrable, it is already said it is not new.

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Microstrip cannot support TEM mode



- Discontinuity in medium around top surface, air-dielectric interface.
- Let us assume TEM wave propagation.
 - in dielectric, $v_p = \frac{c}{\sqrt{\epsilon_r}}$
 - in air, $v_p = c$
- At interface, the two phase velocities cannot match
- So, TEM not possible.

- Exact field analysis \rightarrow hybrid TE – TM
- For this substrate (i.e. $d \ll \lambda$) \rightarrow quasi-TEM fields (i.e. similar to TEM)

Now, the point is there is a discontinuity. Unlike strip line you see we are creating a discontinuity medium that we have a dielectric here we are making it here dielectric here. So, suppose for argument sake let us assume that there is TEM wave propagation. So, TEM wave is propagating here and its inside this dielectric is velocity, phase velocity is given by c it is this c means that $e m$ waves the velocity of light and that divided by square root of epsilon r , but when it will get to here its v_p will be simply c .

Now, these 2 at interface the 2 phase velocities cannot match. So, definitely since we have a discontinuity of medium here we this structure cannot support a TEM wave. Strip line was fully the whole thing was inside dielectric. The central conductor was fully inside dielectric that is why TEM wave there is no problem both sides seen same surface velocity in both the sides could have matched. But here TEM not possible and obviously due to this the field structure will be a hybrid of TE and TM modes

But definitely that analysis is not so simple. So, people have come out that if we make and particularly the substrate is made very thin; that means, if we make this very very less than lambda people will say that you have, since you have a very thin layer of dielectric though there is discontinuity here ignore that. So, that is the field lines it has been seen that the field lines also behave like you can see that more or less if you have very thin layer and by good amount of dielectric here then the strip line, field lines and these field lines are almost same in this region not above (Refer Time: 28.49). So, that people say that this also is a quasi TEM field similar to t e m. So, whatever we do for TEM analysis we will do that; that means we will apply the Laplace's equation. So, this I said somewhere hybrid needed to support them.

Now, quasi TEM approximation for thin substrate and people find out an effective dielectric constant. Effective dielectric constant because as we said that there is a discontinuity in the medium, but people say so that total field lines etcetera are in an equivalent simply assume that instead of 2 dielectric c r and that a there is an effective dielectric homogeneously distributed; obviously, that effective.

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Quasi TEM Microstrip Field

- For thin substrate (width \ll wavelength of signal)
 - An effective dielectric constant ϵ_{eff} can be defined
 - Then the entire structure becomes homogeneous
 - The field becomes similar to TEM
 - It is called quasi TEM

$$1 < \epsilon_{eff} < \epsilon_r$$

ϵ_{eff} is a function of d & W

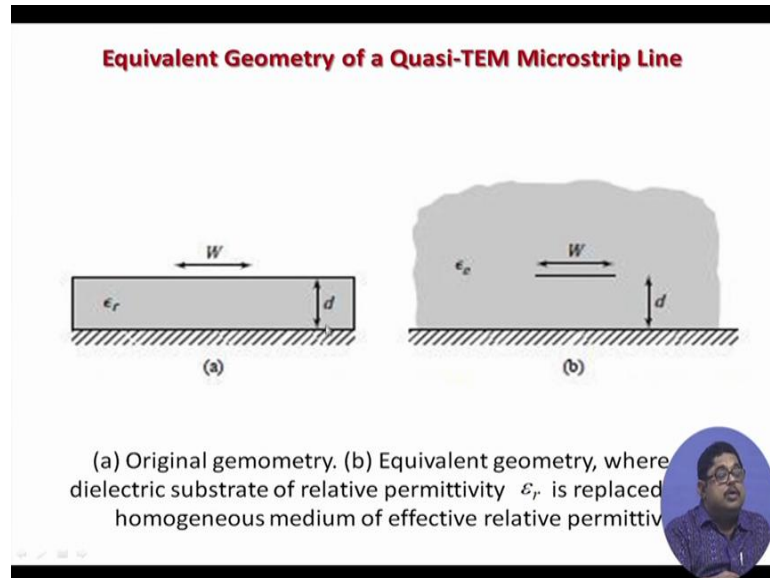
$$\beta = k_0 \sqrt{\epsilon_{eff}}$$

$$v_p = \frac{c}{\sqrt{\epsilon_{eff}}}$$

So, that is attributed effective dielectric constant, its value will be something between 1 and epsilon r. There are various formulas people have come out with based on certain empirical reasoning the values this epsilon, epsilon effective that can be found out then the entire structure becomes homogenous, the field becomes similar to

