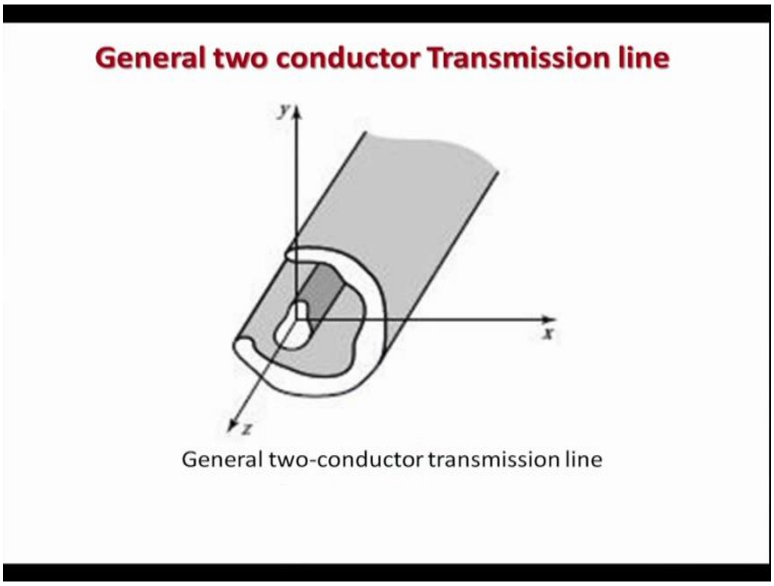
Basic Building Blocks of Microwave Engineering
Prof. Amitabha Bhattacharya
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 06
Coaxial line

Welcome to the sixth lecture of this course on basic building blocks of microwave engineering. In last five lectures we have developed the microwave transmission model, we saw the concept of modes, and then we have seen what are the fundamental modes. There are three fundamental modes; TM T TM, and also we have we have characterised them, and then we have found how to deal with loses in microwave circuit's.

Now that concept today we will apply on the transmission structure, which carries the microwave signal; that is the microwave transmission structure. Now loosely sometimes we call this structure transmission line, but in microwave engineering, we give a particular connotation to the word transmission line. Basically a transmission line is a line, which supports TM line; though actually TM line can also support other T TM lines, like a coaxial line it supports, TM mode T mode TM mode, but generally when we say sometimes the meaning of transmission line means, a structure which supports only TM mode of propagation. And similarly we say wave guide is a transmission structure which supports known TM modes; that is either T or TM modes, but in real practice a wave guide cannot transport TM mode, but the reverse; that means, a two conductor line that can support all the things. So, that is why we give special name to each of the structure, and see what type of mode propagates to that and find out the field structure and other parameters, associated with that mode propagation.

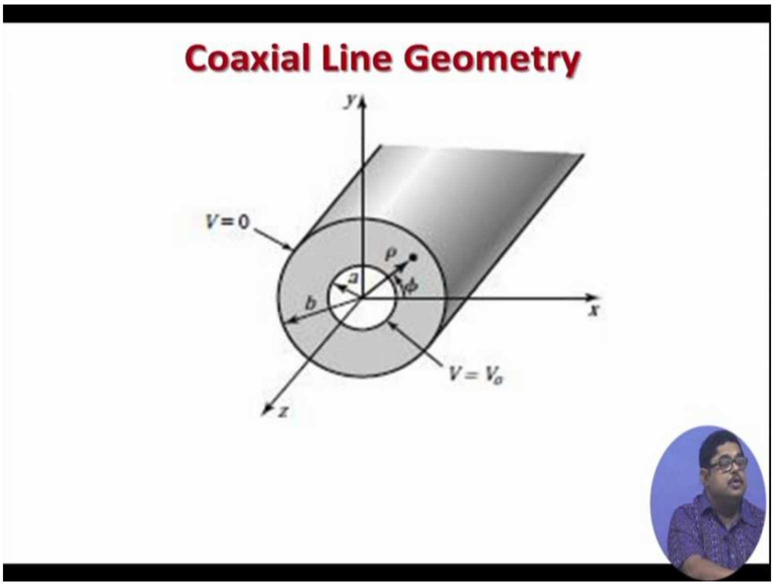
(Refer Slide Time: 02:56)



General two-conductor transmission line

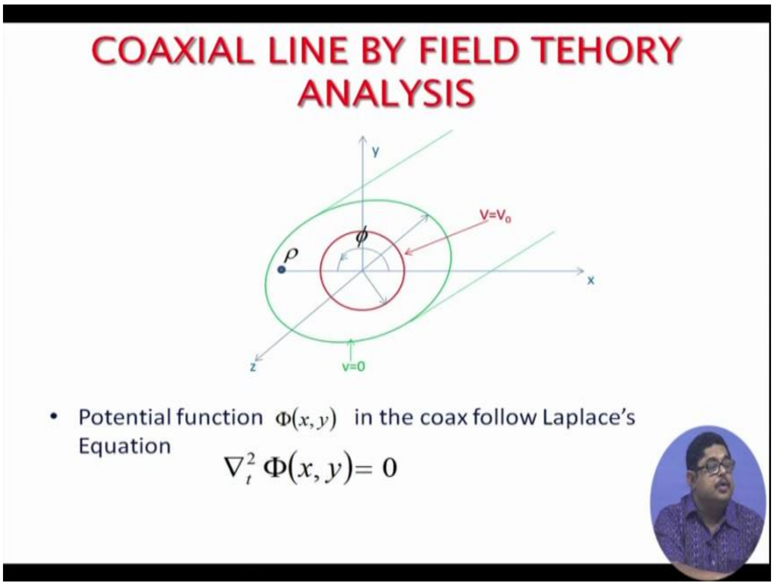
So, the first in that today will see a coaxial line. A coaxial line is, you can see it is a general two conductor transmission line. We have two cylinders of infinite length, because transmission line means it is of a quite good amount of length, in terms of the wave length. So, we have two conductors, and if the two conductors are placed coaxially; that means, their axis is same, but; obviously, their radius is different than that is called a coaxial line. So, in the diagram you can see a two conductor transmission line, there is a inner conductor and coaxially with that inner conductor there is an outer conductor. The axis is generally it is convenience or customary to take the axis of this coaxial system to be z. So; that means, the transverse plane will be x y plane, or we will use the cylindrical coordinates for a this type of cylindrical structure and in that kind it will be row phi plane, and z is the longitudinal plane.

