

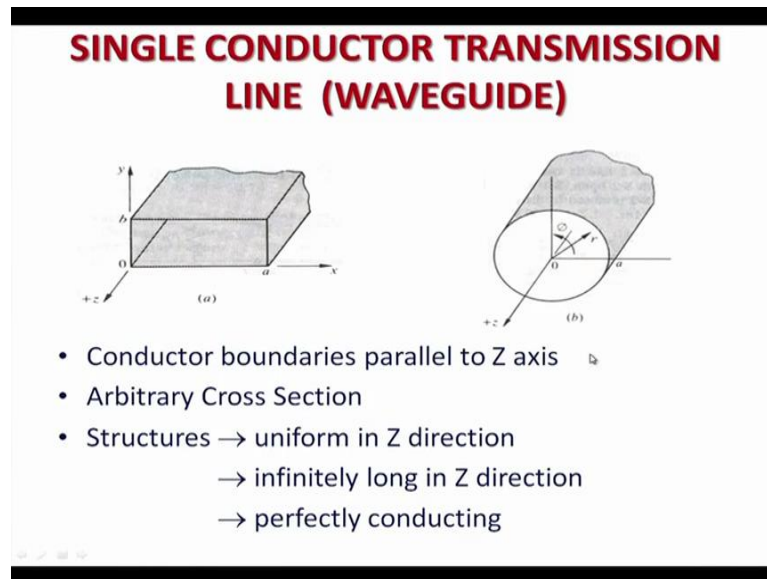
Basic Building Blocks of Microwave Engineering
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Lecture – 07
Rectangular Waveguide

Now, in this seventh lecture, we will see the another high frequency transmission structure; that is wave guide as I already said that, wave guide is, it propagates non TE M mode that is both TEM and TM modes propagates through wave guides. Now wave guides means it has a, its cross section is uniform. Now that cross section, actually it is a cylindrical structure. The cross section may be rectangular, may be circular, and may be elliptic, depending on that we say whether it is a rectangular wave guide, whether it is an elliptic wave guide, whether it is circular wave guide etcetera.

So, it is a very popular one rectangular wave guide like a hollow rectangular pi. So, that we will see support some interesting modes, and that is why it is popular, that its field structure etcetera is very convenient to extract that type of mode in rectangular wave guide is very popular. So, almost in all the applications of microwave engineering in high frequency, particular in the Giga hertz range, rectangular wave guide is used, but will see that sometimes we need a circular cemetery that time circular wave guide is also used, that will see in the next lecture.

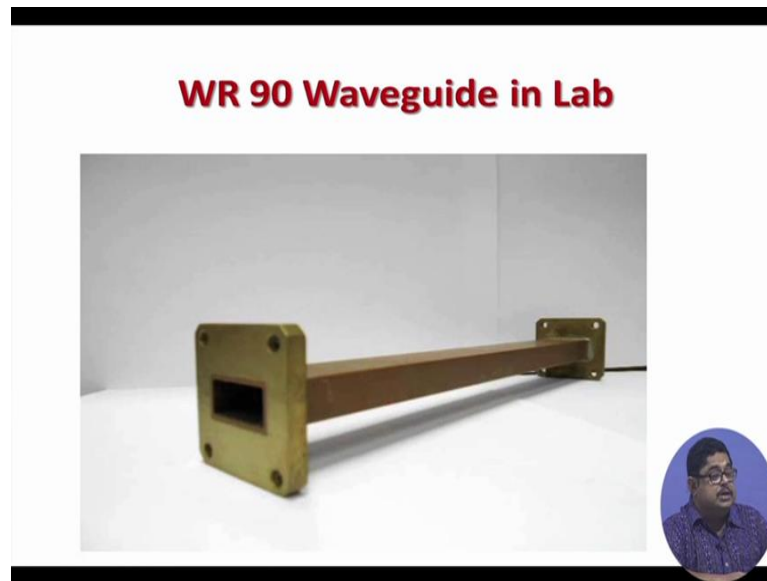
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So, rectangular wave guide it is a single conductor transmission line as the wave guide says. So, you see this, this is a closed metallic structure, conductor boundaries parallel to z axis. This is our custom you can take it to anyone, but generally we say that wave will propagate in the z axis direction. So, conductor boundaries they are parallel to z axis. now arbitrary cross section, but generally we work with by it can be theoretically any arbitrary cross section, but generally work with as I said rectangular structure or cross section or circular cross section etcetera.

The structure uniformed z direction, in z direction I do not have any variation, and they are infinitely long in z direction. infinitely long means that it is far away from the source, far away from the load, so that those discontinuity etcetera not there, so a pure mode is propagating through this, and this metals are perfectly conducting, you know that non real metal is perfectly conducting; that is why we will see that they will any real metal that, since it is not ideal metal, ideal conductor. So, there will be some loss in the conductor

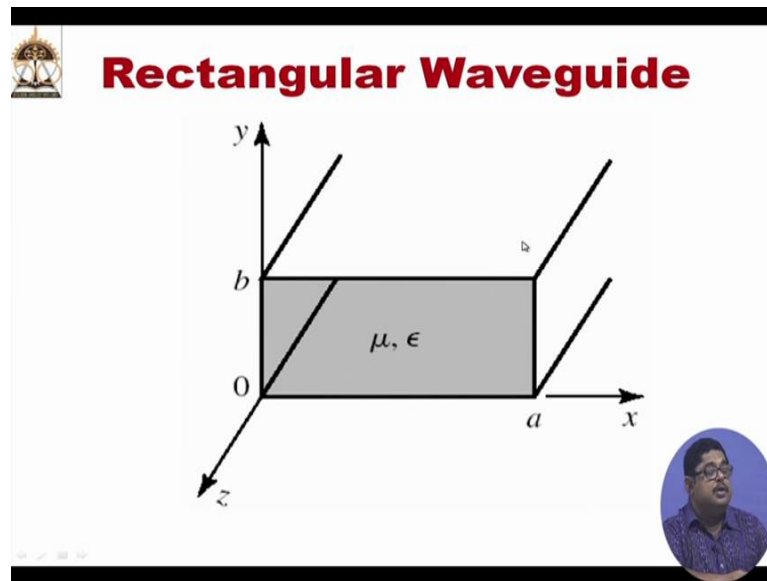
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Now, this is a, generally this is a in a standard, wave guide rectangular guide standard is called W R 90, actually this WR are various things that 36 or WR 70 etcetera. Now this WR 90 is actually used in radar band, x band, x band is a 1 of the radar brand, generally military radars operate. And also in laboratories etcetera we used this WR 90, because this x band 8 to 12 Giga hertz, this dimensions of the wave guide there depending on the frequency they are chosen, and this is quite convenient dimension that is why WR 90 is the standard, and WR 90 wave guide means an x band wave guide. This is a picture you see within this, this halo metallic pipe is there, this is a larger one, and actually the rectangular cross section is this.

