Basic Tools of Microwave Engineering
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Lecture – 09
Broadband Impedance Matching Network Design

This lecture will be on Broadband Impedance Matching Network Design. As in the last lecture, we have pointed out single stuff, double stuff matching, etcetera they are narrow band designs scheme, but nowadays with the improvement of technology, wide bandwidth signals are coming. Wide bandwidth systems are necessary. So, impedance matching network in micro frequencies they need to also increase their bandwidth. How to attain that bandwidth briefly we will touch upon in this lecture.

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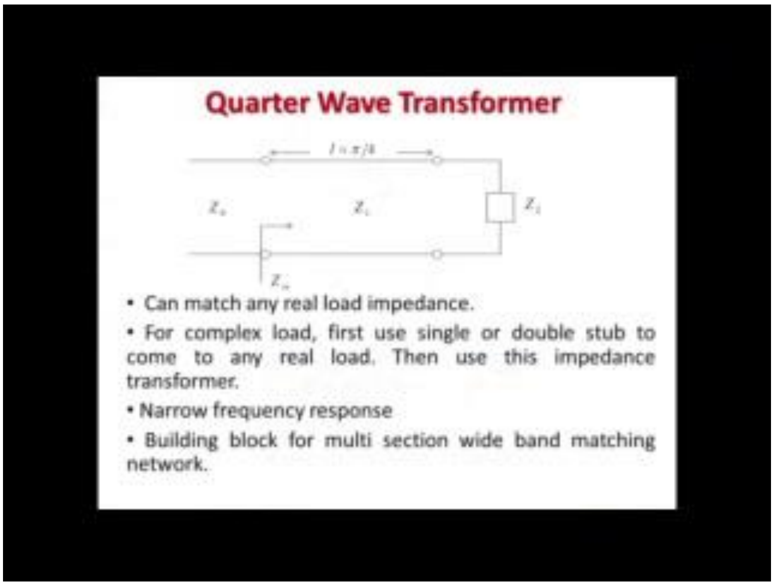

The element by which that is done can be done, one of the simple schemes is called Quarter Wave Transformer it is basically a section of a larger scheme. So, in itself it is not a broadband transformer, but with it you can achieve any broadband design that is important. A quarter wave transformers is shown in this slide, it consists of quarter wave means the length of this is lambda by four not pi by 4.

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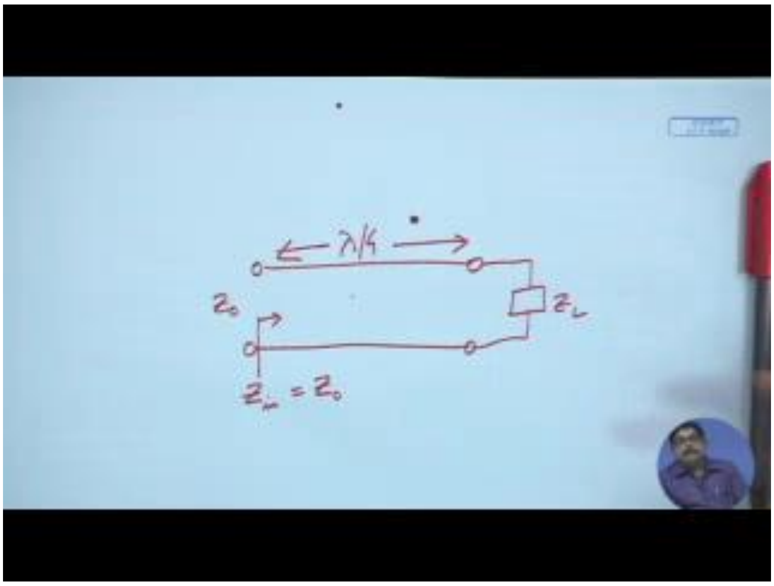

You have a transmission line like this and this length is lambda by 4 and it matches as you see that it matches a load impedance Z L and this side the characteristic impedance usually characteristic impedance of transmission lines are real quantities, load in this case it can match that means, from this side if we see we will be seeing this input impedance that will be Z naught only for real Z L, but we know that if our Z L is not real we can place an stub matching device and by single or double stub whatever you prefer you can come to any real load and then this impedance transformer can be used.

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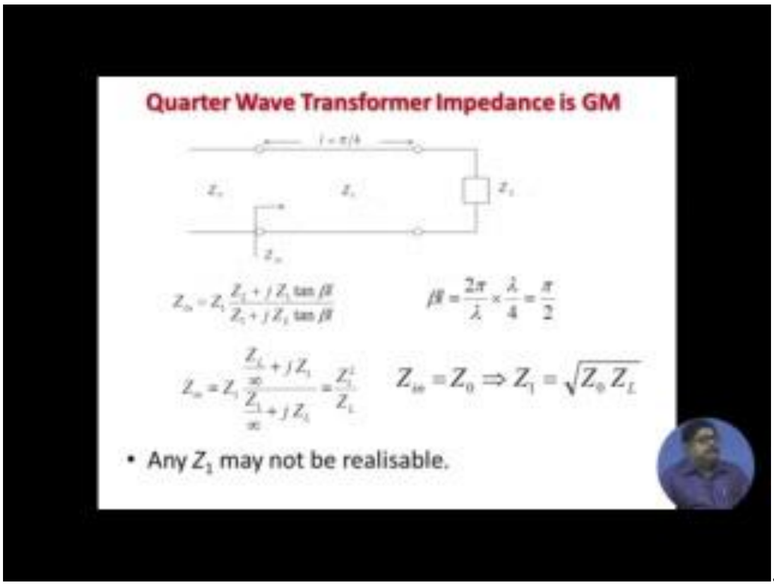

Now, this is the mathematics of quarter wave transformer, if we write the expression for this Z in that input impedance of a transmission line of length lambda by 4 and terminated by Z L then Z in comes something that we equate to Z naught, and from that we solve for what is Z naught, the quarter wave transformer characteristic impedance Z 1. So, that Z 1 turns out to be square root of Z naught Z L that means, you see the geometric mean or the transmission lines characteristic impedance and load impedance. So, by this you can get Z 1 to be any value and, but always whether we will be able to realize it transmission line with value Z 1 that is up to you, that is up to technology manufacture technology, but any value can be attempted.

So, any Z L can be put to any Z naught. So, at design frequency this match gives a good impedance match. So, reflection coefficient is 0, but roughly means if we are off a bit off from that design frequency the reflection coefficient is no more 0. So, there will be a frequency response and this if we plot this frequency response it will be quite narrow band and quite sharp; that means, just if we off the frequency off the design frequency we are not having a very good match.