(Refer Slide Time: 04:24)



So, this is the coaxial line geometry as I was saying that we have shown here, the Cartesian thing, but actually will be describing this coaxial line analysis or fields in terms of row phi. Row is the radial direction, phi is the Azimuthal direction, and z is the longitudinal direction. Generally the outer conductor is grounded; that is why you see it is potential, is shown as v is equal to 0. Usually that is a practice that the outer conductor is grounded, and the inner conductor is given a potential. So, in this case we are denoting it by v naught, where p is potential. So, there is a charge distribution in inner conductor. this inner conductor later we will see pictures, that this inner conductor comes out, and this inner conductor is connected generally to the any load etcetera; like in antenna we connect it to the load, in the source side we connect it to the one end of the source, the other in is a outer conductor generally that is connected to ground.

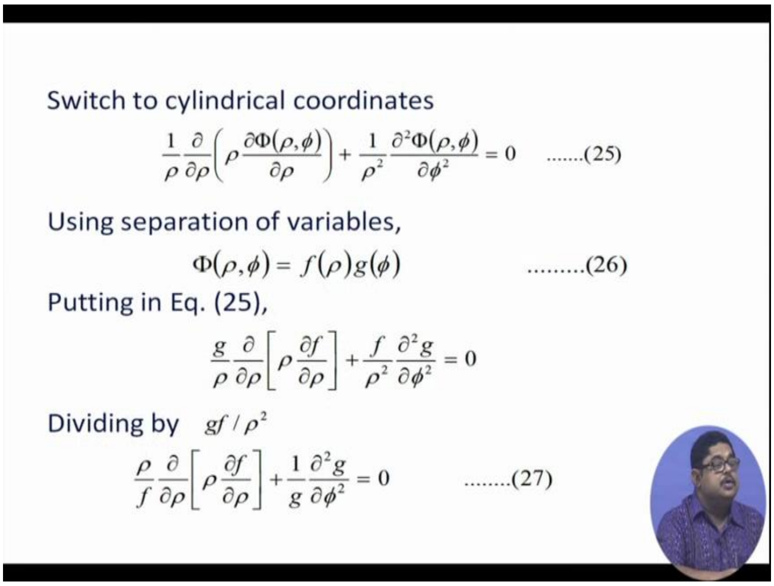
(Refer Slide Time: 05:54)



So, in a single ended system the outer conductor is grounded, and the inner conductor is connected to the active line. Now we will see as we have developed that TM mode analysis. So, coaxial line we know it supports TM mode fundamentally, though it can support higher order modes also, but it is dominant mode is TM mode, and TM modes starts from this value that we have already seen, and we have seen that in case of TM line, the field distribution is quasi static. So, we can though it is a dynamic case, TM is a dynamic TM field variation, but we can still use the concept of the scalar potential function phi, and that follows Laplace's equation in the. It is a transverse Laplacian operator that we have seen. So, that one it follows Laplace's equation.

So; that means, a starting point of this field analysis; that means, derivation of the electrical magnetic fields starts from that inner coaxial line, within the coaxial line structure, this Laplace equation in the transverse Laplacian operator, not the normal or ordinary Laplacian operator does not have t. This is a two dimensional Laplacian; that is why Laplacian operator with a t is reminding transverse Laplacian so that equal to zero. This is a Laplace equation this should be obeyed. So, we will solve this equation, and from this will determine the voltage and current, and from voltage and current we can go back to the electric field and magnetic field etcetera.

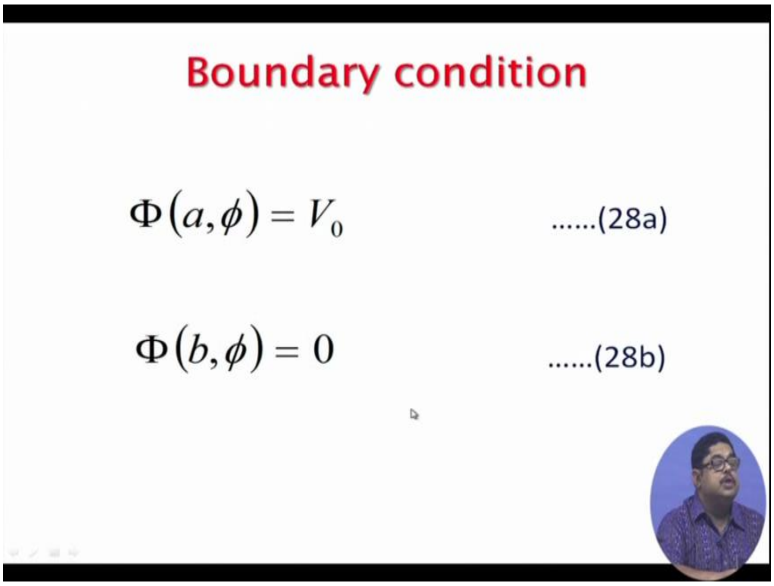
(Refer Slide Time: 07:51)



So, as the structure is cylindrical. So, we can switch over cylindrical coordinates. So, that the transmission line coax, it is geometry falls on the constant surfaces, constant coordinates surfaces of the cylindrical coordinate. So, the Laplacian operators that in cylindrical coordinate will be only have rho and phi variations. So, that is why you see the del - del rho and phi variation.