This is 0.9 inch that is 2.286 centimeter in length, and 0.4 inch that is 1.016 centimeter in width, but this is called flange, this is much larger than that, this is also rectangle. So, here two wave guides or two components, you can connect them together, so there are holes here. So, by this four holes at the four corner two structures get (Refer Time: 04:44), so that this mouth of this wave guide, and mouth of the wave guide which is connecting with it, they are flushed together, so that there is no discontinuity, no air gap there. This flange helps to make connection with two wave guides, so that there is no discontinuity created between the mouths of two wave guides. Now for our analysis this will be the structure you see the generally the length of the rectangle or that is called a.

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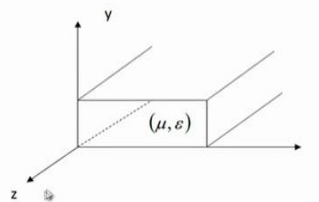


Now, we are taking one of the corners, the left hand corner as the origin, so it is a . You see in some books, or sometimes we takes center as the origin, that time this side is a by 2 , this side is minus a by 2 . Similarly plus b 1 plus b by 2 minus b 2 , but we have taking this geometric this convention that this is our zero. So, this is our b ; that is the width of the rectangle or lower side of the rectangle; that is b , and higher side is a . So, we say that this wall is the broad wall; this wall is the narrow wall.

So, broad wall is of length a , and narrow wall is of length b . So, they are four walls one is the top wall this is metallic, the bottom wall is metallic, side wall is metallic, this side wall is metallic, inside there is hollow thing dielectric it may be air, or may be filled with any completely filled with any dielectric. So, that is why we are saying that inside thing, is a region through which the wave will go that is the dielectric region, if it is not dielectric it is free space. Sorry another thing is that, our propagation direction is z . So, this broad wall that is in the x direction, and the narrow wall that is in the y direction direct now.

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TE Mode



- Hollow pipe is a single conductor line. So, TEM wave not possible. Both TE and TM possible.
- Larger side of the rectangle is of length a and smaller side of b .
- For TE Modes $E_z = 0$
- Corresponding Helmholtz equation $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) h_z(x, y) = 0$
- Applying separation of variables, $h_z(x, y) = f(x)g(y)$

So, we have already seen when we have seen TEM mode, that if you have seen hollow pipe is a single conductor, wave guide is a single conductor in a TEM single conductor, you cannot TEM mode, because you do not have any voltage between the two waves, because all are same structure. So, there potential difference is zero; that is why you can have any transverse electric and transverse magnetic that we have already seen, but both TE and TM possible, whatever I said. Now let us start that for TE modes, converse electric, we known that the longitudinal component of the electric field will be zero. So, if you do that then; that means, we will have to express all the four transverse fields, if they are present in terms of the longitudinal magnetic field, because e_z is not zero. So, in TE modes; obviously, e_z will present, so the transfer component of e_z ; that is small h_z xy . So, will write the Helmholtz equation in terms of that, and then will apply separation of variables, this we have already seen.

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Solution of Longitudinal Component

Putting in Helmholtz equation and dividing throughout by fg ,

$$\frac{1}{f} \frac{d^2 f}{dx^2} + \frac{1}{g} \frac{d^2 g}{dy^2} + k_c^2 = 0$$

So,

$$\frac{d^2 f}{dx^2} + k_x^2 f = 0 \quad \frac{d^2 g}{dy^2} + k_y^2 g = 0$$

Where k_x and k_y are constant and $k_x^2 + k_y^2 = k_c^2$

So,

$$h_z(x, y) = (A \cos k_x x + B \sin k_x x) (C \cos k_y y + D \sin k_y y)$$


So, you know all this mathematics. So, again you are getting two constant, this side also this function of a f , this is the function of g , but it is said that these two functions, they are equal to, their sum is equal to a constant, because my k_c square is constant. So, that is possible only if separately they are equal to two constant so that is b . Now it is solution, you know all these two solutions, again you can see that these $d^2 f / dx^2$; that means, double differential of the function f with respect to x that will be if you take this side here. So, that will be constant into the function; that means, again some cos sin type of thing. So, this is in x , so that is why cosine of and sin of x . Similarly for g you have y so cosine in y variable sine in y variable. So, these are general form that you know.

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Transverse Components

From Eq. 39c,

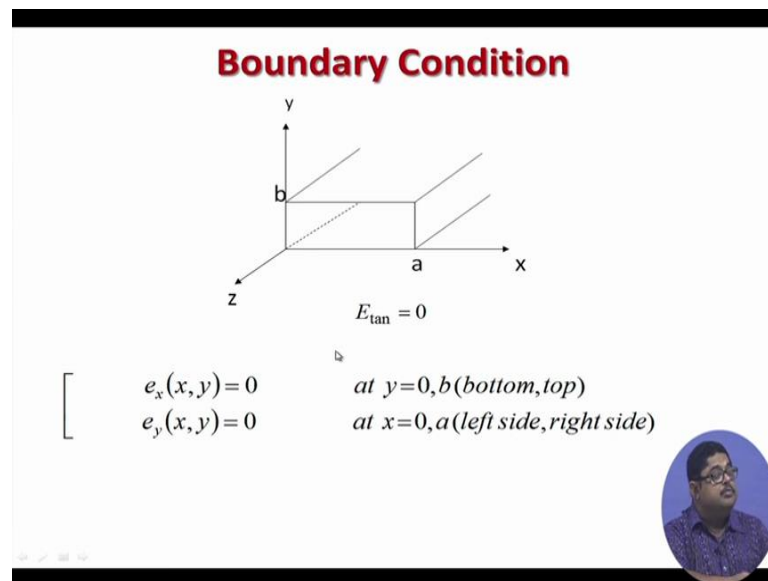
$$e_x = \frac{-jw\mu}{k_c^2} \frac{\partial h_z}{\partial y} = \frac{-jw\mu k_y}{k_c^2} (A \cos k_x x + B \sin k_x x) (-C \sin k_y y + D \cos k_y y)$$

From Eq. 39d,

$$e_y = \frac{jw\mu}{k_c^2} \frac{\partial h_z}{\partial x} = \frac{jw\mu k_x}{k_c^2} (-A \sin k_x x + B \cos k_x x) (C \cos k_y y + D \sin k_y y)$$

Now, you find out the transverse components once you know. So, you see this is the transverse component, sorry longitudinal magnetic field component h_z , we have solved in this; A B C D are constants yet to be determined. So, now, we have already developed remember equation fifteenth, where we have express all the transverse field in terms of longitudinal field, in terms of h_z , because e_z is 0 in this case. So, in terms of h_z we can write, what is the e_x , it will be also involve in this A B C D, and some form of cos and sine, because that depends on the this differentiation etcetera. Similarly we can write e_y in terms of a b c d and this cos and sine function etcetera.

