Basic Tools of Microwave Engineering
Prof. Amitabha Bhattacharya
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

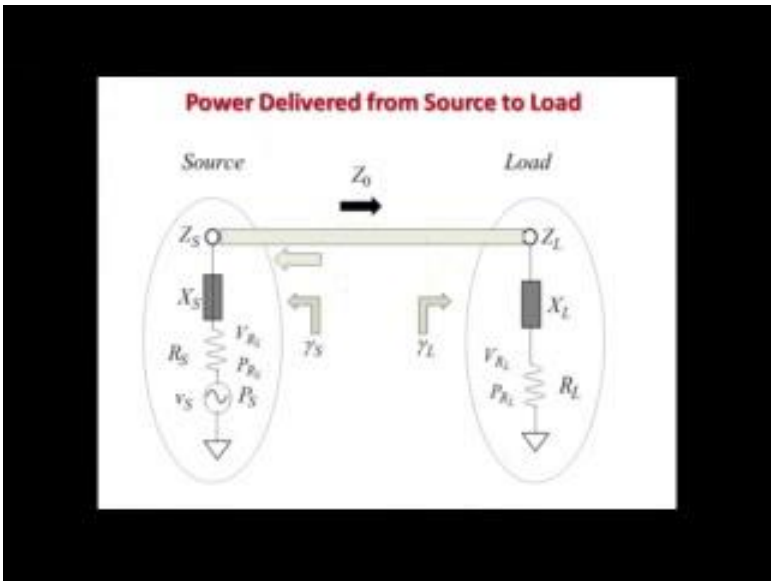
Lecture – 06
Need of impedance Matching at Microwave Frequency

Welcome to the 6th lecture. Now this week we are going to see impedance matching at microwave frequency. In the last module that is the last week we have seen the importance of impedance, how to measure impedance at microwave frequencies particularly we have seen the use of a very elegant tools smith chart, and how to use the smith chart to measure impedance.

Today, we will see that between various parts of a network, if there is an impedance mismatch then lot of power gets wasted and in microwave frequency power is at premium; particularly when we are trying to send power from one place to another place, we should always see that what are the places where the power is getting lost.

So, we try to minimize that lost and for that we need to do impedance matching to prevent that power lost. Now this concept we will see in this today's 6th lecture in detail. So, today's topic is need of impedance matching at microwave frequency.

(Refer Slide Time: 01:50)

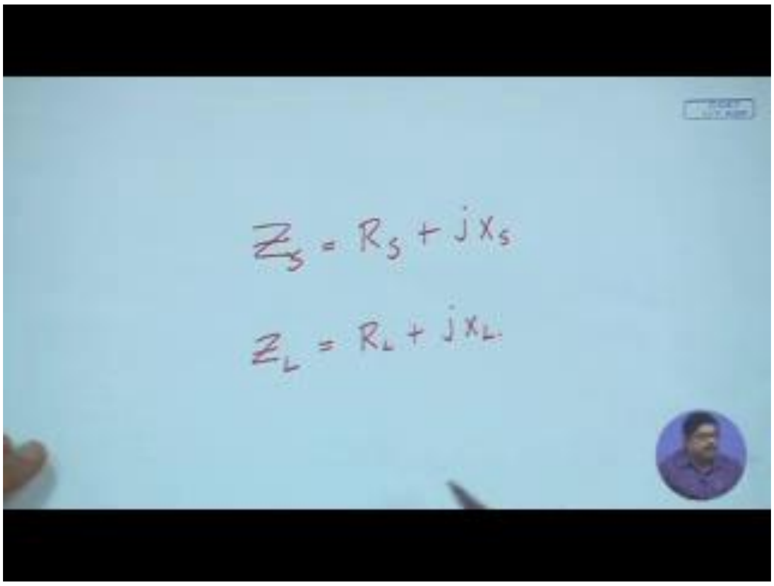


Now, this is the general thing that supposes I have a source, at the left hand side you are seeing and that source is delivering power to the load. Now; obviously, source cannot connect to a load. Suppose a base station of a cellular telephony station that mobile phone base station is trying to send power to various mobile sets that you are having. In that case how that power, the source of that or the transmitter that sense the power to a wave guide and that finally, connects to an antenna. So, that antenna behaves as a load to the transmission line.

So, in the slide you can see that the source it is delivering power to the load. Let us consider in your mental frame that the load is that antenna. So, through that there is a transmission line going. Let us consider the transmission line is having a characteristic impedance of Z 0 the source is having a source impedance also sources have some impedance there is no source without any impedance. So, let us call that Z S and the load that is having the load impedance Z l.

Now, Z S consists of 2 parts; one is the real part that is the R S and another is a reactive part that is the x s.

(Refer Slide Time: 03:29)



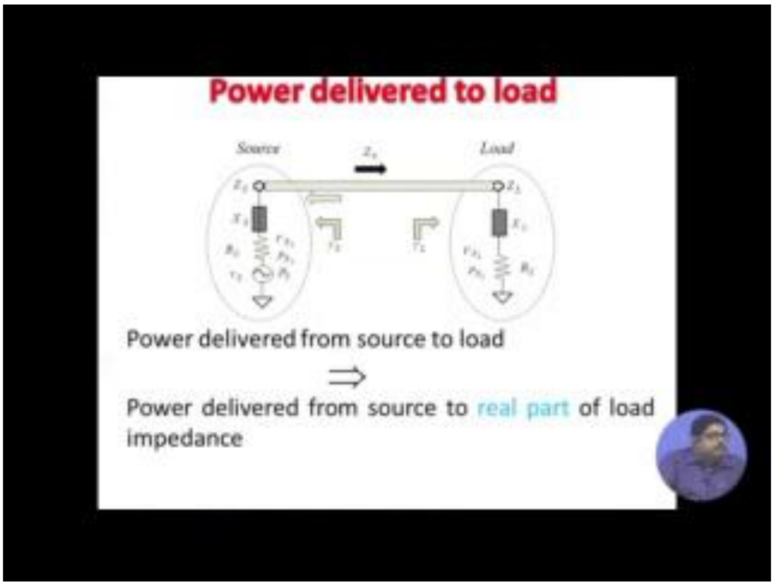
So, we can always write the Z S is equal to R S plus j X S. Similarly in the load side we have R L plus j X L and, across the resistance R S there will be a voltage drop we are calling that v R S across that resistance or in that resistance there is a power drop or power dissipated that we are calling p r s.

