Basic Building Blocks of Microwave Engineering

Prof. Amitabha Bhattacharya
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture – 03
Mathematical Model of TEM mode

Welcome to the third lecture. So, today we will start this mathematical model of TEM mode.

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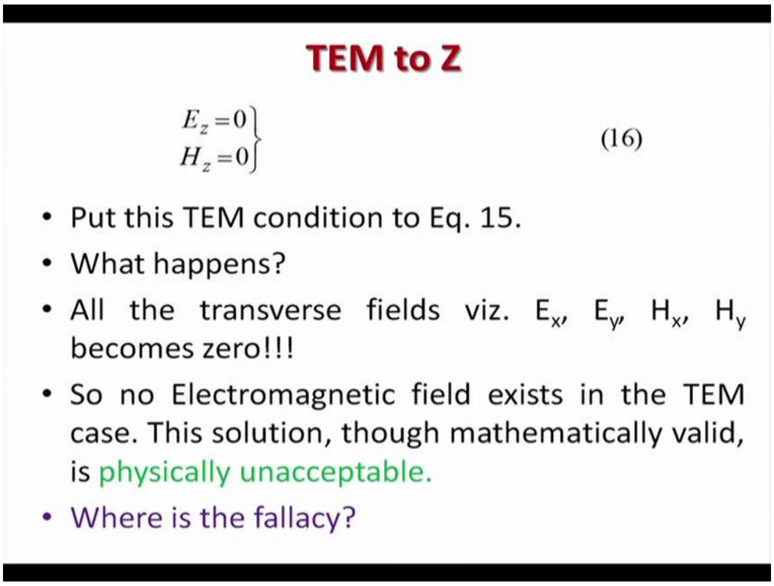

We have already expressed. In equation 15 we have already expressed all the transverse component in terms of two longitudinal component. Now from TEM to z the longitudinal component both goes to zero. Put this TEM condition to equation 15, sorry

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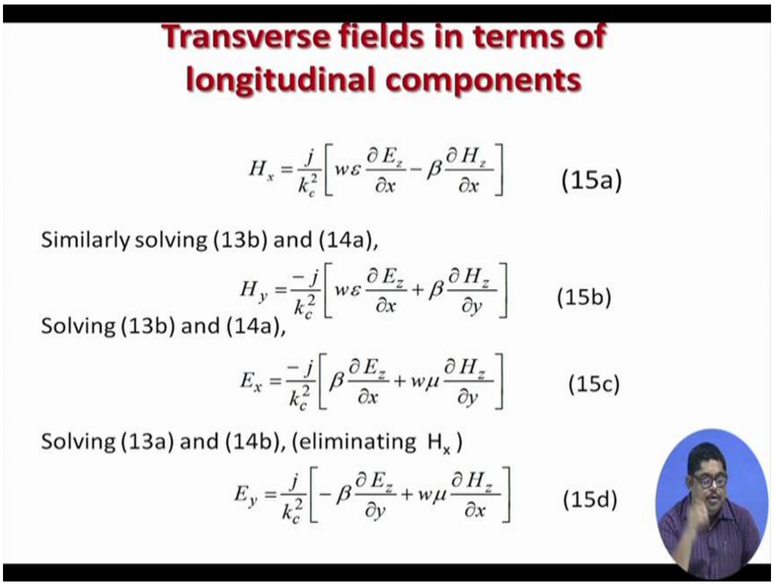

So, you see if you put e z 0 h z 0 e z 0 h z 0 e z 0 h z 0 e z 0 h z 0. So, h x h y e x e y, what happened? All the transverse fields that e x e y h x h y becomes zero. So; that means, there cannot be any field. So, no electromagnetic field exists in the TEM case. Have you made some problem? This solution, it is true that one solution to Maxwell's equation is this that all the fields are zero, but that is a trivial solution that always happens. So, mathematically it is a solution, but physically not possible; that means, TEM mode cannot exist, all mathematics says that. Let us once again see this equation. Now we have said e z h z all are zero, but what is this k c. So, h x will be zero, if this is zero and k c is not zero, but what is k c. If you remember k c is.

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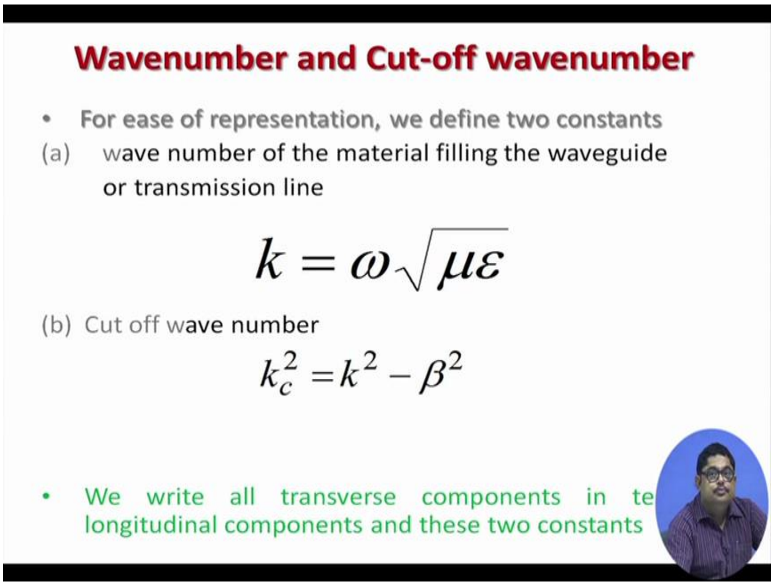

Let us see whether I have these earlier thing, this is k c, you see that k c is, it can be zero also, because k c square is equal to k square minus beta square, if k is not equal to beta, then all electromagnetic fields is zero, but if k is equal to beta. So, you see that when k c is zero then this becomes zero by zero case. So, zero by zero may have some value that you know (Refer Time: 3:45) beta and all those. So, I cannot say that h x is zero always.

If k c is zero that k c is not zero. Let us write that k c is not zero then TEM to z field does not exist, not TEM to z, but k c is equal to zero, it may exists.

