Basic Building Blocks of Microwave Engineering
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Lecture – 02
Mathematical Model of Modes

Welcome to the second lecture Mathematical Model of Modes. So, as I said that we have an engineering model and we need to understand basic modes that are TEM TE TM. Now, we need to develop a mathematical model of those three modes and then only we will be able to go further in the analysis of this whole transmission process. So, let us start with the mathematical model first.

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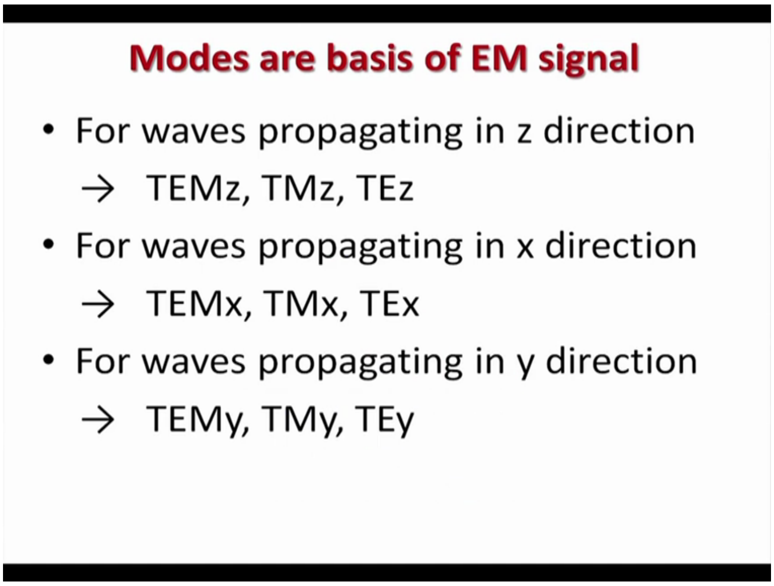

Modes are basis of EM signal and I have already said that all EM signals are basis set of the modes. So, if we have waves propagating in z direction the basis set we call TEM to z, TM to z, TE to z.

Similarly, if the wave is propagating in a x direction, we call this basic set TEMx, TMx, TEx and similarly for y direction TEMy, TMy, TEy. Now, simply if we analyze any of these, suppose generally we all make that the direction of propagation is z.

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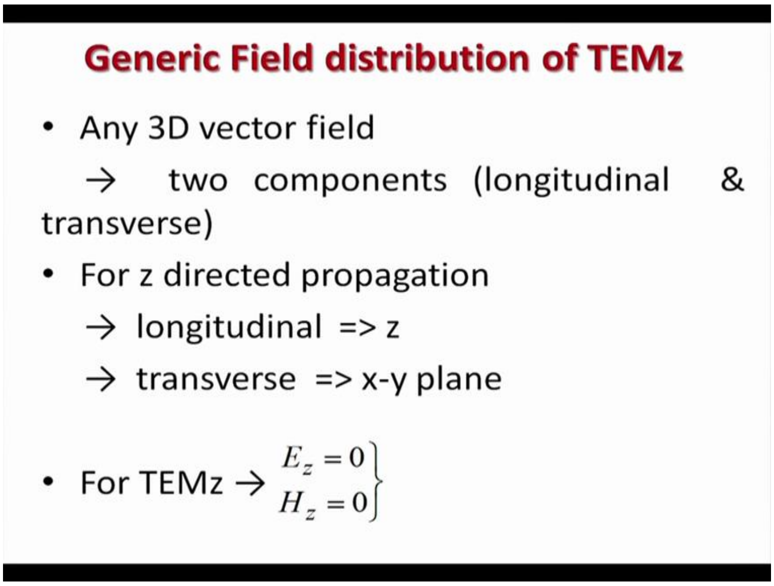

So, that is why we will make the model only for TEM, TM and TEz models if the propagation derives in others you can easily change over to this nomenclature. Now, the generic field distribution of TEMz is a 3D vector field, you know the difference between vectors and field, the vector is directed across theta z direction and also it has an magnitude and the field means it also has a special variation.

So, we can break any 3D vector field into two components, one is along the line of propagation because you see our mode definition that time we have distinguished that what is the direction of propagation and always we are talking in terms of direction of propagation. So, that we call longitudinal direction and the wave is propagating in z direction means the longitudinal direction will be z and to that we have a plane that is called transverse plane. For z directed propagation, we have transverse plane will be x-y plane, longitudinal is z. Similarly, if we have x directed propagation and then longitudinal means x direction and transverse means y z direction like that. So, for TEMz, TEM to z we should have both the electric field and magnetic field which are transverse that means, they do not have any longitudinal component and definition of TEMz is nothing but Ez is equal to 0, the electric field does not have any longitudinal component.

So, that is why Ez is equal to 0 and magnetic field does not have any longitudinal component Hz is equal to 0 and definition of TEM to z is mathematical definition. The boundary condition you can say or the condition for being a TEMz where is Ez is equal to 0 and Hz is equal to 0.