Now as before this potential function we separate into two variables, two functions of single variable. So, the potential function which is a function of both rho variable and phi variable that we are writing as f rho into g phi. This is only f is only a function of rho; g is only a function of phi. So, these if we put in this potential Laplace equation, then we get this equation. These are all mathematical details; this will be uploaded in the site. So, you need not take, you try to understand just we are going the same steps that we have done in previous case also. Then in this case you will have to divide by this, and that is why we get that equation.

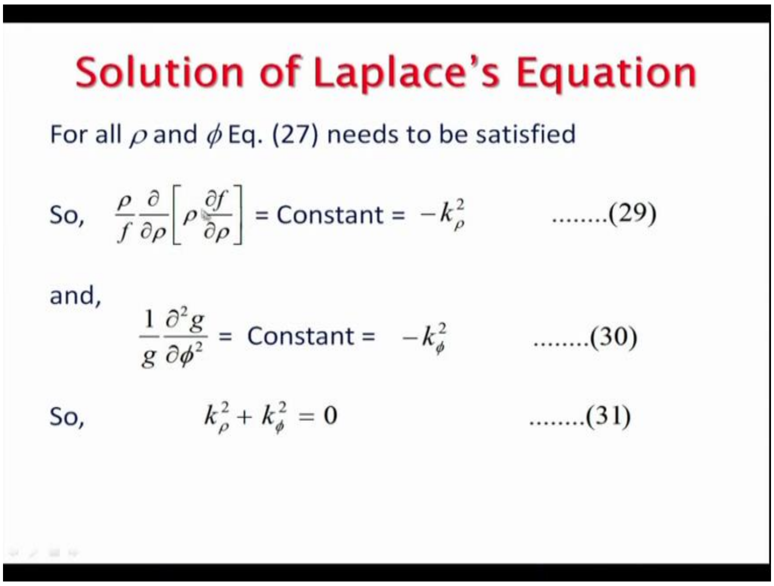
(Refer Slide Time: 09:17)



Now, you subject it to the boundary condition. Now what is the boundary condition? Boundary condition is. Sorry let me see the previous slide. So, what will be the boundary condition? This inner conductor, this red structure, this is a metallic structure. So, we can say that tangential field is here Azimuthal direction; that means, in the uni-vector along the phi cap direction. So, that means, when rho is equal to a, this is generally called a, the radius of the inner conductor so; that means, that rho is equal to a, we will say that the tangential field is zero, because this is a conductor. On conductor we know the boundary condition is tangential electric field is zero.

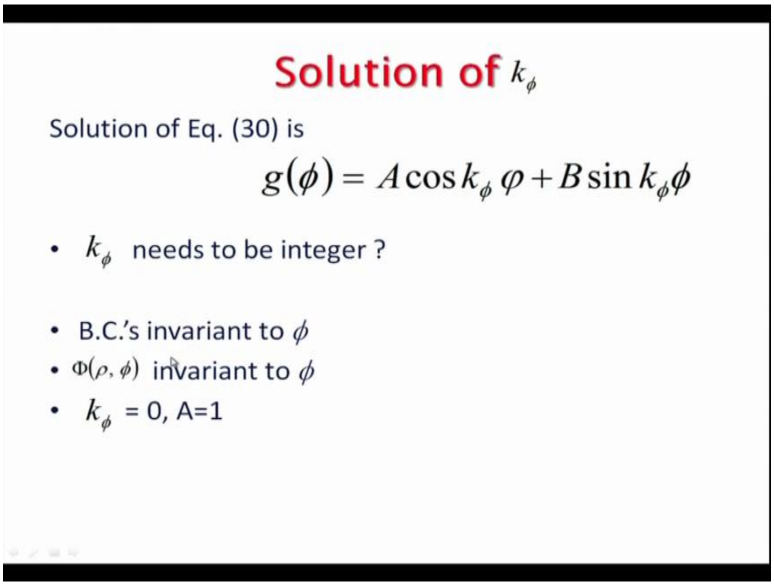
Similarly, here that, or in terms of potential we can say that at rho is equal to a, the potential will be v is equal to v naught, and at rho is equal to b; that means, on the outer conductor the potential is zero. So, since we are here on the Laplace equation potential function. So, potential boundary conditions are that inner conductor, the potential is v naught, outer conductor potential is zero. So, that is written there, that phi this is the potential at a, for all phi's, because everywhere on the surface it is v naught, tangential field, and then this also the potential for any azimuth, but for the outer conductor it is zero. So, these are the two-boundary condition.

(Refer Slide Time: 11:16)



So, you see that these two equations 29 and 30, they are that this left hand. 29 in the left hand side that is a pure function of rho only, but that plus this one that is zero; that is a constant; that means, a function of rho plus a function of phi, their sum is a constant. This is possible only the this slide is equal to a constant that we are naming as k rho square; rho is a sub script, but actually this whole thing is a constant, similarly this is a constant. Now what will be the solution of these one. You see that we are g is a function of phi, we are double differentiating into it phi. So, that double differentiation is equal to this function into some constant; that means, that is double differentiation is also producing a function which is a same function. Now, from our knowledge of differential equation we know that this is possible only if, we have either a cosine or sinusoidal variation.

