Basic Building Blocks of Microwave Engineering
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Lecture - 04
Mathematical Model of TE and TM Mode and Impedance Concept

Welcome to this fourth lecture Mathematical Model of TE and TM Mode the two remaining and also we will discuss the Impedance Concept in this one. So, already you have seen the model for TEM mode, now we already have seen the TE and TM conditions, now we will just put that and in TE and TM, it is much easier because if you remember that equation 15, it was an expression of all four transverse fields in terms of longitudinal field. TM wave there was a problem that since everyone was becoming 0 by 0 and k was also kc was also becoming zero, but here there is no such problem. So, transverse electric field let us see that we know the condition Ez is zero and Hz is not zero.

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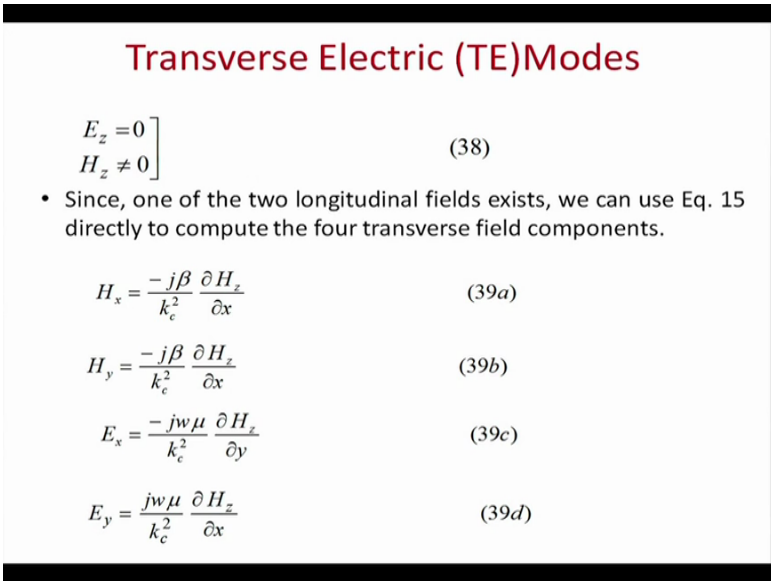

Since, one of the two longitudinal field exist, we can use equation 15 directly just unlike TM case which created some problem by becoming 0 by 0. So, you can find out what is Hx, what is Hy, Hx in terms of Hz, Hy in terms of Hz, Ex in terms of Hz, Ey in terms of h z. So, in three cases if you can find Hz you can find all of these and Hz from Helmholtz equation because Helmholtz equation of a waved equation that is same.

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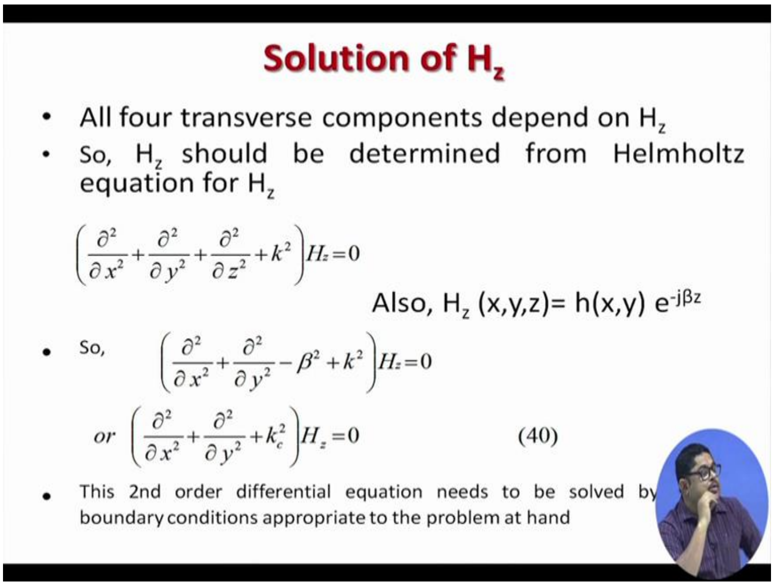

All four transverse components depend on z. So, Hz should be determined from Helmholtz equation for Hz. Helmholtz equation for Hz is this, now also we know that Hz can be written as this transverse component is a magnetic field transverse component into we know the z variation where are getting a z direction. So, a to the power minus j beta z and by putting that these becomes, this is c k c square and this is a second order differential equation and by applying proper boundary condition that will be required.