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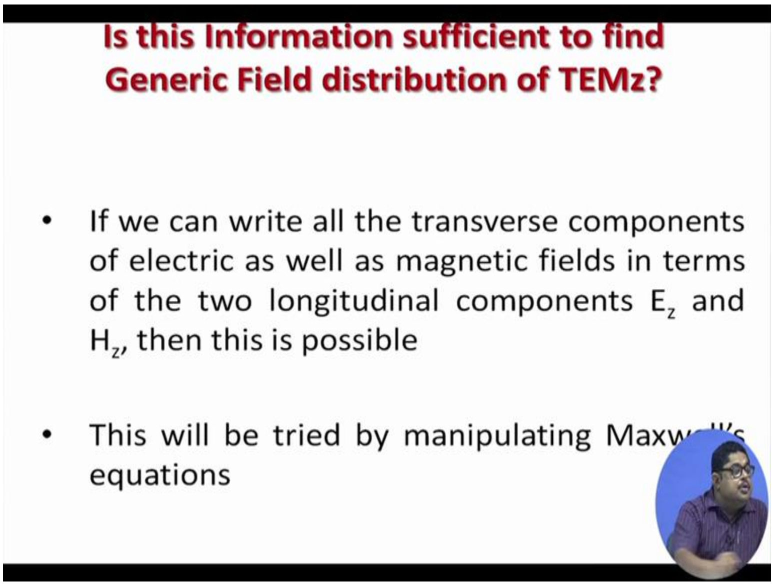

Now, is this information sufficient to find generic field distribution of TEM z? The answer is yes if we can write all the transverse components of electric as well as magnetic fields in terms of the two longitudinal components Ez and Hz, then this is possible because by being TEMz, we know the values of Ez and Hz.

So, if we can write all other field components that means, Ex Ey and Hx Hy in terms of Ez Hz, then we can solve. So, this will be tried and obviously we will have to start from Maxwell’s equation because all solution and all electromagnetic fields are solutions of Maxwell’s equation. We will have to try to manipulate Maxwell’s equation so that, we can write the Ex Ey Hx Hy in terms of Ez and Hz. We will see the definition of Te wave later.

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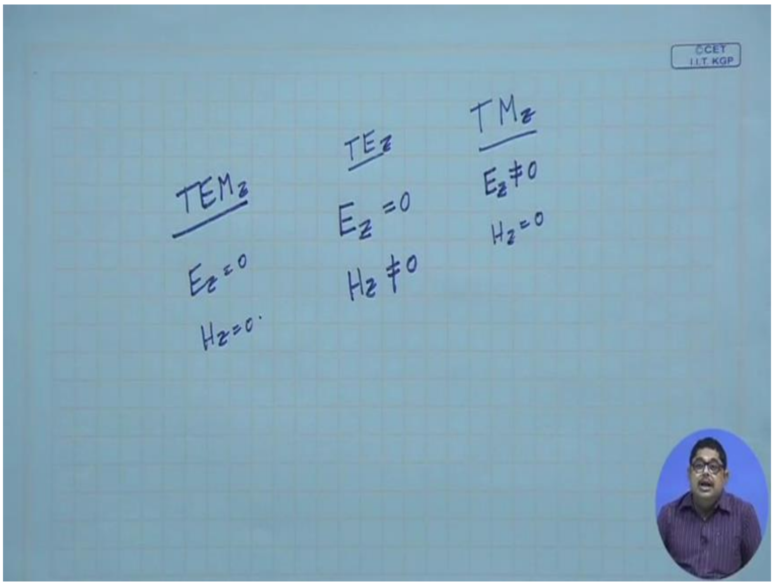

But now I am trying introducing Te wave, it means that the electric field is transverse. Electric field transverse means it will have Ez is equal to 0 and it does not have magnetic field transverse.

So, magnetic field can have Hz. So, Ez is equal to 0 and Hz not equal to 0, because if Hz is equal to 0 it will again become TEM wave. So, this is the definition of TE. Now, the definition of TM, TM will be that Ez not equal to 0 and Hz equal to 0, also let me write TEM here. So, TEM is Ez is equal to 0, Hz is equal to 0 and actually these are all z. In all these cases, basically I know the condition on Ez and Hz and if I can do this manipulation and write all transverse fields in terms of longitudinal fields that will help me to find out the fields for all these three cases. So, that we will spend some time here in this lecture probably we will totally take this time to manipulate Maxwell's equation and to write the transverse components.

Obviously, there will be some mathematics, but you know without mathematics you cannot learn in engineering. So, if you are not getting any trouble in this mathematics, we will be posting you some of the written version of this doc files for these things and you can brush up your knowledge if not, you can always discuss in the forum, we will try to show you how to do that and these are all simple and are taken from mainly David

Pozer’s book on microwave engineering, it is a very famous book. So, still if you have any problems understanding, you come to the forum we will discuss that.