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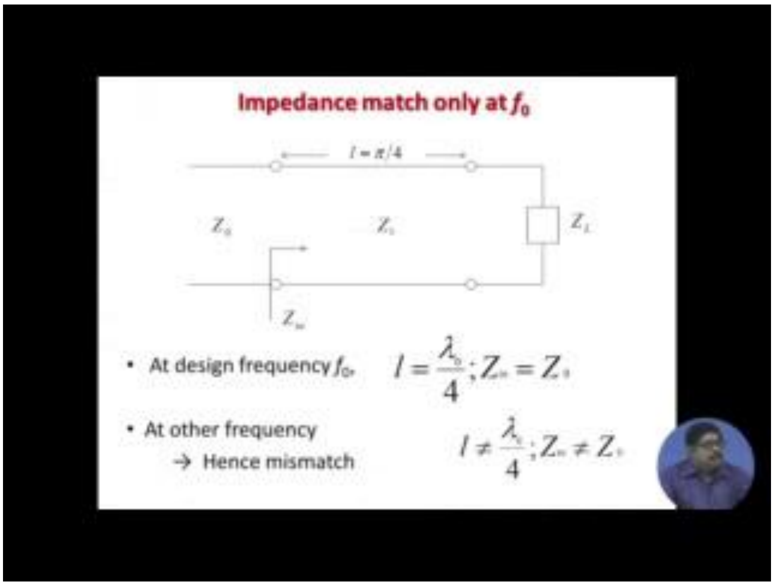

So, this L is lambda naught. So, we are calling this particular lambda for which we design that means, for a particular frequency the corresponding wave length lambda naught that we are denoting as lambda naught. So, at other frequency there will be mismatch lets us expression for that mismatch.

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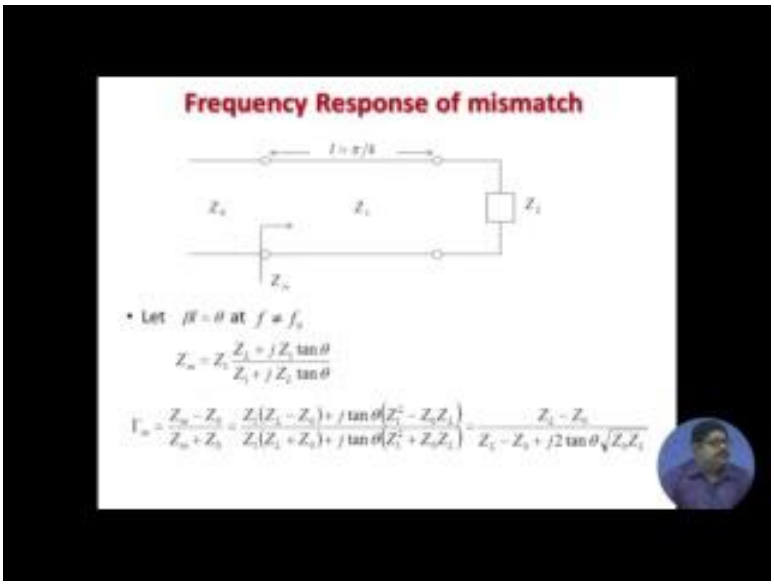

So, you see this is the mathematics, again as we have seen in case of the stub matching etcetera that beta L, the electric length of the transmission line that should be expressed as a function of f. So, that we are done and written it here, what is the reflection coefficient that we are looking at from the input port of the quarter wave transformer that is the calling the gamma in, you see this is mathematics this will be easy to understand and we are considering that we are not at a designing frequency. So, f is not exactly at f naught because if we design exactly at f naught then reflection coefficient gamma in is 0, but we are slightly off. So, we have some value that is given by these.

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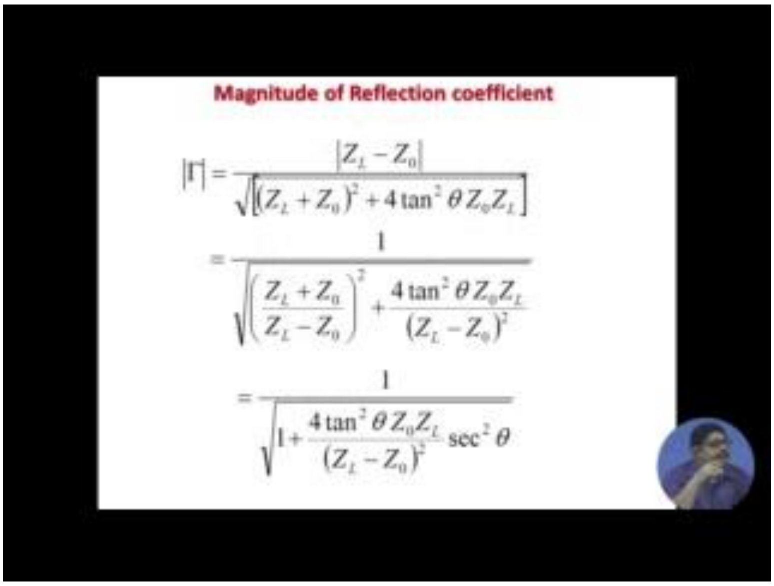

Now, this is what is the previous expression was the complex reflection coefficient. Now, we are interested in mismatch that is based W r. So, that is depend that is scalar quantity. So, the magnitude of the reflection coefficient is sufficient to get W r, that is why we take the magnitude of the reflection coefficient, and that turns to be these ultimately you will see that we some mathematical manipulation it become a some trigonometric relations.