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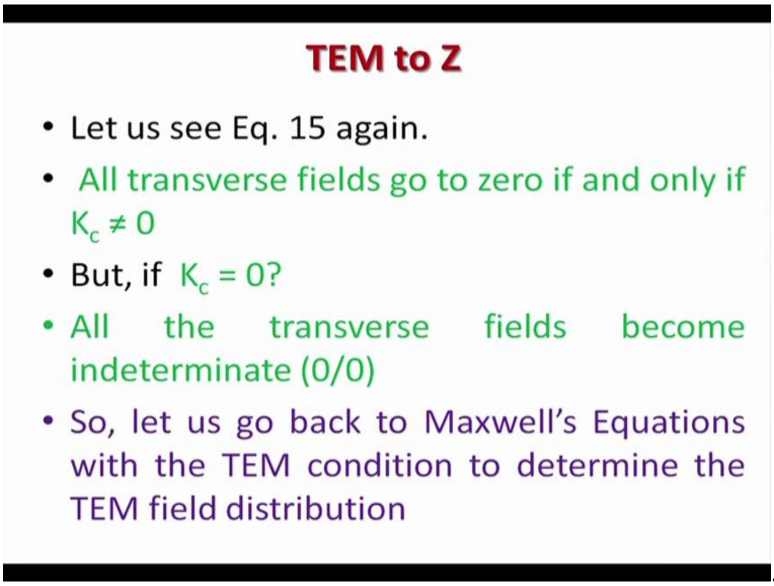

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So, let us we have to go back and see module carefully. So, that we do, since will need to go back to Maxwell's equation with the TEM condition, to determine the TEM field distribution. So, this is the first if you remember in modeling TEM z that do not falter on these, that do not say that if ratio numerator by denominator. You have learned in your school days 6, 7, but we forget many times.

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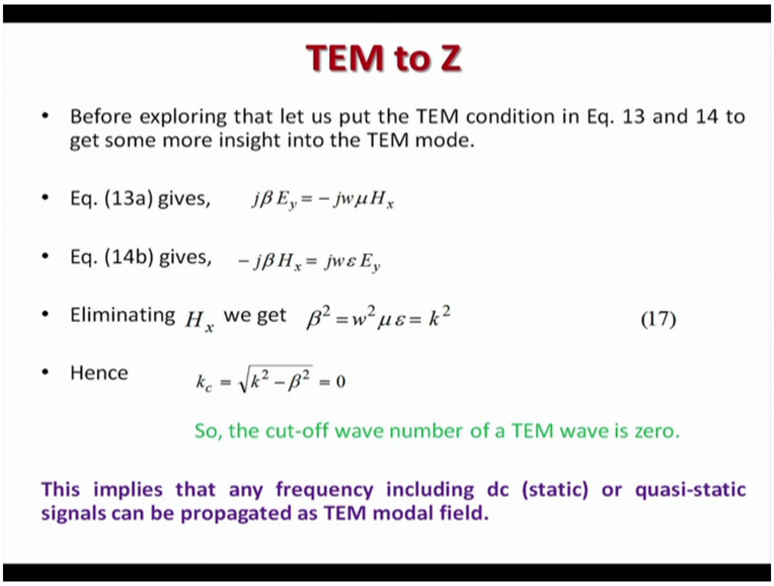

So, from equation 13 you refers to notes, we can write this for TEM mode, and this we get that, you see from Maxwell's equation it is coming that k square may be equal to beta square and in that case k c becomes zero. So, it is a possibility for TEM mode this is possibility that k c is zero; that means, what we say the cut off wave number of TEM wave is zero. Now, this is very advantageous what it says as I said while explaining the cut off wave numbers, physical significance; that it is some, it says that the cut off will happen in particular number, so that cut of wave number is the frequency is reciprocal of these.

So, what it says that all possible things; that means, starting from k c is equal to zero to k c any value, you can have TEM. This implies that frequency even a d c or very near to d c small frequency signals also, can be propagated as TEM modal fields. So, for TEM mode k c is equal to zero, k is equal to beta; that means, wave numbers is equal to the phase constant. For other modes it is not true, but this is the only mode where this is true, and that is why we need to revisit that we are getting all the transverse fields as zero by zero case. So, we will have to go to the basic equation Maxwell's equation and find that

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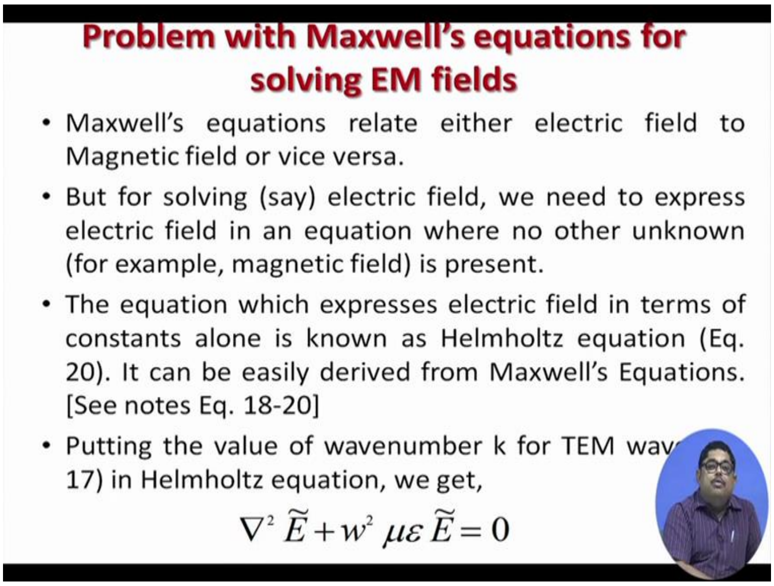

Now in Maxwell's equation there is a problem, that you see Maxwell's equation the first all delays del cross e is equal to that del b del t. second equation del cross h is equal to j plus del d del t. So, basically you see electric field is related to magnetic field, magnetic field is related to electric field, but both are unknown to me. So, from any of these equations I can solve for neither electric field nor magnetic field. So, we need to do some manipulation. So, as I said Maxwell's equation relate either electric field to magnetic field or vice versa, but for solving we need to express electric field in an equation, where no other unknown. for example, magnetic field should not be there in that equation, then only I can solve, because in an equation you can solve when one unknown is there all others are known, but if electrical magnetic field both are there, both are unknown. So, I cannot solve.

Now, who does that you all know that if we take the call of the first equation and put it into the second equation, or call of second equation and put it in the first equation of the Maxwell's equation, we get equation where either electric field is there or magnetic field is there, and that equation takes these shape del square e plus omega square, sorry plus omega square mu epsilon e is equal to zero. This is called wave equation or Helmholtz equation, because Helmholtz is a famous scientist he first gave this solution. So, you can easily derive it as I said, and also you can see our notes equation eighteen to twenty we will show you these.

So, this equation can be solved, because you see del square e plus some constant plus some constant into e is equal to zero. So, it is a second order differential equation second order, deferential equation in e. Also all wave equations like all the our h our potentials they also equal the vector potentials, they also equal like this you know that we have a magnetic vector potential, that will obey these. So, we know how to solve secondary differential equation that is why in math's we were taught that, so we will be able to show that.