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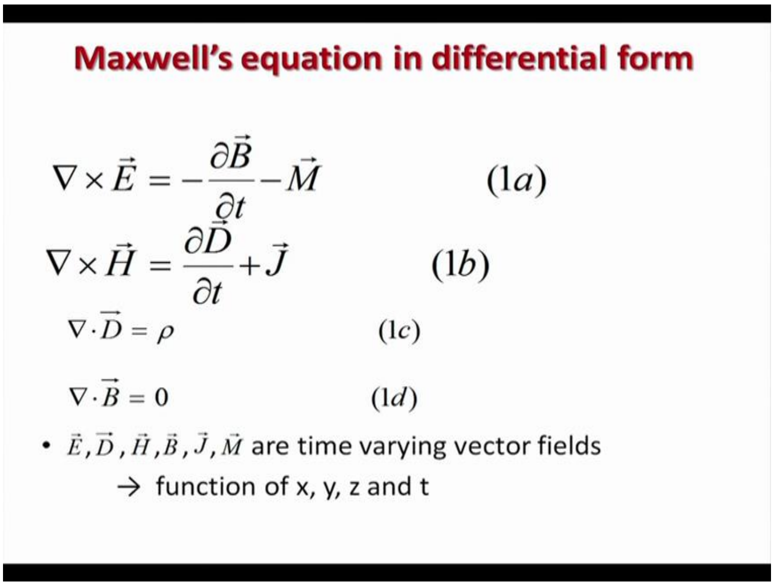

So, first let us see the Maxwell’s equation in differential form and the Maxwell’s equations are in both forms either differential form or in the integral forms, but here we are starting because we are a source. So, source means that we will have to start from points and that show in the differential form you know all this. Here, one thing in the first equation you see we have a magnetic current. Now, no one has still detected magnetic current, but if we do not put it actually if there is any aperture type of source that means there is some conductor then there is an aperture then that generates electric fields and so that, we equivalently defined a magnetic current.

Here, you see that in this one, you see there are the field quantities at any point we will get these fields and these are also field quantities but these are the impress sources that means, the source comes here explicitly. Now, in this one the conduction current source, although we know that when there is a really a time varying charge that is flowing then we have a conduction current and Maxwell shows that there is a magnetic current also that means as I said the dou the charge uncharged flow, they are the till now detected only at, but unless and until we do this we do not get a symmetry. So, we have M which is a fictitious current, but it can be actually represented in terms of electric fields.

So, we have that and write this and these are the Maxwell's equation and E D H B J M molar time varying vector fields which are all functions of all the special coordinates in rectangular coordinates we are saying this x, y, z and t, t is the time without time varying we do not get electric magnetic signal.

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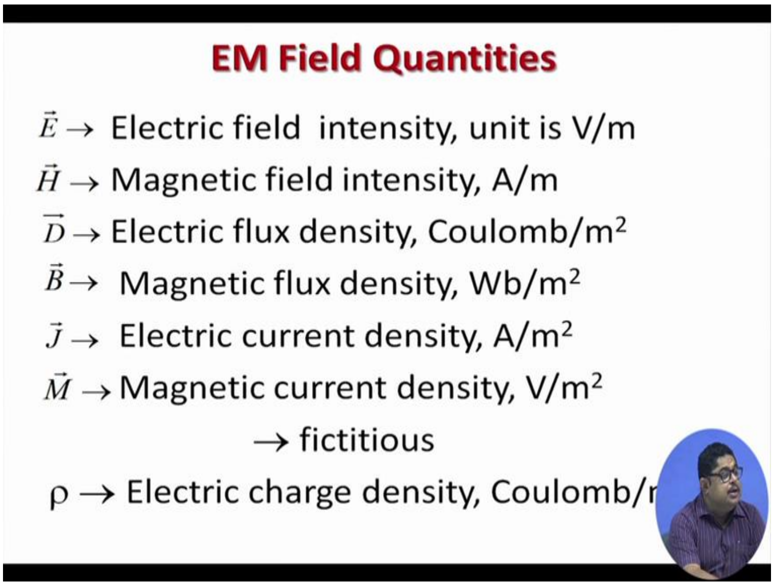

Others are plane things, now here we have listed all the definite physical quantities and there units. So, that later there would not be any confusion because in the earlier books these are definitions of Pozar, but in earlier books there where many times the H and B etc their interchangeably was used and this notations now almost in scientific community, this is the standard notation by which you expressed this magnetic current density which is volts per meter square.

As the unit suggest you know that in electric J conduction current this is electric current density, this conduction electric current density and there is a magnetic current density as I said is example created by any aperture antenna like a horn antenna. So, in that horn antenna there is on the aperture, that means on the horn mount there is a electric field which we can equivalently define and magnetic current for that and that we put in Maxwell's equation so that, sources become symmetric otherwise the magnetic current is not there. So, on the electric current is there which does not makes the things symmetric, also you know that Electromagnetic wave apart from Maxwell's equation if you want to solve the electromagnetic wave because electromagnetic wave propagates to medium.