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Now we can also do it for other two things. Now let us put the boundary condition. What is the boundary condition, that you see in the top wall and bottom wall it is a conductor. So, in the conductor there are; that means, this plain is which plain, it is y z plane. So, electric conjuctival electric field on this bottom wall and top wall should go to zero. Now conjuctival electric fill means, on the top wall let us say conjuctival field means, either it will be an ex field or a ez field, but in a TE mode field is not there.

So, boundary condition is ex at this one is zero for y is equal to zero; that is the top wall. Sorry y is equal to zero is the bottom wall, and y is equal to b is the top wall. Similarly we can say that what will happen to the side wall; left side and right side. They are also metallic conductors, so there also tangential thing will be zero. What is tangential? They are; obviously, one z component, but we do not have any z component. What is the other component? y component so ey should go to zero there, where at left side wall and right side wall. So, put this boundary condition if we now put into the, those solution that we got; that means, ex ey hx hy whatever we got.

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Determination of Constants

$$e_x = \frac{-jw\mu}{k_c^2} \frac{\partial h_z}{\partial y} = \frac{-jw\mu k_y}{k_c^2} (A \cos k_x x + B \sin k_x x) (-C \sin k_y y + D \cos k_y y)$$

$$e_x(x, 0) = 0 \Rightarrow D = 0$$

$$e_x(x, b) = 0 \Rightarrow k_y = \frac{n\pi}{b}, \quad n = 0, 1, 2, \dots$$

Also,

$$e_y = \frac{jw\mu k_x}{k_c^2} (-A \sin k_x x + B \cos k_x x) \left(C \cos \frac{n\pi}{b} y \right)$$

$$e_y(0, y) = 0 \Rightarrow B = 0$$

$$e_y(a, y) = 0 \Rightarrow k_x = \frac{m\pi}{a}, \quad m = 0, 1, 2, \dots$$


So, in that for e_x if we put this, then we have put this and force that to that zero. So, $e_x \times 0$ is equal to 0, that implies that this is, this one, it total thing is zero means that you cannot have any other thing except saying that, so D is equal to 0. So, by that you can find out what is e_x , and also for b you can put that, it will take this value k_y . So, similarly if put the other boundary condition, that gives you b is equal to 0 and k_x takes some discrete values, those are given by this like k_y .

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Solution of fields

So, $H_z(x, y, z) = A_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta_{mn}z}$

$$E_x = \frac{j\omega\mu m\pi}{k_c^2 b} A_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$
$$E_y = -\frac{j\omega\mu m\pi}{k_c^2 a} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$
$$H_x = \frac{j\beta_{mn}m\pi}{k_c^2 a} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$
$$H_y = -\frac{j\beta_{mn}m\pi}{k_c^2 a} A_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$


So, we can write here all the five fields. Generally we have six fields; two transverse for electric field, two transverse magnetic field, one longitudinal electric field, one longitudinal magnetic field, but in case of key field, one longitudinal e field that is E_z is 0 so we are writing H_z, E_x, E_y, H_x, H_y . So, all this constitute the solution of the field. So, that is field analysis that you have now come out from that basic wave equation, you can find out this. So, that is why we have developed those modes and TE modes characterization, because now we can see it becomes very easy to find out the field.

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Propagation of TE wave

- TE waves propagate in z-direction inside waveguide when phase constant β is real, nonzero positive

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

- This demands $k > k_c$

or

$$k > \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

or

$$\omega\sqrt{\mu\epsilon} > \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$


Now, also that time we have developed what is beta. Beta is this. Now we have seen that this k_c , sorry this k_c is here that $k_y^2 + k_x^2$ is k_c , k_y^2 is by boundary condition they have been fixed to some discrete values, k_x also is to discrete value.

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$$\omega\sqrt{\mu\epsilon} > \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$


That's why beta will be this discrete value that beta will be k^2 minus this, this. So, these are the numbers m and n , they are the mode numbers, T E M N mode. So, now, you see that we won the TE wave to propagate, then beta should be a real quantity, as we were saying that otherwise it will become an (Refer Time: 14:45) mode. We want the propagating mode so beta should be real. Now beta should be real means, this under square root thing should be positive; that means, k should be greater than 0, or k should be greater than this value. So, we know the value of the k that is the wave number that was a constant that was. If you remember that time I put k is equal to $\omega \sqrt{\mu\epsilon}$. So, that should be greater than this.

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TE waves have Cutoff

- Wave is cutoff when $k \leq k_c$
 or

$$\omega \sqrt{\mu\epsilon} \leq \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$
 or

$$f \leq \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$
- So, there is a **cutoff frequency** given by

$$f_{c_{mn}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$
- All the signals with frequencies below this frequency cannot propagate through a waveguide.
- All the signals with frequencies higher than this frequency can propagate in a waveguide.



Now, this on the other hand if k is less than k_c , you see propagating when k is greater than k_c . Now k less than k_c that time the condition is this. Now ω is nothing, but angular frequency so you can find the corresponding frequency. So, see that below, if f is below this value, this value is given by a means the number, then the wave cannot propagate, wave will not be able to pass to the wave guide. So, this is the stark constant to the TEM line, as we have seen that any frequency can pass to a TEM line, but in a wave guide both for T and TEM mode we see there is a cutoff. So, on that frequency we have calculated that f is less than equal to this.

So, if we take that equal to value, so; that means, frequency, all the frequencies below this cutoff, there will be wave will not be able to propagate. So, you need to have a frequency above this, and then only the wave will propagate. So, that is why this wave guides are called a high pass filter. So, low frequencies are not pass, but all the high frequencies after the cutoff is passed, and this is given by this. So, all the signal is frequency is below this cutoff, cannot propagate to wave guide. All the signals frequencies higher than cutoff frequency can propagate in a wave guide.