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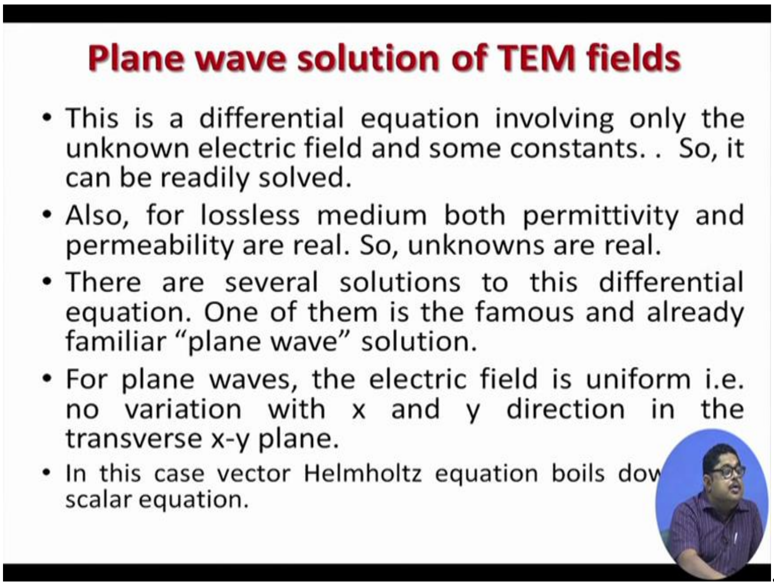

So, now also for lossless medium; that means, if we initially we will try to find this TEM fields in lossless medium, then there will be a lecture on loss how to incorporate loss etcetera. So, for lossless medium we know that lossless means basically sigma that is infinite; that means conductivity is infinite. So, in that case permittivity permeability both is real. So, this unknown omega square is definitely real quantity, and this one is a product is a real quantity. So, this whole constant is real. So, we can always solve for these.

Now there are many solutions to these; one of them is the, already we have seen familiar equation that is plane wave solution. So, that Helmholtz equation for TEM wave, this one, this as various solutions, please remember that; plane wave is one such solution. We will first see that plane wave solution, and then we will see the general solution for this. So, plane waves will satisfy these. Now what is plane waves the electric field, uniform plane waves solution. Sorry I should have said already familiar uniform plane wave solution. Uniform plane wave means electric field is uniform that is no variation with x direction and y direction in the transverse plane. So, what will happen to that vector Helmholtz? This is a vector equation these facade, but these are basically is a vector; obviously, it is a field. So, this is a vector equation, it has got three components, but fortunately in this case that will become a scalar equation in TEM case.

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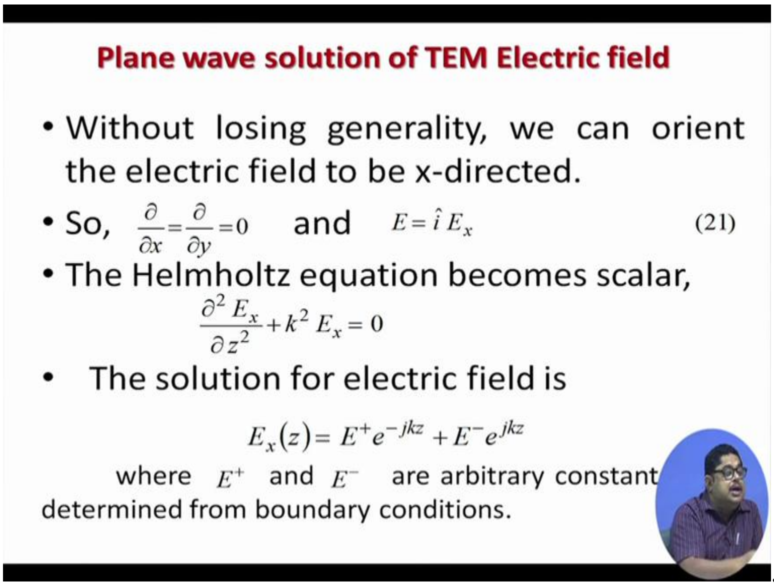

So, for uniform plane wave let us say the electric field is x-directed. So, we are writing e is equal to x directed field, and uniform that is why del del x del del y is equal to zero. As I said uniform plane wave means, in the, they does not have any x y variation it is x directed, but variation x y is not there; that means, del del x is equal to del del del x is equal to zero and del del y is equal to zero. So, put that into Helmholtz equation, this equation, because these del square means del 2 del x 2 plus del. So, if you put that it becomes simply this. So, we know how to solve this equation this Helmholtz equation, if solution will be x is equal to some constant into e to the power minus j k z, because it is del z square that is why x will be e to the power minus j k z plus e to the power plus j k z with some two constants e plus and e minus.

Now e plus and e minus we will have to determined, those are to be determined from the boundary condition. So, as we said that time we have got the phasor, this is the phasor though we have put the tilde, but this is a phasor no time. So, to put time I can come back. You see here what I have left to do, this whole thing I have left multiply with t to the power j omega t, take the whole things total multiplication that real part, that we have done and that becomes these.

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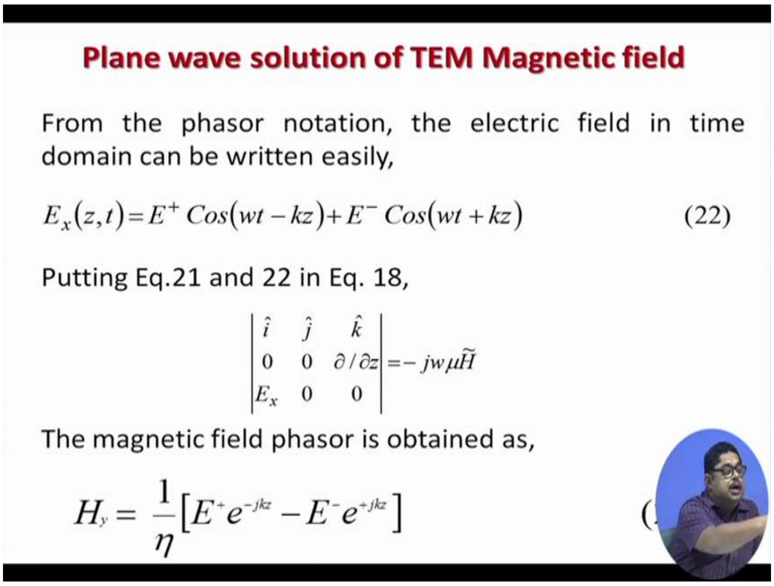

So, plane wave it is e x is this. So, I know the electric field, let us put into the magnetic field to find, magnetic field put it in to Maxwell's equation.