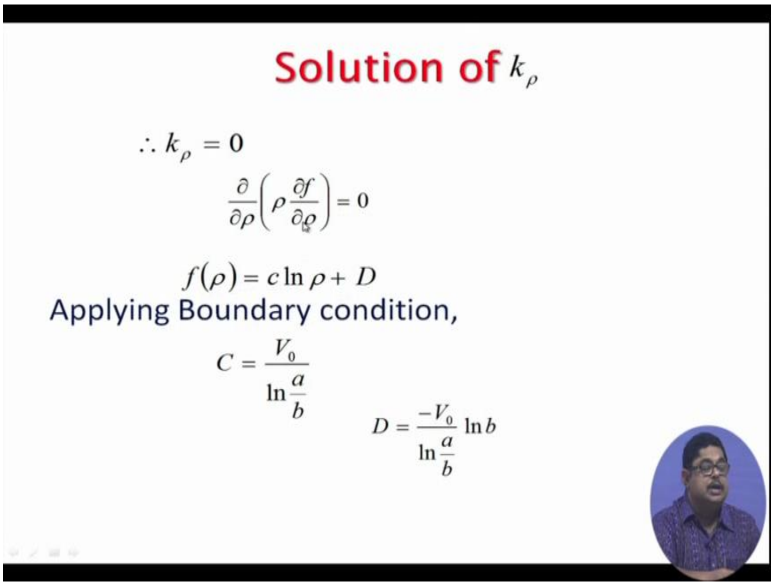
(Refer Slide Time: 12:39)



So, that is why the general solution of this equation will be, that g phi is a cosine function plus a sin function. Now this phi you know that a phi; that means, in a cylindrical coordinate the Azimuthal coordinate, phi that is an angle, so that as a periodicity of 2 phi. So, any value of phi, if I take if I add 2 phi to that it will be of the same value. So, we demand that g phi, also should be periodic in phi with a period 2 phi. So, that is possible if this constant k phi becomes an integer, because then only cos of some value, some integer into the angle. So, if I change that angle by 2 phi. So, that will be multiplied by k phi. So, 2 phi n, some n is an integer. So, k phi should be something like an integer, and then only this is possible.

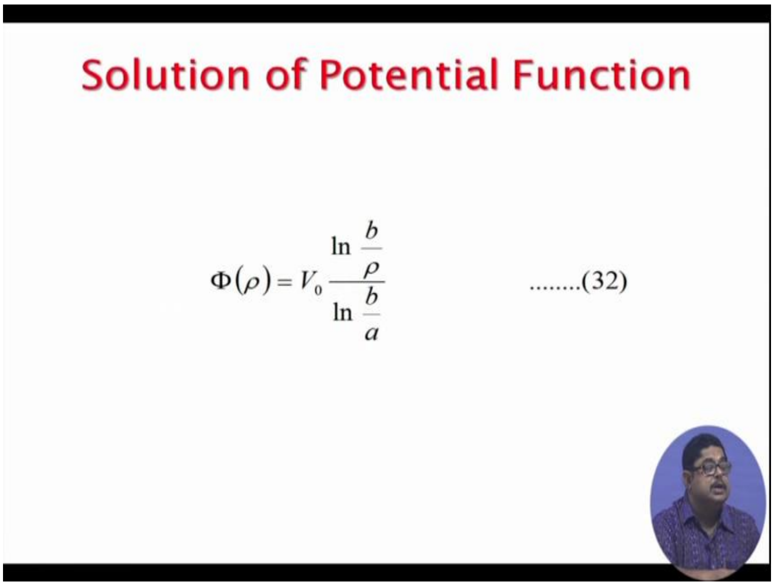
So, this is the first thing that k phi needs to be an integer. Then we have seen that boundary condition of the phi function that is a potential functions v and v naught. They are on inner conductor for any value of phi, the potential is v naught. Similarly on outer conductor the potential, for any value of phi is zero. So, boundary conditions are invariant to phi. So; that means, we demand that the potential function should also be invariant to phi. Now how that is possible, a potential function in variant to phi, is not possible, because it is a cos and sin function. now only way it can go there that is, if k phi becomes zero, but if it is k phi becomes zero then cos of this, that will be a. So, a should be equal to 1 and sin k phi if it is zero, then; obviously, this whatever may be the value of b this whole thing will be zero.

(Refer Slide Time: 15:07)



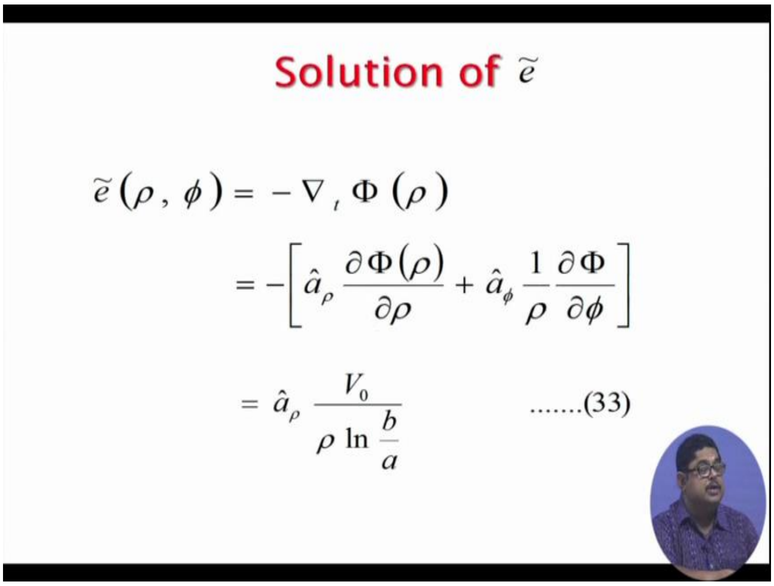
So, where the solution is that a is equal to 1 k phi is equal to zero. Now we come to the solution of k rho the determination of k rho now. So, here we put that value and from that we, this is a second order differential. So, you can, you know how to solve it, it will come as the f function of rho that will be some logarithmic function. Now applying boundary condition we get the value of the two constants c and d that will be like this, this c and d was our solutions.