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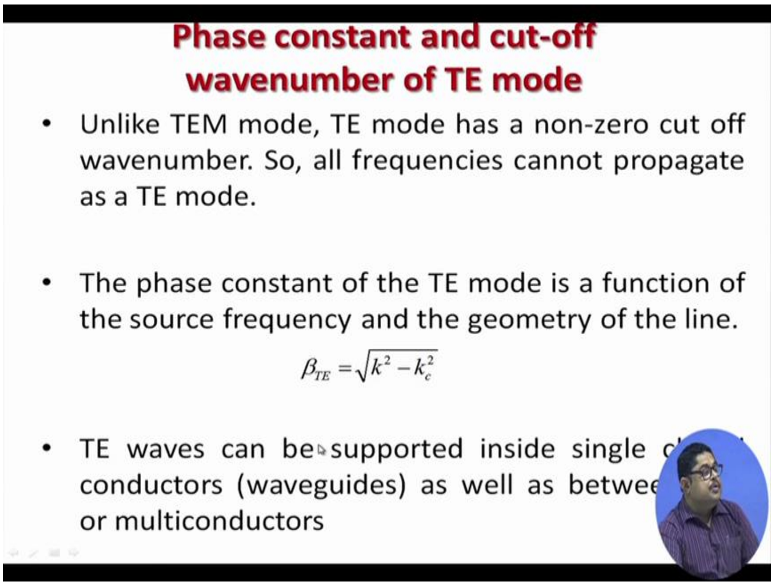

You can always solve for this Hz once you know Hz, you know all the four terms and the other things, phase constant and cutoff wave number those two the beta and the kc. So, unlike TEM mode, TE mode has a non-zero cutoff wave number. So, all frequencies cannot propagate as a TE mode. It has some finite value and phase constant is a function of the source frequency and the geometry of the line because this k c that depends on the geometry of the line, k depends on the k is what k is a constant omega square mu epsilon. So, it depends upon the material and frequency and TE waves can be supported inside single closed conductors –waveguides; no problem as well as between two or multi conductors and in a coaxial line in a multi conductor line, TE modes can always exist, but above a certain frequency that you can always find out from the given case that from which frequency you will start having TE modes, but it is supportive.

So, you see this was all about TE mode just to if I go back, you put that condition in that you find all the page then Hz is written as a second order differential equation and you have the structure that you are having, put the appropriate boundary condition on Hz and get like if you have in a wave guide.

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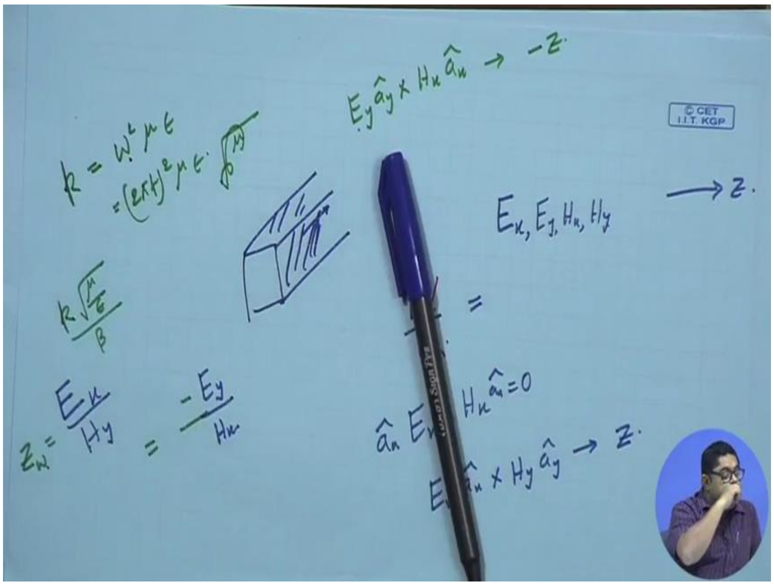

So, what is Hz? I know the boundary condition here and I know it is a metallic thing. So, Hz will be tangential to it. On this phase it will be 0. Similarly, on the bottom phase also it will be zero, now here it is a normal component on this transverse plain, now we will have to see that here suppose in this wall, this is also a metallic wall and Hz is again a transverse field here. So, that will be also zero.