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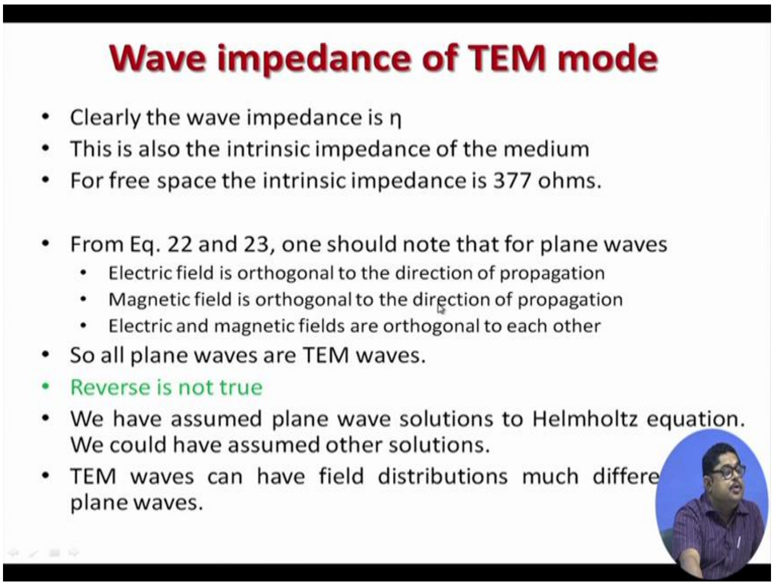

So, you put it and magnetic field becomes h y, it was x it is a e h y in this form equation twenty three, and the ratio of this two, the electric field by magnetic field that you can see from. This is the earlier one 21 equation 21 is the, sorry these equation, these equation it is a electric field phasor divided by the magnetic field phasor that will give you the wave impedance to beta. Now, we know that this is free space propagation. So, it cannot it is value is 377 ohm or 120 pi. From equation 22 and 23, you should note that for plane waves electric field is orthogonal to the direction of propagation, direction of propagation is z electric field is z.

So, orthogonal magnetic field is a h y, z is the direction of propagation, so magnetic field is orthogonal to the direction of propagation and electric, and magnetic fields are also orthogonal to each other; obviously, e x and h y. So, they are also orthogonal. So, they satisfy plane waves we have solved them. Now we see that they satisfy the criteria of TEM modes. So, all plane waves are TEM waves. We have seen how plane waves are created. We have seen if you have any infinite sheet of current you can have plane waves, or from any point source if you are far away you have plane waves. Far away means a reasonable distance away, it becomes plane waves.

So, all plane waves we know they are TEM type of waves. So, they transverse electromagnetic wave, and you have seen it is structured what is the e x h y, h y is like this e x is like this, or if you want the real time signal e x is like this, h y will be you can easily write that it will be also cos same thing on the e plus by eta e minus by eta cos omega t and. So, you know it is structure, but this is not tem, plane waves are TEM waves, but TEM waves not necessarily are plane waves, all TEM waves are not plane waves. So, reverse is not true. So, what are the other solutions? Plane wave solution was one solution; that means we have assumed uniform variation etcetera that may not be true. So, TEM waves can have field distribution much different from plane waves.

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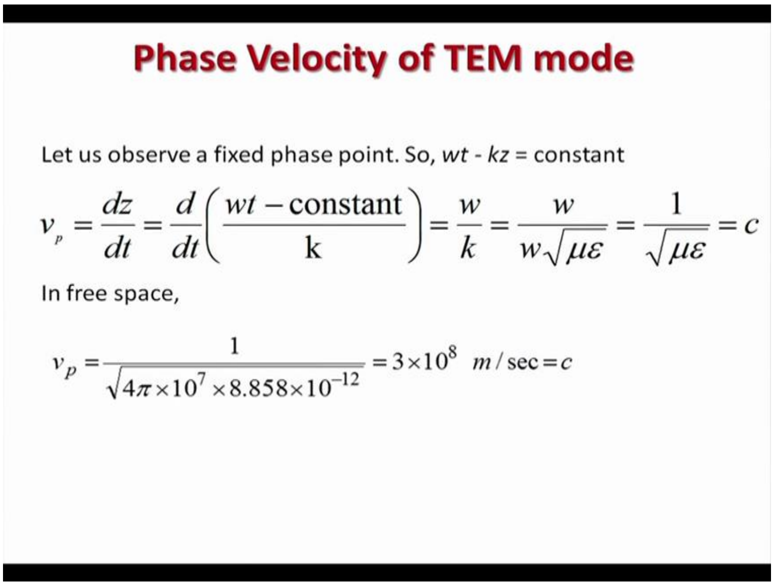

So, also let us see what is the phase velocity of the TEM mode. So, for t, you see that phase velocity. So, what is the equiphase point e to the power? So, here you see omega t minus k z is a phase, so equiphase points. So, we know let us observe the fixed phase point; that means, omega t minus k z that should be constant.

Now, you know the definition of phase velocity, phase velocity v p is d z d t. So, in place of z, what is z? Omega t minus constant by k d d t of that, so that becomes omega by k, k we have already seen, the k was a constant we defined that constant is omega into this, so omega by omega into this. So, one by this, but what is this, this 1 by a; if you put the value we have already seen the values that becomes c is the speed of light. In free space if you put those values omega epsilon it becomes 3 into 10 to the power 8 meter per second that is a speed of light. So, phase velocity of this TEM mode; that is light. So, TEM modes they propagate, they are phase velocity they propagate with light speed.