The voltage source is delivering a power p s out of that p R S is getting dissipated in the r s, voltage source is having an open circuited voltage of v s it is an ac source; obviously, because you cannot transmit any d c source and let us consider that similar thing is happening that, when the power is reaching the load the across the resistive part of that load impedance; that means, across R L we have a voltage drop which is v R L a power that is getting dissipated into that real part that is p R L and in general it is very difficult to have this Z S and Z L equal.

So, that generally this Z s, Z l, Z 0 because they are part of different designs a source designer or a transmitter designer fixes up some Z S to deliver maximum power outside he tries to make his Z S as low as possible. Similarly a load he tries to get that whole part it is when objective is to make the R L maximum. Now Z 0 the characteristic impedance of the line it may be wave guided may be a coaxial line thing that characteristic impedance depends on the material and also the various geometrical shapes that various conductors they are taking.

So, based on that there are various characteristic impedance. So, there is always a difference of impedance levels between this Z s, Z L and Z 0.

(Refer Slide Time: 06:05)



That is why let us see what happens. So, try to consider that the power from the source is getting delivered to load. Here I want to clarify one point, that when we say power delivered from source to load basically we mean power delivered from source to real part of load impedance; that means, basically the power that we deliver to the R L of the load. So, please remember these because the power that we get delivered to x l the reactive part of the load that is not useful power, actually that is a reactive power.

So, no practical work or much practical work cannot be done with that; that is why when we say power delivered from source to load we mean power delivered from source to real part of load impedance that is why we are highlighted that real part. So, this is just a clarification. So, you see that due to those various impedance levels; when the power is getting flowed to the load. Now from a characteristic impedance of Z 0 in the line it is seeing a load impedance of Z l. So, whenever there is we have seen that the wave sees 2 different impedances; impedance discontinuity there is a reflection that reflection we are calling gamma L in terms of power, this is power reflection coefficient gamma l.

Similarly, if any power is reflected from the load it is going to the source side to the same transmission line, and again after reaching the source it sees that there is a mismatch because source is having Z s, but the power transportation mechanism the transmission line is having the characteristic impedance as Z 0. So, again there will be a reflection that we are calling gamma s, the power reflection coefficient at the source.

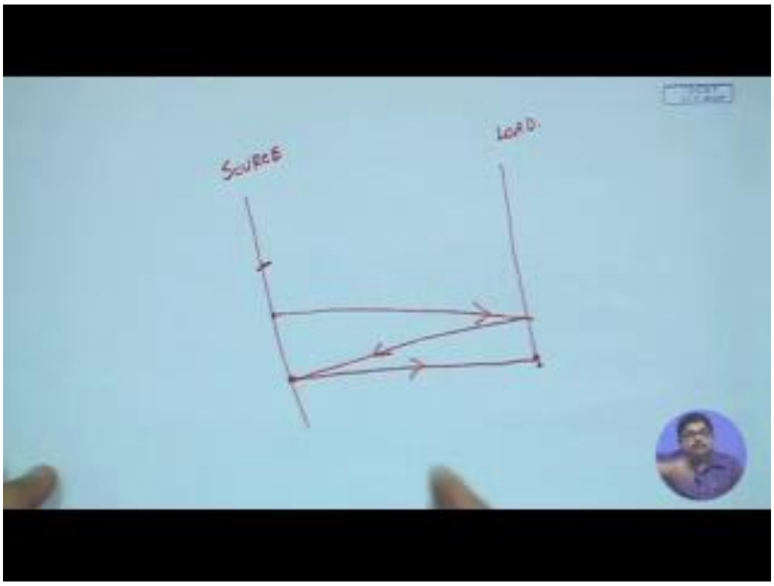
(Refer Slide Time: 08:18)

Now, actually what happens power is continuously bounces to and flow from load to source? So, let us try to see that suppose the source and load they are separated by a distance and the travel time of the wave to that distance, let us call t d. So, this; the wave is sent from the source to the load. So, after T d time it goes to load then, as there is a mismatch at the load some part of the power get reflected some part of the power stays at the load.

Now, that reflected power again comes back to the source; that will again see a mismatch because there is a gamma s, a mismatch at the source. So, again some power that will be reflected and go back to the load, again that will face another reflection at the load that will come back etcetera. So, there is an infinite search waves going back and forth.

So, at any instant, if I see that what is the total power that is coming at the load it is consisting of infinite number of previous reflection, the most recent reflection that happened only.

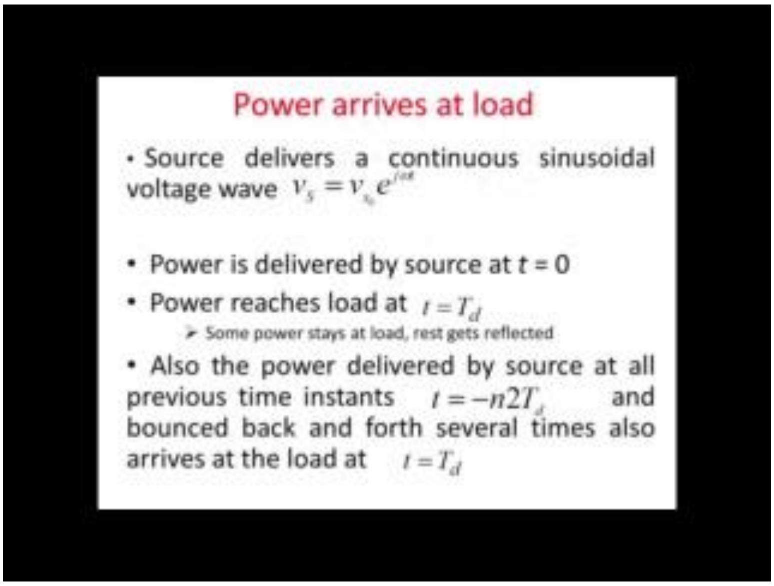
(Refer Slide Time: 10:07)



Suppose; if I look at these that this is the load this is load and this is source, then you see that suppose now I am here. So, one wave has come just T d time before. So, this source has delivered and it has come here, but this time also another one is coming, 3 T d time before there was another wave.

