

Basic Building Blocks of Microwave Engineering
Prof. Amitabha Bhattacharya
Department of Electronics and Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 14
Microwave Resonator

So, welcome to the 14th lecture of this series. Now here we will see another basic building block that, you saw in your electronics throughout electronics engineering course. And LC circuit is fundamental. Because it is the only circuit which can produce from noise it can produce ac signals of any frequency. And then if you want to resonate the circuit at a particular frequency you can tune it. So, by changing the c value you can tune it as already the receiver does. You can amplify; obviously, it is a resonance circuit. So, at resonance signal is very high power is very high. So, you can use it. Suppose you are produce some small signal that you want to sustain, that you want to give some good amount of value that you can use it as this. So, this resonated circuit how we do it in micro wave that we will see today.

(Refer Slide Time: 01:36)

Microwave Resonator

- Microwave Resonators are tunable circuits
- Used in oscillators, amplifiers, wave meters, filters
- At the tuned frequency
 - Impedance is real
 - Average energy stored in electric field
 - =
 - Average energy stored in magnetic field
- Total energy = 2 (electric energy stored in resonator)
- Total energy is maximum at resonant frequency.

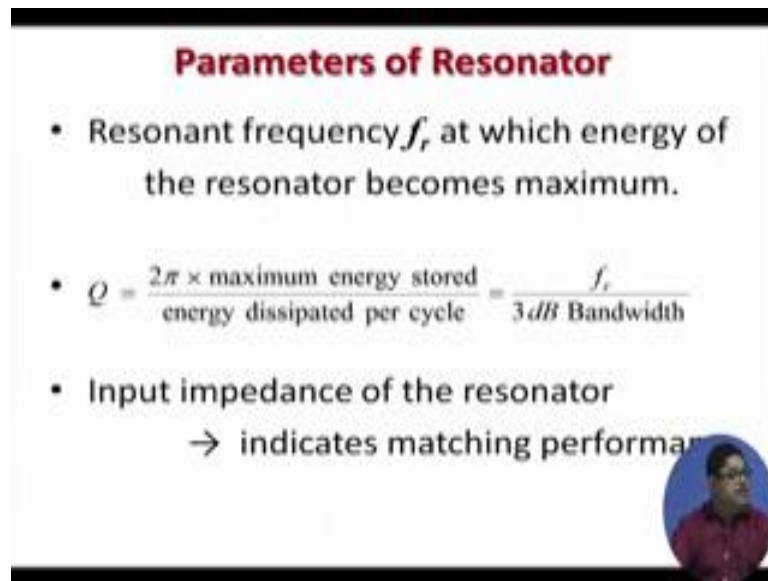
So, microwave resonator is a tunable circuits, used in oscillators, oscillators need because if you have some signal. Now you need to sustain that that is called oscillator.

Then amplifiers you need to make these LC circuit tuned amplifiers you know. They are very sharp is there cut off. So, that is why their Q is very high. Then in wave meters which measures the frequency of the wave that all of you have seen in your microwave benches there is a generally you cylindrical wave meter. So, if you want to find out frequency what you do, you put a cavity or a basically a resonator circuit and go unchanging it is tuning frequency.

Now, if there is any applied signal at some frequency, when the frequency of the signal and the frequency of the resonating frequency of the cavity are equal suddenly, the whole circuit will show that it, it has picked up the signals because it will amplify the signal. So, that is the principle of wave meter, then any filter you want to tune, I want to amplify certain signal you want to reject certain signal. So, you can use these LC devices to amplify or reject. That all of we know that whether a various types low pass high pass and all those types. So, they require all these LC things.

At the tune frequency we know. That impedance is real the reactants part goes away. So, impedance which is actually is some of resistance plus reactants. That reactants part of the impedance goes away at the tune frequency. That is the definition of resonance. And that time average energy stored in electric field becomes equal to average energy stored in magnetic fields and. So, total energy is twice of electric energy stored in a resonator. Total energy is maximum at resonant frequency. That is why you get very high sound when you tune a, suppose you going in a car you are finding the fm radio. You go on distribute moving the knob. Nowadays you put a push button, but inside there is actually is changing the C value by changing the knob. Actually I in earlier days you used to change the C value suddenly, you get a very high song or very high lecture. So, you know that you are tuning is perfect. Now you have getting that signal because energy is maximum.