So, enforce this that will give you the Hz values. Depending on the problem, you will have to do that and then you can always find this beta TE and kc etc. Beta TE you can find as we have already shown.

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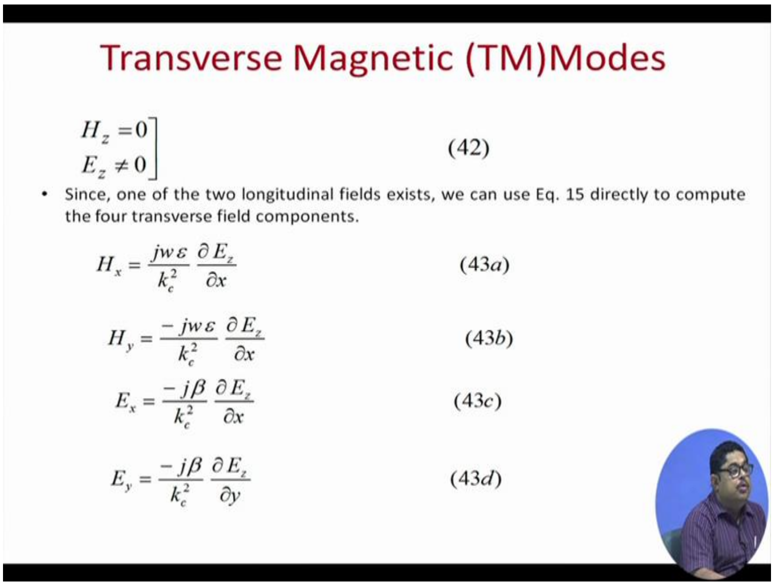

Now, similarly is transverse magnetic modes, you have the condition here Hz is 0, Ez is not 0. So, again put it into equation 15, then you get 43 equation 43 that all Hx Hy Ex Ey in terms of Ez. Solve Helmholtz equation for Ez and it is again once you solve Ez, you can find out the beta mode again, it has a cutoff number and phase constant. So, TM waves also can be supported inside single close conductors' waveguide as well as between two or more multi conductors.

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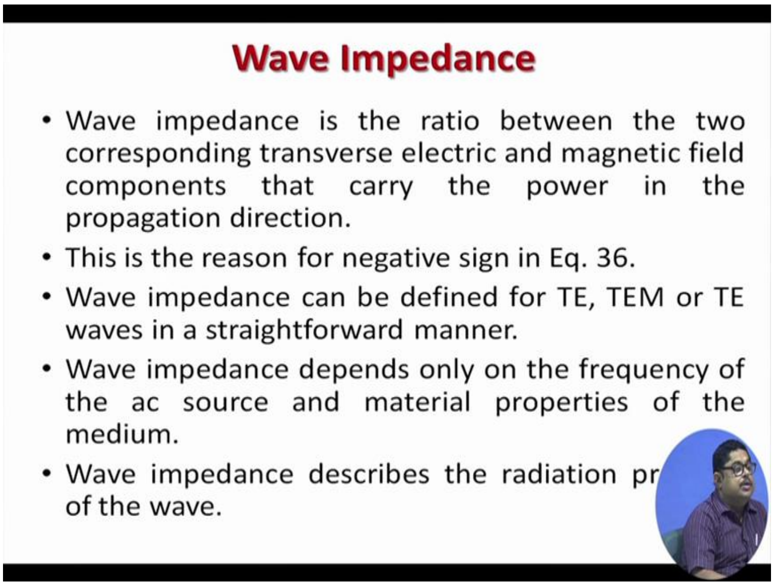

So, in all structures you can have this, but above the certain frequency given by the cutoff frequency. Now, let us come to the concept of wave impedance not characteristic impedance. Wave impedance is a characteristic of the wave which describes the radiation property of the wave, it is the ratio between the two corresponding transverse electric and magnetic field components that carry the power in the propagation direction, you see suppose the wave is propagating in z direction and let us say that I have all these Ex, Ey, Hx, Hy components.