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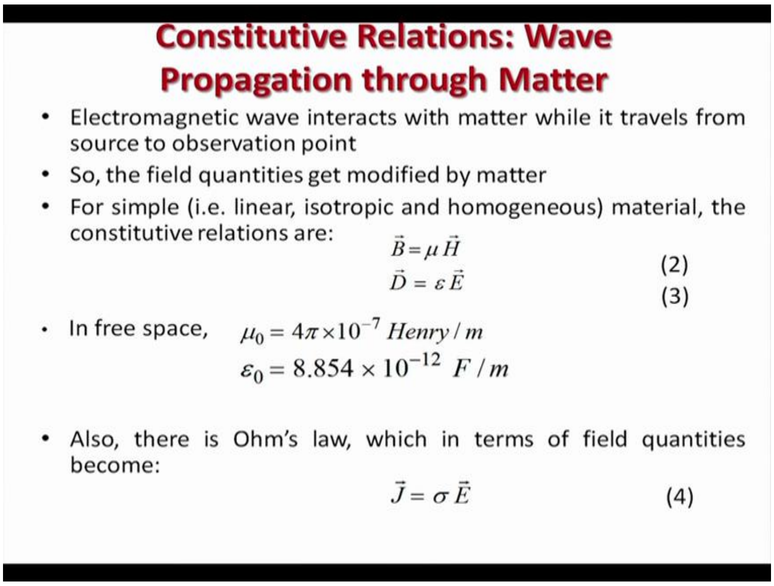

So, it interacts with matter and matter all material properties also should be factored into field quantities which get modified by matter. For simple material, simple means in M theory you learn that simple material means it should be linear isotropic and homogeneous. The relation between the two quantities are shown in or the four quantities that are shown in the Maxwell's equation. Here we have E, here we have D, similarly here we have H here we have B. So, we need to relate them when we will reduce them to one equation. So, that relation is called constitutive relation, B is related to H by the material property permeability. The electric flux density D is related to electric fields by permeability. Now, these are materials property which really materials get characterized by this epsilon mu and also another one which is coming here the conductivity.

So, these three are any electrical characterization of any material and you can distinguish between two materials by finding these values. Now in free space that means, you know the concept of free space which is a standard material. So, there this mu naught that value you know, but you should always remember this value and it is a unit and similarly the free space permeability epsilon naught that is 8.854 etc 10 to the power minus 12 Fahrenheit per meter. Also, you require ohms law and that means the relation between conduction current density and the electric field that is J is equal to this. So, this you see that wave propagation to matter you need to know mu epsilon and sigma. This J will tell you that how matter are interacting with the electromagnetic wave.

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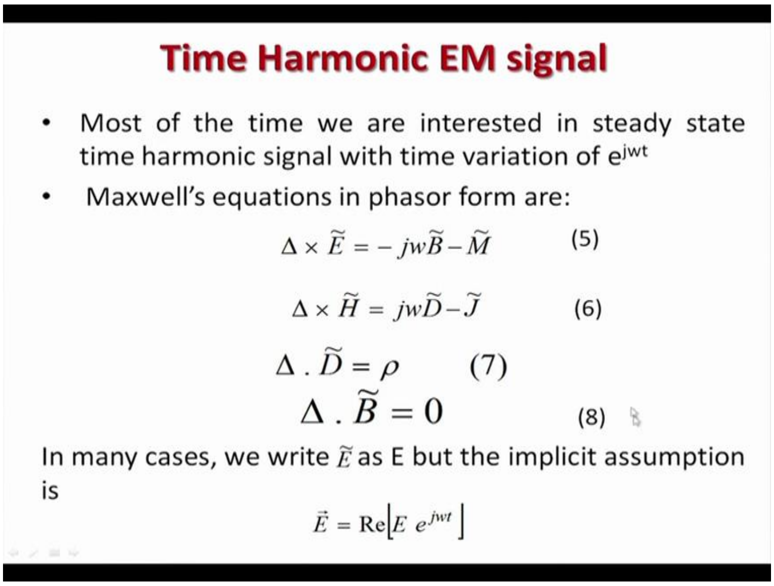

So, now let us try to find the time harmonic EM signal, time harmonic means most of time we are interested in steady state and in steady state actually Fourier has proved that any electrical type of signal that can be broken into various exponential basis thing. So, we say that e to the power j omega t that is the Fourier basis. So, in the time variation, we have already seen that if the z variation of any z propagating wave that is e to the power minus k z, but time variation now we are saying that in the steady state, not in the transient state, in transient state it maybe something else, but in the steady state all signals can be expressed as time variation of e to the power 1 j omega t, e to the power 2 j omega t.

We assume that our field has time variation e to the power j omega t that is called Time Harmonic EM signal that means, my variations will be e to the power j omega t, e to the power 2 j omega t, e to the power c j omega t all are harmonic that harmonic to omega. So, in such cases if the time variation is this, we know that we can drop the time variation by switching over to phasor notation. So, you see that Maxwell's equation now we can write by noting the time variation is this, that is why there are two del del t operations in the Maxwell's equation. This in the first two equations there are two time derivative that will become J omega so that you see the del cross c is this. So, in phasor form, all this tilde means phasor. Later, many times will drop this tilde, but always it will be phasor because time derivation generally we do not deal with, we drop it here, but if we require it can be always a phasor, this is phasor and it can be always written, sorry this is phasor and this is the actual real signal.