(Refer Slide Time: 04:34)

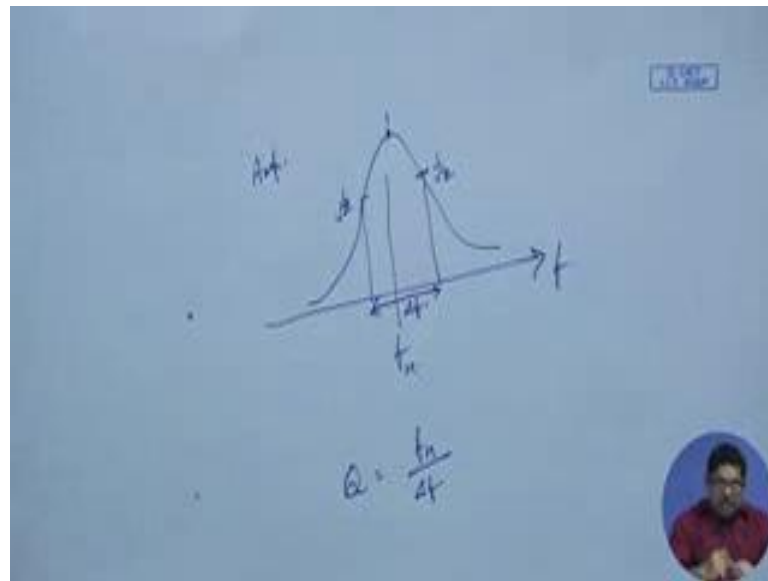


Parameters of Resonator

- Resonant frequency f_r at which energy of the resonator becomes maximum.
- $Q = \frac{2\pi \times \text{maximum energy stored}}{\text{energy dissipated per cycle}} = \frac{f_r}{3dB \text{ Bandwidth}}$
- Input impedance of the resonator
→ indicates matching performance

Now, what are the parameters of a resonance circuit of the resonator; obviously, the resonant frequency f_r at which energy of resonator becomes maximum. Then also there is Q. Q of a resonator is what. Q of a resonator is the how much energy it maximum you stored, by energy dissipated per cycle. 2π is the total length of which it is going and. So, Q means that I will normally require that in a resonator energy stored will be high and energy dissipated will be very low. So, Q we demand very high. And it can be prove that all resonators they have a curve which is very sharp.

(Refer Slide Time: 05:28)



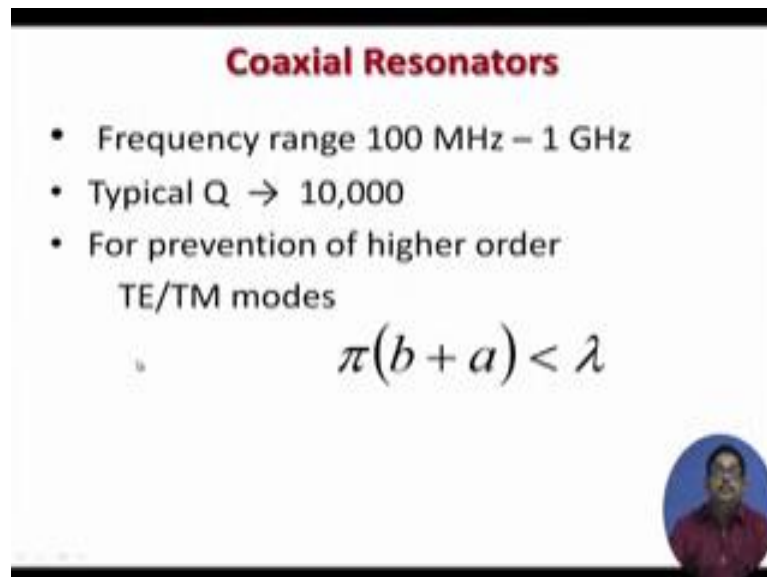
So; that means, their frequency response is something like this. This is frequency and this is the amplitude of the signal. So, they are we know these at the maximum amplitude comes at the f_r frequency and these. So, this sharpness is also defined by from this if I fall down to the amplitude falls down to $1/\sqrt{2}$ of this peak.

So, if this peak is 1. I am falling to $1/\sqrt{2}$ both side now this difference is called the 3 db band width or Δf . So, definition of q is f_r by Δf . So, if I have a really high sharply tuned value; that means, my Δf is very small, 3 db band width. So, that is another factor that what is the q of this circuit; that means, how good is this resonator circuit it should have quite a good amount of q .

Also what is the input impedance of the resonator? Because you see that we are saying that it is impedance becomes real now of a pure LC circuit there is the impedance is 0. Because at resonance the, but you cannot have a pure LC circuit. So, there should be some resistance now that resistance is important because when resonance takes place that time that resistance is to the outside or to the connected circuit that impedance posed is this resistance value of the whole network. So, for impedance matching your other parts they should also have that same resistance then only they can match their. So, the input impedance of the resonator that indicates matching performance what matching is to be

done that depends on that.

(Refer Slide Time: 07:41)



Coaxial Resonators

- Frequency range 100 MHz – 1 GHz
- Typical Q → 10,000
- For prevention of higher order TE/TM modes

$$\pi(b + a) < \lambda$$


So, if you specify these 3 parameters of resonator. Then it is you know for engineering purposes all the information is known. Now in micro wave region we or lower the microwave region, we have coaxial resonators it is frequency range is hundred mega hertz to up to 1 gigahertz as I said. That up to 1 gigahertz you will see is the normal low frequency electronic circuits, but generally after hundred mega hertz you cannot have the lumped elements you have the distributed element that is why coaxial line resonators; that means, transmission line concept and above 1 gigahertz microwave concept wave guide concept that will come.