So, that came to the load, that came here and again coming back here; another 2 T d time before there was another wave that was emitted from source that suffered, two such reflections and came back here. So, all these at a time all these previous ones separated each by 2 T d they are coming near. So, if you look at the slide you will understand that one wave with just T d away, the wave which was emitted by the source t T d away, from this time then 5 T d away, 7 T d away, all are coming at the time.

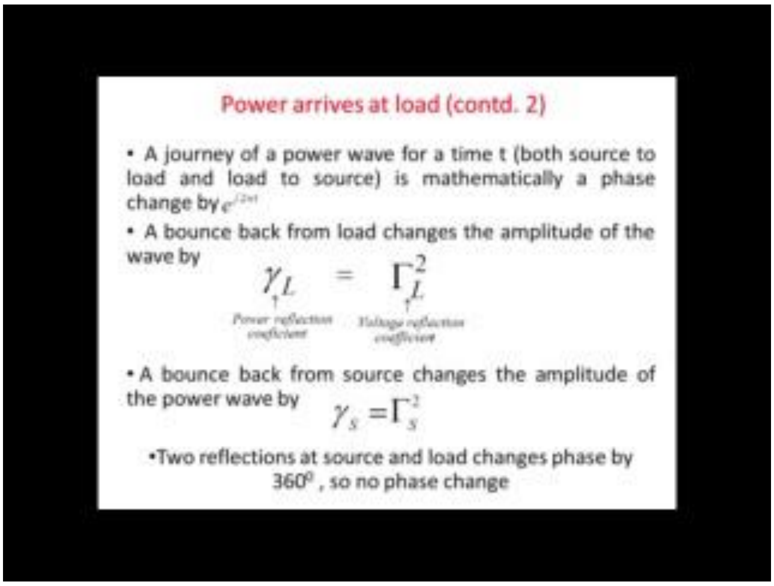
(Refer Slide Time: 11:33)



At any time if I want to find out what is the total power reaching the load that will be summation of all these infinite previously reflected waves. So, that is why we will I write that when power arrives at load basically all these things happen, but let us say source delivers a continuous sinusoidal voltage wave, we call it v s the amplitude part is of the voltage source is v s o and; obviously, with time we have a time harmonic variation any sinusoidal source with a phasor representation we can write as a e to the power j omega t.

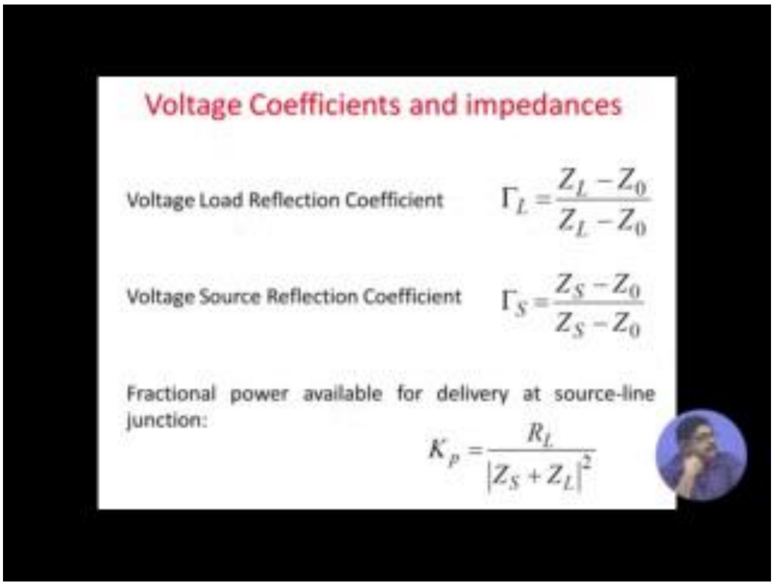
Now, power is delivered by source at t is equal to 0. Let us consider that t is equal to 0 is the time. When it has been we are counting our time presently. So, power will reach the load at t is equal to t d, after T d time some power stays at load rates gets reflected also the power delivered by source at all previous time instants given by t is equal to minus n 2 T d away. So, minus 2 T d minus; 4 T d minus 5 T d all those ones having several reflections they are also arriving at the load at that time t is equal to T d.

(Refer Slide Time: 12:57)



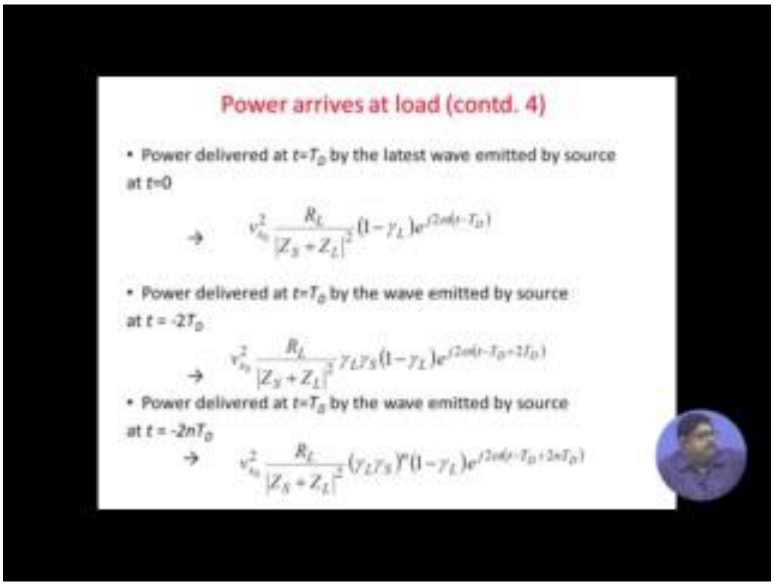
A journey of a power wave for a time t whether both source to load or load to source is mathematically a phase change by e to the power j 2 omega t this 2 please note because it is a power wave. So, in a voltage wave or current wave it is e to the power j omega t, but in a power wave this is e to the power j 2 omega t. So, a bounce back from load changes the amplitude of the wave by gamma L as we have seen and that gamma L power reflection coefficient is related in a square fashion with the voltage reflection coefficient, same thing for source power reflection coefficient.

