

Basic Building Blocks of Microwave Engineering
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Lecture - 13
Port Microwave Power Divider/Combiner

So, welcome to this thirteenth lecture. Here as I was saying in my last lecture, that I can get my wish list as we have prepared for this power division and combiner. All can be satisfied if I introduce one more port, four ports. Still now we are seeing the minimum. So, if you have minimum ports, you cannot have all the wish lists satisfied, but if I go to four ports. So, four port microwave power divider combiner I can satisfy that, and finally we will see that the when I satisfy everything; that means, I can do anything that is called magic. So, that magic device will be called magic T, we will have T, T naught T t.

But then it will be both; I can use it as power divider, I can use it as power combiner, it will be loss less, it will be matched, it will be reciprocal. So, that device is magic t once you get that, we will know that power divider combiner I do not have any problem, but all the ways you know we do not have our that best device at an. So, then we go and do with other things which are there.

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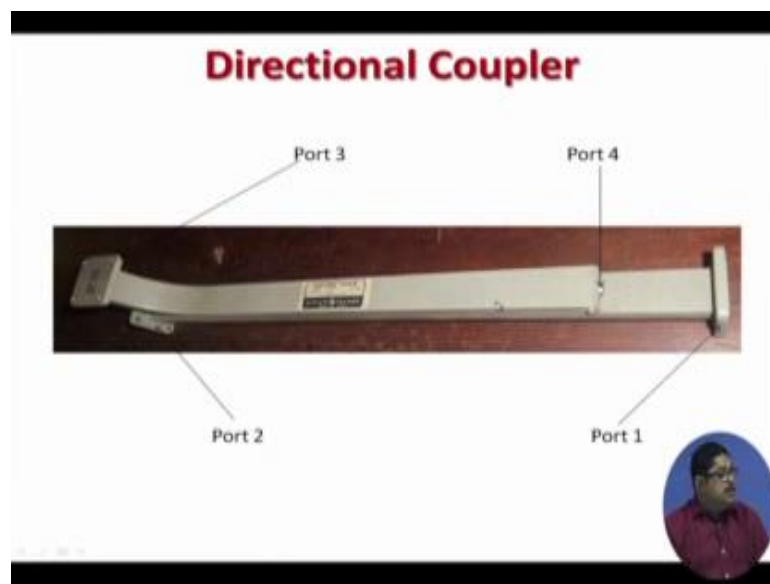
Directional Coupler

- 4 port
- Lossless
- Matched at all ports
- made of passive, isotropic material → reciprocal



So, directional coupler, let us switch about to four port, and first will call a device, very important device is directional coupler. It is four port, it is again we see that, that wish list; loss less, matched at all ports, made of passive isotropic material, reciprocal. So, in four port we are placing that wish list, let us see what happens to our scattering matrix; whether we can fabricate that, and if fabricate various fabrication, one of that is directional coupler, which one till now I have not said, but will see the development one by ones; one of that is directional coupler.

(Refer Slide Time: 02:56)



So, let see this is a directional coupler, all of you are familiar with in your microwave labs that, this is directional coupler you see it has four ports, but generally in labs etcetera many times we use only three ports, this four port is there, it is constructed, but it is not accessed from outside. So, this is a typical directional coupler.

(Refer Slide Time: 03:24)

[S] of Directional Coupler

$$[S] = \begin{bmatrix} 0 & \dots & \dots & \dots \\ \dots & 0 & \dots & \dots \\ \dots & \dots & 0 & \dots \\ \dots & \dots & \dots & 0 \end{bmatrix}$$

- Matched at all ports



Now, from our, let us start building our S matrix of a four port wish list device. So, S if I matched at all ports, I can write these diagonal elements S 11 to S 44. They are all 0; I do not know others that is why I am given.