Now, typical q of those 1000, you know I think in the your in low frequency engineering electronics classes, the making q of an amplifier 10 is easy, but after making q 10 it is quite tough to make a amplifier with q more than 10. Those requires colpitts oscillator Hercules oscillator that type of concept, but here you see the demands is your q should be quite high. Because then only your because microwave or high frequency this mega hertz signals they are not. So, abundantly available like low frequencies signals. So, their amplitude is poor. So, you need to push up that q and in that in coaxial resonators we have also seen that in coaxial line higher order modes also come, but we need to prevent

them. So, that prevention criteria I am saying that b and a are the inner radius diameters and diameter radius of the inner conductor and outer conductor. So, these should be made π into b plus a that should be much less than λ , so that the TE TM modes cannot come, because TE TM modes has the cut off. So, you can prevent them provided you stick to this answer. They are sticking to this equation this inequality.

(Refer Slide Time: 09:59)

Determination of Resonant Length of Coaxial Cavities



- Reference plane on the line
- $B_1 \rightarrow$ Loaded susceptance of left section,
 $B_2 \rightarrow$ Loaded susceptance of right section
- At resonance total susceptance = 0
 So, $B_1 + B_2 = 0$
- Take the reference plane to the extreme right line.



Now, how much you know that one benefit actually a transmission line is already an LC circuit. Because it is equivalent circuit is you see it is a series resistant in shunt with a per unit length capacitance. So, LC is there. Only you need to find out for your case that what is the value of the length of the thing. So, that length of the resonant length of a coaxial cavity circuit you need to do like this. So, think of that you have a transmission line that is a reference plane on a line.

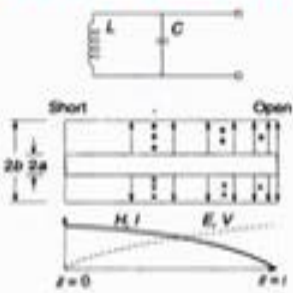
Now, this side if you look you are looking at a susceptance of B_1 , loaded susceptance loaded, we are saying because this line let us say way is terminated by a load of Z_1 , real load similarly here you have a load of Z_2 , from this reference plane if you look at the susceptance you are looking at a loaded susceptance.

Now, at resonance we know that total susceptance of the thing will go to 0. So, B_1 plus

b2 should go to 0. So, now, to design the coaxial resonator, what you do take this reference plane to the right end. Extreme right of the line then enforces this condition you will get a resonators length. So, 3 types of configurations are possible.

(Refer Slide Time: 11:45)

Configuration I: Quarterwave Cavity

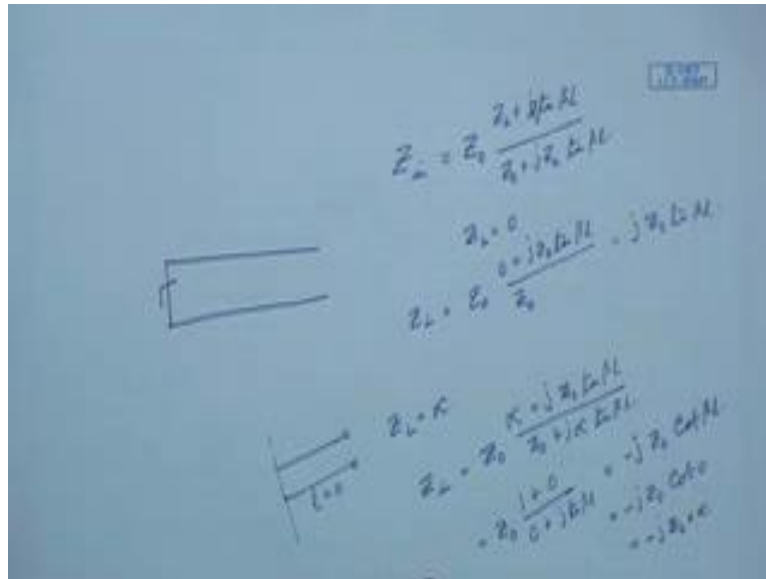


- $jz_0 \tan \beta l = \text{infinity}$ or $l = \frac{\lambda}{4}(2n-1)$
- Parallel resonant cavity

And popular the first one is basically gives you a LC circuit. Where you see this is the transmission line it is left end is shorted; that means, with reference to the previous one basically this z_1 is equal to 0. And z_2 is taken as infinity that is the first configuration.