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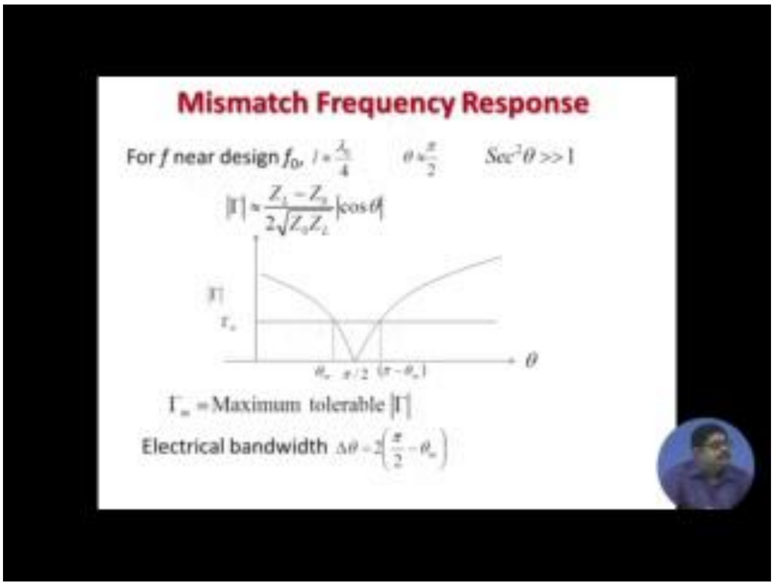

Now, we say that we are not very far away from the design frequency that means f is quite near to f naught and then L also is not very far from lambda naught by 4. So, thing is theta which is beta into L that also is quite near to pi by 2 and so, we can see that reflection coefficient near the design frequency that has a roughly cosine radiation. So, that thing is shown here, that theta versus theta into we plot the magnitude of the reflection coefficient you see the graph. The graph is a cosine sort of graph near the; obviously, if you are far away it is not cosine its exact form is given by the previous exact equation this bottom wall that is the exact one.

Now, again as I say that you can fix up what is the maximum tolerable reflection coefficient magnitude let us call that gamma m. So, if you put a horizontal line from the gamma m then you see 2 points you are getting which is cutting that frequency response. So, roughly if you stay within that theta zone then your transformer is within the acceptable zone. So, you impedance matching is, electrical bandwidth of your transformer we can say is delta theta, which is the design portion is pi by 2 for quarter wave transformer. So, pi by 2 minus theta m and we see the curvy symmetrically cos theta curve magnitude of cos theta is symmetrical means that around the pi by 2, it is having equally that is why are broken into 2 into half of this thing. So, pi by 2 minus theta m is wave electrical bandwidth.

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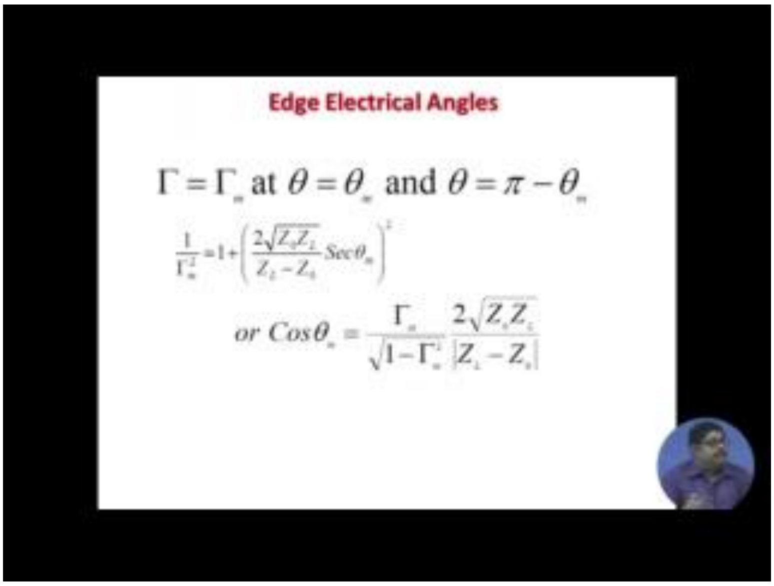

Now, at the two edges of those bandwidth you have one side is theta m we are calling another is phi minus theta m obviously, so that if you put you get that value of cos theta m. So, once this is a relation that theta m and the maximum allowable magnitude of the reflection coefficient they are related by this. If one is specified generally this gamma m is specified that what is the maximum base W r you can tolerate from that? You can find out what is the maximum reflection coefficient you can tolerate, and then you can find what the value of your theta m is?

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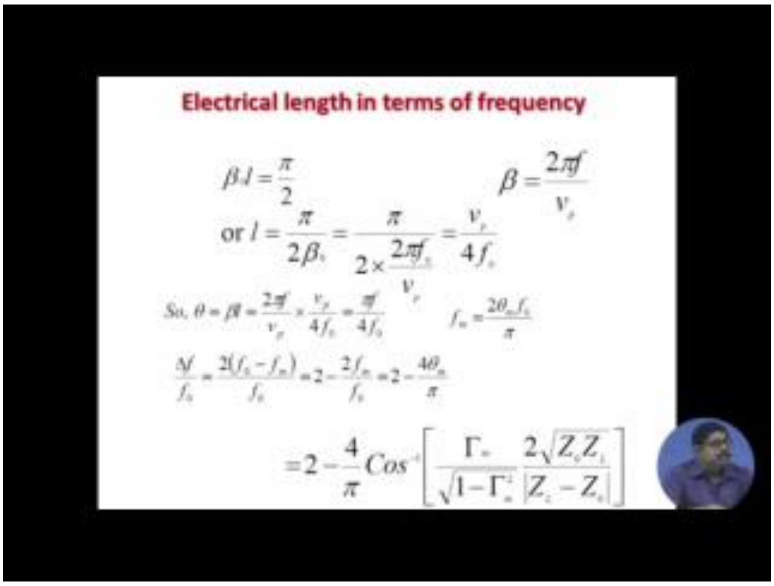

Only thing is generally we do not express the frequency response in terms of electrical length. So, that theta needs to be converted to f. Now, in a transmission line we transverse electromagnetic T m line which is (Refer Time: 10:22) etcetera there that relationship is linear in a web guide and other things, that relationship always may not be linear, but there is a definite relationship. So, based on that you can find, here we are for a T m wave we are finding that relationship. So, this thing I think is self explanatory you can put those values beta, etcetera and from that we see here we have finally, made this part that beta L is expressed as a f by f naught as in the single stub case. Now we write what is delta f by f naught one near this manipulation. So, this is in terms of frequency we can express this.