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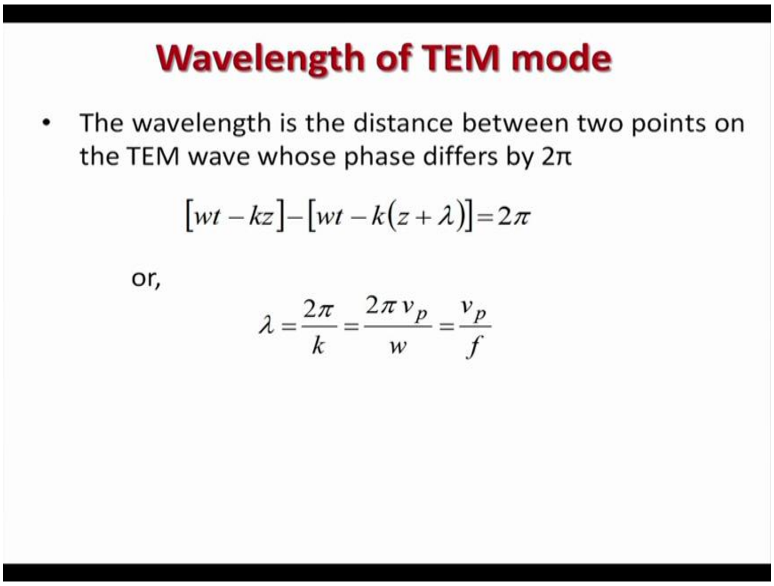

And what is a wavelength of TEM mode, you know the TEM mode any waves wavelength, distance between two points on the TEM wave which space differs by 2 pi. So, 1 is omega t minus k z minus omega t minus k z plus lambda; lambda is a wavelength, and these different should be 2 pi solved that becomes v p by f. So, phase velocity if it is in free space c into 10 to the power 8 meter per second divided by frequency you mean that becomes your lambda. So, that all you know that how to find lambda. So, if I have 1 gigahertz signal, 1 gigahertz TEM waver signal then what will be the velocity 3 into 10 to the power 8 by 10 to the power 9. So, 0.3 meter.

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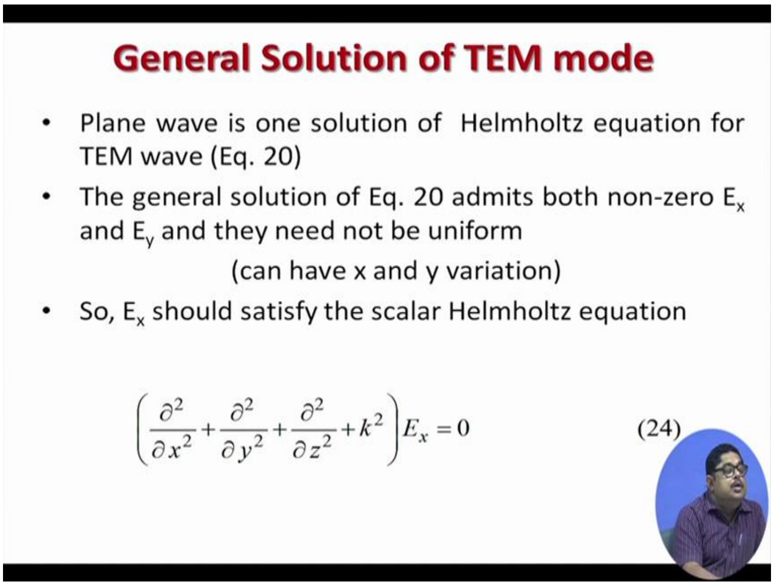

Now let us come to the general solution of TEM mode; plane wave is one solution of Helmholtz equation for TEM wave, that general solution admit's. So, uniform plane wave means we assumed uniform x, but now you can have both e x e y component and non uniform, then in non uniform can I have x and y variation. So, e x should satisfy the scalar Helmholtz equation these del del x square plus del del y square plus del del z square plus k square e x is equal to zero. So, this equation we should solve. Now we know the z variation is given by e to the power minus j beta z. So, del del z square is this. So, by that equation 25 or 4; that means this equation. Now this is 40 x also e y will have some type of equation.

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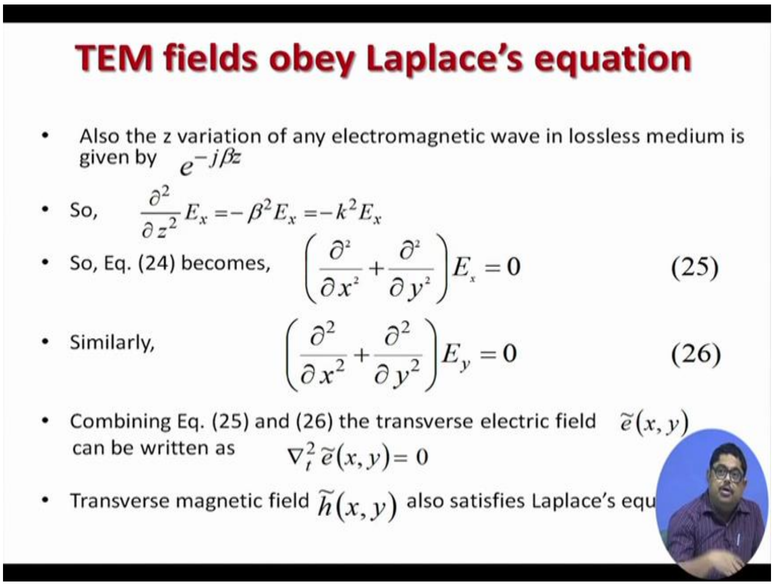

Because the general solution both e x e y possible. So, I have this where is the other part going the k square, you see del del x square that is giving me what e to the power minus j beta z. So, that is giving me minus j beta into minus j beta; that is minus beta square. So, minus beta k square minus beta square, but for TEM wave we have already proved that what is it is beta, where is that gone (Refer Time: 21:31) which light, you see that beta square is equal to k square we have proved from basic Maxwell's equation, it is always 2 per TEM wave.

So, bit minus beta square plus k square that is why that is going. So, we are getting equation 25 and 26. Now, if we combine we can write del t square e x y is equal to zero. Similarly you can do this for magnetic field. And now what is this equation del t square means, del del x square plus del del y square into this. So, this is the Laplace's equation. So, transverse TEM field structure is similar to static fields, because in electro statics we have seen that electric field and magnetic field they satisfy Laplace's equation.