Now, you see here the important part is this corresponding is important, ratio between two transverse electric and magnetic field components. Now, if you say that I will take Ex by Hx, Is that wave impedance? No, this is not wave impedance because it is violating this corresponding electric, transverse electric and magnetic field components that carry the power in the propagation direction. Now, Ex Hx, these two components are not corresponding components and they are not carrying power because Ex cross Hx will be 0. So, it is not carrying any power, Ex means ax direction, it is also ax direction, ax cross ax is zero, but you see Ex cross Hy which is creating power in z direction and they are corresponding, that is why wave impedance will be Ex by Hy.

Now, what are the other ones Ey by Hx. They are carrying power, but I need to carry the power in the z direction now what will happen to Ey ay cross Hx ax, this will carry power in minus z direction, but definition says that carry the power in the propagation direction which is plus here. So, in this case we put a minus sign here. So, wave impedance is written like Zw, Zw is Ex Hy or minus Ey by Hx, there is no other possibility.

So, these definition corresponding that it carries power in this and this is the reason for the negative sign in equation 36 which we have defined where the reason is this that wave impedance carries power, it shows that ratio of the electric field to magnetic field. Wave impedance can be defined for TE, TEM or TE waves in a straightforward manner, we have seen how to find the fields in any particular case if you find out the fields, you find Ex component, you find the Hy component and take the ratio you also take Ey component and Hx component, then put a minus take the ratio that gives wave impedance. So, in a TEM case, already we have seen for a TEM case we have Ex and Hy, you take that ratio that is their wave impedance and wave impedance depends only on the frequency of the ac source and material properties of the medium. Wave impedance depends obviously on the frequency.

So, the change in frequency to 1 mega hertz to 100 gigahertz, wave impedance will definitely change because you know impedance is a frequency dependent term, but it depends on the material properties omega mu sigma etc, but it does not depend on the structure where it is propagating. It describes the radiation property of the wave. Physically, you see for plane waves, the wave impedance is always 377 ohm, now the physical meaning of wave impedance is the strength when a wave is propagating then what is the strength of relative strength of electric field and magnetic field and we say that with wave impedance is higher than 377 ohm, then it is a high impedance electric field and high impedance electric field means electric field is much stronger than magnetic field.

A high impedance field if I want to measure then generally we use an E field probe where as when wave impedance is less than 377 ohm, then we say it is a low impedance field and generally we use a magnetic field sensor to measure that field and if I want measure a field in measurements, I need not measure both the electric and magnetic field because I know when they are propagating in free space they are having the wave impedance, but wave impedance tells us that which one is predominant E field or H field, whose strength is more. So, we try to measure that and wave impedance is a very important property of this, it does not get affected by who is carrying that etc, only the material and frequency of the source that governs up and it describes the radiation property of the wave.

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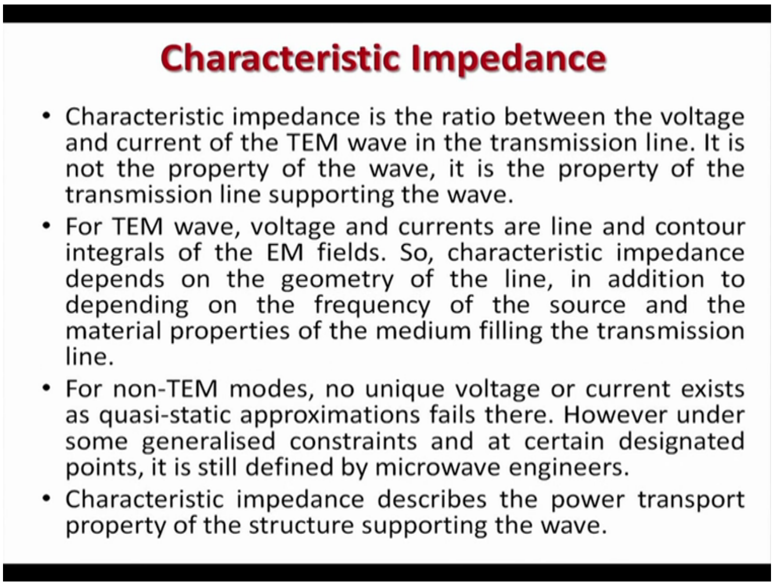