(Refer Slide Time: 15:56)



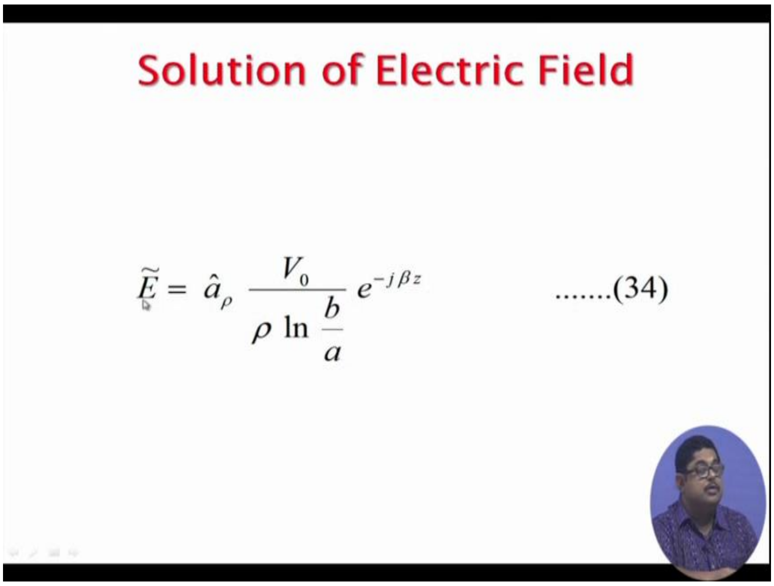
So, c and d we got as this, and from that the potential function can be determined; that is equal to v naught l n b by rho by l n b by a. So, you see that it is a function potential is a function of, it is not a function of phi Azimuthal angle, but it is a function of the radial distance, where is it, because of the presence of this radial thing. So, potential is changing with radial thing that we have seen, that at rho is equal to a, it is taking the value of v naught let us see, that if we put rho is equal to a then this becomes l n b by a this becomes l n b by a whole thing is one. So, phi at rho is equal to a; that is v naught. So, correct. Similarly, another boundary condition is that at rho is equal to v, I should have, this potential function should be zero, put it here, that if I put b it is l n b by b that is 1 l n 1, l n 1 is 0. So, that is why this phi potential function again becomes zero. So, boundary condition is satisfied, and this is our solution of potential function.

(Refer Slide Time: 17:15)



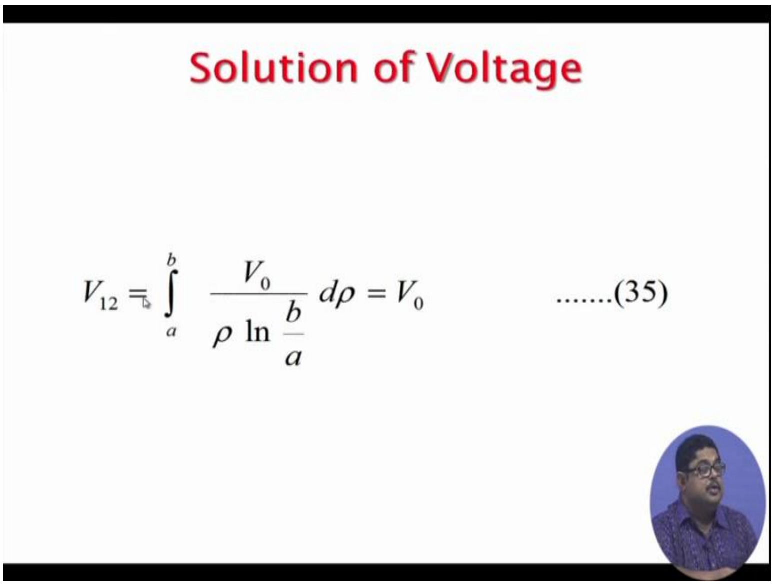
Once we know potential function we can find the transverse selective field. These expressions we have derived earlier in case of quasi TEM analysis. So, you just put that gradient of this potential, is the electric field, transverse electric field. So, if you do that in the cylindrical coordinate we get that electric field, transverse electric field is rho directed. Transverse electric field is rho directed, it is a function of rho, but not a function of phi that is why you see that it is direction is rho, also it is dependent on rho that is why rho is present in the expression, no phi. So, it is always pointing in the radial direction, transverse electric field is radically going out.

(Refer Slide Time: 18:05)



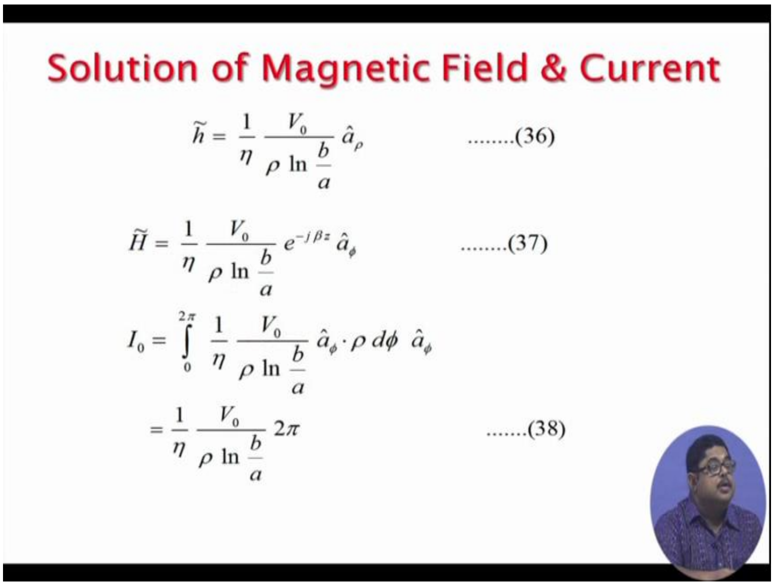
And that was the transverse electric field. The total electric field we need to put the longitudinal variation that we know is wave variation e to the power minus j beta z. So, it is transverse component into e to the power of minus j beta z. This is the solution of electric field phasor.