So, this is short this side is open. So, you see this is the line. So, this side is open circuit this side is short circuit the inner conductor is radius a, outer conductor radius b. So, let us enforce that condition, what will be a shorted transmission line. So, what will be the shorted transmission line, I think just let me show you once. This you should be able to do that I have a transmission line this is shorted or it is.

(Refer Slide Time: 13:03)

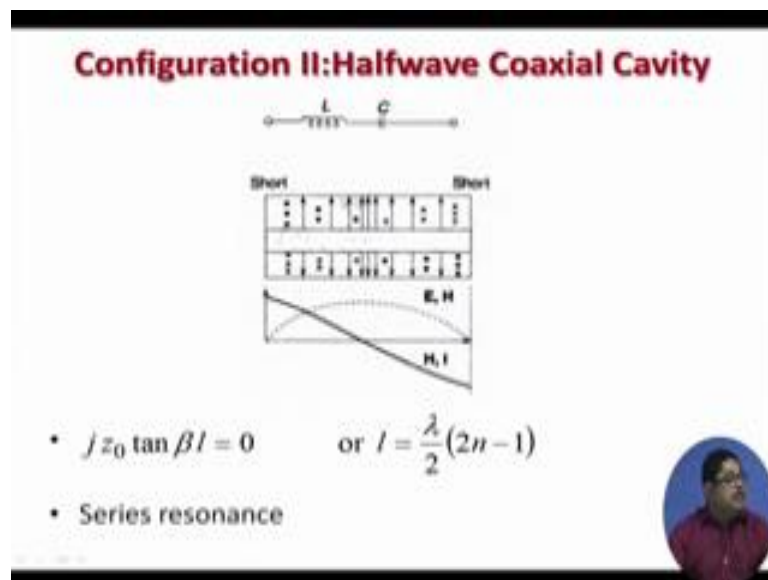


First let me write what is the input impedance of a transmission line is Z_{in} , Z_L , plus $j \tan$. Loss less line, $j \tan \beta l$, Z_L , divided by Z_L plus $j Z_L \tan$, βl . Now what is if I put this Z_L is equal to 0. That is the shorted line. This Z_{in} will be how much. Z_{in} then 0 plus, $j Z_{\text{in}} \tan \beta l$ by Z_{in} . So, that will be $j Z_{\text{in}} \tan \beta l$. Let show every $\tan$ and for the right side you think that there is a, this is the reference plane right side is open; that means, Z_L is infinity and. So, what will be the Z_{in} ? Z_{in} is Z_{in} , infinity plus j , Z_{in} , $\tan \beta l$, by Z_{in} plus j infinity $\tan \beta l$ ok.

So, you can divide throughout by or generally you can write. So, this is Z_{in} , 1 plus that division will make it 0. By 0 plus $j \tan \beta l$. That is how much minus $j Z_{\text{in}}$. $\cot \beta l$; and in this case length is right side, what is the length, length is equal to 0. So, that will become minus $j Z_{\text{in}} \cot$ of 0. What is $\cot 0$? $\cot 0$ is infinity. So, minus $j Z_{\text{in}}$ into infinity so; that means, b what is that b_1 or $2 b_2$. So, I can say b_2 is infinity. That is why it is written that is infinity. You solve these. So, $\tan \beta l$ is infinity. So, the condition is l is equal to $\lambda / 4$ minus 1. This is parallel resonant cavity to understand. How to do it that both side you assume transmission line that is why initially do not assume it be at right side. You write down depending on the terminations what is the b_1 you are seeing susceptance value. B_2 you are seeing from this side of the line then you make the appropriate three. So, this is called quarterwave cavity, because it is

lambda by 4 into some odd number. So, this gives you a parallel resonance circuit.

(Refer Slide Time: 17:15)

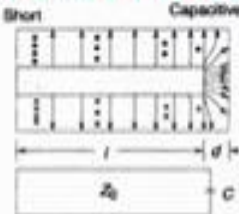


So, if you want to create a parallel resonance circuit. Take a quarter wave cavity; then configuration 2 half wave coaxial cavity. Here what you do you this side short this side also short. So, if as a why this previous one was we called LC. Why because if you see the graphs this v and i graphs here; obviously, this is open means b will be high this is low now these graph comes about that of LC circuit similar parallel LC circuit similarly series LC circuit, you get the b that both sides shorts. So, this this is wrongly said e this will be b correct it as this, instead of h, this will be b, this is eb this is hi.

So, voltage will be like this. Current will be like this; obviously, this is a LC circuit graph and you can write this. So, this is series resonance circuit; if you want to make you make it a lambda by 4 cavity.