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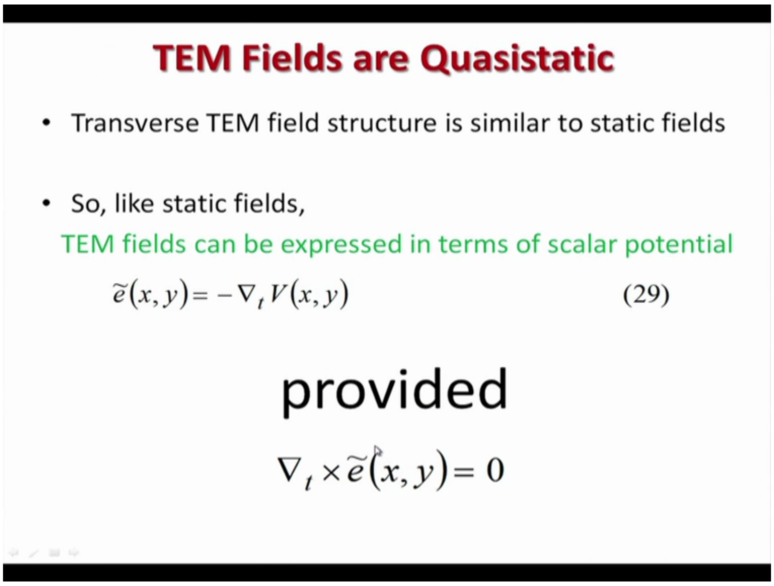

So, transverse field though they are dynamic fields, but they are field structures, because you see this is a fundamental thing that equation, a particular type of equation it is solution is also the same. So, if two equations have, two equations of two variable they have similar equations, then their solution will also be similar, these are fact of mathematics. So, if transverse electric field or transverse magnetic field they are satisfying this equation, means they are satisfying Laplace type of equation. So, and we know that electro static fields satisfy Laplace type of equation. So, solution of them will be similar, at the most they can be differed by some constant etcetera, but their solution is same. So, that is why this TEM fields are called quasi static field like static field, quasi means it is came from some foreign language I do not know, but quasi means similar, so quasi static, like static fields. So, TEM fields can be expressed like the electric field electro static field.

Now, electro static field we know that we can define as scalar potential for that. So; that means, that transverse electric field can be written as gradient of a voltage, but to do that we need to prove one thing. You see that if we write is as gradient of this transverse operator, we will have to prove that the curl given by the transverse operator of this transverse field e x y; that is zero, because that is the basis as electro static field satisfies that curl of that electro static field is zero, that is why we say it is a it is a conservative field. So, it can be written as a gradient of, electric field is written as a gradient of a potential or scalar function, but here you see we are writing it as a different operator. Please remember this is not a curl operator. I am sorry this is not a gradient operator this is transverse gradient operator, it is not three dimensional it is two dimensional.

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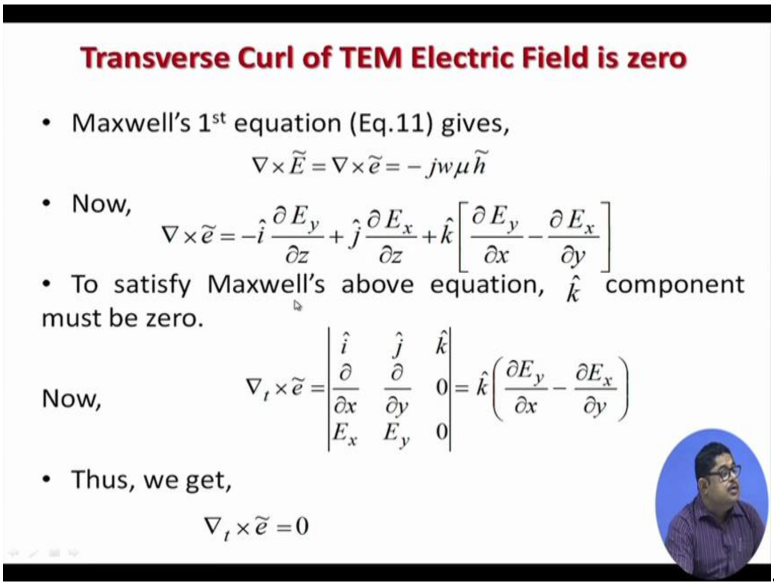

So, we will have to reprove that the curl or transverse curl I should say, transverse curl of transverse field that should be zero, then only we can say this. So, let us see go to Maxwell's equation. This is a Maxwell's first equation, this is e, this is h, this is transverse e please remember this is the three dimensional e we know that can be written as the, any three dimensional vector can be written as a transverse component plus longitudinal component. In case of TEM electric field that a longitudinal component is zero. So, that is why we can write this equal to the curl of transverse operator, and that is these, because h also is only transverse, now you explained. Now you see that this is right side is does not have any j component, left side we have written del cross this transverse electric field. So it has a k component. Now to satisfy this equation this part should go to zero, this part should go to zero. So, k component must be zero. So, we can write that.

Now, let us see what is our required one that transverse curl of transverse field. If you do the mathematics it becomes k of that. now this is zero, so put that here. So, it does not have any k component. So, we can say that del t cross e, the whole thing goes to zero, so that is also zero. So, we have proved from Maxwell's equation we have proved that transverse curl, I emphasize not curl, curl is a three dimensional operator this transverse curl operator, transverse curl of the transverse electric field of TEM wave that is zero. Once I prove this I save, I proved this, so I can write this. So, I am justified in writing 29.

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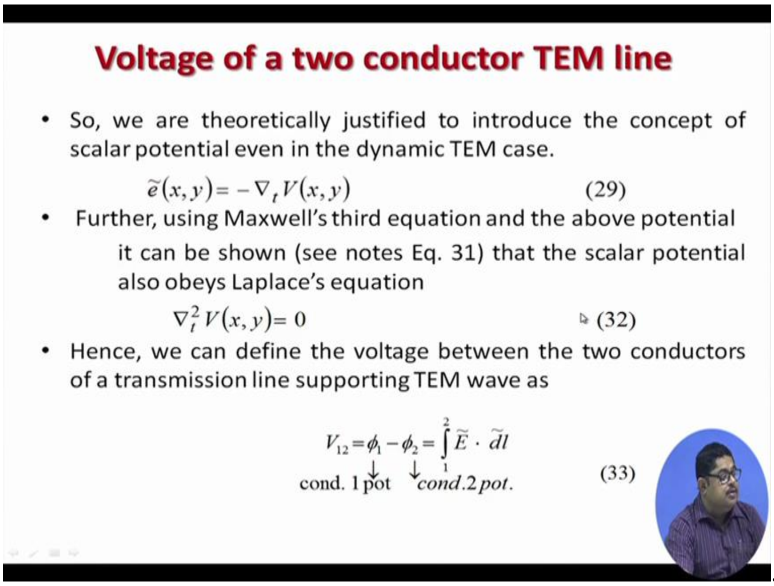

And now we say we are theoretically justified to introduce the concept of scalar potential, even in the dynamic TEM case, no one can stop us. We have proved all the mathematics. So, now, will write e x y as this equation 29 again. Further using Maxwell's third equation and the above potential, these you see notes these are mathematical details, these also obey Laplace equation hence. Once anything obeys Laplace equation we can write it as. So, if a TEM wave is propagating to a two conductor's line we can always define a voltage difference between which is nothing, but the potential of conductor one potential minus conductor two potential. So, it is equal to 1 to 2 e dot d l.

