

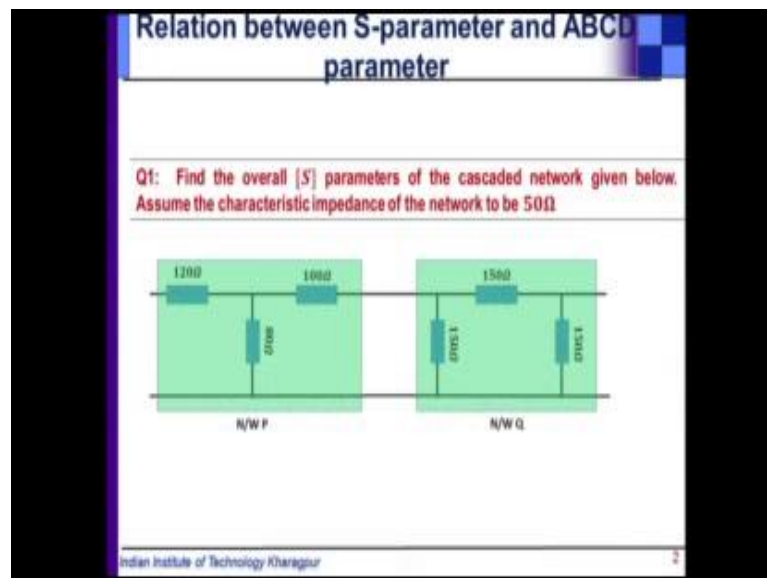
**Basic Tools of Microwave Engineering**  
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**Lecture - 20**

**Tutorial 4: Problem Solving Related to S-Parameters and Signal Flow Graph**

Welcome to the last lecture of this course. In the last tutorial also; tutorial 4 we will solve some S parameter problem and signal flow graph problem, the first problem that find the overall S parameter of the cascaded network given below.

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Obviously that there are 2 networks you see 2 basically 1 tee network cascaded with a pie network values are given characteristic impedance is 50 ohm.

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### Relation between S-parameter and ABCD parameter

**Solution:**

If two networks are connected in cascade then the ABCD matrix of the network can be multiplied to get the overall ABCD matrix of the equivalent network.

So, the ABCD matrix of the equivalent network can be written as:

$$\begin{bmatrix} A^E & B^E \\ C^E & D^E \end{bmatrix} = \begin{bmatrix} A^P & B^P \\ C^P & D^P \end{bmatrix} \begin{bmatrix} A^Q & B^Q \\ C^Q & D^Q \end{bmatrix}$$

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So, it is easier since this is cascade of 2 networks, we can convert to ABCD matrix and so ABCD matrix of equivalent network you know this can be written like this.

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### Relation between S-parameter and ABCD parameter

The ABCD parameters of the network can be calculated by using the following formulae.

$$A = 1 + \frac{Z_1}{Z_3}$$

$$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$C = \frac{1}{Z_3}$$

$$D = 1 + \frac{Z_2}{Z_3}$$

So,

$$\begin{bmatrix} A^P & B^P \\ C^P & D^P \end{bmatrix} = \begin{bmatrix} 2.5 & 370 \\ 0.0125 & 2.25 \end{bmatrix}$$

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So, ABCD parameters of a tee network, you can easily find out, this is your B. Tech level knowledge then. So, the first network tee network you can find the ABCD matrix to be this, then the pie network that either you can convert to tee network and then find the ABCD matrix.

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### Relation between S-parameter and ABCD parameter

Similarly, for the network Q, to determine the ABCD parameters we can convert it to equivalent T-Network

So,

$$\begin{bmatrix} A^Q & B^Q \\ C^Q & D^Q \end{bmatrix} = \begin{bmatrix} 2 & 150 \\ 0.02 & 2.0 \end{bmatrix}$$

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So, these 2 we have found.