(Refer Slide Time: 13:57)



Two reflections at source and load changes phase by 360 degree. So, no phase change with all these things we can now see that, voltage load reflection coefficient this is well known in last week's lectures also we have used these that the load reflection coefficient will be the load impedance minus the characteristic impedance of the line by oh again those same mistake Z L plus Z 0 it will be. Sorry! Please see in here last time also I did not correct it.

(Refer Slide Time: 14:28)



Gamma L is equal to Z L minus Z 0 by Z L plus Z 0 and gamma S is equal to Z S minus Z 0 by Z S plus Z 0 this plus in the denominator is wrongly given here and fractional power available for deliver here source line junction; that means, when I have source. So, the source side you see there was source, there was the Z S part and then when you are giving these to Z l.

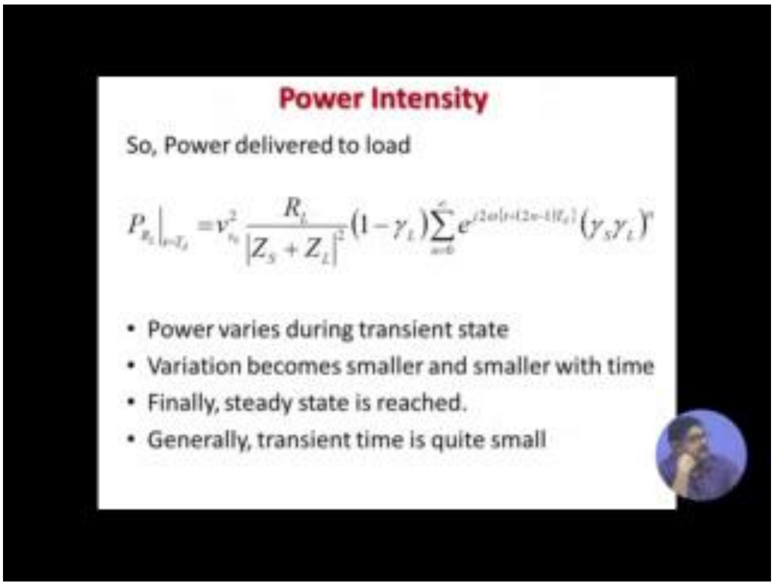
So, this is a voltage divider. So, the fraction of that is delivered to load and; obviously, delivered to load means delivered to the real part of load, which is why R L by Z S plus Z L whole square the magnitude part. Now each wave deliver some power to the load cannot deliver it is full power at any instant power delivered to the load will be is equal to mainly actually it is infinite number of those waves they are some, but we are separating that just recently emitted wave and all other multiply reflected waves.

So, that is why we are making it as sum of two parts; one part is the first part power delivered by the wave emitted by source at T d time back, just one journey time back plus power delivered by infinite number of waves emitted by source at n T d time back. Only the difference between these two groups is the first one is the first reflection that is coming. Whereas, for others there are multiple such reflections have already taken place.

So, we can say that with these we can say, power delivered to load will be changing with time. Because it is a time function that waves are coming and going. So, it will be changing with time. So, we can write an expression for that the power delivered by the latest wave emitted by source; that means, which was emitted t is equal to T d back at t is equal to zero that we can write this expression you can all know that this is the expression of power and for a wave. Similarly power delivered by the other at t is equal to minus two d is the second expression.

You will easily understand that at load it is first taking one gamma L fractional power. Then again at source it is taking one gamma S fractional power. So, that is why multiplication of that and we have written the general expression at the third expression, that the wave which was emitted minus two n T d back from our reference time. So, that when it is reaching the load T d time after the expression is like this, because it has suffered small n number of reflections. So, we can write the expression like this.

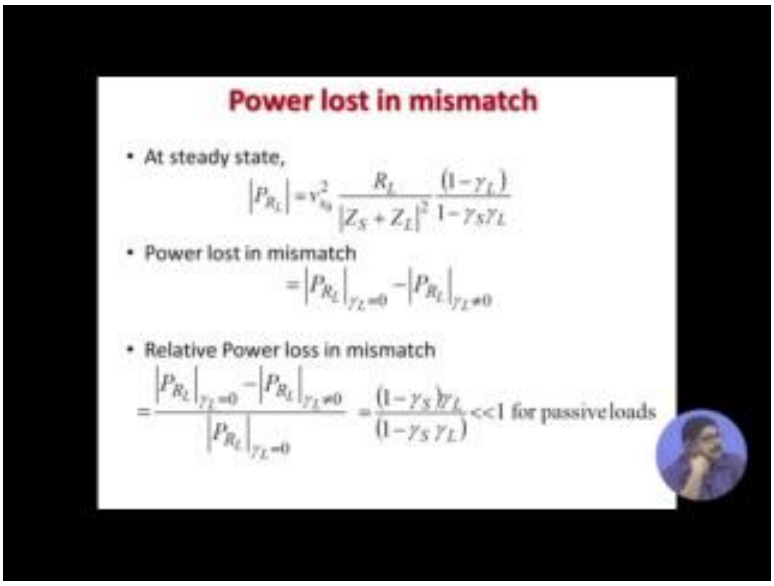
(Refer Slide Time: 18:00)



So, now we can sum the power intensity will be sum of all these and that we can write as infinite geometric series. So, we see that this thing is a thing expressed with time. So, with time there will be various values of this power that is why we say power varies during with time. So, the time when power is varies that we call transient state, but as time progresses; you see that the whole thing is getting multiplied by gamma S gamma L to the power n. Now both gamma S and gamma L they are fractions; that means, less than 1. So, as we go on increasing the value of the small n, the amount of power that is reaching that is becoming smaller and smaller.

So, we say variation becomes smaller and smaller with time. Finally, a state is raised after that practically there is no variation; obviously, there is a variation, but that is so, small. So, that time we call steady state is reached, and generally the transient time is quite small. So, after a short time that depends on the time constant of the whole network the whole thing goes to steady state and we have a power which is more or less for all practical cases constant.

