Basic Tools of Microwave Engineering
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Lecture – 18
Signal Flow Graph

Welcome to this lecture on Signal Flow Graph.

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This is our third tool. We have said that, we will introduce 3 main tools of microwave engineers in this course. First was Smith chart; the second was scattering parameter; the third is signal flow graph.

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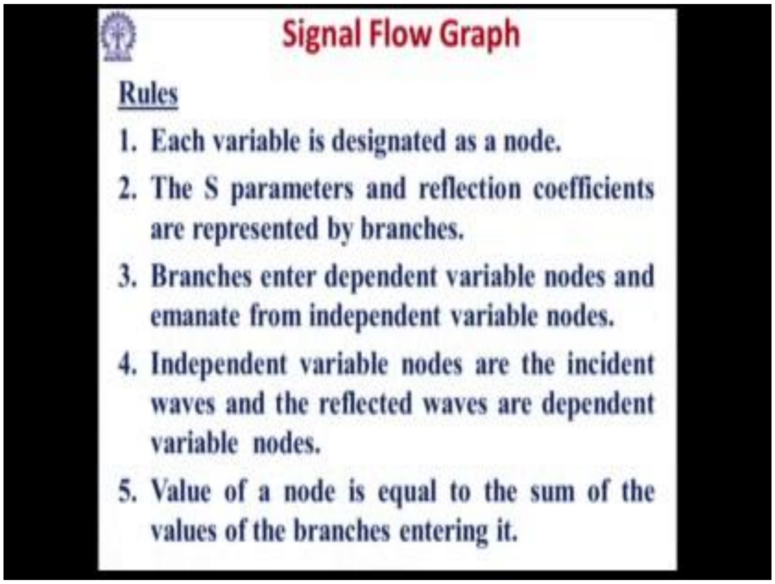

Now, what is a signal flow graph? It is I think, many of you have used these, in your study of control theory, we have used signal flow graph, where, if you have a block diagram, and various blocks, they are, they have some interrelationship; if you want to find the transfer function of the whole thing, that means any linear network, if you are having, and you know that, various branches, etcetera, they are connected that there is some interrelationship between various blocks then what is overall transfer function, that you can find here. The same thing here; we see that, if we know the wave is propagating with, in various ways, between various parts of a circuit, now, from one part to another part, if you want to find the transfer function, or the gain then this signal flow graph is very much needed.

And, in our case, we will see that, when a network gets very much complicated, like, we will see the network analyzers' calibration procedure, there, the network is quite complicated; but that time, if you know signal flow graph, we can very easily find out the things. Similarly, in various microwave characterization, particularly for amplifiers, etcetera, we have various power gain definitions.

Now that sometimes creates a lot of problems, if we do not use this tool of signal flow graph. But, signal flow graph can very easily, very elegantly, help us in such cases, to find the gain, or transfer function of, between one part of network to another part. But, you know that, as in case of Smith chart, or scattering parameters, we have seen that, if you want to use a tool, we need to learn its rules, because, under certain rules, this whole thing becomes elegant. If we do not follow those rules then the elegancy tends to be a black mare. So, that is why, these rules are introduced here that, each variable, that means, variable means, in a network, you have seen that, it can be either V 1 plus, or V 2 plus, or V 1 minus; so, those are designated as a node in a signal flow graph.

The S parameters and reflection coefficients are represented by branches. So, S parameters, any S parameter, they are, or reflection coefficient, they are, they should be represented by a branch. Branches enter a dependent variable node, and emanate from independent variable nodes. So, obviously, branches can enter a dependent variable node; and, an independent variable node, that makes, that behaves as a source of the wave etcetera. Independent variable nodes are the incident waves; so, you see that, which are incident that means, the excitation waves are coming from the source of the waves. They are the independent variables; and obviously, depending on the networks behavior, you get a reflected wave, that is a dependent variable node. Value of a node is equal to the sum of the values of the branches entering it.