(Refer Slide Time: 03:48)

Directional Coupler is Reciprocal

[S] matrix symmetric

$$[S] = \begin{bmatrix} 0 & S_{21} & S_{31} & S_{41} \\ S_{21} & 0 & S_{32} & S_{42} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{34} & 0 \end{bmatrix}$$

6 unknowns



Now, it is reciprocal, this four port device that should be reciprocal. So, invoke

reciprocity. So, matrix is symmetric. So, now, I give the nomenclature, this. So, you see how many unknowns now I have; one is S_{21} , then S_{31} , then S_{41} , then S_{32} , then S_{42} , and then they are already over S_{34} . So, six unknowns are there. So, I need to solve these six unknowns to determine the S matrix completely.

(Refer Slide Time: 04:47)

Directional Coupler is Lossless

- $[S]$ is unitary
- $(\text{Col } 1)^* \cdot (\text{Col } 2)^* = 0$

and

$(\text{Col } 3)^* \cdot (\text{Col } 4) = 0$

Multiply by S_{41} $S_{31}^* S_{32} + S_{41}^* S_{42} = 0$

Multiply by S_{32} $S_{31}^* S_{41} + S_{32}^* S_{42} = 0$

$$\begin{array}{r} - \qquad - \\ S_{42} (|S_{41}|^2 - |S_{32}|^2) = 0 \quad \dots\dots(a) \end{array}$$



Now, let us directional coupler we want to be loss less. So, apply unitary property, I have said. So, you will have to do it yourself, because these are mathematics things, teaching is difficult, you will have to do then only understand, otherwise everything.

(Refer Slide Time: 05:12)

Unitary Property

One solution of equation (a) and (b) is

$$S_{41} = S_{32} = 0 \quad \dots(c)$$
$$[S] = \begin{bmatrix} 0 & S_{21} & S_{31} & 0 \\ S_{21} & 0 & 0 & S_{42} \\ S_{31} & 0 & 0 & S_{34} \\ 0 & S_{42} & S_{34} & 0 \end{bmatrix}$$

4 complex unknowns



So, by this you formed some equation, then again by that you some equation. Now one solution of that is, as I said these. So, these two became zero, for that solution. So, this is the how many unknowns now. I have reduced two of those unknowns, so this four complex unknowns.

(Refer Slide Time: 05:33)

Unitary Property (Contd)

- $|S_{21}|^2 + |S_{31}|^2 = 1 \quad \dots(d)$
- $|S_{21}|^2 + |S_{42}|^2 = 1 \quad \dots(e)$

(d) & (e) $\Rightarrow |S_{31}| = |S_{42}| \quad \dots(f)$

Also, $|S_{31}|^2 + |S_{34}|^2 = 1 \quad \dots(g)$

(d) & (g) $\Rightarrow |S_{21}| = |S_{34}| \quad \dots(h)$

6 real unknowns



Now, I have reduced two of those unknowns, these four complex unknowns. Now unitary property again invoked. So, in terms of that you see I have got these equations, where I have got six real unknowns. Due to this actually one is reducing that is why now you have six real unknowns.

(Refer Slide Time: 05:59)

Choice of Phase Reference

Let,

$$S_{21} = S_{34} = \alpha$$

$$S_{31} = \beta e^{j\theta}$$

$$S_{42} = \beta e^{j\phi}$$

- α, β real
- θ, ϕ to be determined from unitary property
- 4 real unknowns



So, let us fix the unknowns that S_{21} , let us call alpha and we also take the phase difference of alpha 2 is zero, so it is alpha. Now S_{21} and S_{34} they are not connected; that is why I can take this unknown also, let us take value alpha, so that it becomes minimized. now S_{31} is some other thing, because S_{21} so; that means, I am giving power to port one, the amount, the voltage that is developed at 2 is alpha, voltage that is developed at three is beta. Similarly this is S_{42} means actually is 2 4 also. So, S_{34} is alpha; that means, power I am giving in port three. So, power will be also going to port two, that we are calling beta and this.