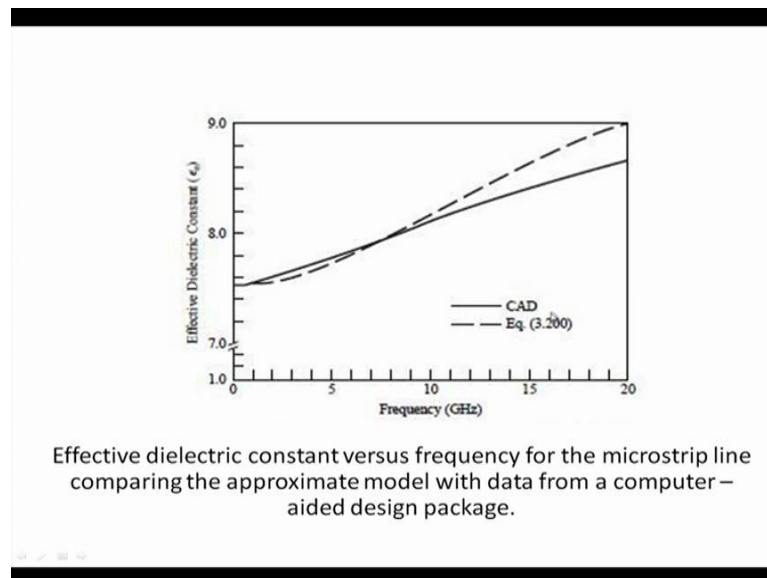
TEM, quasi TEM. And this epsilon effective is a function of d by separation of the height of the micro strip from the ground plane and width of the micro strip. These are for TEM waves we can write beta will be something like this and v_p is something like this.

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So, equivalent geometry of the quasi TEM micro strip line you see here also after some distance we will terminate this up to electrical walls. This is our micro strip patch and this is the same. So, this is the equivalent thing that here we have a discontinuity, here we say that throughout we have a new dielectric. So, there is no discontinuity between a epsilon r and a so everything is inside this. So, this is something like a strip line sort of structure, but here there are no ground planes, but this structure that I have a whole thing surrounded.

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So, and that effective dielectric constant various formulas people have found and compared. And like in some book I think it is (Refer Time: 31:40) book in this equation they have given some effective dielectric constant values. So, if you compare that with the actual dielectric constant calculated by a CAD program you see that almost they are for almost practical cases can be said to be same. So, those formulas are quite good, empirical formulas.

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Approximate Analysis

Conductors at $\pm a/2$
 $a \gg d$

Laplace's equation
B.C. $\nabla_t^2 \phi(x,y) = 0, \quad |x| \leq a/2 \quad 0 \leq y \leq \infty$

$$\phi(x,y) = 0 \quad \text{at } x = \pm a/2$$
$$\phi(x,y) = 0 \quad \text{at } y = 0, \infty$$

Two different regions with two different dielectric
So, charge discontinuities at strip.
So, separate $\phi(x,y)$
Solving by separation of variables & applying B.C.

Now, approximate analysis follows like what we have done for the strip lines. You put Laplace's equation inside this confined zone. So, again there will be this boundary condition that in the side walls the potential function should go to 0 and the top and bottom potential function goes to 0. So, again there is that is this micro strip will have some charged distribution.

So, there will be a charged discontinuity. Also, there will be the discontinuity, displacement vector. So, from that you can find out by Gauss's law what are the surface charges? From surface charges you go to the charge, total charge. And also from solution of this potential function by applying boundary condition you find out what is the potential function from that you find out what is the voltage between this center conductor and the ground and then you find out the capacitance and you find out the characteristic impedance.


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$$\phi(x, y) = \begin{cases} \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{a} \sinh \frac{n\pi y}{a}, & 0 \leq y \leq d \\ \sum_{n=1}^{\infty} B_n \cos \frac{n\pi x}{a} e^{-\frac{n\pi y}{a}} & d \leq y \leq \infty \end{cases}$$

Potential continuous at $y = d$.

$$\therefore A_n \sinh \frac{n\pi d}{a} = B_n e^{-\frac{n\pi d}{a}} \quad \text{for } y \text{ in air}$$

To evaluate A_n & B_n , surface charge discontinuity to be revoked
 To do that, $E_y = -\frac{\partial \phi}{\partial y}$

$$E_y = \begin{cases} -\sum_{n=1, \text{ odd}}^{\infty} A_n \left(\frac{n\pi}{a} \right) \cos \frac{n\pi x}{a} \cosh \frac{n\pi y}{a}, & 0 \leq y < d \\ \sum_{n=1, \text{ odd}}^{\infty} A_n \left(\frac{n\pi}{a} \right) \cos \frac{n\pi x}{a} \sinh \frac{n\pi d}{a} e^{-\frac{n\pi(y-d)}{a}} & d \leq y \end{cases}$$


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$$\begin{aligned}\rho_s &= D_y(x, y = d^+) - D_y(x, y = d^-) \\ &= \epsilon_0 E_y(x, y = d^+) - \epsilon_0 \epsilon_r E_y(x, y = d^-) \\ &= \epsilon_0 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} A_n \left(\frac{n\pi}{a} \right) \cos \frac{n\pi x}{a} \left[\sinh \frac{n\pi d}{a} + \epsilon_r \cosh \frac{n\pi d}{a} \right]\end{aligned}$$

This is a Fourier series in x.