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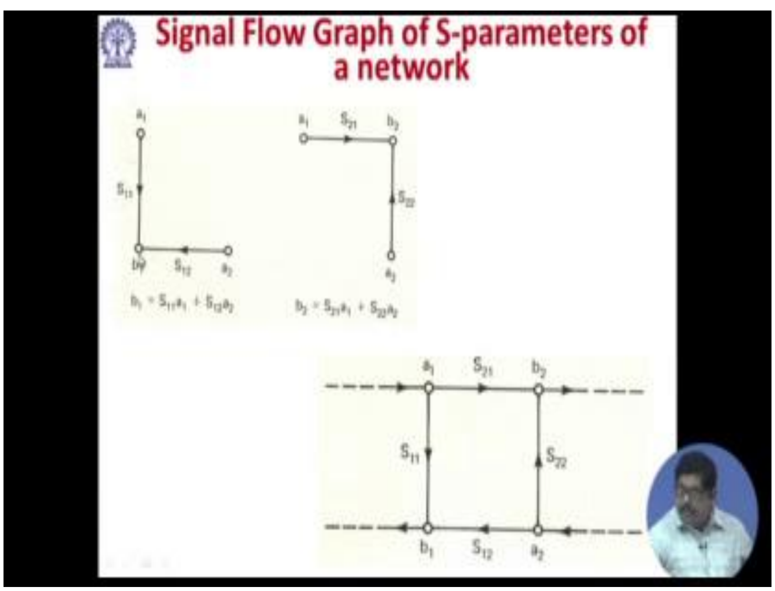

So, you see that, you see here. Since there are other things involved, so, instead of V 1 plus, V 1 minus, etcetera, here, any plus quantity is called as a, and minus quantities are called as b. So, basically, you can think that, this is your V 1 plus; this is your V 1 minus. Now, in a two port network, how V 1 minus comes?

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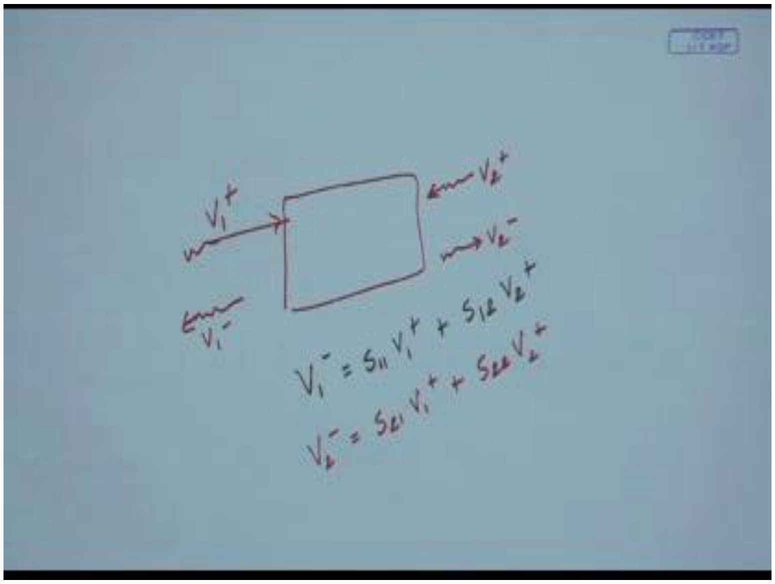

If you see this thing that, in our two port network, I have V 1 plus; I have V 1 minus. I have V 2 plus; I have V 2 minus. Now, how V 1 plus comes? V 1 minus, due to this V 1 plus, I can get some V 1 minus, that we call as S 1 1 V 1 plus; plus, due to this V 2 plus also I can get a V 1 minus; that we call as S 1 2 V 2 plus. So, V 1 minus, that is the V 1 here; so, from a 1, this V 1 plus, I am getting S 1 1; this is S 1 1 a 1; from a 2, that means V 2 plus, I am getting s 1 2. So, this is the representation of V 1. As the last rule said that, value of this is sum of the branch values that are coming here.

Now, branch value, this branch S 1 1, a 1 into S 1 1, plus a 2 into S 1 2 will be one; so, these two are entering; if another one was entering, another plus would have been there. Similarly, the dependent variable in the second port V 2, which was actually V 2 minus, that we can write as V 2 minus is equal to S 2 1 V 1 plus, plus S 2 2 V 2 plus; that has been done. So, finally, we can combine the two ports that, in the port one, I have V 1 plus V 1 minus; so a 1 V 1; in the port two, I have V 2 a 2.