So, alpha beta real and theta and phi are two angles, which will be determined from unitary property. So, you see that I have reduced that those 1 2 3 4, four unknowns. They are now expressed in terms of two real one, 2 and 2 4. So, four unknowns' four real quantities we will have to solve them. Now by unitary property I have to say. So, you finally, come to this conclusion where theta plus phi they should be equal to phi; theta

and phi the two unknowns, theta and phi they should be, having this relation should satisfy to satisfy unitary property.

(Refer Slide Time: 07:56)

Determination of θ, ϕ

- $(\text{Col } 2)^* \cdot (\text{Col } 3) = 0$
- $S_{21}^* S_{31} + S_{42}^* S_{34} = 0$
- $\alpha \beta e^{j\theta} + \beta e^{-j\theta} \alpha = 0$
 $\theta + \phi = \pi$
- $\theta = \phi = \pi / 2 \rightarrow \text{Symmetrical Coupler}$
- $\theta = 0, \phi = \pi \rightarrow \text{Antisymmetrical Coupler}$



Now, this is symmetrical coupler. Now there are two ways by which I can satisfy this, either I take theta and phi both to be equal, that choice is called symmetrical coupler. So, both theta and phi they are equal to pi by 2, these are called symmetrical coupler and otherwise I can take theta as zero phi is pi that is called antisymmetrical coupler.

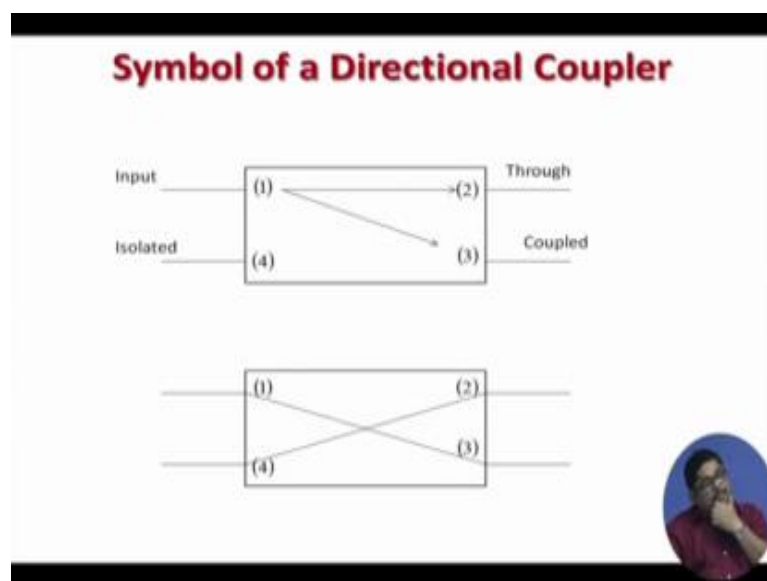
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A Directional Coupler

$$S = \begin{bmatrix} 0 & \alpha & \beta e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta e^{j\phi} \\ \beta e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta e^{j\phi} & \alpha & 0 \end{bmatrix}$$


So, a directional coupler its S matrix of that four port device that you are developing that is like this. Now theta and phi that choice I said, so based on that this will be either called in symmetrical device or antisymmetrical device. Symmetrical means theta is pi by 2 phi is pi by 2 theta phi by 2, and antisymmetrical device means theta zero. So, these two are zero, well theta and this theta and phi is pi.

(Refer Slide Time: 09:26)



Now, both of these choices that is called a directional coupler, now symbol of a directional coupler is like this, both these symbols are used you see what we say that if you give in power to 1, the power will go to 2 and 3, power 1 go to 4. So, this is called input port, this is through port, this is coupled port, this is isolated port. The same thing also some people like this terminology, this symbol that from input port you are going to couple port, but main power is going here. Similarly here this is couple the main power is giving here. So, this is the symbol I think you should know this symbol very well.