(Refer Slide Time: 18:26)



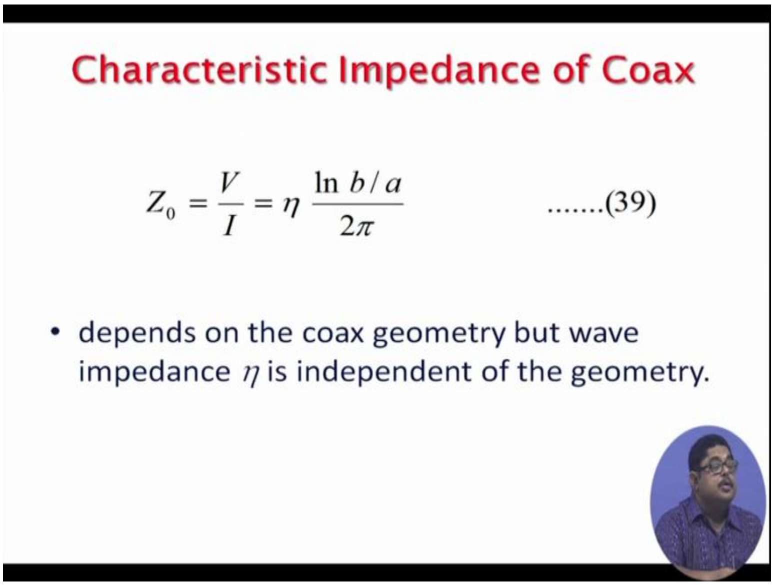
Now, once we know electric field phasor, we can find the potential difference between the two conductors v 1 2. So, from a to b if we go, then we know v 1 2 that will be the minus integration of e dot d l. So, if you do that integration, the v 1 2 the; that means, the difference between the, or potential difference of the inner conductor, with respect to the outer conductor that will be v naught; that is obvious from the thing also and also from the field theory, from our field solution also it is coming.

(Refer Slide Time: 19:12)



Now, similarly you can find out the magnetic field. We will have to find the transverse component of h as in equation thirty six. Then you find out what is the total electric field phasor; that means, this is the transverse components h tilde. So, you will have to take the z variation e to the power minus j beta z, and that full magnetic field phasor is written like this. Now once you know magnetic field, you know how to calculate the current, because from the magnetic field we can easily find the current, and that will be given by this. So, this is the current is on a loop. So, you will have to take that loop direction, and that becomes. So, it is nothing, but you see the, it is v naught by rho l n b that thing. So, 2 pi into this; that is 2 pi into the magnetic field that is equal to phi naught.

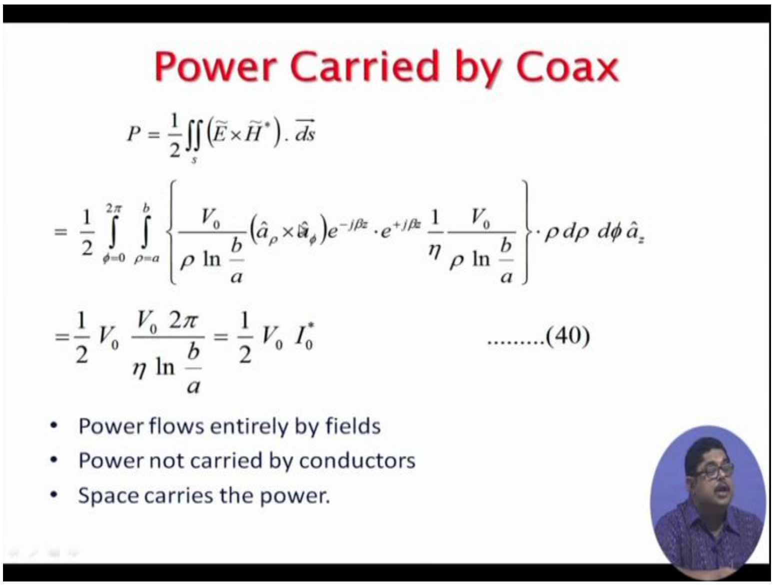
(Refer Slide Time: 20:28)



Now, characteristic impedance, characteristic impedance, this is a transmission line. So, it will have characteristic impedance, and characteristic impedance is the ratio of voltage by current in a TM line. For T TM we have seen the characteristic impedance does not have a meaning that we have already discussed, but we have wave impedance for all, but let us calculates in coaxial line, we will have to calculate the characteristic impedance that v by I and that is eta l n b by a by 2 a.