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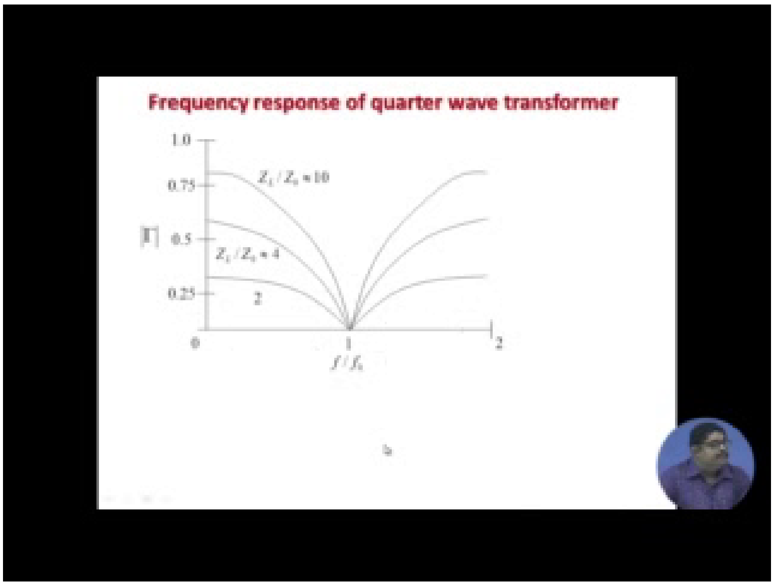

So, based on that theta value so that we have put here and you see that the actual things that the frequency response for various values of Z L Z naught that means, the source side and load side if they are very much separated like Z L by Z naught, Z L is 10 times Z naught you see it is a very narrow band frequency response. So, that quarter wave transformer is very sharp here, but if you come here that is only Z L by Z naught is four you have a wider bandwidth you take any particular value here you see that as the frequency response is less sharper more flat then your bandwidth is increasing. Now, this is the key idea for broadband design that if I have the two sides that means, load side and source side impedance's they are not very far away then I have a wider bandwidth from the same section.

So, in many real cases the load and source are separate away. So, you will get a very sharp frequency response, but instead of one section if I have multiple sections then the whole thing can be improved that each multiple section is seeing a molar change of load and source impedance and then its match will be broadened. So, the overall sum total match that will be much broadened very near to 100 percent bandwidth sort of things.

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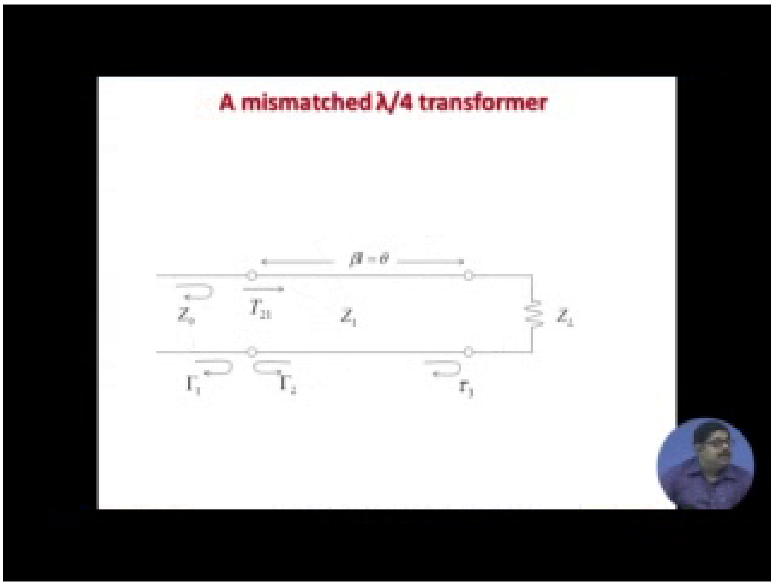

So, that is the idea and that is why we see again that same mismatch transformer, there are the wave as comes then sees a mismatch here is a mismatch transformer. So, some portion goes here, some portion comes here then again this here it comes again here it sees a match mismatch. So, it comes back with a reflection coefficient gamma 3, then comes here again here some portion comes back reflected some portion goes here that they are calling is gamma 1, etcetera.

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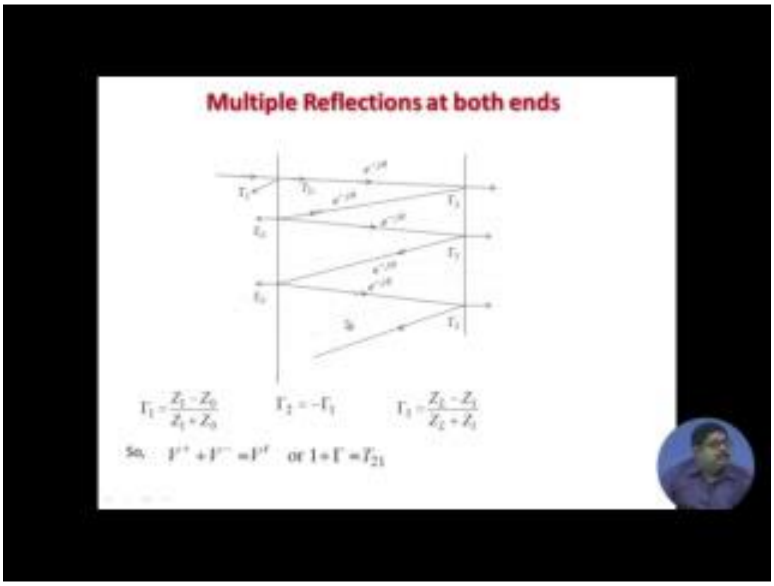

So, this one can be pictured like this one where coming then. Some portion deflected and some portion transmitted coming here then coming here, this is called multiple reflections. As we are seen here in case of power there are multiple reflections of power is similarly, here also you see lot of reflections taking place. So, we can put our electromagnetic knowledge that everywhere we know that this continuity and impedance that will give rise to a reflection coefficient.

Now, this conditions that gamma 2 that will be is then because this gamma 2 is occurring at this place gamma 2 is here, gamma 1 is here. So, the impedance level just opposite in these two cases that is why gamma 2 is equal to minus 1 gamma 3 is this. So, and you have the voltage where its continuity. So, from that you can see this comes from the power continuity.