(Refer Slide Time: 10:25)

Characterisation of Directional Coupler

$$\begin{aligned}\text{Coupling (C)} &= 10 \log \frac{P_1}{P_3} = -10 \log |S_{31}|^2 \\ &= -20 \log \beta (dB)\end{aligned}$$

$$\begin{aligned}\text{Directivity (D)} &= 10 \log \frac{P_3}{P_4} = 10 \log \left(\frac{|S_{31}|^2}{|S_{41}|^2} \right) \\ &= 20 \log \frac{\beta}{S_{41}} (dB)\end{aligned}$$


And also those directional couplers are coupled by 2. They are characterized by two parameters, the one of them is called coupling. It is the coupling is equal to the ratio of power p_1 by p_3 .

So, now coupling power we require that, basically y directional coupler is required, why at all I need to couple power. Suppose some power is flowing in a line, between transmitter and receiver. Now without disturbing this whole thing I want to sends that I want to measure how much power is flowing. So, for that what is done, basically in all microwave things you know that. If you open anything and see to measure the power, actually we are disturbing the whole thing, the power mechanism by which it is flowing it will be disturbed, or you will be exposed to a very high field, if you want to go

somewhere and measure power, but if you put a directional coupler. So, the maximum power will go to the through port, but in the couple port if fraction, known fraction of power is coming, so you can put your device and measure, so you know that fraction by that you can send the main line how much power is going; that is the concept of coupler, power coupler.

Now, couplers also need a directivity; that means, when I am giving power I required that in the coupled port there should be power, some power going; obviously, in through port the power is going that is the main thing, but at the port where I will be sensing or measuring, that port I will the coupling power. So, p 3 should be there, and I also demand, basically this is in the isolated port; that means, other port should not be disturbed. So, port four should be isolated from this power reached. So, power is going to port two, power will be coupled to port three. No power should be coming to p port, but how much I am coupling and how much I am giving to p port, that is expressed by a ratio directivity.

(Refer Slide Time: 13:05)

Characterisation of Directional Coupler

$$\begin{aligned} \text{Isolation (I)} &= 10 \log \frac{P_1}{P_4} = -10 \log |S_{41}|^2 \\ &= -20 \log |S_{41}| (dB) \end{aligned}$$

$$\begin{aligned} \text{Insertion Loss (IL)} &= 10 \log \frac{P_1}{P_2} = -10 \log |S_{21}|^2 \\ &= -20 \log \alpha (dB) \end{aligned}$$



You see also I have another parameter that is isolation I am coming back to directivity, because the name of this coupler could have been without directional coupler, but directional coupler is therefore, because it have some directivity.

So, I will come back there, but before that I define. What is isolation what is isolation; obviously, I want as I was saying that isolation we want to be infinite; that means, when I am giving some power to p_1 I require that, I should not have any power coming to port four. So, this ratio, but in real life you do not have that, so there will be some power coming, I measure that, isolation if it is less I say it is very bad, like all those power dividers we have measured, that time we said four. Four means it is half is. So, four is not a very high value, we require infinite isolation, but four we said no; that means, one fourth of the power you are coming to that isolated port.

So, that now power. So, it is a power ratio in terms of S parameters, you can say it is twenty log, because S parameter is a voltage ratio. And insertion loss, you know that how much power is getting loss, because my actual p_1 power I am giving p_2 should get that, but due to this whole coupler with job is to sense the power etcetera, it is taking some power. So, how much is that that is called insertion loss p_1 by p_2 and in terms of S parameter I can write like this. Basically whatever we have assumed there, so from alpha knowledge you can say this. Remember this ten twenty explanation you understand, you yourself satisfy that. Always remember that whenever there is a power ratio that should be ten, but whenever there is a voltage or current ratio that should be twenty. So, you see the moment I switch over S parameters, generally the voltage ratios that in my frequency that is voltage wave ratio the reflected wave by a, but that is voltage or current, not power that is why twenty log.

Now, let me go to what is the concept of directivity. You see what I want. I want that; obviously, I want this port isolated. So, isolation checking is not the part of directivity. Now directivity means I want that OK, it is a reciprocal device. So, if I keep power here, the power will be going to 2 power will be going to 3. Now similarly if I keep power here, then that power will go to where, it will go to port four, and it will be coupled to port one, and it will be isolated to port three. So, reciprocal device means. If the power instead of giving at port one if I give to port two, the power should go to port four as well as port one. Main two power will come here, couple power will be coming here, and the this one will be isolated, but I want to put it direction of sense here, because my main thing I am interested about that, if some power comes from here, how it can come. If this port two is matched then there will be some reflected power.