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Dominant Mode in a Waveguide

- Mode with lowest cutoff frequency is called **dominant mode**

$$m = n = 0 \Rightarrow E_x, E_y, H_x, H_y = 0$$
- So, TE_{00} not possible.
- Since, $a > b$, lowest cutoff frequency occurs when $m=1, n=0$

$$f_{c_{10}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{1}{2a\sqrt{\mu\epsilon}}$$

- This mode is called **TE_{10} mode**



Now, there are many modes depending on the value of m and n, you seen that when we say this. So, various values of m and n; m and n can be any number, any integer - m and n both are starting from zero and integer. So, there can be various modes TE modes, but out of that, the one with lowest cutoff frequency is called dominant mode. So, let us say that lowest one will be m and n both are zero, but if m and n are zero, if we go back to the field equation. Here if we put m and n zero you see by putting n is equal to zero E_x is zero, by putting m is equal to zero E_y is zero, by putting m is equal to zero, by putting m is equal to zero.

So, if m and n both are zero, here also you see that, if n is equal to zero. So, sine zero this is also zero. So, all the fields E_x, E_y, H_x, H_y all are zero. So, that is not a physical solution.

So, we say that this is not acceptable. So, m is equal to n; that means, T 0 0 mode TE m n, you see the first of script is m, the second script is n, TE 0 0 not possible in a wave guide. Now, since a is greater than b that is our convention. So, lowest cutoff frequency occur for m is equal to 1 n is equal to zero, because m is related to a, if you look at that m pi by a, so that will give you the frequency, and that frequency is nothing, but 1 by 2 a mu epsilon. Mu epsilon are the parameters of the thing, if it is they are wave guide, these are the mu epsilon or the free space values, and a is the broad dimension of the wave guide. Now this dominant mode is called TE 1 0 mode.

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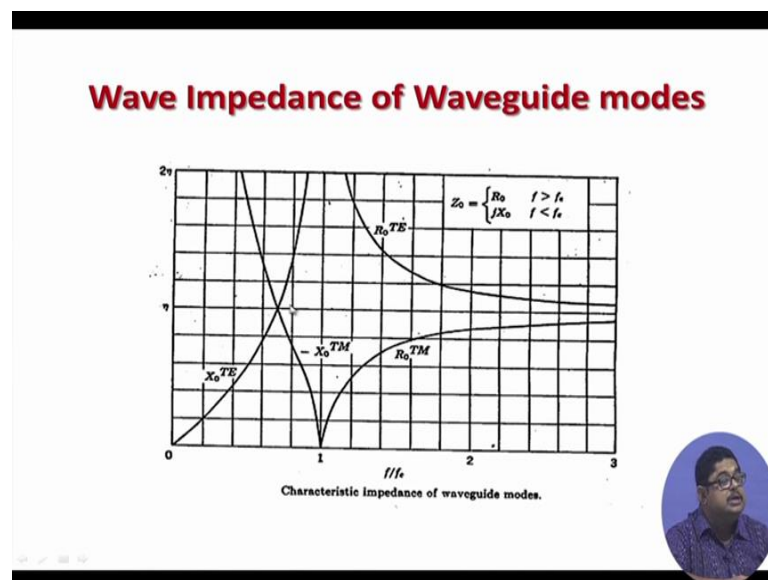


Is a very interesting mode, and if you have a wave guide where more than one mode is propagating, we call it wave guide is over moded. So, in over moded field, power is transported by all the modes which are propagating. So, you need to separately exit the modes. Similarly if the modes are taking power, you need to extract the modes separately. So, that creates an engineering problem. Also, when you have one type of mode extraction the other power that is carried by other modes that gets wasted, generally we try to make a wave guide non over moded, now the wave impedance. Remember it is we cannot have characteristic impedance here because we cannot define p and i. Characteristic impedance is a low frequency concept, but this wave impedance that is always there, as we have already discussed earlier that wave impedance is the

fundamental property of the wave. So, that will be $k \eta$ by β , various modes will have various β s. So, actually we should have written β_{mn} . Similarly here $Z_{TE_{mn}}$, so that various TE modes T_{mn} modes they will have, different wave impedance.

Now wave impedance Z_{TE} is real when β is real, because k is a number its real number for simple material, η is real number intrinsic impedance of free space equal to 120π or 377 ohms. So, if β is real Z_{TE} is real. Now β dl means we have seen that when, the wave is propagating or we have a propagating mode. So, by looking at wave impedance you can see that, whether you have a propagating mode, if you have a β imaginary, then Z_{TE} will be imaginary and you can say that I have an evanescent mode. So, I cannot transfer power to the evanescent mode. Evanescent mode will die down after sometime, because its β will actually, instead of wave propagation $e^{-\beta z}$ to the power minus $j \beta z$, it will be $e^{-\beta z}$ to the power minus j again $j \beta z$, so that will be a loss $e^{-\beta z}$ to the power, it will be contributing to the attenuation of the wave. So, that is why after sometime it will be propagating.

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Now, this is the picture of wave impedance. So, you see x axis is f by f_c , if we are above f_c then the both TE and TEM modes, they are impedance, wave impedance is real that is why it is shown as r naught, but when they are less, then the impedance is reactive, and

for T mode this is the variation, up to here you see that just before the cutoff, it is shooting up to a very high reactants. Similarly just after cutoff then it is gradually decreasing, and coming to this value intrinsic impedance, this is eta, so intrinsic impedance value. So, when you are further away from cutoff; that means, quite high from the cutoff frequency, you approach the impedance of eta.

Guided you see this is from a book, I do not remember the book; it is characteristic impedance of wave guide mode. Now this wrongly written this is not characteristic impedance. This is talking of wave impedance; that is why I have given this heading as wave impedance, it is from some good book, but they are writing characteristic impedance. This is basically the electric field to magnetic field ratio; this should be called wave impedance, not characteristic impedance. This is loose stock, but you should under know that it is the wave impedance concept.

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Guided wavelength and phase velocity

- Wavelength inside guide is larger than free space wavelength

$$\lambda_g = \frac{2\pi}{\beta} > \frac{2\pi}{k} = \lambda$$

- Phase velocity is larger than the free space velocity of EM wave

$$v_p = \frac{\omega}{\beta} > \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$



Now, similarly guided wavelength and phase velocity can be easily calculated, we have already seen. So, since beta is greater than k wavelength inside guide, is larger than free space wavelength. So, if you a 10 Giga hertz signal that is wavelength is 3 centimeter, but when its passes through any wave guide in the T E mode, then its wavelength is more, may be 3.5 centimeter or something. Also the phase velocity you see, we are

showing here that why it is so, because this values you see k actually for TEM mode β is equal to k that we have already seen.