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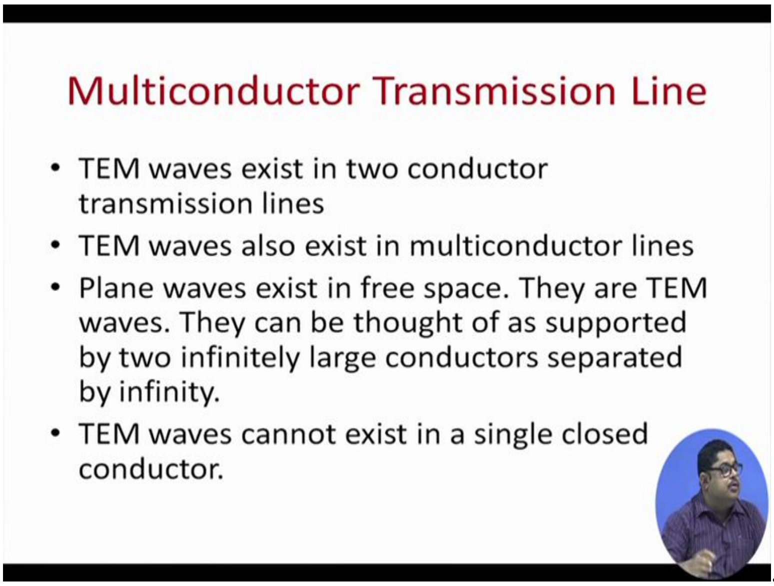

So, you can if you know the electric field we can find what is a b 1 to as a line integral from first conductor to second conductor. Now this so we know the structure. So, we have told you how to do it.

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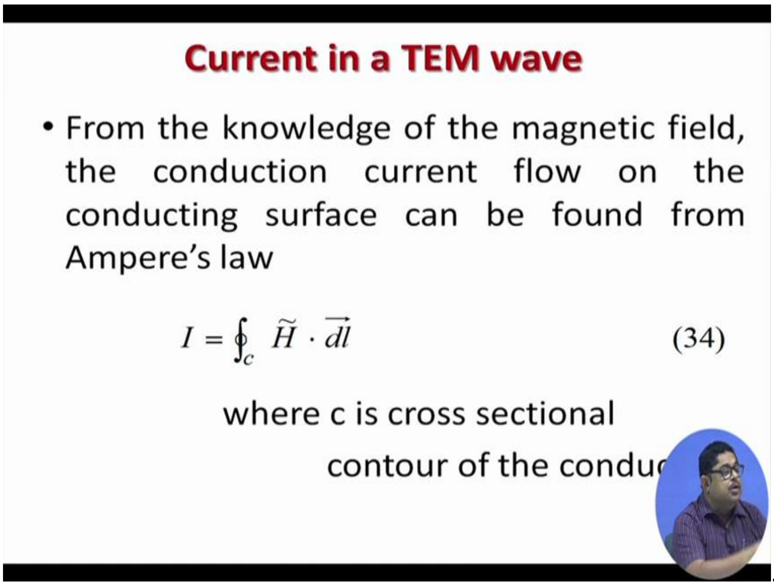

And also there is a. So, similarly we can also define a current from the magnetic field by having a contour integral, along any closed path. So that means we can define the TEM wave by voltage and current definition. Now multi-conductor transmission line, so TEM wave exists in two conductor transmission lines, TEM waves also exist in multi-conductor lines this can be proved. Plane waves exist in free space we have seen that if you are far away from any radiating source you get plane waves. They are TEM waves, they can be thought of as supported by two infinitely large conductors separated by infinity.

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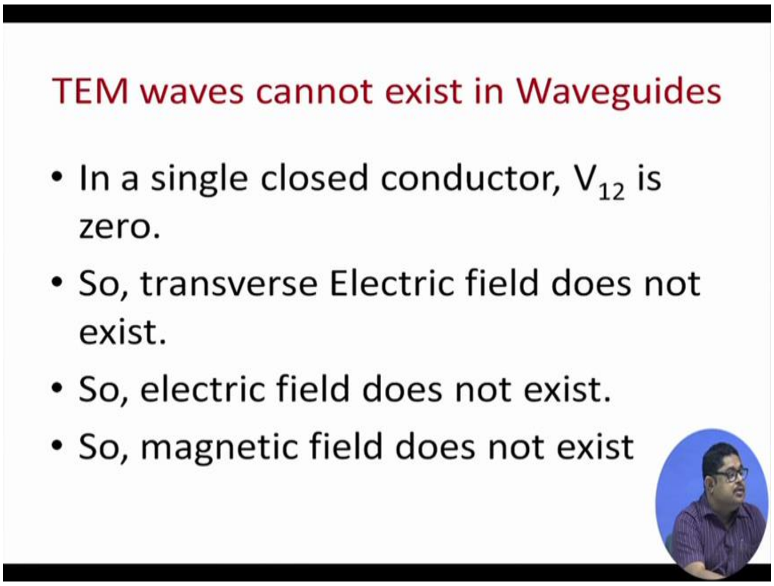

But TEM waves cannot exist in a single closed conductor. Instead of two conductors if you have a single conductors TEM wave cannot exists, why. And a single conductor example is a wave guide; you see a wave guide is a pipe.

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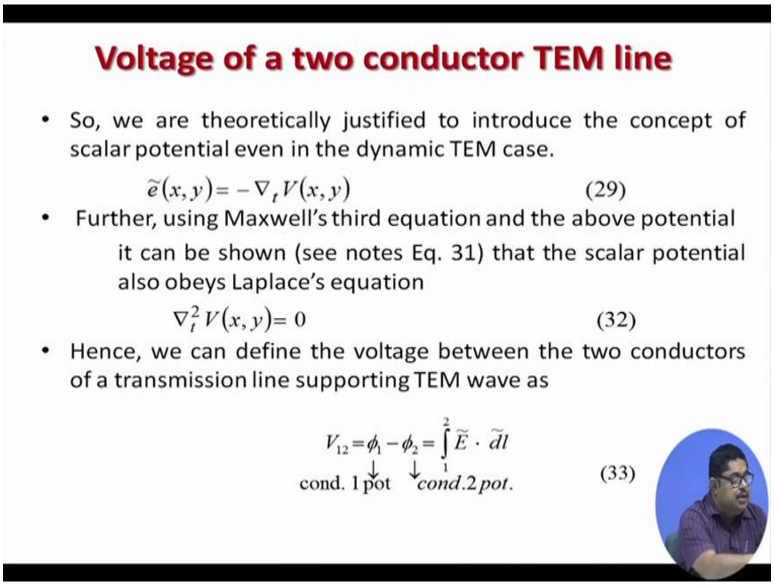

So, it is a single close conductor, why? because in the single close conductor that equation you see that if you are a single conductor; that means, your potential of one conductor and potential of two conductor they are same. So, v 1 2 is 0, the moment v 1 2 is 0, what happens, the scalar potential is zero. So, it is divert gradient that is zero; that means e becomes 0. once e is 0 h is also 0, so note TEM wave.