So, actual real signal is real part of this phasor into e to the power j omega t. You multiply e to the power j omega t with the phasor and then take the real part that will be your actual v. We will solve Maxwell's equation in phasor form, then whatever solution we will get of that phasor electric field phasor, we will just multiply it with e to the power j omega t, the whole thing real part will take and we say that is the real electric signal, all signals are real and there cannot be any imaginary signal, but we deal with for convenience to get rid of the time variation we come to phasor. So, there is no time variation, then when you finish we come there.

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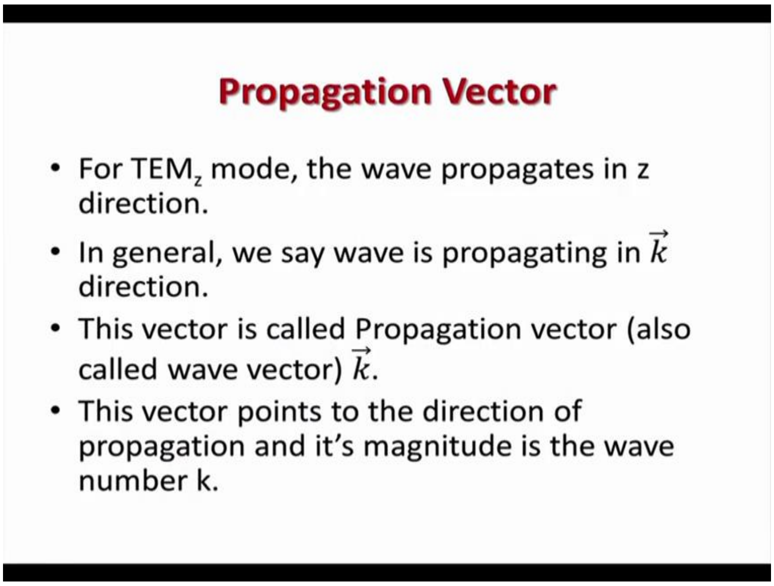

Now, this is a thing that for TEMz mode, the wave propagates in z direction.

So, that is true, but to make our thing generally we have already seen in the beginning that wave may propagate in x direction also, wave may propagate in y direction also. So, to make the thing most general, we will now say that the wave is propagating in k direction, k in most of our cases will be z, but there can be instances of x directed field also, y directed field also or in any general field is breakable or can be expressed in terms of z component, z directed field, y directed field and x directed field and these vector k is called propagation vector, also sometimes called wave vector. So, vector means we should define its magnitude and its direction, its magnitude is the wave number k which equals k which is color quantity wave number and its direction is in the direction of wave propagation.

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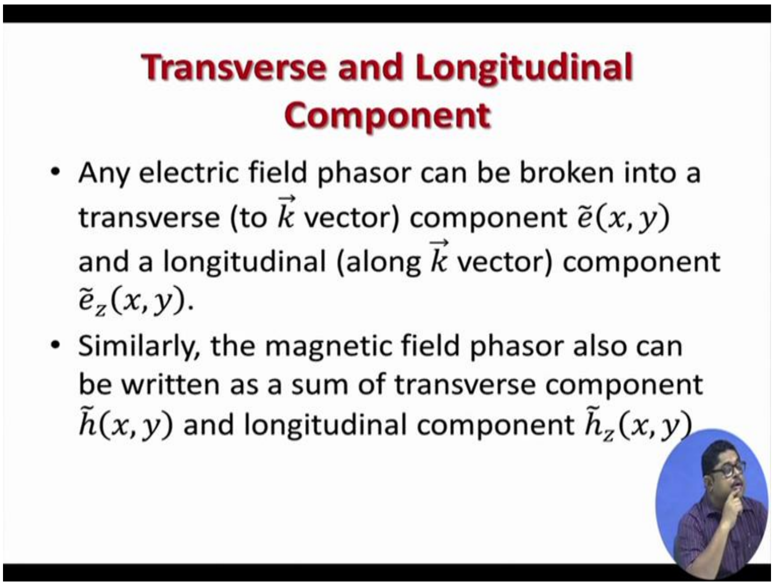

Now, in Transverse and Longitudinal Components, any field quantity electric filed phasor can be broken into a longitudinal and transverse component. The longitudinal component with this new propagation vector terminology, it is along k vector, though you see we have still writing here ez, but along k vector and the other component is the transverse component which we write as e x, y. So, similarly the magnetic phase phasor also can be broken into the transverse component h x, y and longitudinal component along k direction that is hz x, y.

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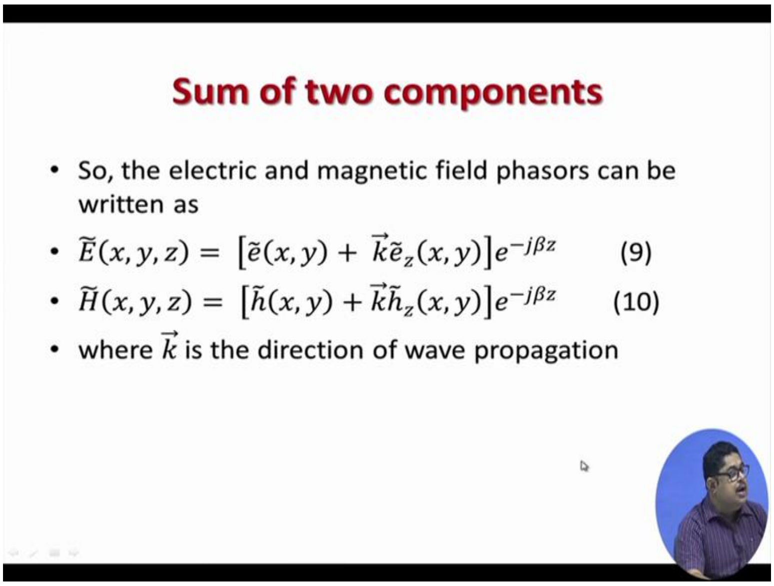