And since this β is less than k ; that means, that in TE mode the phase constant β is less than this wave number k , we get that λ_g the guided length is greater than free space wavelength. Similarly for phase velocity, for the same reason phase velocity is larger than these. Now what is this velocity. This is if it is free space one by this, actually if you put this value, because these are constant, this will come to be c into the power 8; that is the speed of light. So, inside a wave guide the TE mode phase velocity is larger than the velocity of light. So, you can have a fast wave, we call it any wave, we is faster than the velocity of light; that is fast wave. So, inside wave guide you have fast wave; that is why wave guides are called fast wave structure.

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Dominant Mode Field structure

The electric and magnetic fields for TE₁₀ mode are

$$E_x = 0$$

$$E_y = \frac{-j\omega\mu\pi}{k_c^2} A_{10} \sin \frac{n\pi}{a} e^{-j\beta_{10}z} = \frac{-j\omega\mu a}{\pi} A_{10} \sin \frac{n\pi}{a} e^{-j\beta_{10}z}$$

$$E_z = 0$$

$$H_x = \frac{-j\beta_{10}a}{\pi} A_{10} \sin \frac{n\pi}{a} e^{-j\beta_{10}z}$$

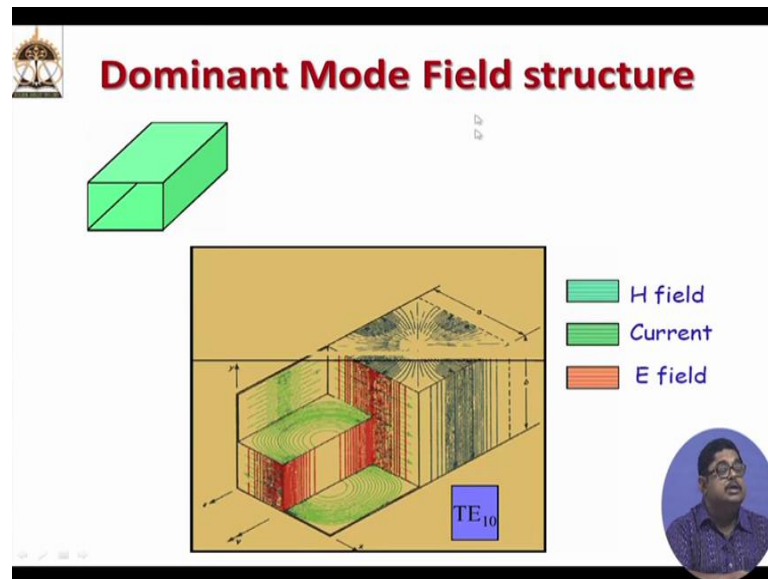
$$H_y = 0$$

$$H_z = A_{10} \cos \frac{n\pi}{a} e^{-j\beta_{10}z}$$



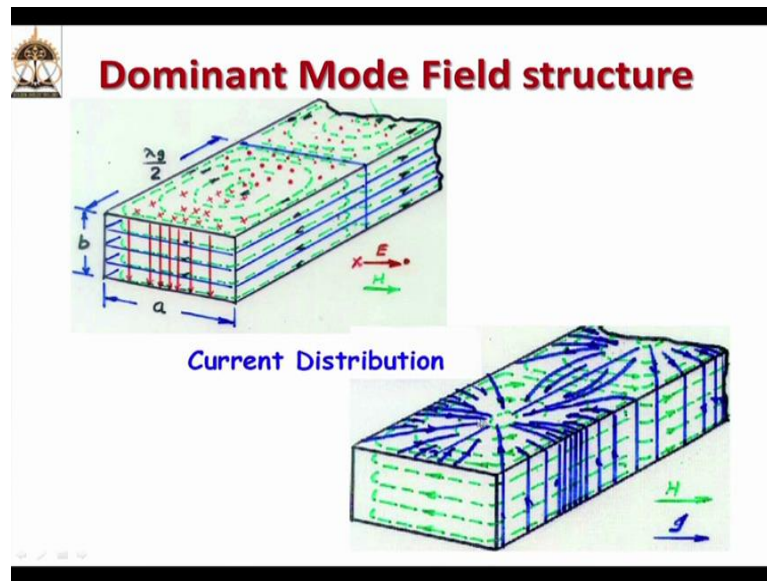
Now, dominant mode if we TE 1 0 mode, if you know it E_x is zero everywhere, that means it does not have an x component you seen it only have a y component, and that y component is. There is the mistake it will be sine $n\pi x$ by a E_z is equal to zero, H_x this also should be in πx by a . And it does not have any longitudinal component, similarly H does not have any y component, but H has a H_z component.

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So, let us see the structure, if we see the structure, the structure looks like this. This is the 3 d structure, this is red things are, you see if I take this cross section this, this is the cross section there you see electric field, at the center it is very high, its electric field is vertical or from top wall to bottom wall it is coming electric field, and you know field lines when that dense means its value is very high. So, at the center its value is very high, at the sides its value is very less. Whereas, magnetic field you see this is on the top wall the magnetic field is like this, you see and in the central portion there is no magnetic field. Also in the side wall you see at the center there is the very dense or very high value of magnetic field, gradually at the side it is there. This whole structure repeats after every $\lambda/2$ and currents also we have seen.

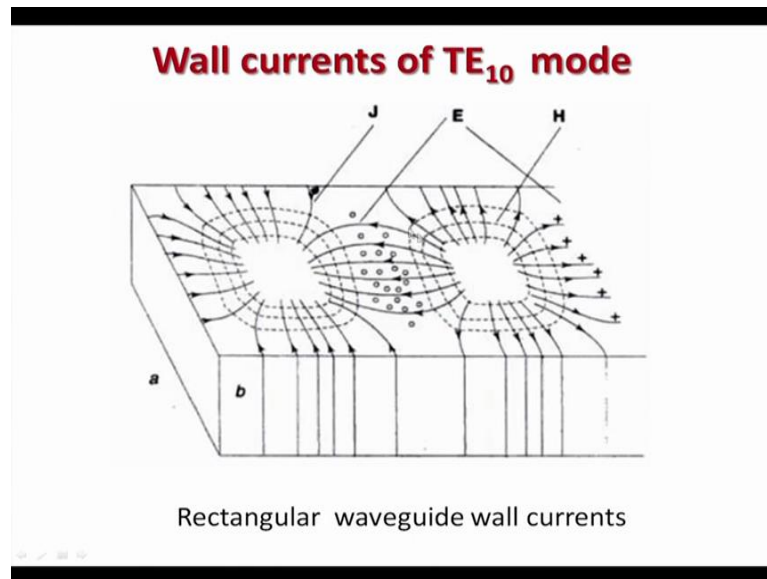
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Let us see some better structure for better understanding. So, you see electric fields, they are from top wall to bottom wall they are flowing whatever I said. The green ones are magnetic fields. So, magnetic field is like this, and you see that the whole thing is getting repeated magnetic field in the, along the longitudinal direction after very $\lambda_g/2$. And current distribution if you see that this is the current distribution but this one is a bit wrong actually. Central portion there is no current, because you see that the magnetic field is like this.