(Refer Slide Time: 18:29)

Configuration III: Capacitive End Coaxial Cavity

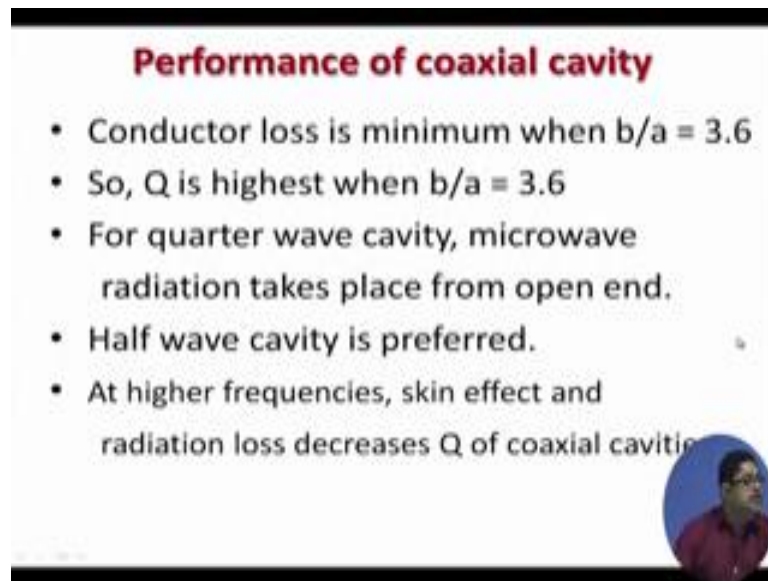


$$j z_0 \tan \beta l = -\frac{1}{j \omega_r C} \quad \text{or} \quad l = \frac{\lambda}{2\pi} \tan^{-1} \left(\frac{1}{z_0 \omega_r C} \right)$$

where z_0 is characteristic impedance of the coaxial line
 C is the gap capacitance between the central conductor and the shorting terminal

Now, there is another that you want to make, some other value. So, what you do that here you give a gap. So, this transmission line here you put a short, here also you put a short, but here you make a gap in the line, that will you give a capacity of gap that is, why you see that this side will be like this, but this side will be some capacitance omega. So, from that you find out, what is the length of the transmission line these d. So, so you give a d here z_0 , is C is the gap capacitance between the central conductor, and the shorting terminal. So, depending on what dielectric, these there you can find out this C value, and then you can design this capacity when derivative.

(Refer Slide Time: 19:27)



Performance of coaxial cavity

- Conductor loss is minimum when $b/a = 3.6$
- So, Q is highest when $b/a = 3.6$
- For quarter wave cavity, microwave radiation takes place from open end.
- Half wave cavity is preferred.
- At higher frequencies, skin effect and radiation loss decreases Q of coaxial cavity

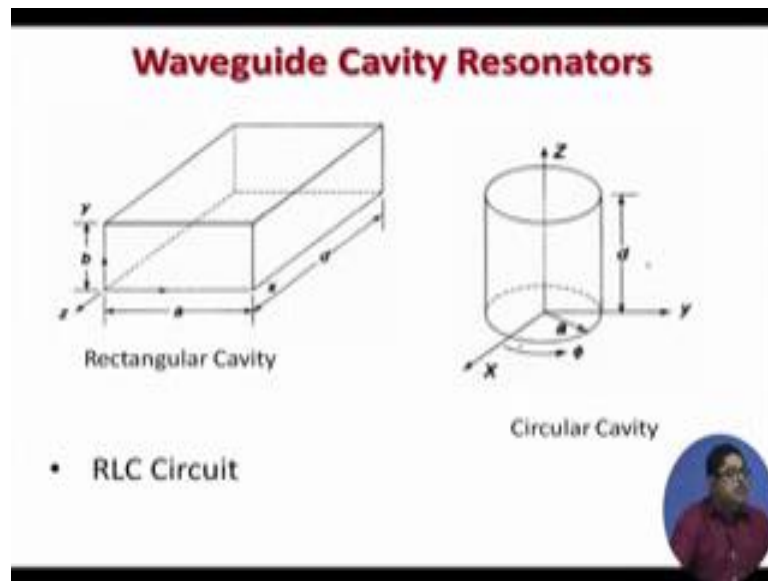
A small circular video inset in the bottom right corner shows a man with glasses and a red shirt speaking.

Now, performance if you compare. Now conductor loss is we have already seen that when we saw the coaxial connectors, that conductor loss in a coaxial line is minimum when b by a is takes the value 3.6. So, q is nothing, but q 's denominator is that loss, or energy dissipated so; obviously, highest q will get in a coaxial line, when b by a is 3.6.

So, we can choose the value of b by a to be 3.6. That will be your highest q . So, from any particular design you can find out how much q you can get. Now for quarter wave cavity one problem is there that since it is one end is open. So, energy may leak out from there. So, microwave radiation may takes place from open end. So, that loss is becomes more. So, that is why q suffers, but that is why half wave cavity is generally preferred over quarter wave cavity because it is both ends are shorted. So, the wave is confined there the leakage of microwave radiation is not there.