So, we can write these phasors which will be having E x, y, z that is breakable into this is the transverse component plus k ez ex y then we know, but this is the two direction and what is the z variation of this electric field e to the power minus j beta z because the wave is propagating in the z direction.

Similarly, h is also breakable into this and this equation numbers also will maintain so that it is similar to the nodes that we will upload. So, please you can refer to that how they are coming and that is why we are taking some numbers either you see that 5 6 7 8 then 9 10.

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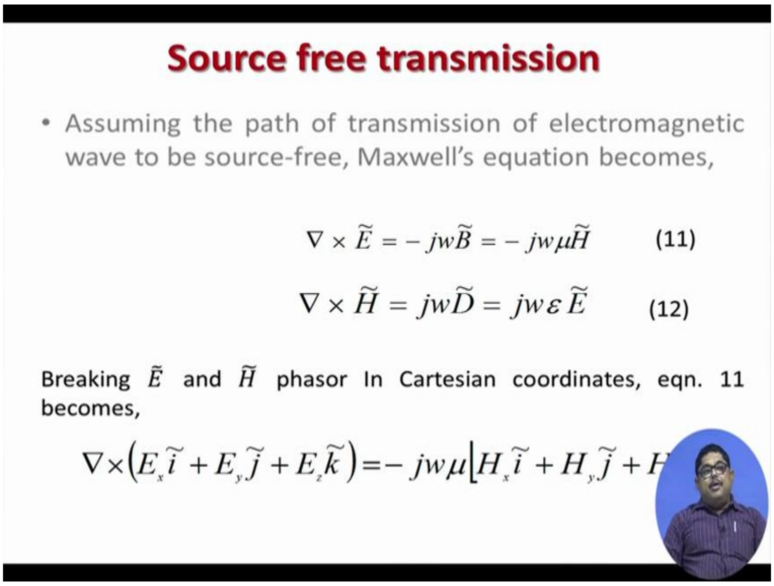

Now, we are trying to model the micro wave transmission and let us assume that we are far from the source which is away, but we are several wave lengths away from the source so that transmission is taking place. In time also we had studied in, special direction also we are studied that we are far away from the source which is not effecting us because near the source something else happens, but when the wave has started taking energy. So, that is why we assume that source pre means J and M that we introduced, those are not there because those where the sources, now we get rid of them and say that del cross E is equal to this simple, so 11 12. Now, to see del cross E is related to H magnetic field. Similarly, magnetic field is related to electric field. So, this equation 11 if you break let us in Cartesian coordinates we get these. So, if we take this (Refer Time: 22:39) then there will be three components, you can write them as these.

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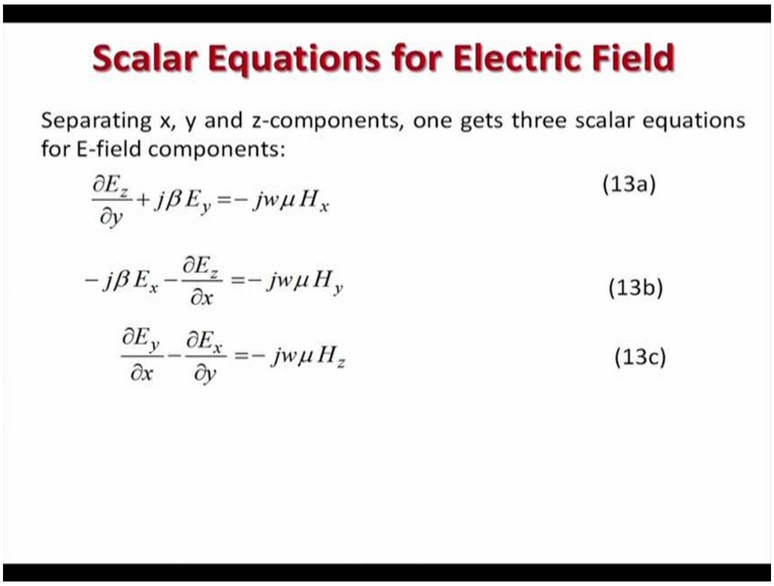

This equation gives you equation 13 which is three in parts so you have three, but here also we see that we have not succeeded in doing what was our objective, we will manipulate so that, the transverse component all are expressible in terms of a longitudinal component.

But, you see that here this equation if I take here I will get these, but here you see that this is what I can do all the electric and magnetic field jumped up transverse fields can be expressed here.