Now, it is done elegantly that, V 2 has been placed here, instead of putting it here, so that, you can get a very, a 1 to b 2, what is going is S 2 1; a 1 to b 1, what is coming is S 1 1. Similarly, from b 2, from a 2, you are getting whatever to b 2, that is S 2 2, and a 2, whatever you were getting here is S 1 2. So, a two port can be represented in a signal flow graph with its S parameters as these. So, this is a two port representation of a, two port representation, two port network with its S parameter can be represented in the signal flow graph like this.

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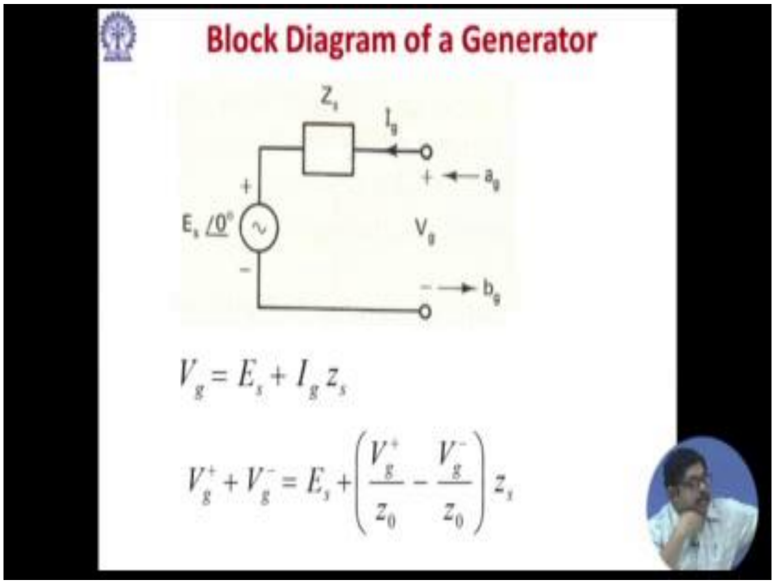

Similarly, you see a block diagram of a generator. So, we know that, V g, the total voltage that is open circuit voltage you are getting, that will be E s plus I g into z s. Now, V g, you can break into again two parts, that V g plus and V g minus; V g plus you call a g, and V g minus you call b g; so that, again is written here.

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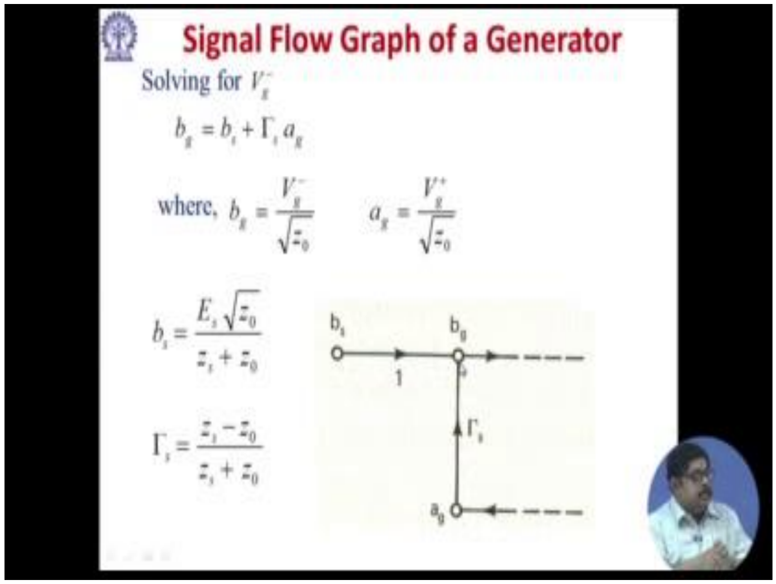

And, by solving, this is the generator that, you are having a V s; from that you are getting b g, and in a g, you are putting that value, that reflection coefficient of the source that is coming as gamma s.