So, central portion, there are no currents, longitudinal current is not there at the center, other places they have longitudinal currents; that is why what happens you see since the magnetic field is not there, we can cut a slot there. If we cut a slot a very small thing, we can probe inside the wave guide. Actually that is the way by which the field inside wave guide is measured. We have already discussed in an earlier NPTEL lecture, that how to measure impedance, how to find fields etcetera, and you can see that lecture. So, there we all use in a wave guide, in a slotted wave guide, we have a slot cut in the middle that we can do, because of this dominant mode it does not have any current here. So, we do not disturb anything the magnetic field is here.

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Here you can see in a better way, you see the magnetic field, if I go along the center they are no magnetic field. So, if I cut a slot, I am not disturbing any magnetic field, because otherwise you see that if I cut a slot and disturb the field, basically I am disturbing the whole structure. So, what I am measuring is not the proper, but due to this fine variation that, at the center portion there is no magnetic field, and also you see that at this portion there are currents are minimal. So, I can easily cut a slot, and I can probe what is happening.

Another beautiful thing is if I do it at the center, we have already seen that in this cross section, the electric field is maximum. So, when I am cutting the slot I am not the disturbing magnetic field here, here; obviously, electric field cannot be here because it is the TE mode. So, note a longitudinal electric field, but here at the center the electric field is maximum. So, in all the cross section the central field is maximum, electric field. So, the beauty is, when I am when I am sensing the maximum with maximum sensitivity sense, because electric field is maximum there; that is why the slot is cut at the middle in a rectangular wave guide, which is working with TE_{10} mode.

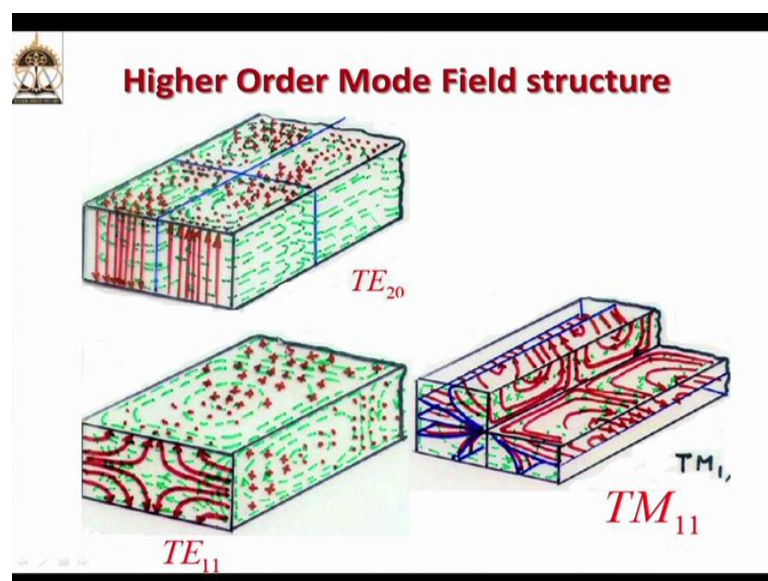
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Salient Points of TE_{10} Field Distribution

- Electric field is vertical, maximum at center, zero at side walls, varying cosinusoidally along x direction
- No Electric field variation along y direction
- Displacement current due to E_y vertical
 - Magnetic field encircles this D
- Two component H_x & H_z
- H_x is maximum at centre, zero at side walls.
- H_z is maximum at side walls, zero at center
 - Longitudinal slot at center won't disturb magnetic field,
 - E field is best for sensing at center
- H_x corresponds to longitudinal current at top & bottom (opposite direction)
- H_z corresponds to transverse currents at top and bottom & vertical currents at sidewall.

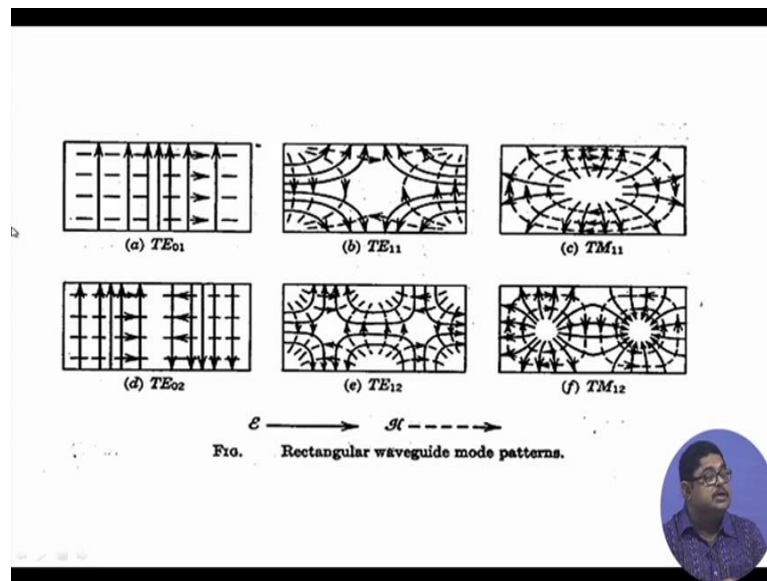
So, electric field is vertical maximum at center zero at side walls, and varying cosinusoidally along x direction this when here. h field has both H_x and H_z component that we have already seen, H_x is maximum at center, zero at side walls etcetera. So, the longitudinal slot at center won't disturb magnetic field, e field is best for sensing at center, because it is high.

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So, these are other modes structure, and there you are that there are magnetic field is, there are, you see if I put the cut a slot here there will be magnetic field gets disturb, because this magnetic field is there. Suppose I cut it here there is a magnetic field here, I cut it here, so there is only for TE 1 0 mode we can cut a slot there; that is why for all experimental purposes or for any sensing TE 1 0 mode is preferred.

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Now, these are some other field structures also you see that, there are some field structures which are not uniform, but this is a uniform, our dominant mode TE 1 0 field electric field that is uniform. So, we require uniform fields, and then we can have various other things, and wave impedance, cutoff wave number is this, phase constant is this.