Now, after 1 gigahertz of problem with this coaxial cavities is that, skin effect etcetera becomes more and, that is why we have seen yesterday, that conductor loss will be more and radiation loss will be more. So, you see what we have written that skin effect will increase the conduction loss, also radiation loss will increase both of these effect will decrease the q of coaxial cavity. So, that is why at higher frequencies people do not use coaxial cavity.

(Refer Slide Time: 21:24)



So, microwave resonators are made of waveguide resonator, in waveguide we can easily form the cavity. You see you have a wave guide just this is a wave guide now this side is open another side is open it is a transmission line, but if you put on these 2 sides, 2 electrical walls that becomes a cavity. So, already in a wave guide 4 metallic walls are there the top wall bottom wall side wall left side and right side.

(Refer Slide Time: 21:38)



Now, also you make these 2 fronts and what is that front and rear these 2 walls if you make you make a cavity. So, in a this is called rectangular cavity similarly in a circular wave guide circular wave guide this total side wall is metallic single conductor now top and bottom you make 2 metal plates that will make a circular cavity.

(Refer Slide Time: 22:36)

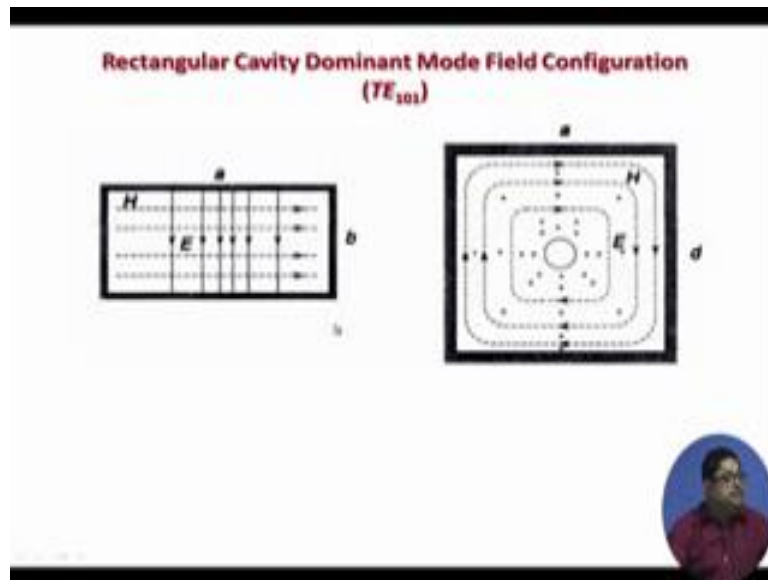


So, both of these are the RLC circuit. So, you see this is the cavity. In micro wave laboratory all of you used this. Now this is called frequency meter the on these there are frequency marks are there. What happens when you move this cavity we move this cavity now basically the length of the cavity; that means, top and bottom. So, this cavity it is top that is getting changed. So, the length of the cavity is getting changed the resonance frequency depends on these d that you will see and. So, the frequency is getting changed.

Now, the signal is coming this device probably clipstone. This clipstone this is creating this some signal. Now that signal is generally cavity is not passing. So, here the waves wr is meter is showing that not much deflection, but when the resonance frequency of the cavity matches with the signal it takes power and that time that power. So, here there is a deflection and you understand that. So, from the graduation on the frequency meter you find out. So, now, the clipstone is producing, so and so signal; so and so frequency

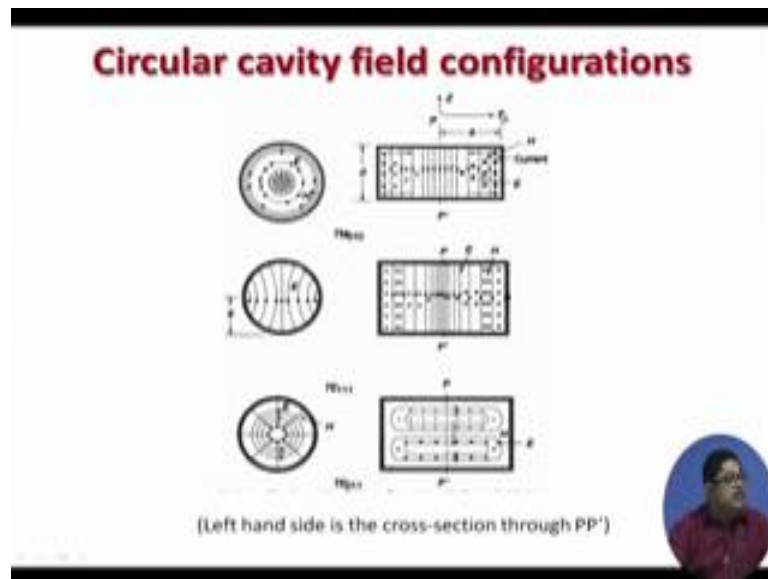
signal.