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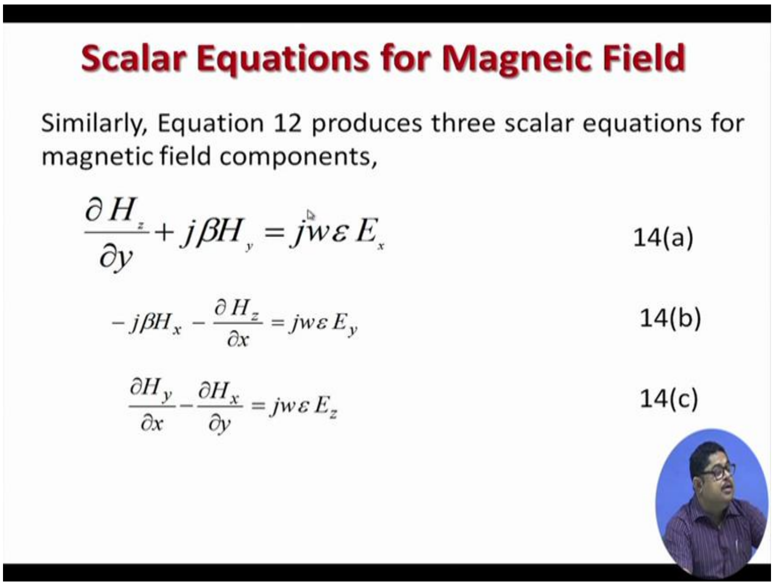

Similarly, magnetic field you can again break that into three parts, but still it is not doable we have not got our answer because here you see electric and magnetic field together. Suppose, I want to solve this first equation, then the problem is I have Ex and Hy and that is expressible in terms of Hz, I can put the Hz condition because longitudinal condition I can put, but then E and H are mixed up. This is the problem of Maxwell's equation.

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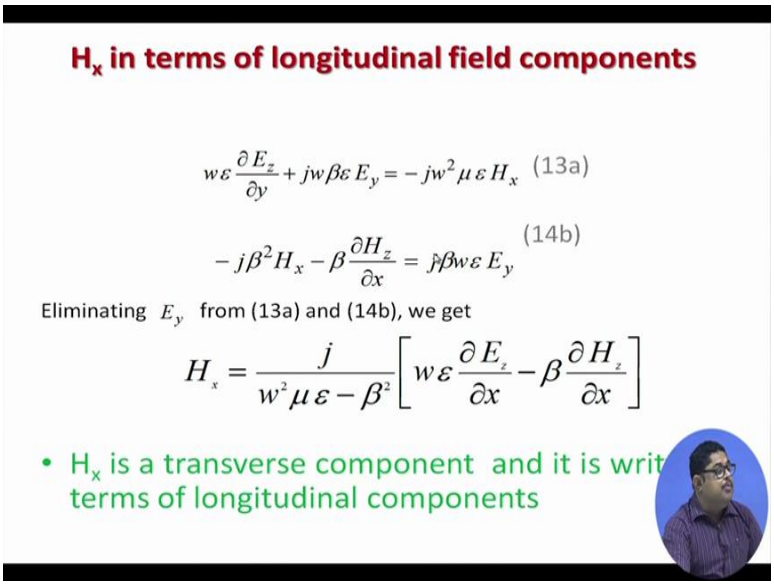

So, here we have shown that if you eliminate then Hx from these two equations 13a and 14b and if you eliminate y from 13a you get Hx. So, now, by this we can know that Hx is a transverse component and it is written in terms of longitudinal component.

We will see that still it is a problematic of getting a solution, suppose this E and H if they are together, now both are unknown to me. So, we need to do something later, but now first we express like this one by one, we will express this Hx Hy Hz Hx Hy Ex Ey that means, transverse component in terms of Ez and Hz. So, this is an equation Ez and here this denominator, you see these are all constants omega is a frequency, these two are material parameter and beta is the propagation constant.

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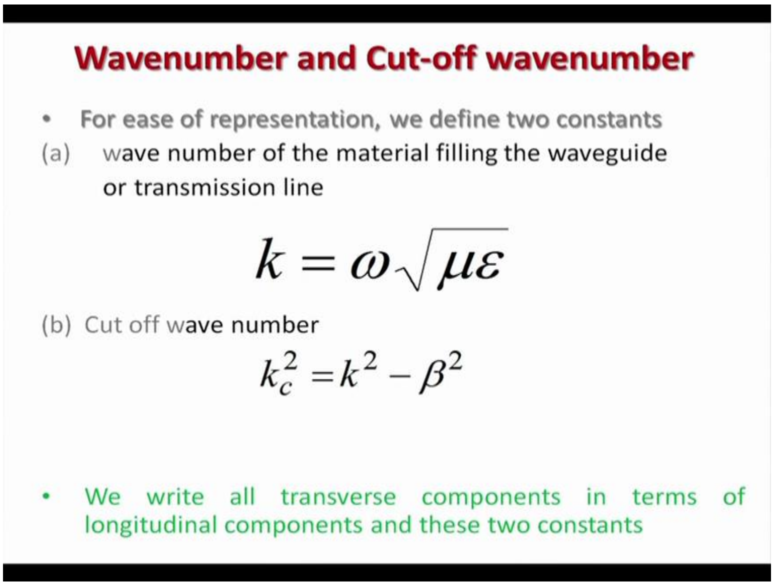

So, that also depends on the materials and these will give some name and that name or wave number and cut off wave number, wave number already we are familiar that is the propagation vectors magnitude so that is, omega into this and will also define a cut off wave number k square minus beta square. So, we write all transverse components in terms of longitudinal components and these two constants.