Now, that reflected power will come where. That reflected power come to these p 4, this port four. I want from this coupler additional constant apart from coupling and power division etcetera I want that suppress these part, because these reflected power is unwanted. So, if there is by chance any mismatch in the load side of this port two, because that is not in my hand, that power will come. Now do not cheat my forward power like that reflected power. So, you suppress that. That is why directional couplers they are made from reciprocal device, but due to their proper design this w power that is coming back from there that gives suppressed, directivity is a measure of that parameter.

So, you see that is why what it is measuring; p 3; that means, in the forward path p 3 is going. Now ideally is o p 4 is not getting there, but then if there is any mismatch in the through port, then that will come to port 4. So, how much it is coming there, I want this ratio to be very high also, so that the p 4 is suppressed with respect to p 3, both are the through powers, but one is due to reflection, another is due to my intention. So, I want to boost my power that is why the coupled power which is coming to p 3 that should be coming to, that should be much higher than p 4; that is why directivity of a directional coupler also I want to be high.

Now, this is expressed in terms of p 3 means basically is S_{31} , p 4 means S_{14} , but we call it due to symmetry is S_{41} , actually should be S_{14} . So, if you do that in terms of that beta 3 means it is this ratio.

(Refer Slide Time: 19:37)

Interrelationship Between Parameters

$$I = D + C \quad (\text{in dB})$$


And then if you just see that S parameter ratio and since this is loss less device, you can easily prove that I is equal to d by c, but remember this is in terms of db; otherwise a relation will be a multiplicative relation. So, I is equal to d by c provided I d and c are expressed not in absolute scale, but in db.

(Refer Slide Time: 20:01)

Directivity

- Directivity measures coupler's ability to separate forward and backward waves



Now, directivity measures couplers ability to separate forward and backward waves that reflected wave is backward wave. So, high directivity means it will be able to do the moment it does.

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Ideal Directional Coupler

$$D = \infty$$
$$I = \infty$$


So, ideal directional coupler I should have, isolation is high, but also it should distinguish the d and the forward wave and reflected wave. So, again I am repeating this is an important concept of directional coupler that, when I am giving power to this power is going here. For sensing I am taking that power to three. I do not want power should come here, and in reality very small little isolation is quite high 120 140 db isolation can you can make vibrate.

Now, when this in the through port there is maximum power is going here; 99 percent more that also I want here. Now here there can be some mismatch. So, that power will be coming, and the moment that power coming. So, that power will come that, when power will go to in input, but I want that, it should be this 2 port. So, it will be coupled to this port also. Now I want that that the device should be such that, this device should be such that this part should be isolated. So, how to measure that? That measurement I say that this p 3 whatever is getting coupled here, and compared to that whatever is coupled here that should be suppressed. So, that is why I demand that p 3 should be much higher than

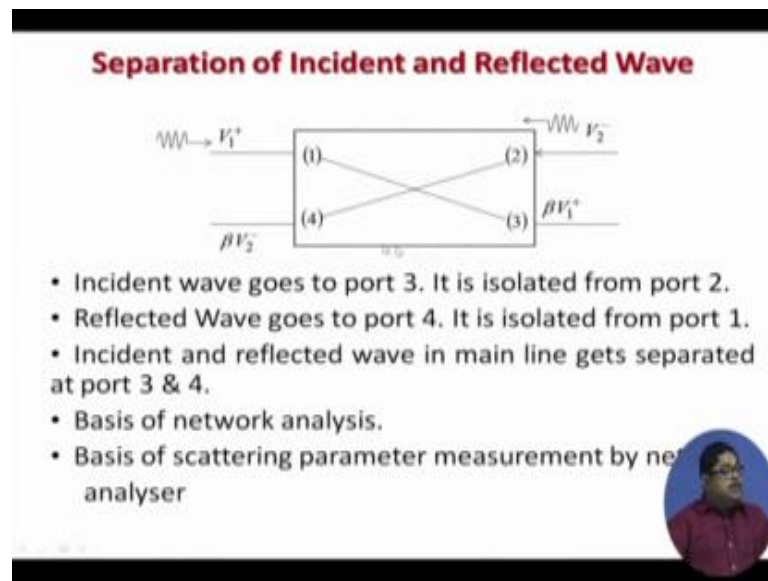
p 4.