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Wave Impedance of dominant mode

- The cut off wave number $k_c = \pi / a$
- Phase constant $\beta_{TE_{10}} = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}$
- Wave Impedance $Z_{TE_{10}} = \frac{k\eta}{\beta_{TE_{10}}} = \frac{\eta}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}$

For a WR-90 waveguide, at 10 GHz, the wave impedance is 505 Ω

Wave impedance you see it is a less is this. So, if you do lambda by 2 a. Now wave impedance, this is intrinsic impedance this 377 ohms, but this TE 1 0 mode its impedance, wave impedance is much larger than this intrinsic impedance, so for a WR 90 wave guide at 10 gigahertz if you do, because I know what is the value of a, 10 gigahertz means 3 centimeter. So, this wave impedance is 505 ohm at 10 Giga hertz. Power flow we have done this calculations, and for propagating modes p 1 0 is non-zero real power.

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Power flow of dominant mode

The power flow along the guide for TE_{10} mode is

$$P_{10} = \frac{1}{2} \operatorname{Re} \int_{x=0}^a \int_{y=0}^b \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \cdot \hat{\mathbf{z}} \, dy \, dx = \frac{1}{2} \operatorname{Re} \int_{x=0}^a \int_{y=0}^b E_y H_x^* \, dy \, dx$$

$$= \frac{w \mu a^2}{2\pi^2} \operatorname{Re}(\beta) |A_{10}|^2 \int_{x=0}^a \int_{y=0}^b \sin \frac{2\pi x}{a} \, dx \, dy = \frac{w \mu a^3 b}{4\pi^2} |A_{10}|^2 \operatorname{Re}(\beta)$$

For propagating modes, P_{10} is non zero real power

for non-propagating modes, P_{10} is zero

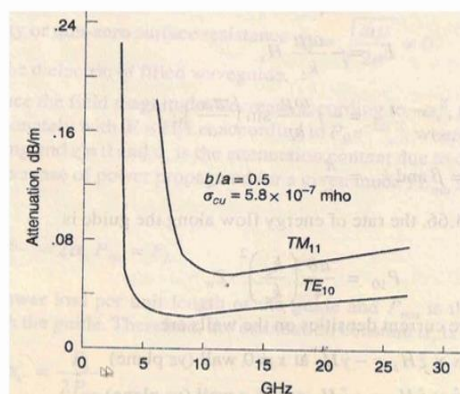
- Dielectric loss

$$\alpha_d = \frac{k^2 \tan \delta}{2\beta_{10}}$$

- Conductor loss $\alpha_c = \frac{R_s}{a^3 b \beta_{10} k \eta} (2\pi^2 b + a^3 k^2)$

So, also dielectric loss, we have already found the expression, this conductor loss if you do. Actually all this mathematics how to arrive at this expression that will be given in your notes which will be uploaded, you can see from there, but basic things we have already in the first five lectures we have given you, how to calculate that these are all applications of that.

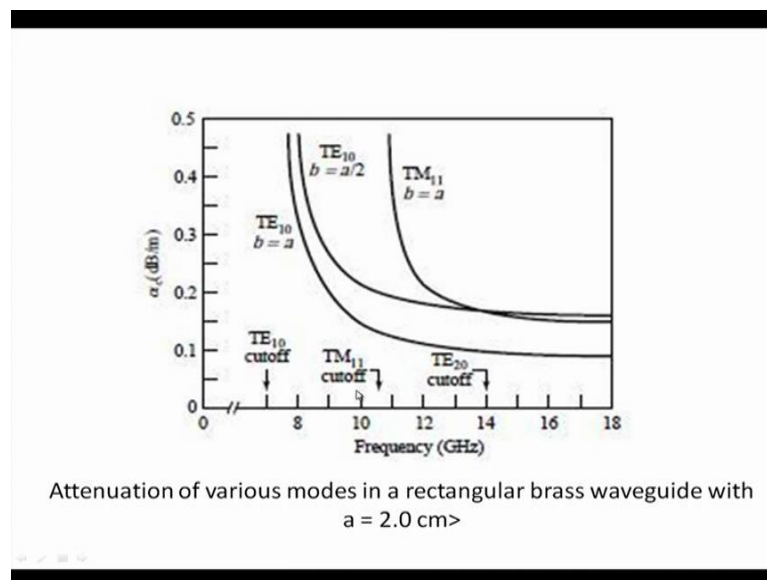
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Attenuation of dominant mode in a rectangular waveguide

So, if you see that at attenuation of the dominant mode. You see this is the dominant mode. So, essentially just though the mode starts after this, roughly near about 5.6 gigahertz, but attenuation is changing very fast, but after some time the attenuation stabilizes and it is quite small; that is why it will use it from 8 to 10 gigahertz; though the cutoff starts at 6 gigahertz, but near that the attenuation is varying, but after sometime attenuation stabilizes. So, it can use, but you cannot use it greater than 12 gigahertz, because after 12 gigahertz you will see that the next higher order mode; that is TE₀₁ mode, not TE₁₀ mode. TE₁₀ mode also have, this is TE₁₀ mode, this is the next mode that comes then 1 TEM modes comes.

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So, this is the thing that first TE₁₀ cutoff takes place as a I said roughly 6 gigahertz, then TE₀₁ actual this are all the thing. This is other modes will start coming, and then TE₂₀ cutoff that comes, but TM₁₁ cutoff that comes around 12 gigahertz; that is why after gigahertz people do not use it. So, this is called, these are structure that you can have this diagram, to find out what is their attenuation as well as what is the cutoff.

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TM Mode

$$H_z = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) e_z(x, y) = 0$$

- With the help of separation of variables,

$$e_z(x, y) = (A \cos k_x x + B \sin k_x x) (C \cos k_y y + D \sin k_y y)$$

Boundary Conditions $\rightarrow$

$$\begin{cases} e_z(x, y) = 0 & \text{at } x=0, a \\ e_z(x, y) = 0 & \text{at } y=0, b \end{cases}$$

Now, similarly how did you will see TEM mode here H_z is equal to zero. So, here the Helmholtz equation will be in terms of the longitudinal e field e_z . Again with the separation of variable, again the boundary condition is, in the side wall what is the electric field, tangential component that will be in the side wall? It will be e_z , and in the top and bottom wall it is e_z .