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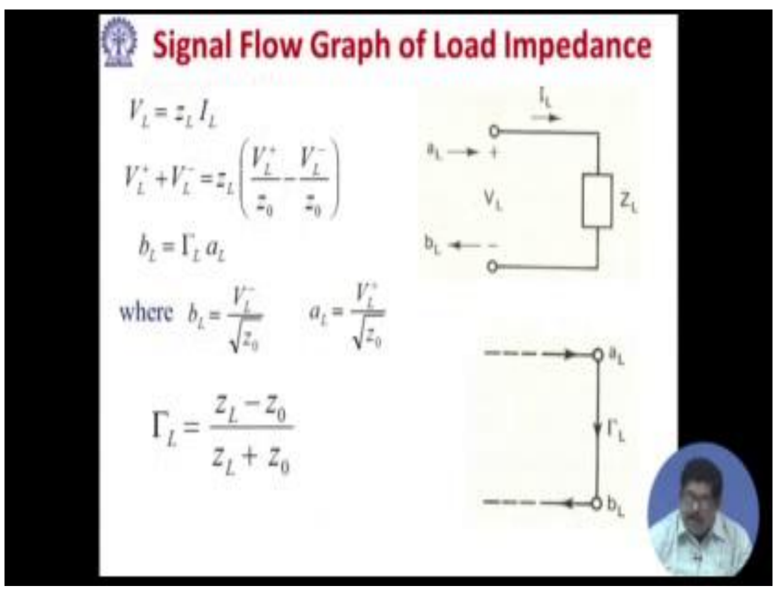

Similarly, signal flow graph of load impedance is load, there is one incident, one reflected wave; so, their relationship is a load reflection coefficient.

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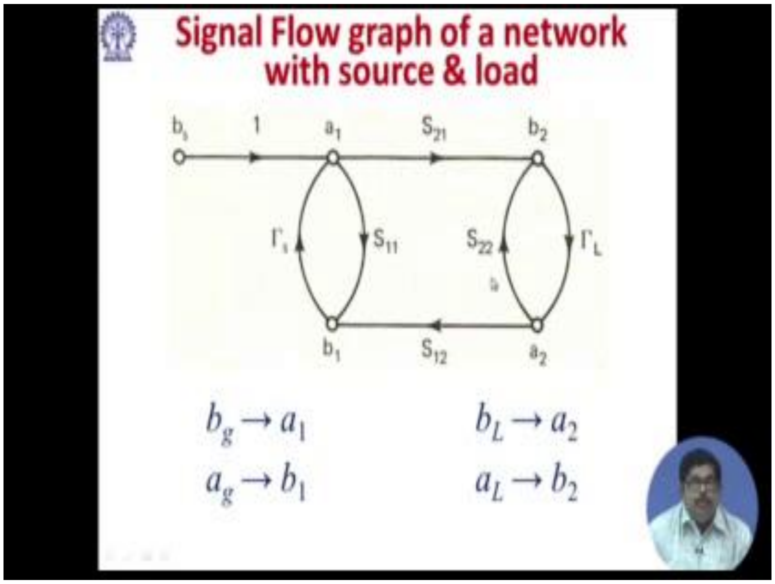

Now, so two port network, loaded, and you can say generated two port network, or source two port network; so, you have here, there will be a source; between the network and the source, there is a reflection coefficient gamma S. There is a load reflection coefficient gamma L. So, this is the complete network. You know that, sometimes, we find the S parameters after finding, given a particular load impedance and source impedance, we want to find, what are the various values; so, that time, this network is used.

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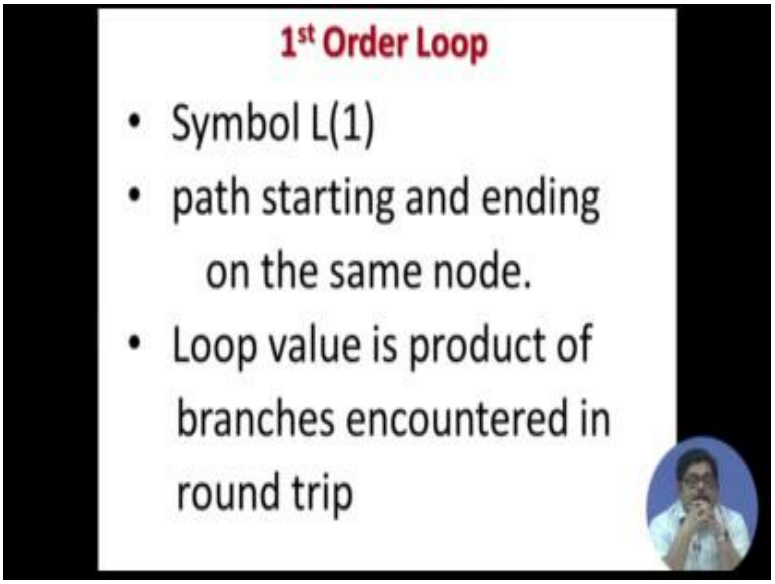

Now, there are some terminologies in the signal flow graph. The first thing is called first order loop. Its symbol is L 1. Now, what is a loop? Loop means, any branch, or a combination of branches, which is starting from a node and making a round trip, and coming back to the same node. Now, that is what is written, a path starting and ending on the same node. It may be a single branch that is called self loop. It may be various branches, after that, it is coming back; that is a standard loop. This first order loop, why it is called first order? That means that, any loop you find, that we will call a first order loop.