In the applications as I said that if I want to measure etc sensing applications, wave impedance is the important thing. Now, characteristic impedance I have already said earlier that characteristic impedance, what is the concept from two port network that is basically impedance so that, both sides you see impedance match. For non-symmetric network, it is image impedance and for symmetric network, it is characteristic impedance. So, if you terminate by characteristic impedance you see that the input impedance is same and that is why you are from the source to the line that power transfer is same. Similarly, if you put the source impedance to be equal to characteristic impedance of the line, then at the load side that will matched. So, the maximum power transfer will take place and it is not the property of the wave, it is the property of the transmission line supporting the way.

For a network characteristic impedance it is important, for the wave from an antenna and one (Refer Time: 15:33) of characteristic impedance. Antenna has input impedance which is a separate concept, but it is not characteristic impedance, the wave that antenna radiates, we calculate its wave impedance, but for line we say of characteristic impedance. For TEM wave, voltage and currents are as we seen for TEM wave the fields are quasi-static. So, the voltage is the line integral, the current is the contour integral of the EM fields and now we know that line integral of anything depends on the line. So, it depends on which path I am taking, contour integral also which contour I am taking.

Characteristic impedance is nothing but ratio of voltage and current, ratio of line integral to contour integral. So, it is a function of the geometry of the line that is why different structures of different characteristic impedance. We obviously depend on the frequency of the source, martial properties of the medium filling the transmission line, but apart from that it also depends on the structure. So, how much is the radius? What is the cross section? In all these characteristic impedance depends. Now, problem is characteristic impedance cannot be defined for TE and TEM mode, only for TEM mode it can be defined because it is a quasi-static field we can define it B and I. We have already seen that non-TEM modes, no voltage current exist as quasi-static approximation is not valid there the Laplace's equation is not valid.

However, in microwave engineering you will see that we always refer a voltage current description. So, we microwave engineers try to define even for TE, TEM modes also we define some amount of voltage and current, but that is besides the thing, but since I cannot say that we cannot do that and high bar 100 some generalize constants at certain and that certain designated points only, we call it ports where only we define that some voltage current, equivalent voltage current to this TE and TEM waves, but otherwise these non-TEM modes. So, the characteristic impedance is we do not define for TE and TEM modes, we define voltage current etc, but characteristic impedance in those cases also, we do not define or defined for with respect to standard things sometimes we defined arbitrarily also.

Characteristic impedance is not so important for TE or TEM type of things. Now, what characteristic impedance physical significance as I already described that it describes the power transfer property of the structure supporting the wave that means, how much power transfer you can make because if you know the characteristic impedance of the line you can choose your load impedance, source impedance properly so that, you know that whether you are able do efficient power transfer. So, characteristic impedance is very important information when you design your whole transmission and reception structure that how to choose your load impedance and how to choose your source impedance so that, whatever power you are producing you can put it to the line and also you can extract the line from the air. So, for that characteristic impedance is important.

It is part of the training of you that you should understand whether to find wave impedance or characteristic impedance, remember that whenever I have a wave which is not guided by any structure it is going, I want to describe some property of the wave of which the definition is impedance. Impedance is a very important concept you have in an earlier NPTEL lecture we have seen lot of impedance how to match impedance, what is the physical concept, but actually impedance is whole structures information, it is a ratio of electric and magnetic field of voltage and current. So, whatever may be the electric field and magnetic field because they are part of the (Refer Time: 20:32) and they can change, but the impedance is the property of the structure. Radiating structure has impedance and it would not change whatever electric field you produce, it will have that thing.