(Refer Slide Time: 24:01)



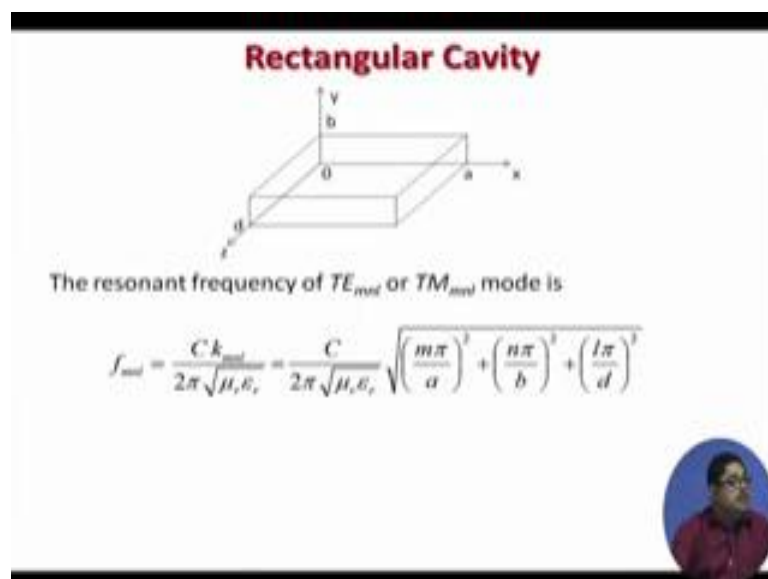
Now, this is the rectangular cavity dominant mode. You see similar to t 1 mode, but here since it is a 3 dimensional structure. Is not a 2 dimensional structure, because wave guide means we have that inside in z direction it has infinite direction, but the moment you have the z direction instead of z direction you have 2 separate 2 metal plates separated by d. 2 metal plates separated by d you see field structure is almost same transverse plane ab a is like this h is like this. And here you see that this there is a full the magnetic field is like this. So, this is t 1, 0 1 mode. 1 0 is the last part is another 1 that is a dominant rectangular field cavity configuration.

(Refer Slide Time: 24:57)



Now, circular cavity this is the field configuration. You see circular thing dominant mode was TM_{11} . So, this was the mode similar mode is TM_{01} , but; obviously, these 2 are also preferred as we said that they have circular symmetric field structures.

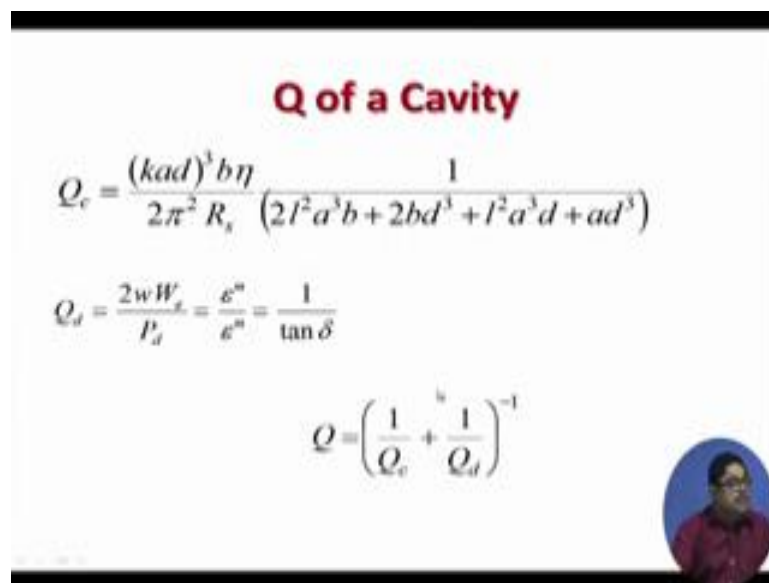
(Refer Slide Time: 25:30)



So, this you see that they are better field structure. That why a circular wave cavity also

that non dominant mode also some times, used for rectangular cavity the resonance frequency of TE or TM mode that is given by this, this is c means this, is on the sorry this should not be equal capital c, this is small c which is the velocity of light, 3 into 10 to the power 8 meter per second. These are and they again depend on this. So, you here you put that 1 0 1.