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So, this is the ideal directional coupler. Now we have seen in case of impedance measurement etcetera how to do that. Also you should always know how to characterize any directional coupler. The characterization means, you find out those parameters of directional coupler; that is you are the coupling, because if you do not know coupling you will not be able to say the how much power is going in the main circuit. Also you should be able to find out how much the isolation is; so that in the isolated ports how much you are disturbing your neighbors. then also you have a directivity that forward path of power that should be un disturbed, or that should be go on a, but reflected power should be suppressed.

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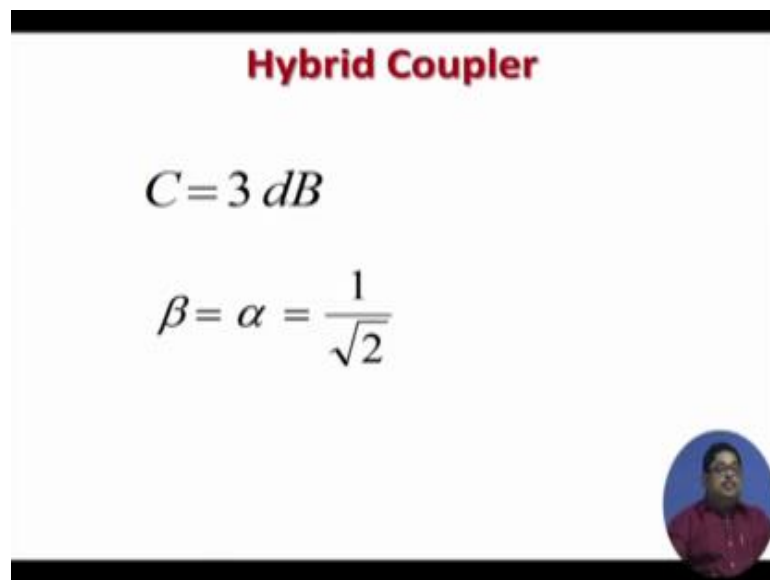
Now, but with the invention of this directional coupler we got another thing, that incident wave, you see separation, incident waves can be separated, because in general in any structure the incident wave and reflected wave they are together, but sometimes, suppose I want to measure the S parameter that time I want to separate them, how to separate that. That device is again you see in directional coupler. So, made it you will understand the v 1 is here. So, as I am saying the main power is coming here, some couple power is coming here. Couple power we are calling beta v 1 plus, and this one the v 2 is coming here.

So, but then if there is any reflection that v 2 is going here, and from that whatever the v 2 is coming, that we are calling v 2 minus. So, now, reflected wave goes to port four as I said. It is isolated from port one. Incident and reflected wave in main line gets separated at port 3 and 4. You see incident wave that goes here, and reflected wave that goes here. So, if you sense in how what is the value of p 3, or what is the sorry not power, what is the voltage at p 3, at port three, you get incident wave, you know beta, so you can only calculate. Similarly the coupling is same, so what is v 2 minus. So, beta b 2 minus you sense here and.

So; that means, this whatever you give a signal here, just here you can give that how

much of it consist of v_1 plus and how much of v_2 plus. Now this is useful, because when directional coupler came in microwave network, people invented the S parameter measuring device; that is the network analyzer. network analyzer we saw in earlier NPTEL lecture, but the basis of that is network analysis, that this is due to its directional nature, the directional coupler can separate the incident and reflected wave as we shown here and. So, this is the basis of scattering parameter measurement by network analyzer, basically network analyzers whenever they give some power, they put a directional coupler there and by that easily by seeing its port three and port four they can, extract what is the value of that incident wave and reflected wave.