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### Relation between S-parameter and ABCD parameter

Now, the overall ABCD matrix of the given network is:

$$\begin{bmatrix} A^E & B^E \\ C^E & D^E \end{bmatrix} = \begin{bmatrix} 2.5 & 370 \\ 0.0125 & 2.25 \end{bmatrix} \begin{bmatrix} 2 & 150 \\ 0.02 & 2.0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A^E & B^E \\ C^E & D^E \end{bmatrix} = \begin{bmatrix} 12.4 & 1115 \\ 0.07 & 6.38 \end{bmatrix}$$

So, we can now convert the overall ABCD matrix to its corresponding S-matrix to get the overall S-parameter of the cascaded network:

Hence,

$$[S] = \begin{bmatrix} \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} & \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \\ \frac{2}{A + B/Z_0 + CZ_0 + D} & \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \end{bmatrix} = \begin{bmatrix} 0.56 & 0.47 \\ 0.44 & 0.29 \end{bmatrix}$$

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So, overall ABCD matrix is this and then this ABCD matrix can be converted to S matrix by going like that. This is to show you the usefulness of the ABCD parameter if you can convert from S to ABCD nothing like that.

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### Numerical on S-parameter of a Directional Coupler

**Q2:** The scattering matrix of a lossless directional coupler is given below. Determine its directivity, coupling factor and isolation.

$$[S] = \begin{bmatrix} 0 & 0.92 & j0.32 & x \\ 0.92 & 0 & 0 & j0.32 \\ j0.32 & 0 & 0 & 0.92 \\ x & j0.32 & 0.92 & 0 \end{bmatrix}$$

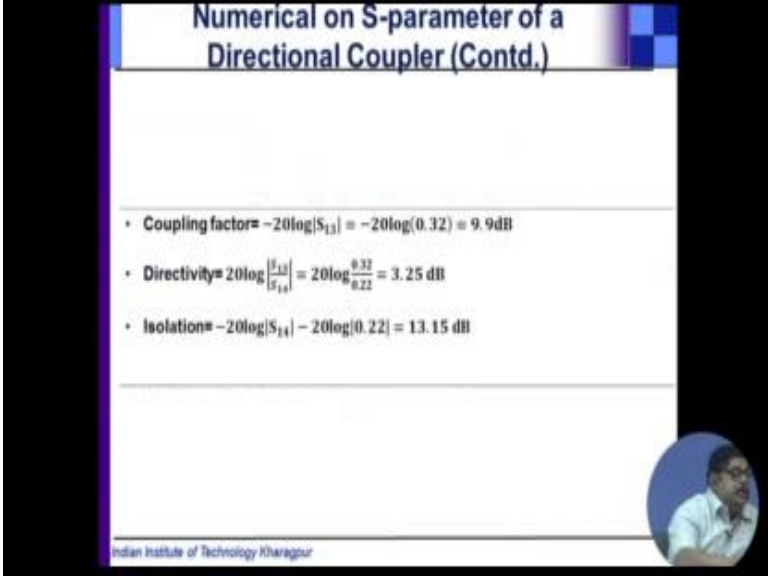
**Solution:**

- The directional coupler is lossless, so by unitary property:  
 $|0.92|^2 + |j0.32|^2 + |x|^2 = 1 \Rightarrow x = 0.22$
- The generic s-matrix of a symmetrical coupler is:
 
$$\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$
- By comparing we get  $S_{13} = j0.32$ ,  $S_{12} = 0.92$ ,  $S_{14} = 0.22$

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Now, let us come to the second problem; the scattering matrix of a lossless directional coupler is given, determine its characteristic parameters that is directivity coupling factor and isolation. You see here deliberately 1 value is not given, the  $S_{41}$  value  $S_{41}$  or  $S_{14}$  whatever you said let us call it as  $x$ , but it is told that it is lossless. So, the  $s$  matrix will be unitary. So, we know that unitary means any column (Refer Time: 03:00) is self conjugate will be 1. So, for the first column we can say  $0.92$  square plus  $j0.32$  square plus  $x$  square should be 1 that reveals the value of  $x$  to be 0.22. Now, in a symmetrical coupler, we know this is the generic value that we have seen and if we compare then we get that the  $S_{13}$ ,  $S_{12}$  and  $S_{14}$  values once we have that.