(Refer Slide Time: 26:05)



Q of a Cavity

$$Q_e = \frac{(kad)^3 b \eta}{2\pi^2 R_s (2l^2 a^3 b + 2bd^3 + l^2 a^3 d + ad^3)}$$

$$Q_d = \frac{2\omega W_d}{P_d} = \frac{\epsilon''}{\epsilon'} = \frac{1}{\tan \delta}$$

$$Q = \left(\frac{1}{Q_e} + \frac{1}{Q_d} \right)^{-1}$$

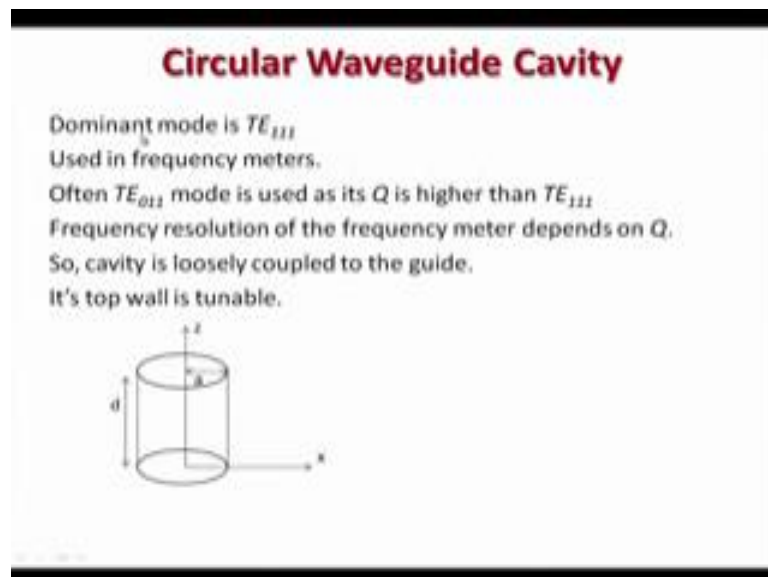
So, basically depends on a, sorry, a and d because 1 0 dominant mode so n, so b will not matter a and d they matter a by pi and d by pi.

So, that will matter the frequency and. So, you need to choose the generally rectangular cavity, they have dimensions a and d. So, you need to choose to give it what frequency you need to do generally the c; that means, if you vary the d, but distance between the 2 walls you change the rectangular cavity also you can calculate what is the q of the cavity, for conductor loss by doing the similar exercise, as we have done for coaxial line. You can find out this is the q for conductor loss. This is the q for dielectric loss this is simple as 1 by tan delta. So, find out dielectric find out it is specification. So, you know tan delta tan delta is generally specified. So, qd finding is not a. So, if you have dielectric let us say that generally it is tan delta is point 0 0 1 so; that means, you will get thousand q with that.

Now, you see you find out depending on frequency it matters because of this k presence. So, as you go higher and higher q_c starts becoming significant, but generally in the microwave region 10 to 100 GHz it is not that much, that time q_d is the main deciding, but if q_c is there when both conductor loss and dielectric loss are present this is the formula.

So, if q_c is significant you take. Otherwise knowing the dielectric and δ you can find what will be the q of the rectangular cavity. So, you can make by choosing proper dielectric, you can make or here if you can find out what will be the thing here.

(Refer Slide Time: 28:34)



So, similarly circular wave guide is dominant mode is instead of TE_{111} , in circular wave guide TE_{011} is used in frequency meters as I said often, TE_{011} mode is used as it is q is higher than TE_{111} , frequency resolution of the frequency meter depends on q cavity is loosely coupled to the guide, its top wall is tunable generally as I said that top wall you vary that varies the d , that will vary the frequency of the thing. Resonance frequency is given by this formula this all you know that all the things.

(Refer Slide Time: 29:09)

Resonance frequency

resonant frequency

$$f_{res} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \quad (TE)$$

$$f_{res} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \quad (TM)$$


So, you see basically resonance frequency is depending on d. These are constant a is the cross section. So, for a given for broad length this is 1 this is circular a is the radius, and this is the oh d. So, l value will be your mode number. So, generally for dominant mode it is l is 1.

(Refer Slide Time: 29:58)

Q of the Circular Waveguide Cavity

$$Q_C = \frac{W_0 W}{P_e} = \frac{(ka)^3 \eta a d}{4(p'_{nm})^2 R_s} \times \frac{1 - \left(\frac{n}{p'_{nm}}\right)^2}{\left\{ \frac{ad}{2} \left[1 + \left(\frac{\beta a n}{(p'_{nm})^2}\right)^2 \right] + \left(\frac{\beta a^2}{p'_{nm}}\right)^2 \left(1 - \frac{n^2}{(p'_{nm})^2} \right) \right\}}$$

$$Q_d = \frac{wW}{P_d} = \frac{\epsilon}{\epsilon''} = \frac{1}{\tan \delta}$$

$$Q = \left(\frac{1}{Q_e} + \frac{1}{Q_d} \right)^{-1}$$


So, d is that the determining factor. This TE and TM the change is on in that p nm dashed or p nm q is there. Equations are given Q d is again that one by $\tan \delta$ these formulas are just similar that is for, so that by this way of land the resonator circuit, how to make resonators LC various RLC devices, in the microwave region. Next, in the next lecture we will see another important thing, that how to make attenuators in microwave region.

Thank you.