So, the network or the structure is properly comes out of impedance and impedance describes that property, your electric field and magnetic field etc can change, but the ratio gives you some insight about the property in case of transport structure, characteristic impedance give that property in case of radiating structure, wave impedance give that property, in case of a wave that how electric and magnetic field of the wave is related, wave impedance tells you in free space that whatever wave you give, but it will be some, its wave impedance will be something. So, whatever electric field you give, what will be the magnetic field? You can always calculate from this property of impedance. Since, impedance is there, it depends on the structure, it depends on material, it depends on frequency. So, all those things are known to us and whatever unknown field is there we know that if you have electric field so much, the magnetic field will have to be so much then power will be this much.

So, all these information comes in impedance it is a very important concept in electronics.

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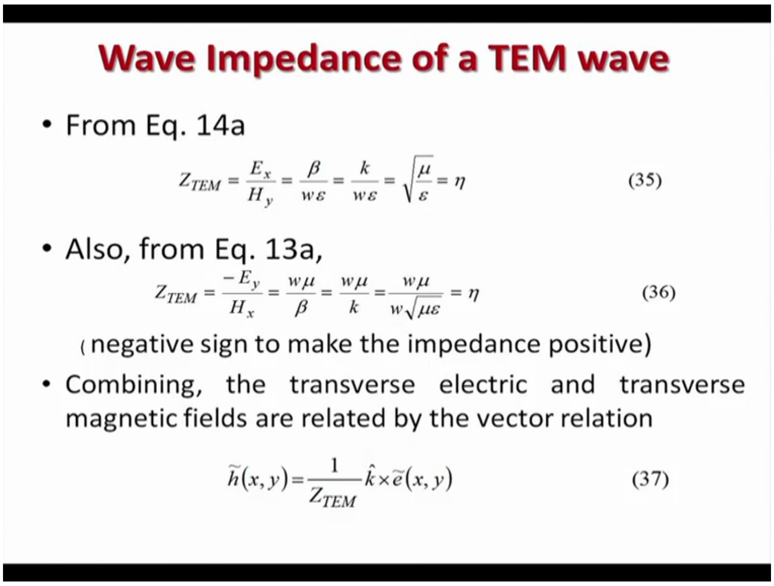

So, now let us calculate wave impedance of TEM wave and you can see that we have found out Ex and Hy, if you put that it becomes beta mu finally, you see that any TEM wave, its wave impedance is always eta, no relation to which structure is making that whether it is a plane wave that is propagating or the in coaxial ability is propagating the wave impedance is equal to only the material properties in this case, you see it is not frequency dependently. In general, wave impedance depends on frequency, but for the TEM wave, it does not depend on frequency and that is why all possible frequencies can pass their wave impedance is same which is same as intrinsic impedance.

Now, if you have a TEM line field with (Refer Time: 23:18), it will be written on that is 377 ohm and if it is filled with something direct whose direct electric constant four, it will be 377 divided by root 4 that is 2. Now, this is Ex by Hy as I explained that you can also take this ratio as minus Ey by Hx and that also is coming to be eta and we can always say that you see transverse electric component and transverse magnetic component of that field and in case of TEM, they have only the transverse component no long original component. So, they are related by this that if you know one you can find others provided you need to know this from the material structure. This whole arc is now made with suppose inside water if you radiate signal. So, water (Refer Time: 24:38) constant is 8T and you can take permeability roughly same as the free space.

So, the wave impedance inside water that will be this 377 ohm divided by root 8T, root 8T let us say 9 then 377 by 9 that will be 9 42 ohm then if I find out what is the electric field, I can tell what is the magnetic field and that is the beauty of wave impedance etc.

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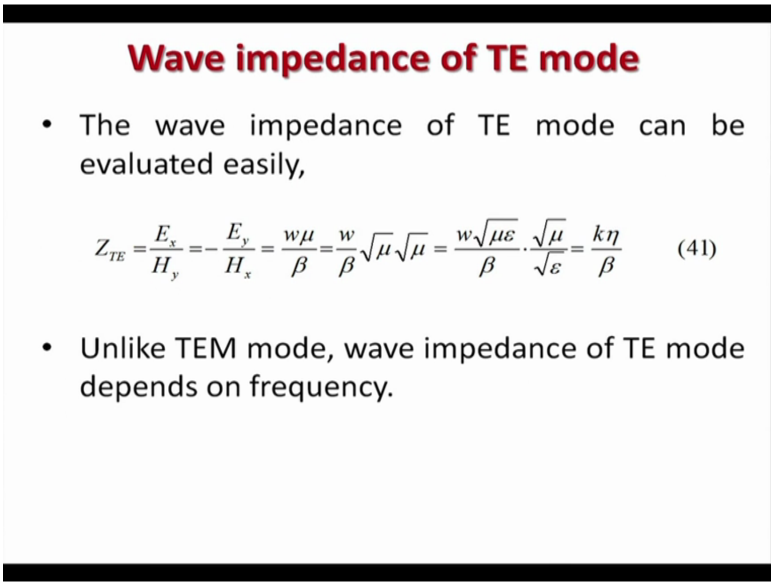