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**Numerical on S-parameter of a Directional Coupler (Contd.)**

- Coupling factor =  $-20\log|S_{13}| = -20\log(0.32) = 9.9\text{ dB}$
- Directivity =  $20\log\left|\frac{S_{13}}{S_{14}}\right| = 20\log\frac{0.32}{0.22} = 3.25\text{ dB}$
- Isolation =  $-20\log|S_{14}| - 20\log|0.22| = 13.15\text{ dB}$

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We have coupling factor, we have given the formulas in the earlier lecture that coupling factor is minus 20 log S 13. So, you can find out it is 9.9 db, directivity is 20 log S 13 by S 14. So, 3.25 db isolation see and as you can verify that in db scale. We know i is equal to b plus c. So, d is 3.25, c is 9.9. So, their sum is 13.15 and that is equal to i.

So, you can say that this directional coupler is not a very good directional coupler that though it is coupling power is 10 db, but its directivity is very low; that means, its it cannot suppress the reflected wave very much and its isolation is also poor 13 db isolation is not always very good just because its directivity is poor. If directivity instead could have been 30-40 db you say isolation also could have been increased a lot.

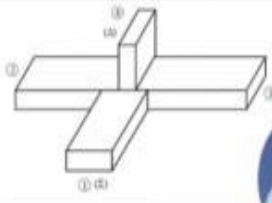
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### Numerical on S-parameter of a Hybrid Tee Network (Contd.)

**Q3:** A waveguide hybrid shown in the figure below has the following properties:

- It shows no reflections when all the ports are matched.
- When some power is applied to port-1, 40% of the input power comes out from port-2 and 40% comes out from port-3. Also the signals at port-2 and port-3 are in phase.
- When the power is applied to port-4, 40% of the input power goes to port-2 and the signal at port-3 is  $180^\circ$  out of phase with that at port-2.
- The network is reciprocal.

Determine the S-matrix of the network





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Now, let us come to a magic tee problem, it is said that it shows no reflections when all the ports are matched when some power is applied to port 1 40 percent of the input power comes out from port 2 and 40 percent comes out from port 3 also the signals at port 2 and port 3 are in phase when the power is applied to port 4 40 percent of the input power goes to port 2 and the signal at port 3 is 180 degree out of phase with that at port 2 the network is reciprocal determine the S matrix of the network.

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### Numerical on S-parameter of a Hybrid Tee Network

**Solution:**

- The network consists of four ports and shows no reflection when all the ports are match terminated.


$$S_{ii} = 0 \quad \forall i = j \text{ where } 1 \leq i, j \leq 4$$

As the network is reciprocal, therefore,

$$S_{ij} = S_{ji} \quad \forall i \neq j \text{ where } 1 \leq i, j \leq 4$$

considering the  $180^\circ$  phase shift

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & -S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & -S_{24} & S_{34} & 0 \end{bmatrix}$$



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So, it is said that 4 ports and no reflection when all the ports are matched that means you can easily say that the 4 diagonal elements of the matrix will be 0 as network is reciprocal. So, you can reduce and also if you remember that 180 degree. So, 1 of the thing you can take as minus.

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**Numerical on S-parameter of a Hybrid Tee Network (Contd. 2)**

- Now from the unitary property of the S-parameter
 
$$|S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$
 Now  $|S_{12}|^2 = |S_{13}|^2 = 0.4$ , therefore,  $S_{14} = S_{41} = 0.45$  and  $S_{12} = S_{13} = 0.63$
- Similarly it is given that  $|S_{42}|^2 = |S_{24}|^2 = 0.4$ , hence by unitary property we get that
 
$$|S_{14}|^2 + |S_{43}|^2 + |S_{42}|^2 = 1$$

$$\Rightarrow 0.45^2 + |S_{43}|^2 + 0.4 = 1 \Rightarrow |S_{43}| = |S_{34}| = 0.63$$
- Again with the help of unitary property in 2<sup>nd</sup> row we can solve for  $|S_{32}|$ ,  
 Therefore,  $|S_{32}| = |S_{23}| = \sqrt{1 - (0.63^2 + 0.63^2)} = 0.45$

By replacing the obtained values in generic s-matrix we get

$$[S] = \begin{bmatrix} 0 & 0.63 & 0.63 & 0.45 \\ 0.63 & 0 & 0.45 & -0.63 \\ 0.63 & 0.45 & 0 & 0.63 \\ 0.45 & -0.63 & 0.63 & 0 \end{bmatrix}$$

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Now, you can apply it is the lossless device. So, unitary property, from that you can determine S 14 and S 41 and also S 12, S 13 also it is said 40 percent going to some where means that power wise you remember S matrix is voltage. So, S 42 square is equal to S 24 square is equal to 0.4. So, you get those values then unitary property in second row you can solve for S 32. So, S 32 is the by that you can solve for all the values. So, enough information is given in the problem to get you those values remember that symmetrical means s i j is equal to s j i unitary properties that gives you all the things.