(Refer Slide Time: 19:29)



Now, let us see at steady state what is the power lost in mismatch? That is shown like this, you can easily calculate because if you sum this infinite series for infinite terms you know that a geometric series the series may be infinite members may be infinite, but their sum is finite. That sum if we put, then we get the power that is delivered to the real part of the load at steady state that is a finite thing that is not infinite, and from that we can find out that if there was no mismatch in the whole thing; that means, if there was no load reflection then, what was the power and with load reflection what was the power?

Their difference is the power that is lost in this mismatch process, due to mismatch how much power is lost? That we have found and then you can find the relative power lost.

So, that expression finally, comes to this that 1 minus gamma S into gamma L by 1 minus gamma S gamma L. So, this is much, much less for passive loads, because for passive loads all these reflection coefficients they are less than 1. We have seen while describing smith chart that if this reflection coefficient becomes greater than 1; that means, we have active some devices present oscillations present etcetera, but in case of this loads, if we assume passive loads then this value is much less than 1.

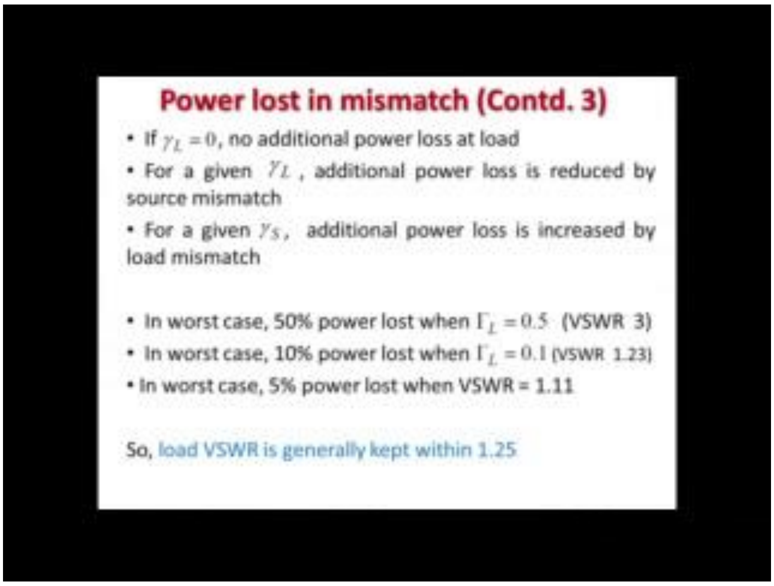
(Refer Slide Time: 21:11)

Power lost in mismatch (Contd. 2)

Γ_S (%)	Γ_L (%)	Relative Power Loss (%)	Γ_S (%)	Γ_L (%)	Relative Power Loss (%)
0	0	0.00	0	20	20.00
5	0	0.00	5	20	19.19
10	0	0.00	10	20	18.17
20	0	0.00	20	20	16.67
50	0	0.00	50	20	11.11
0	5	5.00	0	50	50.00
5	5	4.76	5	50	48.72
10	5	4.52	10	50	47.37
20	5	4.04	20	50	44.44
50	5	2.56	50	50	33.33
0	10	10.00			
5	10	9.55			
10	10	9.09			
20	10	8.16			
50	10	5.26			

Now, here is a table from that actually we have generated this table that we have taken various values of gamma L and gamma S and found out that how much is the relative power lost in percentage. So, you see there are various things, but see the extreme bottom column or the last column of this table. It says that if gamma S is 50 percent means 0.5 and gamma L is 0.5 then, relative power lost is 33 percent; that means, one third of the power is lost in this mismatch process. So, if I am trying to send three watts of power one watt power will be lost due to this mismatch.

(Refer Slide Time: 22:03)

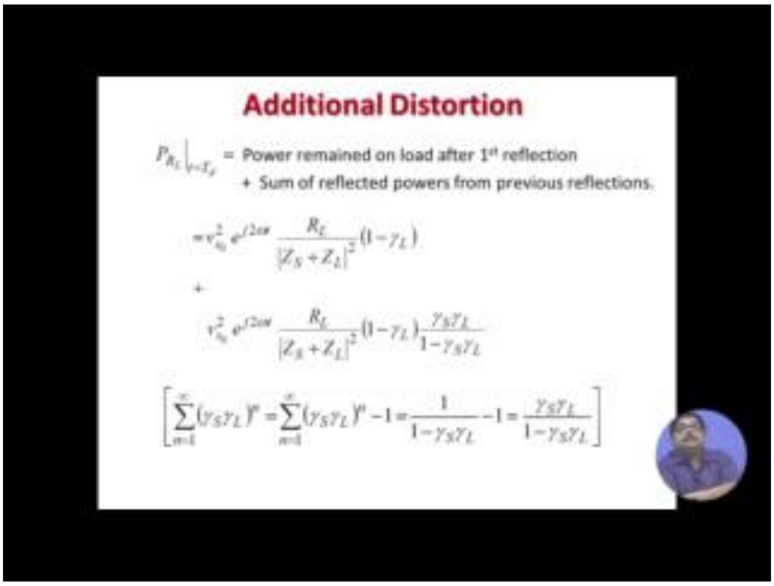


Let us see that what the take from the table are, that if there is no load reflection then no additional power lost at load. So, power is not lost at load. So, that is a good; that means, if we can achieve that gamma L is equal to 0, no load reflection then no power is lost in this mismatch process, but if we have a given load mismatch; that means, given load reflection coefficient then, additional power lost; that means, this lost is reduced by increasing source mismatch; obviously, physically it is correct because if the source is having mismatch. So, again some of that power will come back to the load.

So, that can be also seen from the table previous table that you see we have kept gamma L constant and with more increase in the gamma S you see the power lost actually is coming down. Take any search where gamma L is fixed, but with change of increase of gamma is the power lost that is coming down. So, that is obvious that with increased of source mismatch, we are getting again some of that power is coming back to the load. On the other hand for a given source mismatch; that means, given gamma S this power lost is increased, if we increase the mismatch because more mismatch in the load means more power is coming to the source. So, that is the deterioration of the power lost.