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Solution of Longitudinal Field

Also we want,

$$\begin{cases} e_x(x, y) = 0 & \text{at } y=0, b \\ e_y(x, y) = 0 & \text{at } x=0, a \end{cases}$$

Applying

$$e_z(0, y) = 0 \Rightarrow A = 0$$

$$e_z(a, y) = 0 \Rightarrow k_x = \frac{m\pi}{a}, \quad m = 1, 2, 3, \dots$$

$$e_z(x, 0) = 0 \Rightarrow C = 0$$

$$e_z(x, b) = 0 \Rightarrow k_y = \frac{n\pi}{b}, \quad n = 1, 2, 3, \dots$$

$$E_z(x, y) = B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta_{mn} z}$$


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Solution of Transverse Fields

The transverse components are :

$$E_x = \frac{-j\beta m\pi}{ak_c^2} B_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$

$$E_y = \frac{-j\beta n\pi}{bk_c^2} B_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$

$$H_x = \frac{j\omega \epsilon n\pi}{bk_c^2} B_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$

$$H_y = \frac{-j\epsilon m\pi}{ak_c^2} B_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta_{mn}z}$$



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Dominant TM Mode is TM₁₁

- Phase constant $\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$
- So, $f_{c_{mn}}, \lambda_g, v_p$ are same as TE modes
- The field expressions suggest that if either of m or n is zero, all the transverse components are zero.
- TM₀₀, TM_{0,n} and TM_{m,0} modes non-existent and dominant TM mode is TM₁₁
- Dominant modal wave impedance

$$Z_{TM_{11}} = \eta \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2 - \left(\frac{\lambda}{2b}\right)^2}$$


So, you force that by that you get those values, and again those k_x and k_y are some discrete values. So, transverse components you can write once you determined k_z , you can write E_x, E_y, H_x, H_y phase constant, same expression. So, cutoff frequency λ_g, v_p is same. Now field expression suggest that if, sorry here it suggests that if either m.

Here you see that this is the big different from the transverse magnetic field, and if either m or n is zero, all the transverse component are zero.

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Cut of frequency

- Dominant TM mode cut off frequency is

$$f_{c_{TM_{11}}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$
- Comparing with dominant TE mode cutoff frequency

$$f_{c_{TE_{10}}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{\pi}{a}\right)^2}$$
- $f_{c_{TM_{11}}} > f_{c_{TE_{10}}}$
- So, the dominant mode in a rectangular waveguide is TE₁₀

So, TM₀₀ TM_{0n} and TM_{m0} modes cannot exist, because all the fields go to zero. So, only thing is dominant mode; that means, the first one that can start is, that it should be both one. So, dominant modal wave impedance is given by this cutoff frequency this. Now if we compare with TE mode, because this is dominant TM mode TM₁₁ and this you compare that TM₁₁ is this. This; obviously, it has extra package of this, that is why the cutoff frequency for TE₁₀ mode, is less than cutoff frequency of TM₁₁ mode. So, among all the combined T_n TM mode that TE₁₀ is the dominant mode

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WR-90 cut-off frequencies				
Mode	m	n	(GHz)	
TE	1	0	6.562	
TE	2	0	13.123	
TE	0	1	14.764	
TE, TM	1	1	16.156	
TE, TM	1	2	30.248	
TE, TM	2	1	19.753	

For WR-90 waveguide $a = 2.286$ cm $b = 1.016$ cm

Now you can see that TE 1 0 mode starts at that is why I was saying, that TE 1 0 mode for WR 90; that means, this as I said 0.9 inch and 0.4 inch or this centimeter values, TE 1 0 mode start at 6.56 gigahertz, TE 2 0 mode starts at 13 gigahertz, TE 01 mode starts at 14 gigahertz; that is why before down set off TE 20 mode, you should not go beyond that; that means, that is why we generally WR 90 is operated up to 12 gigahertz, it is operated up to 12 gigahertz. So, that TE 20 mode does not come here. Now you see that T TE modes come, then the next one that TE 11 and TM 11 they come together, then TM modes start propagating.

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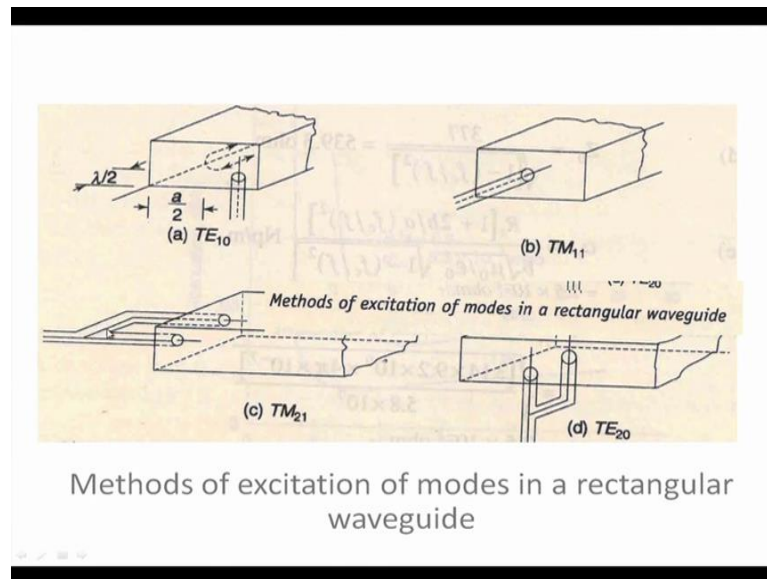
Table $(f_c)_{mn}/(f_c)_{TE_{10}}$ for $a > b$

Dimension		$(f_c)_{mn}/(f_c)_{TE_{10}}$							
a/b	TE_{10}	TE_{01}	TE_{11} TM_{11}	TE_{20}	TE_{02}	TE_{21} TM_{21}	TE_{12} TM_{12}	TE_{22} TM_{22}	TE_{30}
1	1	1	1.414	2	2	2.236	2.236	2.828	3
1.5	1	1.5	1.803	2	3	2.500	3.162	3.606	3
2	1	2	2.236	2	4	2.828	4.123	4.472	3
3	1	3	3.162	2	6	3.606	6.083	6.325	3



And this are again the, if you change the dimension you see a by b, we call rectangular wave guide, but what happens if we make a by b 1 that is a square wave guide. Now in case of square wave guide you will see that, it is over-moded wave guide, when TE 10 is propagating that time also TE 01 propagates to it. So, that is why there are problems because this two waves are different orientation of electric field so, but if you want that, in some application you want that electric field should be in the both the direction, there square wave guide also is used.

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Now, these are the methods of excitation of various modes. So, as various modes are different field structures you need to excite, the field structures differently like TE_{10} mode generally what we do at the central position, we put that coax if central conductor we connect to the wave guides, wall where electric field is dominant, and then it starts electric field from there. Whereas the in case of TM_{11} , we from the side wall which I try to excite, like that there are if you look at the electric field and magnetic field structure you get the clue, that how to excite the field. These are just you to know that this type of thing you see, that at the central portion you put a coaxial cable, and connect the central conductor to the wave guide bottom layer, you get a you make that potrabation and get the excitation of modes.

So, I think that is all about wave guides, rectangular wave guides. In the next lecture we will see that circular wave guide geometric.

Thank you.