Assume, ρ_s on microstrip line is distributed uniformly

$$\rho_s(x) = \begin{cases} 1, & |x| < W/2 \\ 0, & |x| > W/2 \end{cases}$$

Like Fourier constants, A_n is evaluated (orthogonality of $\cos \frac{n\pi x}{a}$)

$$A_n = \frac{4a \sin n\pi W/2a}{(n\pi)^2 \epsilon_0 \left[\sinh \frac{n\pi d}{a} + \epsilon_r \cosh \frac{n\pi d}{a} \right]}$$

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Voltage of strip w.r.t. ground plane,

$$V = - \int_0^d E_y(x=0, y) dy = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} A_n \sinh \frac{n\pi d}{a}$$

Total charge / length on the center strip,

$$Q = \int_{-W/2}^{W/2} \rho_s(x) dx = W \text{ c/m}$$

Static capacitance / length of microstrip

$$C = \frac{Q}{V} = \frac{1}{\sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{4a \sin \left(\frac{n\pi w}{2a} \right) \sinh \left(\frac{n\pi d}{a} \right)}{(n\pi)^2 W \epsilon_0 \left[\sinh \frac{n\pi d}{a} + \epsilon_r \cosh \frac{n\pi d}{a} \right]}}$$

So, that is shown here. So, finally, you get this is the value for capacitance with quite tough looking lot of expressions. But the idea that is why you try to understand that idea is simple I try to show it, value is definitely no one expects you to remember this.

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Empirical Formulas

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/w}}$$

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\epsilon_{eff}}} \ln\left(\frac{8d}{W} + \frac{W}{4d}\right); & \frac{W}{d} \leq 1 \\ \frac{120\pi}{\sqrt{\epsilon_{eff}} \left[\frac{W}{d} + 1.393 + 0.667 \ln\left(\frac{W}{d} + 1.444\right) \right]}; & \frac{W}{d} \geq 1 \end{cases}$$

And now the empirical formulas that an effective should be put use. You see that depends on; obviously, the dielectric constant of the micro strip, but also on this d by w ratio. This d by w ratio is important or d by w or w by d. So, depending on whether it is less than 1; that means, w is less than d w greater than d less than that the whole formula that determines these are the empirical formulas that characteristic impedance will depend on this w by d and this.

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Inverse Relation

For given z_0 & ϵ_r ,

$$\frac{W}{d} = \begin{cases} \frac{8e^A}{e^{2A} - 2}, & \frac{W}{d} < 2 \\ \frac{2}{\pi} \left[B - 1 - \ln(2B - 1) + \frac{\epsilon_r - 1}{2\epsilon_r} \left\{ \ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right\} \right], & \frac{W}{d} > 2 \end{cases}$$

where

$$A = \frac{z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left(0.23 + \frac{0.11}{\epsilon_r} \right)$$

$$B = \frac{377\pi}{2z_0\sqrt{\epsilon_r}}$$


So then to find out for design purposes when z not is given and epsilon r is known, you need to determine what is w or w by d then d is known then people will ultimately find w . So, people have inverted that this is the investigation where these are again constant etcetera.

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Microstrip losses

Considering quasi-TEM line,


$$\alpha_d = \frac{k_0 \epsilon_r (\epsilon_{eff} - 1) \tan \delta''}{2 \sqrt{\epsilon_{eff}} (\epsilon_r - 1)} \quad Np/m$$

[Filling factor = $\frac{\epsilon_r (\epsilon_{eff} - 1)}{\epsilon_{eff} (\epsilon_r - 1)}$ fields partly in air, partly in dielectric]

$$\alpha_c = \frac{R_s}{z_0 W} \quad Np/m \quad \text{where} \quad R_s = \sqrt{\omega \mu_0 / 2\sigma}$$

(surface Resistant of conductor)


Mostly, $\alpha_c \gg \alpha_d$ for commonly used substrate.



And alpha d the attenuation constant due to dielectric loss is given by this and here this micro strip people create define a filling factor. That means how the fields are part linear and partly in dielectric that you have seen. So, how much they are filling the homogeneous thing that is given by this filling factor and the conduction part is simpler to understand we pick the and that also. And r is surface resistance, surface resistance of this conductor that is given by this. So, once you know the conductivity of the material you can find that is and. So, losses etcetera micro strip that also now people have paying attention and micro strip.

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Material	ϵ_r	$\tan \delta$
PTFE / glass	2.2	0.0002 – 0.0005
RT / Duroid 5880	2.26	0.001
Alumina	9.6 – 10	0.0002 – 0.0005
Sapphire	9.4	0.0001
GaAs	11-13	0.0016
Si	12	0.015



Some common substrate material that is used for micro strip is PTFE glass. So, you can see and tan delta it is important because we have seen that it directly affects the dielectric loss. So, people try to go for better and better low dielectric loss things. So, RT Duroid is one such thing 0.001 loss. Then these are things but these also gives you epsilon r around 2 3 so that is also designable.

So, these are planar lines, their analysis is almost quasi TEM or TEM analysis and they are not under heavily use. But one thing is they cannot carry much power. So, these are for low power application, but for high power applications you will have to still use that wave guide or metallic things which can carry much power.

Thank You.