Now, see the worst case thing 50 percent power lost when gamma L is equal to 5, when if we have gamma L 0.5 then, half of the power is lost in now gamma L 0.5 means VSWR is 3, similarly if gamma L 0.1, 10 percent power is reflected you see this is gamma L is the voltage reflection coefficient. So, our power reflection coefficient is different that will be 0.25 we can see that thing that 0.25. So, here you see gamma L 0.5. So, you see for VSWR 1.23. We have 10 percent power lost and in worst case when we are keeping VSWR to be 1.115 percent power is lost. So, load VSWR is generally kept that is why it is our drive to keep the VSWR of the load within 1.25 percent.

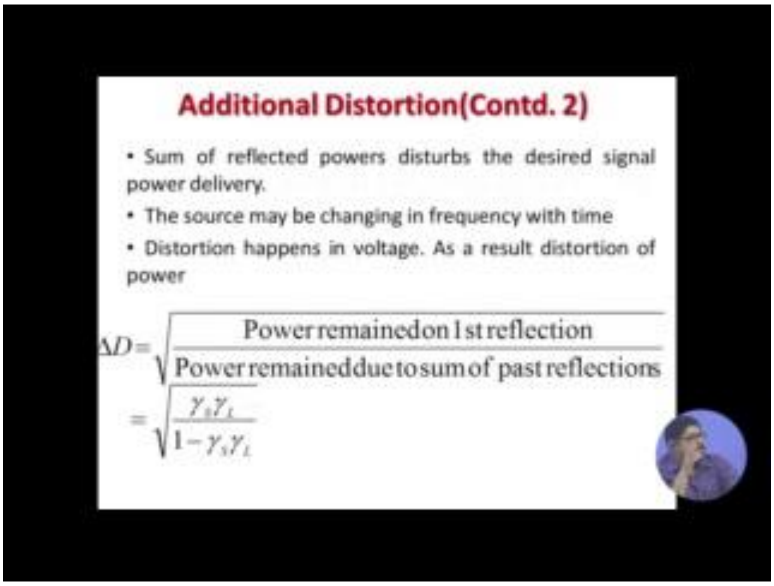
(Refer Slide Time: 25:25)



Also apart from this lost there is a distortion associated, because as you see that at any time total power is power what is coming due to the last emitted source plus previous sources. Now sources they generally any oscillator it has some free running. So, after some time it changes it is frequency; that means, the waves which were generated earlier maybe there their frequency was different from what we have.

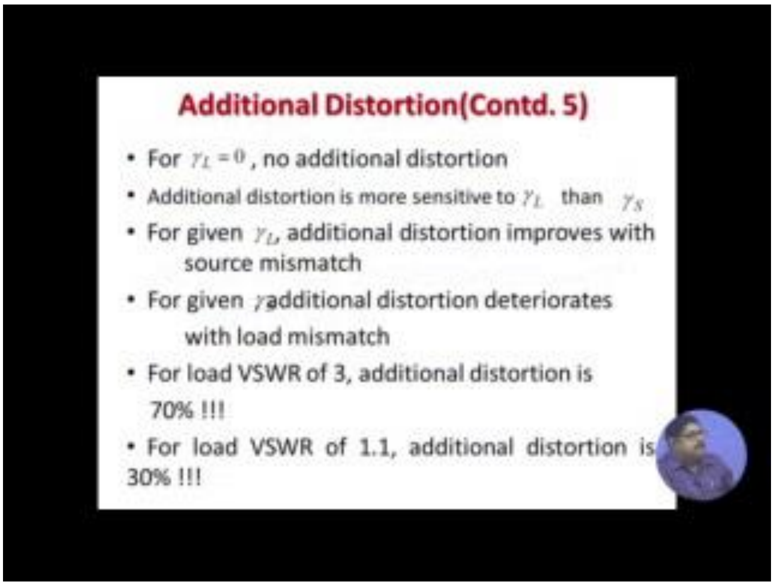
So, when finally, we are getting the signal that time we are getting a distortion in may get a distortion in the signal because, so many waves which are emitted at different times they are getting some. So, some of them may be having different frequencies than others though all of them are emitted from the same source, but sources also any oscillator is not a stable one for that you need a costly oscillator. So, otherwise they generally dipped a bit, frequency changes.

(Refer Slide Time: 26:44)



So, here we have tried to find out to see all the mathematical details are explained in the thing. So, what is that distortion we are getting and remember that distortion in power it is meaningless distortion always happens in the voltage, so ultimately we have found that what is the distortion power remained on first reflection, divided by power remained due to sum of past reflections and that expression we have given.

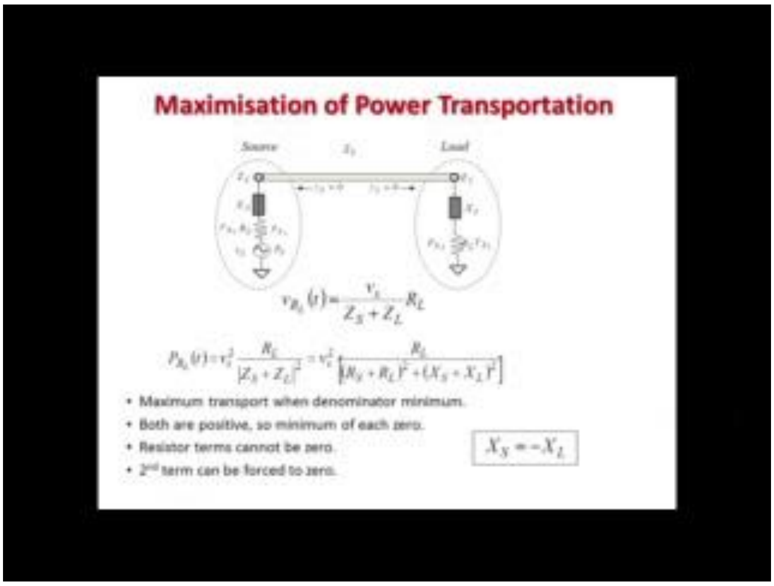
(Refer Slide Time: 27:08)



Now, again we have made a table like that, you can see the table and see that for again gamma L is equal to Z no additional distortion. So, again additional distortion is more sensitive to load mismatch, the same thing now see the worst case values that for load VSWR of 3 additional distortions, may be 70 percent. On the other hand if VSWR is 1.1

additional distortion is 30 percent. So, you see there is always a high chance of distortion, if you have good amount of mismatch between the load and the transmission line.