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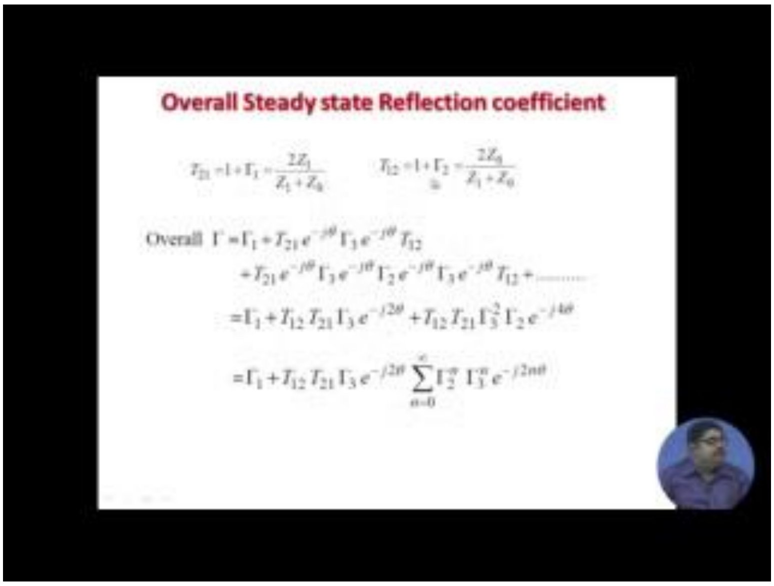

So, from that you can do this what is the overall reflection coefficient that is sum of the first reflection then the second wave going to load and coming back then the sum of that will be going? So, multiple reflections that come to these ultimately again it comes like a infinite series as we have seen odd series. So, all these cases are the same type of thing that waves they are going and coming back.

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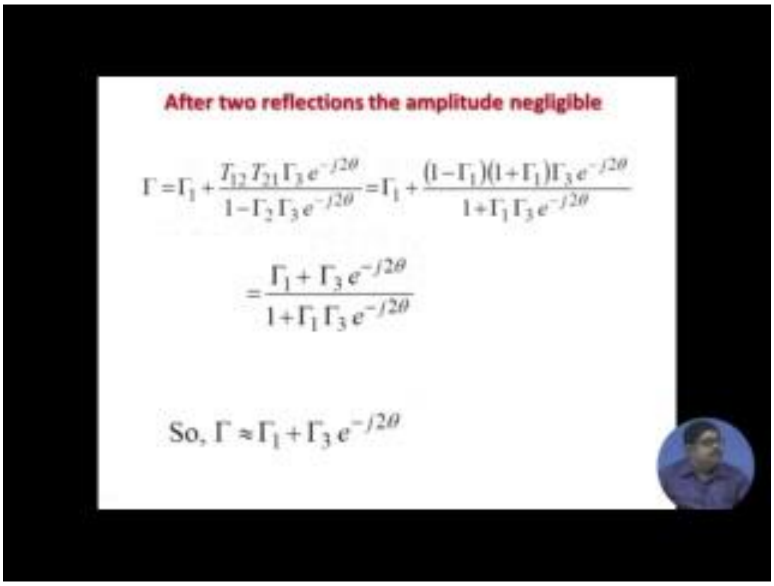

So, if you do that finally, we will see that gamma after two reflections finally, you can sum that geometric series and it becomes like these. So, the thing is mainly the first reflection is very important and also that second reflection means that one transmission going to load coming back, these two are important others are there, but the value is falling. So, mainly these two determines the steady state.

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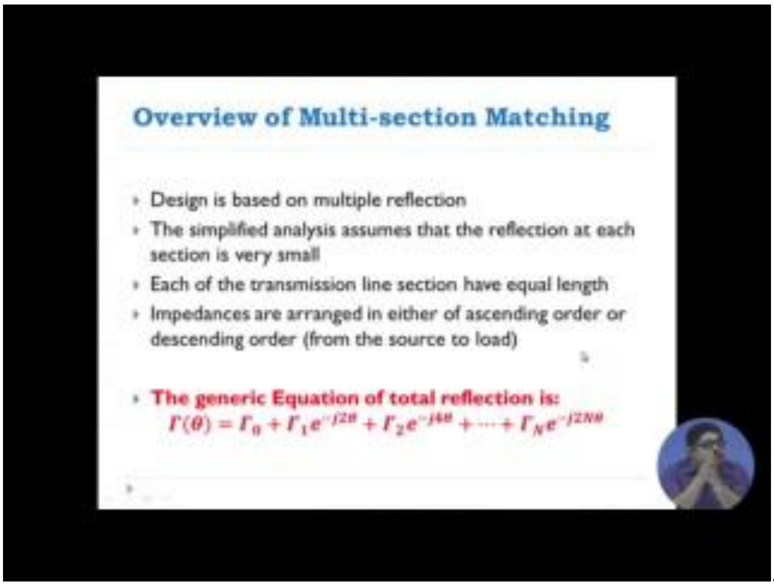

So, multi section matching that design is based on this multiple reflection phenomena of wave reflection at each section is very smaller. As I say that, if you can make the load and source not much different in impedance then the reflection is very small. Also each of the transmission and section have each lengths and impedance's are arranged in either of ascending order or descending order meaning is that, if the source is at higher impedance than load then gradually you may the from source side suppose there are multiple sections of this quarter wave transformer, quarter wave for any equal length transformer. So, characteristic impedance you decrease progressively.

On the other hand, if source is having lower impedance and load is having higher impedance then you gradually go on increasing the characteristic impedance of sections and this generic equation of total reflection we already written here you see this one. So, that just like this we can write that this will be the generic equation.

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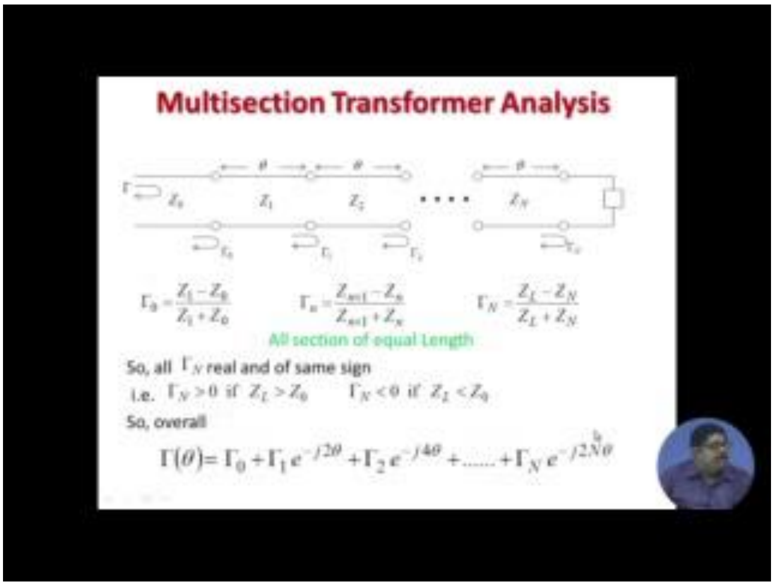

And with this, you get this is the series.