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Hybrid Coupler

$$C = 3 \text{ dB}$$

$$\beta = \alpha = \frac{1}{\sqrt{2}}$$

Now, if this coupling ratio is made three db. So, coupling ratio, what is coupling? The p_3 by p_1 by p_3 , that if you make two, so that coupler is called a hybrid coupler. So, the moment you have that you get the value of beta and alpha, also you take as $1/\sqrt{2}$, so; that means, you are dictating those alpha and beta values.

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Magic Tee (Rat Race)

- Anti-symmetric Directional Coupler
 $\theta = 0, \quad \phi = \pi$
- Hybrid Coupler
 $\alpha = \beta = \frac{1}{\sqrt{2}}$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

And that anti symmetric directional coupler, you see magic T that anti symmetric; that means, theta zero this choice we have see anti symmetric directional coupler. hybrid coupler means alpha beta is equal to 1 by root 2, so S parameter of these devices is this, and these was our magic T. Basically it is a combination of we have seen e plane T and h plane T, if you combine that in a four port device, so 1 2 port you have 3. So, you have three port as an e plan T and four port is the h plane T.

So, in a main line you take 1 e plane junction, you make 1 ha plane junction, you call them three and four ports, and the phase difference is already is there you see, we have seen that e plane T, as a phase in between the port one and port two signals there is a difference of phase of pi, and in case of h plane T there similar phase. So, that is why the phase choice is theta is equal to zero and phi is equal to pi, and both e plane and h plane T are alpha beta they are same as 1 by root 2 they are 3 db fe.

So, basically all that we have combining in a magic T and hence S parameter will take this form with this choice. So, we have chosen alpha beta theta phi those four unknowns. If we choose in this fashion you get a magic T.

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Power Division

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

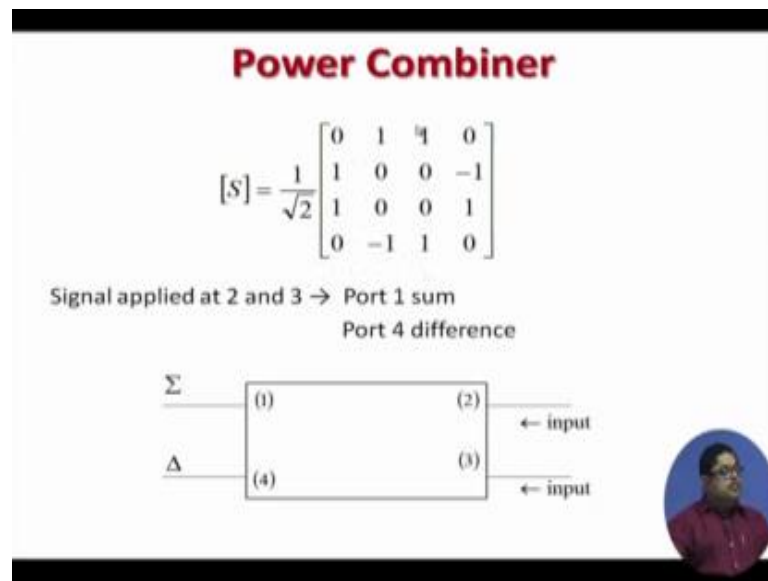
Signal applied at 1 → evenly split into two in-phase between 2nd and 3rd ports. 4th port isolated.

Signal applied at 4 → evenly split into two opposite phase between 2nd and 3rd ports. Port 1 isolated.

So, you see with this structure how it do a thus power division signal applied at port one. So; that means, I will have to look at these signal applied as port one, you see port two and three are getting equal signals. So, evenly split into two phases between second and third ports, and fourth port you see isolated. Signal applied at port four, it is evenly split into two opposite phase signals between second and three, and port one isolated. So, this is the power divider. And power wise you see, that power wise also you are having half of the power.