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### Numerical on Signal Flow Graph

Q4: By using signal flow graph determine the overall reflection coefficient at input ( $\Gamma_{in}$ ) and the overall reflection coefficient at output ( $\Gamma_{out}$ ) for the network given below. Assume Source and load impedances are  $50\Omega$  each.

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Now, the signal flow graph this problem that suppose this tee matrix was given in an earlier problem also you have also loaded thing that means,  $Z_d$  &  $L$  is connected  $Z_d$  &  $S$  is connected. So, find out the reflection coefficient and both at input and output source and load impedance is given to be 50 ohms.

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### Numerical on Signal Flow Graph

**Solution:**

- The signal flow graph for the given network configuration is given below:

- The over all reflection coefficient at input is:  $\Gamma_{in} = \frac{b_1}{a_1}$
- The over all reflection coefficient at output is:  $\Gamma_{out} = \frac{b_2}{a_2}$

The S-matrix of the given network is:

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0.89 & 0.10 \\ 0.10 & 0.89 \end{bmatrix}$$

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So, s matrix of the network we have earlier found out you can just see from there that S matrix of the 2 port network will be like this 0.89-0.10 and the graph we have shown.



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### Numerical on Signal Flow Graph

**Solution:**

By using the Mason's formula,

$$\Gamma_{in} = \frac{b_1}{a_1} = \frac{S_{12}S_{21}\Gamma_L + S_{11}[1 - S_{22}\Gamma_L]}{1 - S_{22}\Gamma_L} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

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Now, the input side we have already in the lecture you can see the input reflection coefficient for a loaded 1 by signal flow graph comes to be this  $S_{11}$  plus  $S_{12}, S_{21}$  comma 1 by 1 minus this.

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### Numerical on Signal Flow Graph

In the similar fashion we can obtain

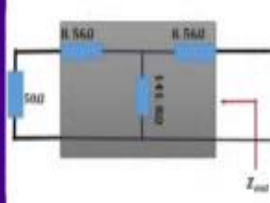
$$\Gamma_{out} = S_{22} + \frac{S_{21}\Gamma_s S_{12}}{[1 - S_{11}\Gamma_s]}$$

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So, gamma out also you can find from this, but to determine gamma L and gamma S you need to determine Z out and Z in output impedance and input impedance.

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### Numerical on Signal Flow Graph



$Z_{out}$

Now to determine the  $\Gamma_L$  and  $\Gamma_S$  we need to determine  $Z_{out}$  and  $Z_{in}$

Hence,  

$$Z_{in} = 8.56 + [(50 + 8.56) \parallel (141.8)] = 50\Omega$$


Similarly,  

$$Z_{out} = 8.56 + [(50 + 8.56) \parallel (141.8)] = 50\Omega$$

So,  $\Gamma_L = \frac{50 - Z_{out}}{50 + Z_{out}} = 0$  and  $\Gamma_S = 0$

Therefore by using previous result of  $\Gamma_{in}$  and  $\Gamma_{out}$  we get  

$$\Gamma_{in} = \Gamma_{out} = 0.89$$



Now, input impedance you see input impedance will be what that it is saying 8.56 in series with this whole thing which is nothing, but parallel of 141.8 and series of these 2. So, that is written here self S matrix that becomes 50 ohm. So, input impedance is 50 ohm and Z out also if you see that it turns out to be 50 ohm. So, basically characteristics impedance 50 ohm Z in Z out is 50 ohm load and source also 50 ohm. So, gamma L will be 0 gamma S will be 0. So, gamma if you put gamma L and gamma S, here 0, it is simply S 11 and it is simply S 22. So, this is 0.89 that is all. So, we have shown you all the problems, etcetera.

Now, I hope that in this series of lectures, we have tried to give you the 3 fundamental tools and various applications of that also, lot of problems have been solved in tutorials. So, it will give you a good foundation in your future professional life.

Thank you.