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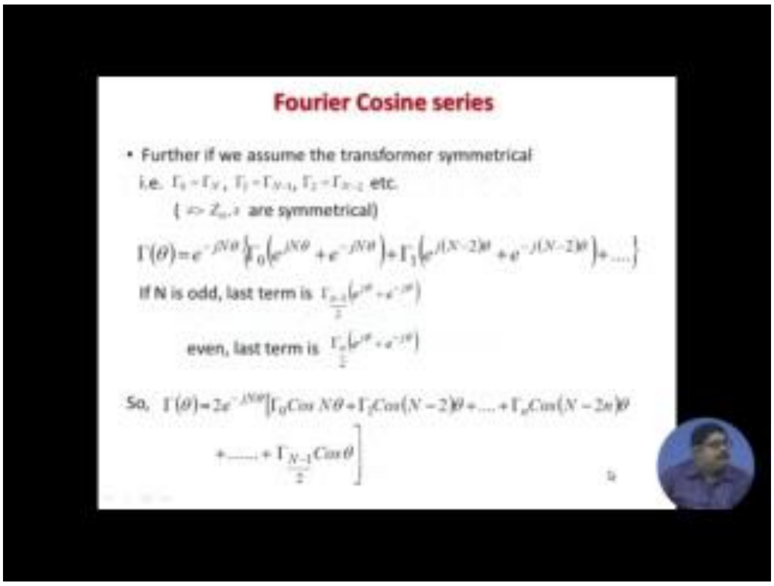

Now, we also assume two more things, one is the transformer is symmetrical, symmetrical in the sense that the reflection coefficient at the two ends the remaining is suppose I have multiple sections.

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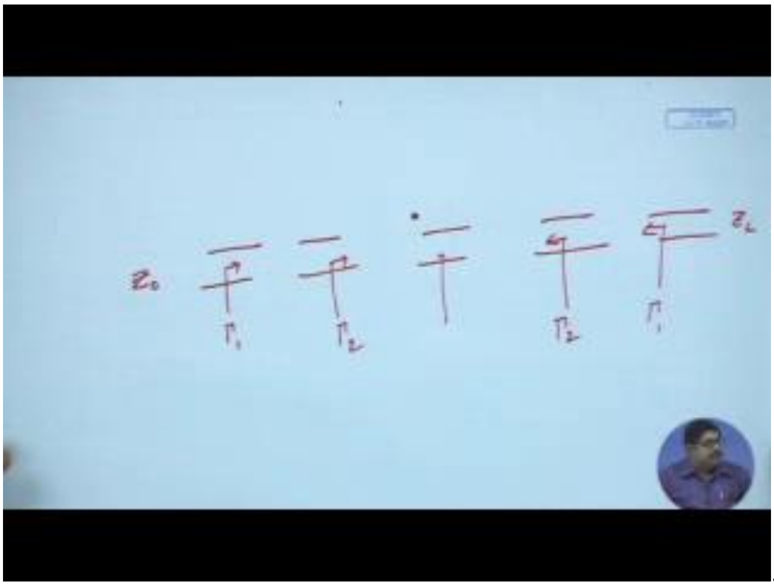

So, this side is source, this side is load. Now, I have some reflection of this section which is (Refer Time: 17:58), but we see that symmetrically from these two side the two impedance's, the two reflection coefficient. This should be same that is called symmetrical network this does not mean that their impedance's are same this means the reflection coefficient remain same.

Similarly, this one if we call it gamma 2 this also will call it gamma 2 and this one is odd man out. So, in a five one, these two are same these two are same this is something else that is called the symmetrical things gamma naught is equal to gamma n racetrack etcetera, but here we have said that Z n's are not symmetrical we do not mean that because this each one depends on this minus this, that minus thing is symmetrical. So, by that you can group this 2 and then it becomes a cosine series you see these series gamma three's magnitude that will be sum into n theta plus cos n minus 2 theta, etcetera.

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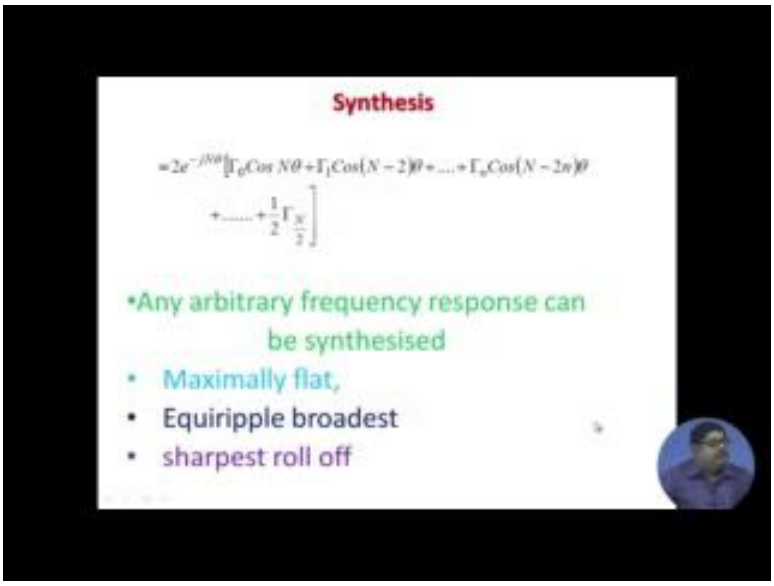

Here you can see that this is nothing, but a Fourier series so; that means, the reflects overall reflection coefficient you can express in terms of reflection coefficient of multiple sections, and this is expressed in a Fourier series we know that any arbitrary function can be represented as sum of a Fourier as a Fourier series. So, we can say that if you we want to synthesize any broadband frequency response of any shape that can be achieved here that is the guarantee of Fourier. So, any arbitrary frequency response you can synthesize.

So, people have tried that we can have maximally flat that means, we have a design frequency point where we are matched where we are now near that I will have very flat as far as possible that type of response is called quarter worth response. Now, people have also saw that I do not want any flat because I can sustain or tolerate certain amount of mismatch. So, I can have ripples that within that zone of interest the reflection coefficient may vary it can have ups and downs, but it should cross a limit and then the overall bandwidth of the impedance transformer can be increased that is called equiripple broadest or optimum design.