If voltage is zero, then you put it here gradient of that voltage that is also zero; that means electric field is zero. Once electric field is zero put it into Maxwell's equation magnetic field is zero. So, in a single conductor v 1 2 is 0. So, e; that means, transverse electric field does not exist. So, electric field is does not exist. So, magnetic field is does not exist. So, TEM waves cannot exist in waveguides, any form of waveguide. waveguide means a single conductor a metallic pipe of any cross section, it can be circular, it can be rectangular, it can be elliptical, any it is cross section is uniform it is a single pipe.

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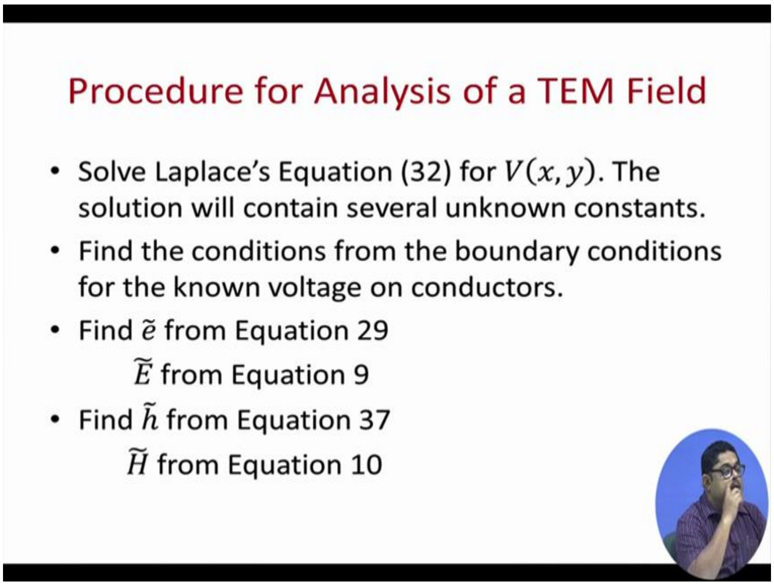

So, they are TEM waves cannot exist. Then current as I already said. So, let us summarizing the two slides what is the procedure for analysis of a TEM field? Solve Laplace equation for v x y, the solution will contains several unknown constants; find the conditions from the boundary condition for the known voltage on conductors.

Then from equation 29 find e, you know equation 29 is that one, this one. So, once you know v take that radiant that will give you the electric field transverse electric field.

Then e from equation 9, you see your notes that if you know e then we have already retained the total field in terms of transverse field by longitudinal, longitudinal is not here. So, transverse I know multiplied by e to the power minus j k z; that is a e. Similarly find h from equation 37 we have a manipulated it bit, and h from equation ten. You see your notes I will upload that. So, from there you can find e and h, and compute v from equation 33.

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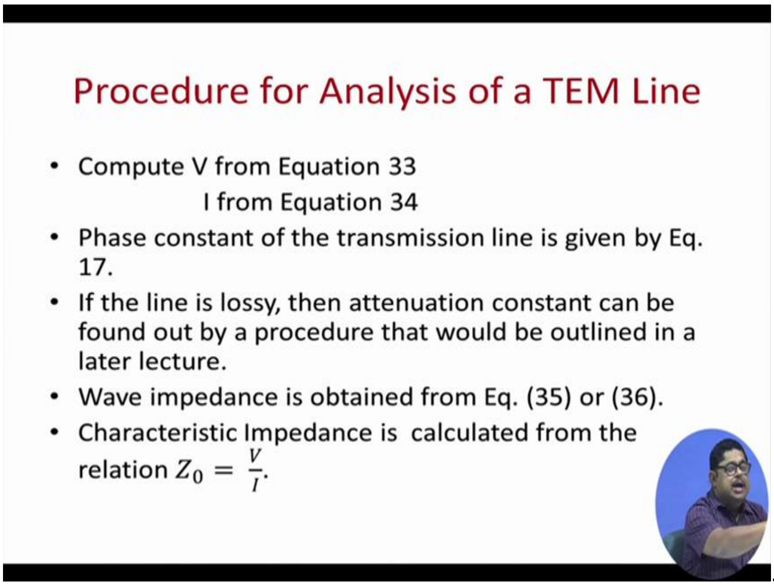

So, you can always find this, from here you can find. once you know e you can find v easily, and I from equation thirty four, then find phase constant of the transmission line is given by equation seventeen let us see quickly equation seventeen. So, you phase constant is coming, once you know k, which is the frequency and material constant so from that you can find beta that is a phase constant. So, phase constant of the transmission line is given by equation seventeen, so that is all.

Now we are assuming lossless, if lossy things we will seen in a separate lecture how to find this all these things and we have seen the impedance is equation 35 or 36 let us or that way come later. Impedance also we will discuss in another lecture and characteristics impedance of any TEM line. Please remember this characteristic impedance is, you have found v you have found a I take the ratio that is characteristics impedance. Now why it is called characteristic impedance, basically characteristics impedance is defined for a symmetric through put network. Any symmetric network, what is characteristics impedance, that actually any non symmetric, also any general through put network that as image impedances. What is image impedance, that suppose I have a source, I have a source resistant? Now I have some load, so that.

So, from the source site the input impedance is matched with the source. Now, I reverse that I put the image of that impedance, it is a pair of impedance that in the source site, and from the output site I see that the, what the output impedance; that is same as the load impedance. Now symmetric through put network this pair of image impedance they are equal and that is characteristics impedance. So, what happens to characteristics impedance; that means, if I terminate any to port network with characteristics impedance, then the input impedance seen at the input port is same as characteristics impedance, and that value you can calculate for TEM wave you know v you know i. So, you can calculate z naught characteristic value. Usually a TEM line is specified in terms of it is characteristic impedance.

Now we will discuss characteristic impedance and other impedances in the separate lecture. So, I think we have computed this, how to analyze, what is the mathematical model is now complete that you know. So, start from Laplace's equation solve for this, find out e, find out h, then you find out v and I you find out beta that is all, and also you can find out the wave impedance that will see later and characteristic impedance. So, that completes the analysis of TEM mode.