So, when I am exciting at port one I am getting half power in port two half power in port three. Similarly if I give it in port four I am getting half power here half power here. So, power divider magic T will do.

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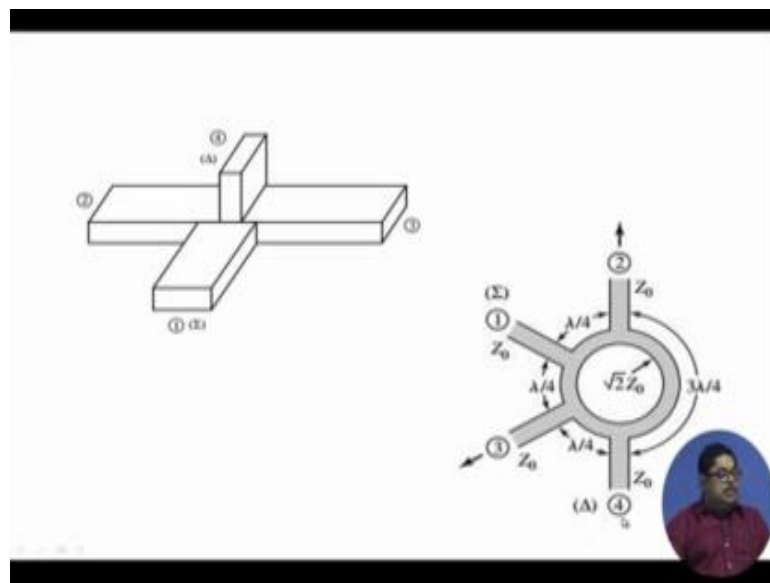


Now, let us see this power combiner. Suppose signal applied at 2 and 3. So, look at this column two, look at these column three. Now what happens to port one; port one is I am giving signal applied to port two. Let us see port one is getting 1 by root 2, signal applied to three port one is getting 1 by root 2, what is the total sum 2 by root 2, so 2. So, port one in terms of voltage it is sum of the two signals, in terms of power also it is twice the power, so port one getting sum. What is happening to port four? You see I am giving here two, so signal is, I will look at this column, what is at port four minus 1 by root 2, what is at, I am giving signal at port three what is here plus 1 by root 2 total signal zero, so port four is difference.

So, now I can say that, the if I give input at 2 and 3, these are the two input ports of this four port device port one is a summer, port four is a subtractor. So, this is called power combiner. Now this is very important of very modern radar. Modern radar has a thing calls mono pulse comparator. Suppose I send one pulse, and I want to sense the angle or exact location of an object. What they do. One signal is sent, and from the reflection I take that in the, because actually 2 1 signal is sent, but actually two beams. So, from the two different positions to offset beams are sent, and from that two reflection comes; one is in azimuth plane, two in the elevation plane, and then they make sum and difference. So, they put a magic T that will give you some signal, and different signal, and that sum

and difference if we process by signal processing with an accuracy up to order of 1 by 100 or 1 by 1000 of a degree I can say what is a position. So, without this magic T that is not possible. So, sum and difference signal, two signals if you send two beams you send of two signals, their sum and difference can be obtained from this magic T in the power combiner mode.

(Refer Slide Time: 32:44)



So, you see this is whatever we have discussed two three. So, 4 is the sum, 4 is the difference and 1 is the sum. So, this is a web guide version, this is a planar circuit version. You just need to all are, there are four ports solar characteristics impedance same. the three of them are having a path links difference of lambda by 4, one of them having 3 by 4, you can easily find out that if you give powers at these two and three, then one will act as a sum, and four as a difference. This is a magic T wave guide we have used from our student days; to our teaching days we all used this device really.

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Now, we consider is not a magic, but when first time when you come across this is really a magic. You see this same device is a divider, same device a combiner, same device is can be giving you the sum and difference etcetera. So, it is really good that if you understand this passive device then power division is not a problem for you.

Thank you.