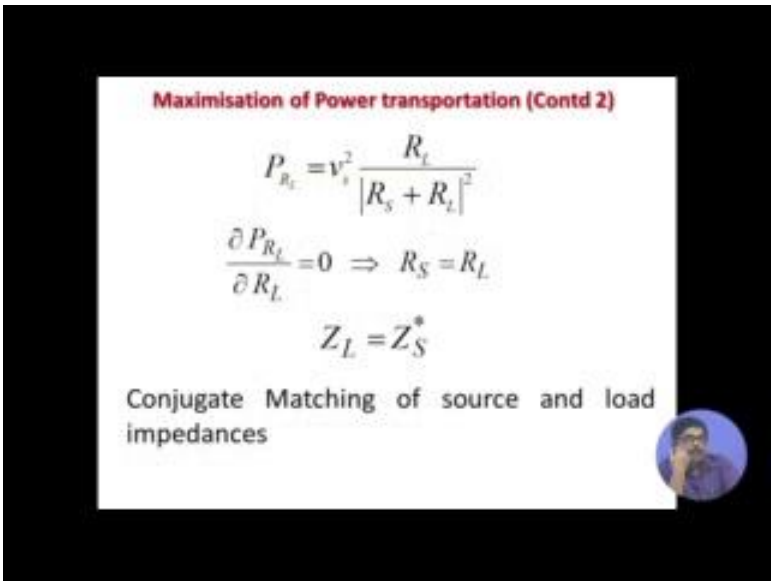
(Refer Slide Time: 27:49)



Now, let us do the try to find out under what condition this power can be maximized. So, we see that the power that is delivered to the load or real part of load v R L that is actually v s by Z S plus Z L into r l. So, from that we have found the power expression and; obviously, if I write the power expression, it comes like that all of these you know. Then maximum transport will occur, when the v R L is maximum; that means, when the denominator is minimum.

Now, denominator you see is sum of two square quantities. So, minimum of any square quantity is 0. So, each of them can be zero, but R S R L they are the resistive part now they are fixed they cannot be changed. So, resistors stumps cannot be 0 generally, because it is very difficult to make a circuit without any resistor both the source and load they will have some resistances, but reactive terms their frequency dependence. So, we can make that x s plus x l their sum we can make 0. So, this we can enforce second term can be forced to 0; that means, we can force x s is equal to minus x l source reactance is equal to negative of load that we will make it minimum.

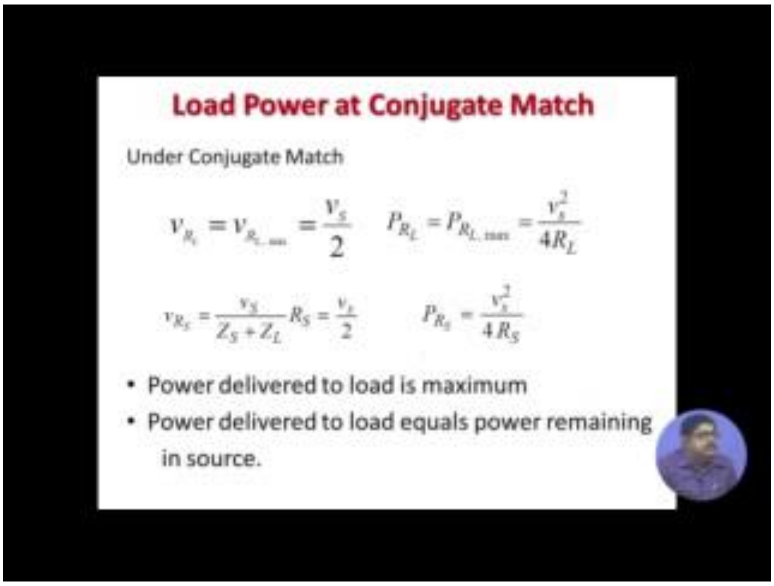
(Refer Slide Time: 29:24)



And that time if we under that criterion p R L will be this, then we can see that whether this is the maximum or minimum further we can differentiate that with respect to R L and that gives the condition that r s; that means, the source resistance should be equal to r l.

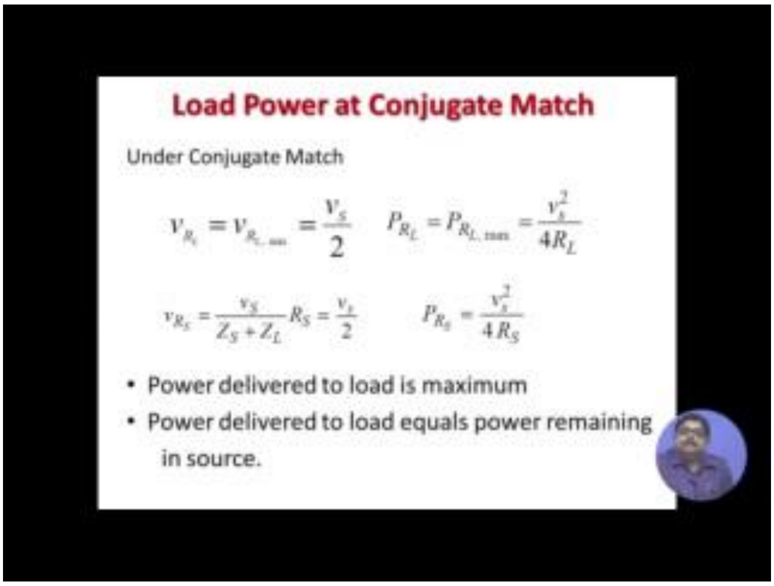
So, source resistance is equal to r l, source reactance is negative of R L so; that means, the source impedance that is complex conjugate of load impedance this you know is called conjugate matching of source and load.