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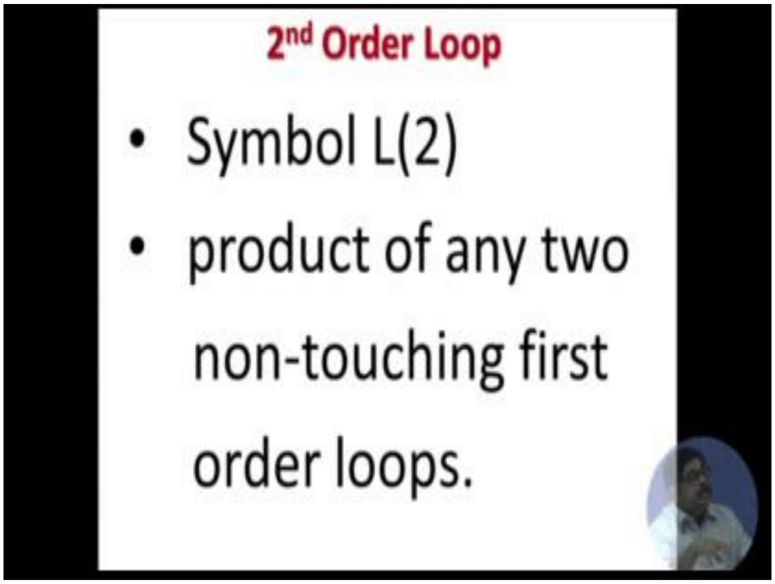

Then, there is second order loop. Second order loop is product of any two non - touching first order loop. First, we have seen loops; all loops we are calling, in general, first order loops.

Now, out of that, second order loop means, two, product of any two non - touching first order loops. So, we will have to identify, who are touching; if they are touching, they are rejected; but, if they are non – touching, two loops then their product is one second order loop. All these pairs, we will have to identify and find.

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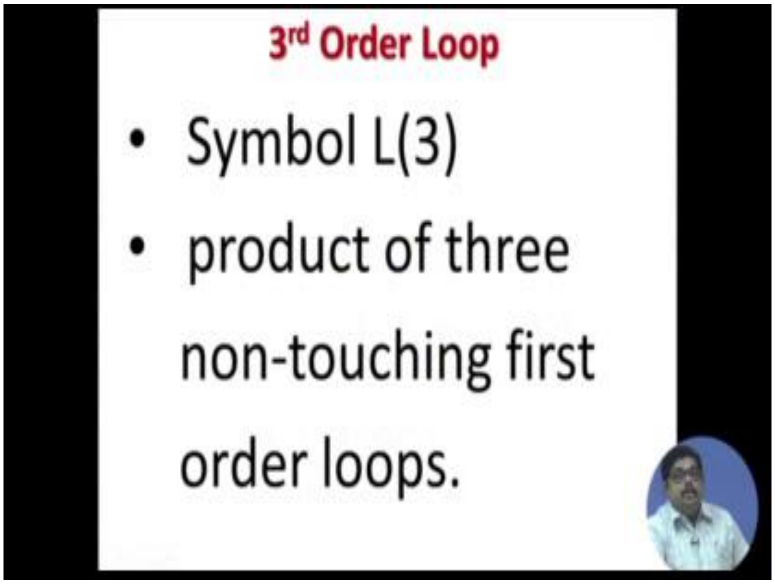

Then, we have third order loop. So, product of three non - touching first order loops. Like that, you can go on any order. So, in a network, as it gets more and more complicated, first, you will have to identify the loops; then, that means, you have identified all the first order loops. Then, you find out, what are the non - touching pairs of loops. Those are second order loops. Then, you identify the triplet of non - touching loops; that is, third order loop. Then, you identify quadruple of first order loops, non - touching first order loops, that is called L 4, etcetera, etcetera.