Now, wave impedance of TE mode again you have expressed so, Ex Ey, Ey I have expressed. So, if you put that it becomes something. This you see that it is a function of k eta by beta and eta was there, but you see now it is k by beta. So, k is a frequency dependent term. Frequency is coming here is unlike TEM mode, wave impedance of TE mode depends on frequency because k is what omega square, if you remember k was our propagation constant that wave number. So, k is omega square nu epsilon. So, through k this frequency is coming omega which is nothing but 2 pi f whole square mu epsilon and frequency is coming here, materials are coming here, eta is square root of this is k then k eta means mu by epsilon and then beta.

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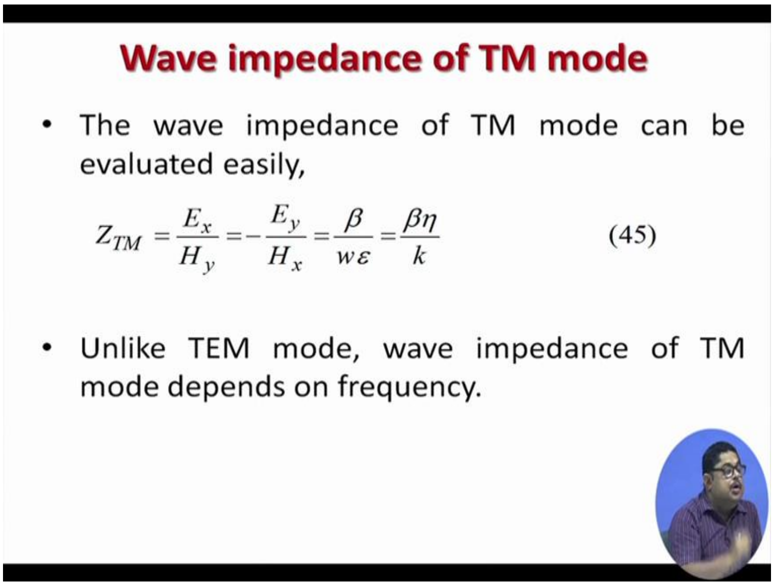

So, in TE mode this is your wave impedance of TEM mode, again two k that means, the wave number the frequency is entering and that is all the impedance that is why you should know when to find out wave impedance, when to put up of characteristic impedance and how to measure impedance, how to match impedance etc, you can see our earlier NPTEL lectures were called basic tools of microwave engineering where we have covered impedance things how to measure particularly because a microwave engineer should know how to measure impedance and you see that lecture series where basic tools sub microwave engineering where that was dealt elaborately.

So, you refer to that if you want measure impedance, if you want match impedance etc and with this we conclude this lecture and only one thing remaining that till now we assumed everything to be lossless and in microwave it is up to certain frequency of it is true, but there will be loss that means, instead of generally we have talked of conductors and dielectrics, now generally an ideal conductor it should have infinite conductivity and that is why ideal conductors are lossless, but any practical conductors, they will have some finite conductivity, nothing is infinite.

So, there will be some loss, similarly an dielectric, it should have 0 sigma, conductivity should be 0, it should not have through it any conduction of chart, but in real life it has some conductivity so, there will be some dielectric loss and that will change this whole thing because there we are assuming in the whole thing if sigma is changed because J and E are related by ohm’s law through sigma and if sigma is not the ideal then the whole structure will change. So, how to in incorporate that change or how to find out what is the loss that takes place in microwave transmission that we will see in the next lecture.

Thank you.