Now, generally chubby shape functions are there by which this design is attempted then someone may say that whatever this thing, but I want that when the I am ejecting that means, going out of my pass band that means, I am after my bandwidth I want that there should be very sharp reflection coefficient change. So, that some of the signals of those frequencies are ejected. Now, those are called sharpest roll off various elliptical functions are there which can; so all that can be synthesized by that.

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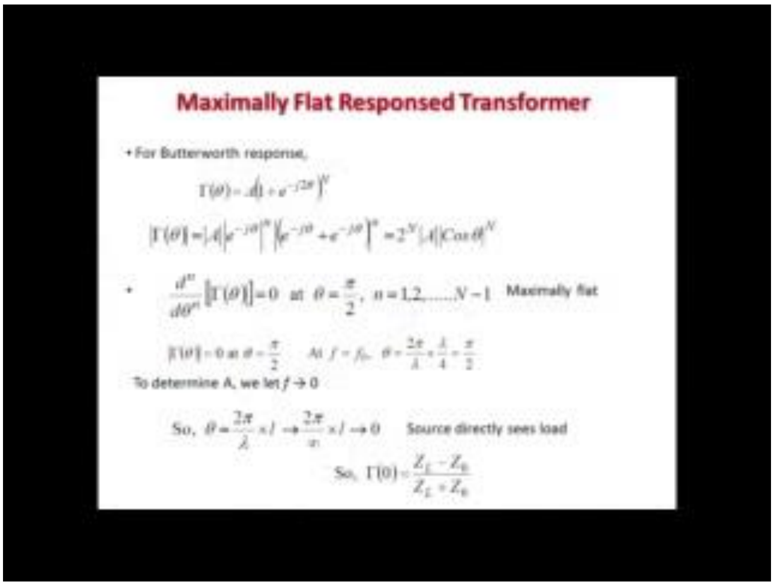

One of that we are showing here others all we cannot take in this class. So, maximally flat response, this is the well known is the quarter worth response, quarter worth response is given like this that the overall reflection gamma theta is a is a constant into you know that rules of the quarter worth function they are on a equal cycle. So, they can be in a complex spin, they can be written as 1 plus c to the power minus and if you have in nth then it is multiplied by nth quarter worth. We are assuming mathematics just to expand and if you find that the overall response, if you maximally flat means up to any differentiation of the reflection coefficient function that will be 0, that means, it will be flat up to nth order.

So, based on that finally, you see that you need to determine to design this and how many sections you will add that will be this n and also what are the characteristic impedance of the multiple sections? So, that you achieve this synthesized reflection coefficient. So, to determine what we do, we put f is equal to 0. Now, theta is electrical length is beta into L beta is 2, pi by lambda.

Now, if we put frequency 0 it is just hypothetically frequency 0 means this web length is infinity. So, two pi by infinity this theta also becomes 0. So, f is becoming 0 means theta is 0. So, we get gamma of 0 now what is the meaning that theta 0 electrical length of the section 0 that means, that there is no transmission line the source is directly seeing the load when that happens what is the reflection coefficient overall reflection provision there are not much differentiation now the source is directly giving to load. So, the reflection coefficient will be Z L minus Z naught Z L plus Z naught.

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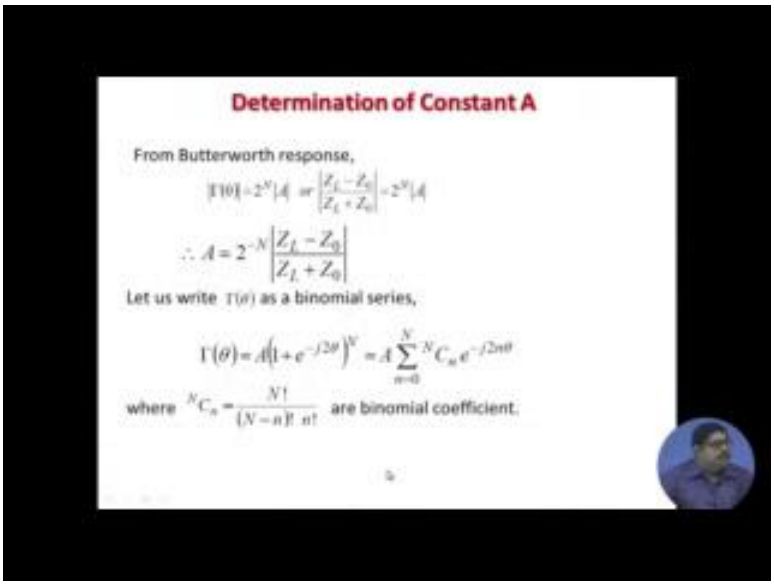

And from quarter worth response, we put it that theta is equal to 0 that gives 2 to the power n a. So, by that we can solve that a will be 2 to the power minus n Z L minus Z naught. Now, let us write this gamma theta as a binomial series that means, this is the binomial expansion. Now, if you just work out this can be written like this in terms of N C m and these are the binomial chain these are the binomial coefficient.

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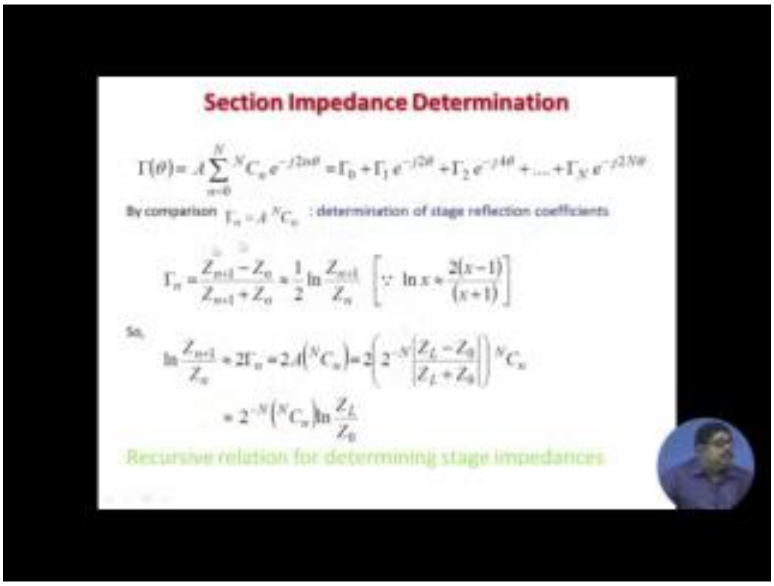

So, by comparison you can find out each stage reflection coefficient. So, you can find out each stage reflection coefficient gamma n and then gamma n can be approximated this is a automatic scheme. So, finally, this characteristic impedance of n plus 1th section can be expressed in terms of inner section this is the difference for determining stage impedance's characteristic impedance's of various stages.