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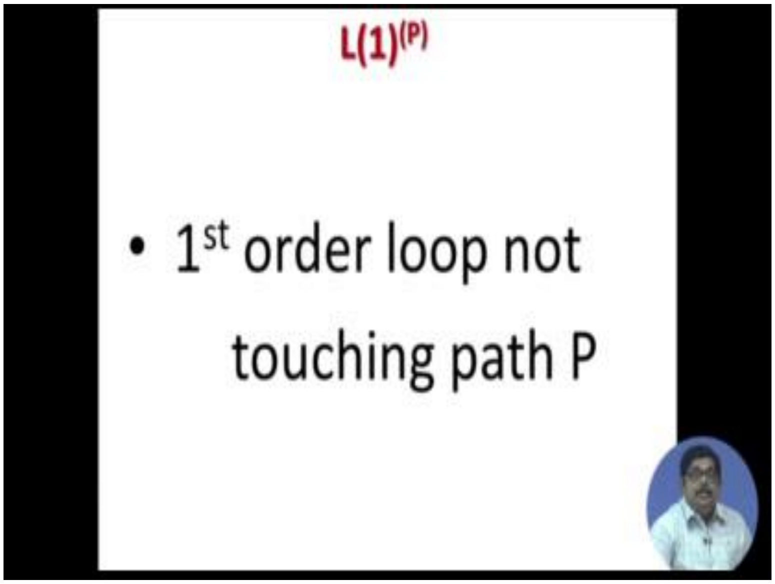

Now, you see this is L 1 with a superscript P. L l is the first order loop, with a superscript P means, first order loop not touching path P. So, there can be several paths in which we are interested, and L 1 P 1 is first order loop not touching path P 1. L 1 P 2, first order loop not touching path P 2, etcetera.

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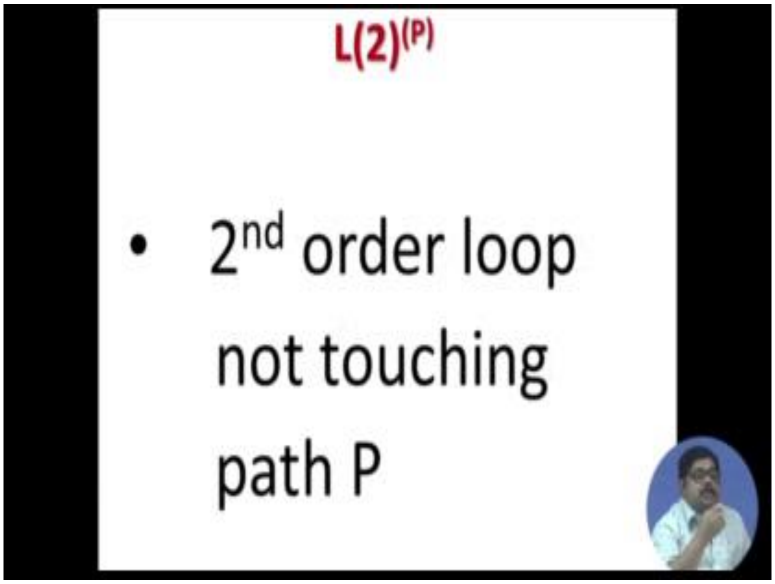

Similarly, L 2 P, second order loop not touching path P, etcetera.

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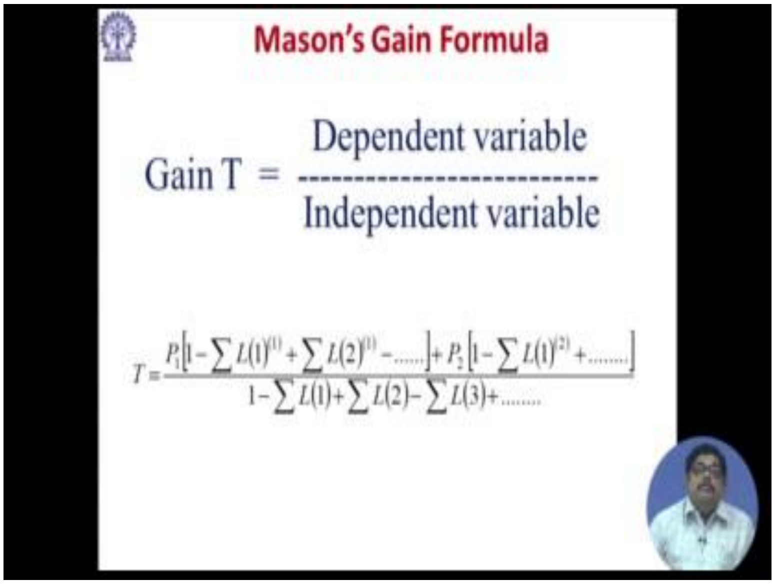