These two will be introducing there. So, for later simplicity the physical meaning of k is the magnitude of propagation vector and this kc will see that it will give us the cut off wave number and from that will be able to find out some cut off frequency, actually will see that in cases there will be propagation will be stopped in certain cases that though depending on the geometry of the structure, there will be some cut off and from these cut off we can calculate that which frequency electromagnetic signals, they cannot propagate and that is why it is call cut off wave number, but that meaning we will attach later. Here, that meaning is not obvious, but I know that is why I am telling you. So, these are the two constant which we defined.

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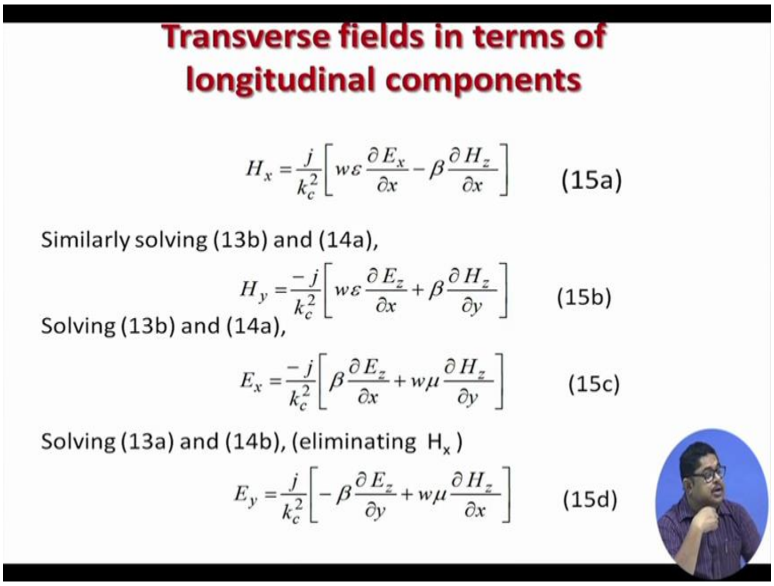

So, in terms of these two constants if we write then there are four such equations, but basic philosophy is all transverse fields which I am writing in terms of the longitudinal components, you see that I have already said these in the nodes you would know that and these you will have to do so that, this is a very simple mathematic by that you will get this four fields I can write. Now, my job will be I will put one by one TEM TEMz condition TEz condition and TEMz condition and find out the field structure.

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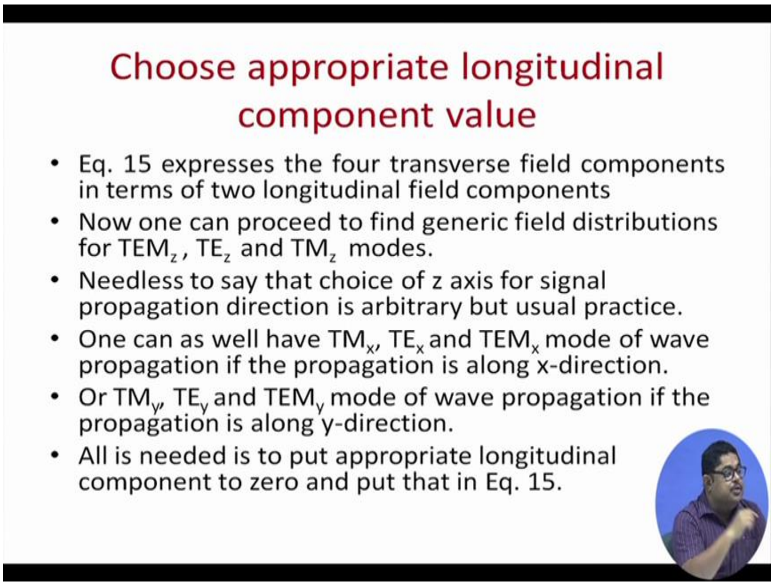

So, this equation 15 is the part that we have manipulated somewhat and got you. So, choose appropriate longitudinal component value, equation 15 expresses the four transverse will components you can put the TEM, TEz values needless to say that choice of z axis for signal propagation direction is arbitrary, but usual practice one can as well have x directed propagation etcetera, all is needed to put appropriate longitudinal component to 0 and put that in equation 15.

So, this was our mathematical model we have developed for the all modes, this model is valid that we could express from Maxwell's equation that all four transverse components possible in terms of longitudinal component.

Next, we will be putting the values and that will be specific to that mode which will be starting from the next class. The next lecture we will see that if we have TEM, what is the field? If you had TEM, what does the four fields becomes? If you have TE what does the four fields become?

Thank you.