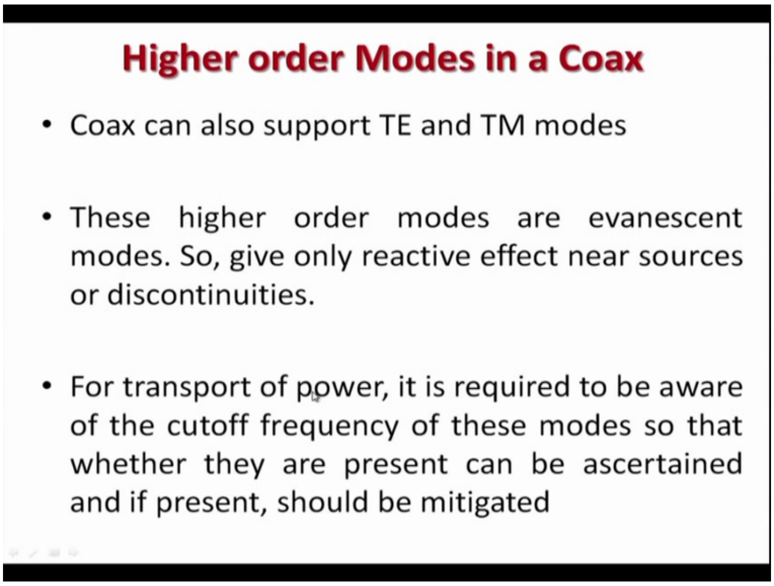
So, you see that eta is intrinsic impedance now that is you can have sum value of it chosen. So, that that is more than 2 into 3.14, roughly 6 or roughly 7. So, more than seven if the l n b by a is, then you get a - that z naught is more than eta, otherwise z naught is less. So, characteristic impedance of coax, you can choose like this, or if the characteristic impedance is specified you can immediately find out that what would be the value of the this l n b by a for a given eta. So, this characteristic impedance depends on the coax geometry by wave impedance, but eta is independent of the geometry; obviously, eta is a intrinsic impedance of free space, it does not depend on the geometry, then what is a power carried by coax.

(Refer Slide Time: 22:15)



We have already seen the, when we did loss analysis how to do, what is power. Power carried by in the electric field is a pointing vector, this side is pointing vector, and then you will have to integrate it over a surface, on which the power is flowing. So, pointing vector into dot d s. So, that if you do in cylindrical coordinate you get it is half v naught I naught square I naught conjugate. So, this power, remember that in the coax, power flows entirely by fields. So, otherwise no circuit, because the where is the power. Power is between that two conductors; the electrical and magnetic field is there, and that field is making the power flow possible, that is wave, wave is taking that power. Power is not carried by the conductors. Conductors do not carry the current. It is not the conduction current type of thing. It is inside the wave inside the, to the medium between the two conductors; that is the either free space or directly, through that the power is going.

(Refer Slide Time: 23:41)

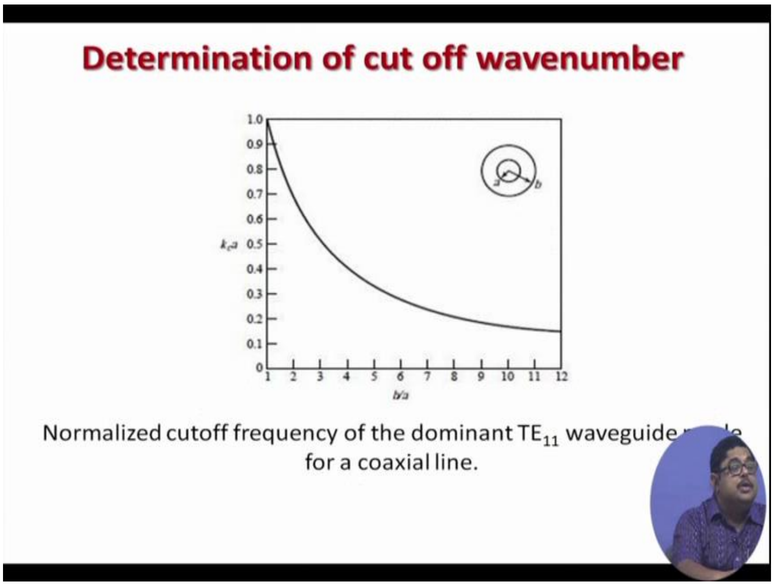


So, that space between the two conductors that carried the power. Now coaxial line can also support higher order modes; like TE and TM modes, but the modes that in a coax, this modes if we see their beta it becomes an imaginary thing. Now if beta is imaginary for any wave we call that evanescent wave, because it actually, instead of carrying the power in the wave form, actually it attenuates the wave. So, evanescent waves, when they are produced, after travelling certain distance they are cut off due to this, because they have a lossy structure. So, evanescent in this, in coax whatever TE TM modes we get, those are evanescent mode. So; that means, they do not take the real power with them, and they give only reactive effect near sources or discontinuities.

So, this evanescent modes are produced, because to produce higher order modes, either when we are at the near the place where the source is put in the power to the transmission line or coax, there is the discontinuity. So, they are these evanescent modes are present, there are this higher order modes are present, but it we are a bit away from the source, roughly phi lambda 10 lambda distance, then this evanescent modes die down, only the TM mode propagates that. Similarly, near the load side what if we within the coax; suppose we join two coaxes together. Now; obviously, any joining by mechanical thing, so that will create a discontinuity, or if the coax suppose one larger coax and one smaller coax, if I let them connecting. So, that will be problem and there will be discontinuity and that time these modes will come.

Now, there are, by doing the mathematics that we have developed in case of TE TM modes you can find the cut off frequencies, you should be aware of that, because transport of power. You should see that when you are trying to send power through a coax, unnecessarily those TE TM modes if they come, they will take some power, which you will be able to extract at the load end, because at the load end we will be extracting the TM mode, because this TE TM mode, they are not able to propagate longer distance to coax. They can propagate in wave guides wave guides are main for TE TM mode propagates. So, there they can wave go for longer distance.