(Refer Slide Time: 30:03)



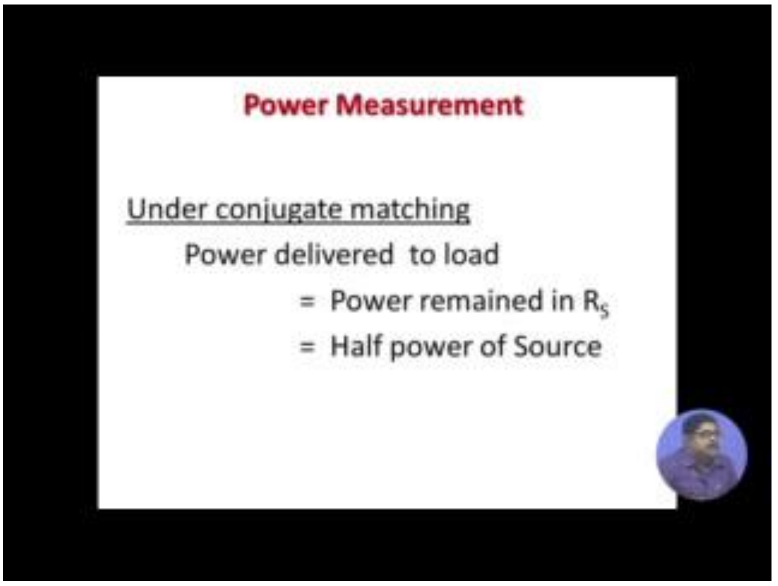
So, if conjugate matching occurs; load power at match. So, under conjugate match, we know what is the voltage developed at the load, that you worked out because that time no reactive part in the total reactance is 0. So, voltage, but two equal resistances this R S and R L equal. So, half of the source voltage is across R S half of the source voltage is across r l.

(Refer Slide Time: 30:46)



So, power we can calculate that will be the v s square by 4 r l. So, power delivered to load is maximum power delivered to load equals power remaining in source. That means, in the as per the source resistance what is the power the same power you can deliver to the load not more than that.

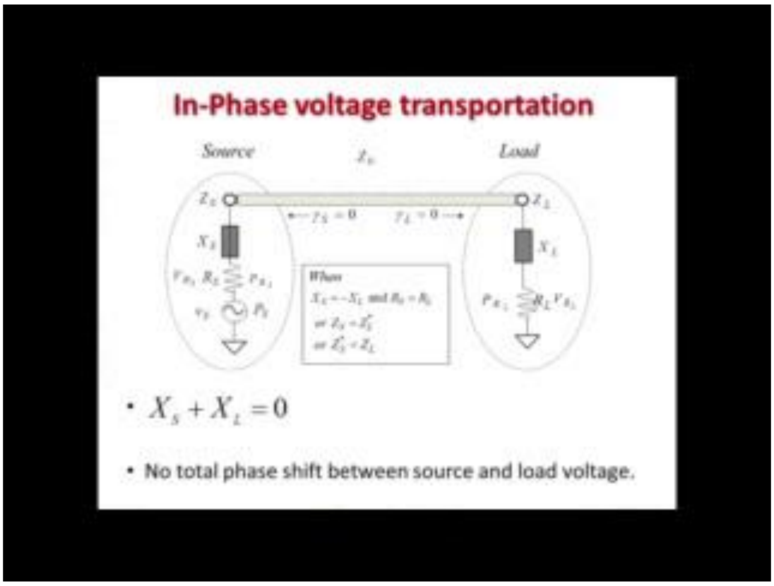
(Refer Slide Time: 30:50)



So, now this is one beauty, this is a byproduct of this thing, t hat under conjugate matching since power delivered to load is equal to power remained in source. So, source is giving half power now even if suppose that case I was saying cellular telephone it is radiating. So, load is not accessible to you, but you can always make a measurement at your source that across the source resistance if you find that half of the power is there; that means, you know that you have got a conjugate match and load is getting half of that power.

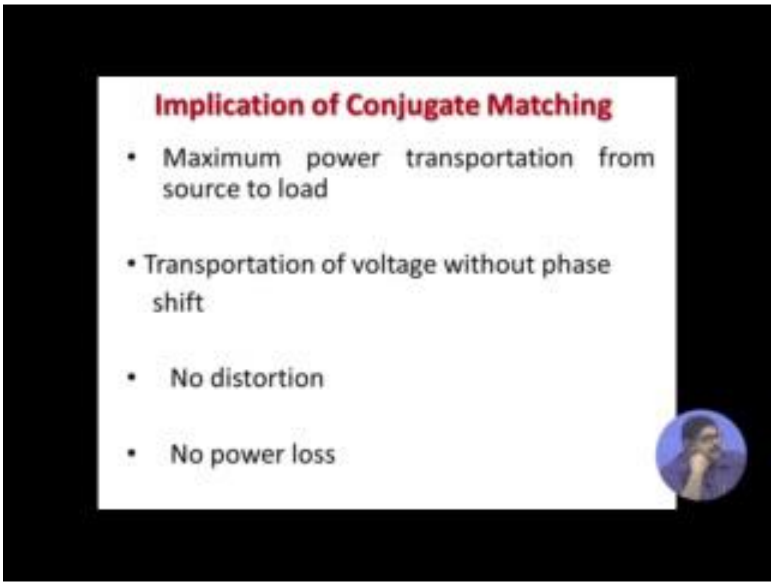
So, this is the basis of power measurement. So, at the transmitter itself we can find out how much power the load is taking basically this is power measurement one of the technique is this that, you find out at which voltage or you can also find out what is the load you find out when you are getting half of the power, because you know the source power. So, let us say source power is one watt the moment you see that across R S you are getting half of that power; that means, half work you know that you have got a match. So, the source is equal to the time conjugate match.

(Refer Slide Time: 32:00)



Now since, the total reactance of the whole network under conjugate match is 0. So, we can say that there is no phase shift between the source voltage and load voltage. So, this is call in phase voltage transformation.

(Refer Slide Time: 32:18)



So, what is the implication of conjugate matching one we have seen that it is maximum power transportation from source to load, transportation of voltage without phase shift also under that case the gamma l, load deflection coefficient is 0 gamma S is 0. So, no distortion will be there no power lost will be there.

So, under conjugate match we can get lot of things. So, that is why the conjugate match is a very desired thing at microwave frequency because lot of difficulties that power lost, distortion in phase transformation etcetera can be avoided if we can do conjugate matching. So, this lecture tried to tell that why conjugate matching or impedance matching is necessary in case of microwave networks, and the ideal impedance matching is called conjugate matching where the load and source they are complex conjugate to each other load and source impedance.

Now, in the next lectures we will see that how to design networks which achieves this conjugate matching.

Thank you.