Now, this is the formula. I think, this formula, many of you have used in the control theory. In electrical control, this is used in many type of block diagram, when we try to find the transfer function from that, generally, this formula is used. It is well known, Mason's formula. Generally, we call that gain; that means some dependent variable by some independent variable; as we have said that, dependent variables are the reflected type of thing in our microwave things; and, independent things are our incident things. So, one dependent variable, that means, some reflected by some incident, that we are calling gain. It may be gain; it may be anything. Basically, it is a transfer function.

Now, that is written by this formula. You see that, what is this formula says that, you find out P. Now, there can be several paths from this independent variable that you are interested, to the dependent variable. Suppose a 1 and b 3; so, between a 1, a 1 is a node; b 3 is a node; a 1 is an independent node; b 3 is a dependent node. You come from that independent node to dependent node; see how many paths you can come like that, those are your paths. They are called, here you see, P 1, P 2, P 3 etcetera. Now, you find out what are L 1s; how many first order loops are there; that means, how many standard loops are there. Now, sigma L 1 means, sum of all first order loops. Then, you identify second order loops, sum of all those second order loops. Then, sigma L 3 is sum of all these third order loops, etcetera.

And, in the numerator, you see this path; path will have some value; value means, the reflection coefficients, etcetera, on that path; each branch will have some reflection coefficient, or some S parameters, so that product of that; so that path value, multiplied by 1 minus this L 1 1, L 1 1 means, all first order loops which are not touching path 1; sigma of that. So, there may be several such.

Similarly, all second order loops not touching path 1, all third order loops not touching path 1, all fourth order loops not touching path 1, etcetera, and the sign, you see, minus, plus, minus, plus, like that. Then, path 1 is closed. Then, repeat that for path 2, again, 1 minus this L 1 2 means, all first order loops not touching path 2. Then, L 2 2, all second order loops not touching path 2 2, etcetera. So, this will be the numerator; so, this will give you the value.

Now, you need some time to get it inside you that this terminology; but, with examples, we will try to tell you.

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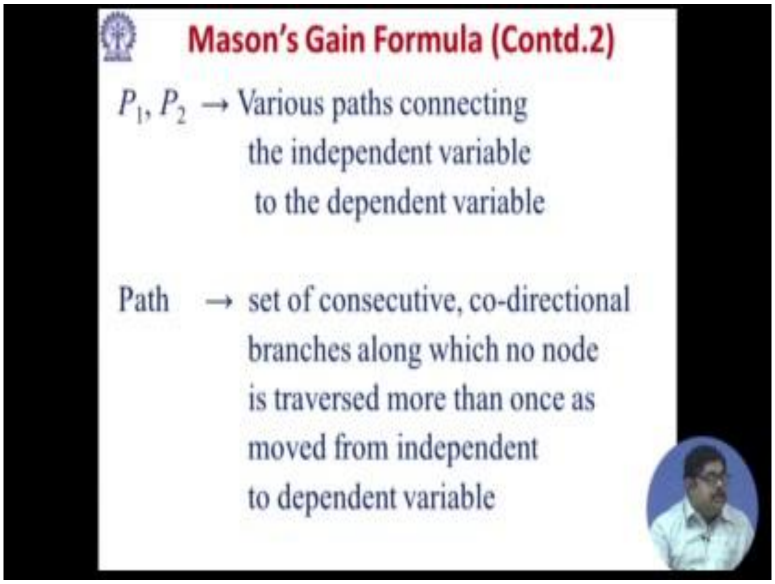

And, P 1 and P 2, I am again repeating, what is P 1, P 2. Various paths connecting the independent variable to the dependent variable of your interest, because, there may be several dependent and independent variables in that; all dependent variables will be b something, b subscript, and all independent variables are a subscript. You are going from one independent variable to one dependent variable.