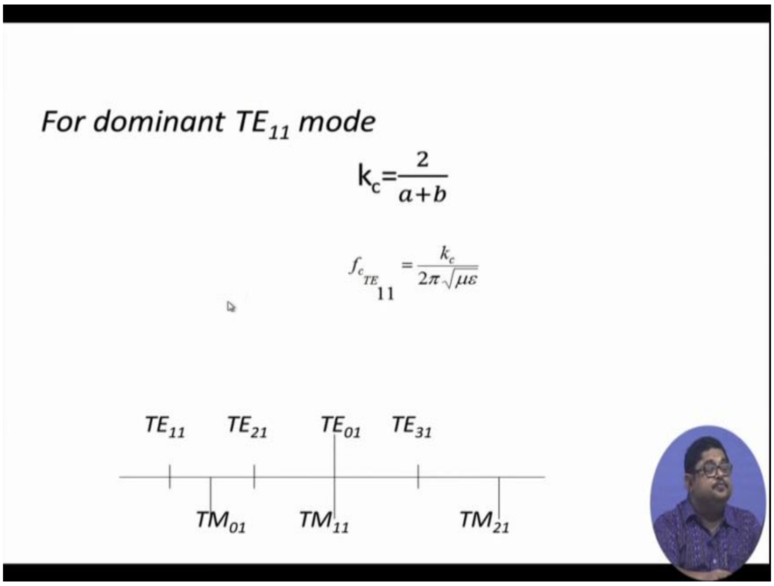
(Refer Slide Time: 26:42)



So, you should be aware that, in your coax these higher order modes TE TM are not present. So, you should design your coax geometry by that way. Now this is the cut off wave number. So, dominant TE 11 mode, this is supposing one of the first modes that come as a high order mode TE 11. So, this is the ratio b by a, outer conductor radius by inner conductor radius. Basically this ratio is very important; you have noticed I think all the voltage, current, fields, etcetera all are coming in terms of this ratio. So, this ratio says that are the frequencies that too, this will take place. So, suppose I have the b y a ratio, generally we take b by ratio something like 3.6, the reason we will see later, but. So, they are I can see that TE 11 modes will come at this value of a.

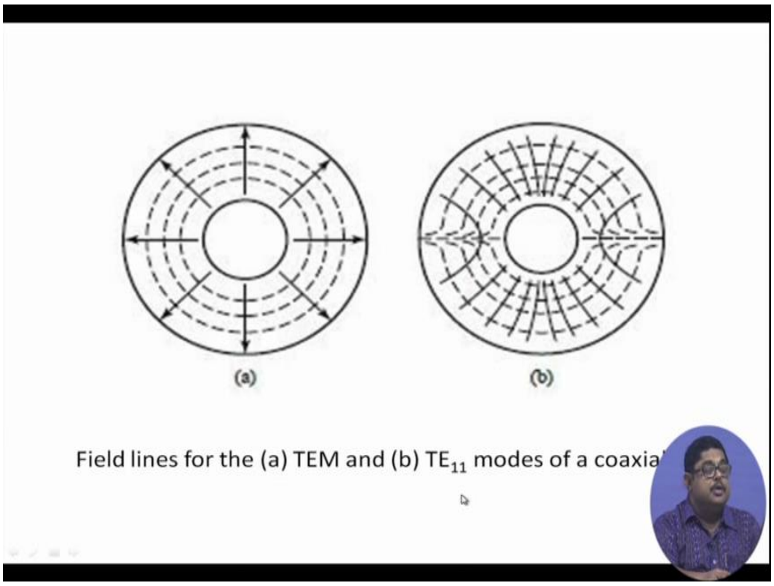
So, we will choose a, so that h does not come; that means, if I choose a value a bit less than this, a value given by this, then I will be able to find out k c is you remember that cut off wave number. So, cut off wave number that depends on both the frequency of operation, medium parameters, as well as for. In case of TM mode the beta is zero base constant, beta is equal to k, but in other cases there is a beta. So, that kc is, they are finite. So, you can calculate that, and it is normalized to that

(Refer Slide Time: 28:26)



So, for dominant mode these are the values k c is equal to 2 by a plus b, based on that you can find out the cut off frequency. So, cut off frequency will be this, and here we have shown that if we go on increasing the frequency. So, where, first we will see a TE 11 mode coax, then it will be a TM 0 1 mode, then it TE 2 1 mode, then here if you go on increasing further here two modes; 1 TE 1 T M. They start propagating and then if you go further your TE T E 1, then you have TM 2 1 (Refer Time: 29:10). So, this shows that which modes are propagating. So, decide on the frequency. Suppose your frequency this; that means, you know that, apart from TM you will also have this all three modes present. You need not to worry about this, because you are here. If you are here then you need not to worry about these two, but you need to worry about this.

(Refer Slide Time: 29:36)



So, this is the TEM field lines, you see the electric fields are this solid lines. So, as we have seen that they are radically. So, this is the inner conductor, this is the outer conductor. You see from inner conductor to outer conductor everywhere it is the radically outward directed, and the magnetic field is, the circular lines so; that means, at this point what is the magnetic field. You need to take a tangent to that, because these are actually lines, steep. Sorry they are called field lines. Now tangential to that at any point gives you the tangents direction is the field direction. So; that means, this magnetic field is azimuthally directed. Whereas, you see the electric field line of a TE 11 mode in the same coax, it is not exactly radial.

So, it has some parabolic shape you see at the two points; otherwise also it is somewhere it also has a curvature at all the point. Whereas, magnetic field (Refer Time: 30:49) same. So, that was a coaxial line we have. You know coax from lot of analysis in the transmission line theory, you have come across. We have seen here from the field theory view point also, the same field structure etcetera. We have seen that this field structure you have earlier in your transmission line class is derived, that this is the electric field and magnetic field, but from voltage current concert here also, we came there and see that the electric field line are like this etcetera. So, this is we have demonstrated that TM mode theory whatever we have developed, that it will apply you can calculate the fields, as well as various parameters like characteristic impedance etcetera.

Thank you.