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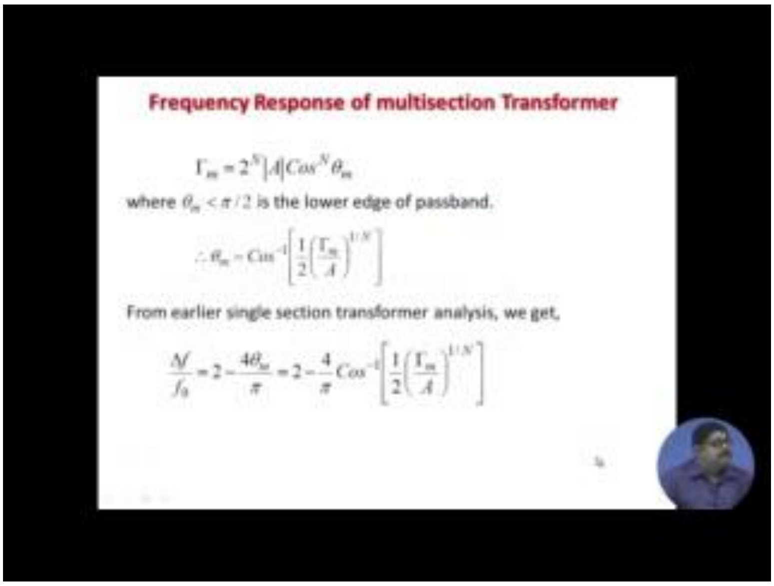

Already, we have seen that this also that we can put a maximum value of the reflection coefficient that gives us this expression. So, theta m is this. So, single section transformer analysis we have found delta f by f naught 2 minus 4 theta m by phi theta m value we can put. So, all the formulas are here you just go through these are self explanatory if you see the slides.

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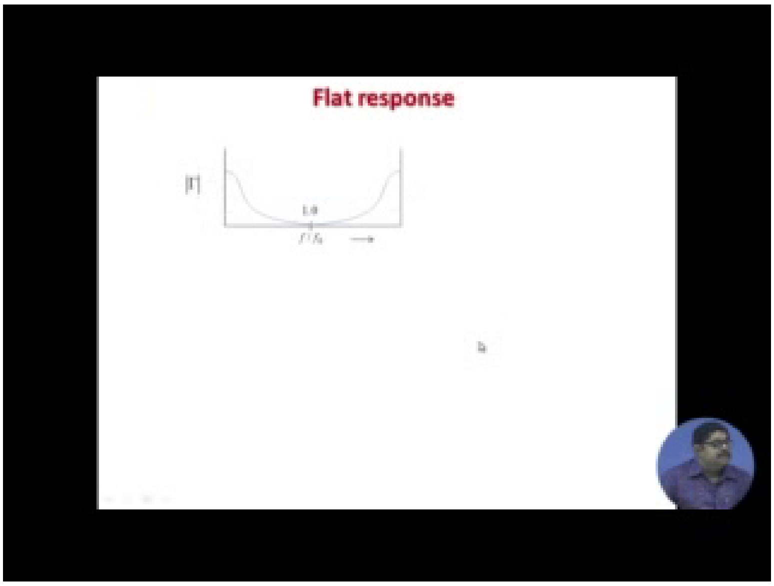

And the response you get very flat like this also this is quarter worth response.

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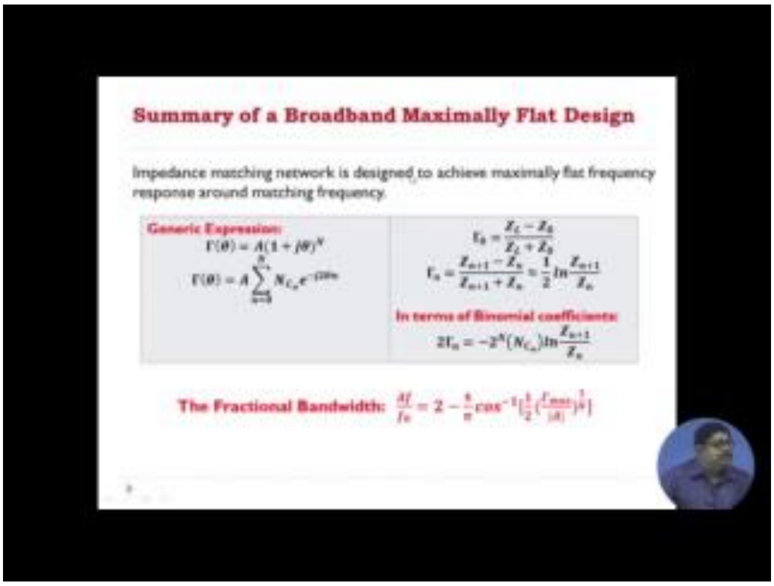

So, summary of these is impedance matching network is designed to achieve maximally flat response the generic expressions you have seen. So, finally, we see that you will have to find out, what is the gamma ends and then here this is actually half of this. So, in terms of binomial coefficient, next lecture will give an example of doing this expressions will use and the fractional bandwidth like before will calculate the properties of fractional bandwidth that we will see that the whole game is changed, and if we go on adding more and more sections we can achieve higher and higher bandwidth.

So, 30 percent, 40 percent, 70 percent, 90 percent bandwidth these are achievable only price you are paying is more number of sections etcetera and the key idea of physically you can think of this that we are not changing the source to load impedance at one go, gradually we are changing the impedance's. So, characteristic impedance of each section is slightly higher than the; let us say source is at lower impedance and load at higher. So, will have to go form lower to higher, gradually the first section will be its characteristic impedance is a bit higher than the source the next one slightly higher, the next one slightly higher. So, that nowhere we are generating a high reflection coefficient.

So, gradually the wave is allowed to match with the as progressively it is going it is nowhere seeing a very high jam that is why the reflection is low. So, more or less various frequencies there passed easily to this transformer that is the physical idea I think with this we come to the conclusion and this is the broadband. So, by this whole module we have seen that this impedance matching, what is the need and then how to achieve that? First we have seen it by passive two part designs L matching that is called then, we have seen by distributed lines that is called stabs, how to do that and then how to improve the bandwidth. So, that the impedance matching is realistic.

Thank you.