And, path, set of consecutive co-directional branches, along which no node is traversed more than once, as moved from independent to dependent variable. This is important that, you can take any path from going to the independent variable to dependent variable, but no two paths, no two nodes should be traversed twice; that means, it should be completely a different path. That means, through a loop, you cannot come to a new one. Always, because, if you traverse through a loop, basically, you are traversing the node twice, paths cannot contain any loops. Path value, product of branch values in the path.

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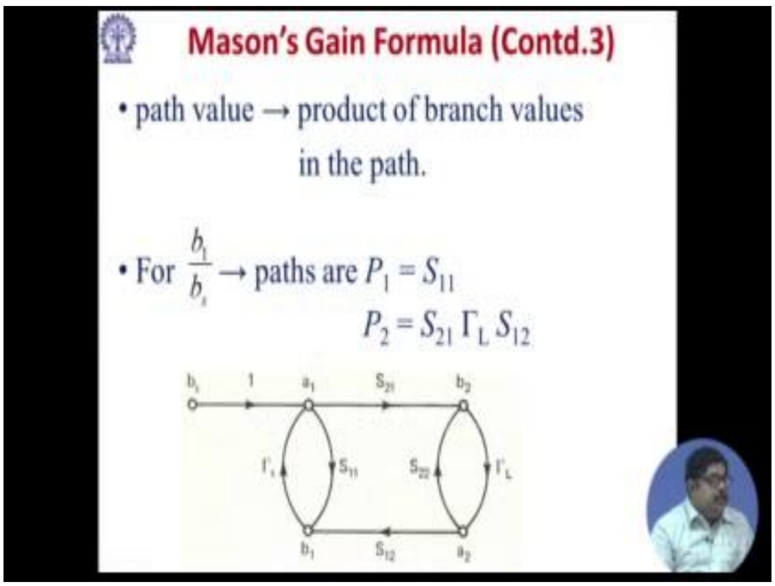

Now, let us take this example that, suppose, we want to find b 1 by b s. No, you cannot find b 1, b s; b 1, b s, ok. So, from here, though it is written as b s, actually, it is a s, because, it is an incident thing. So, from here to here, I want to find. What are the paths? You see, one path is here; then, this one is one direct path, that is why, let us call this as P 1 is S 1 1. And also, I can come here; then, I can go here; then, I can come here; I can come here. So, this is another path P 2. It is 1 into S 2 1 into gamma l into S 1 2. You see, there are no other paths here. This is a path; but, this is not a path, because, arrow is not here. I will have to come here, and this one, I cannot take; only thing is this. So, there are only two paths in the network, for coming from b s to b 1.

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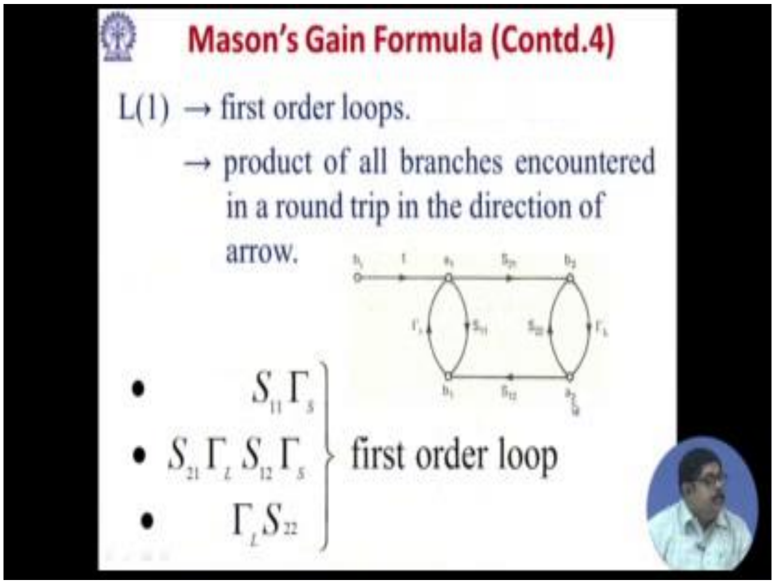

Now, let us first find what are the L 1s, first order loops. So, while coming from here to here, what are the loops? One loop is definitely this, that we are calling S 1 1 gamma S. Then, another loop is obviously this S 2 2 gamma L. Now, there is another loop here, you see, I am starting here, coming here; here, this is a loop; so, S 2 1, gamma L, S 1 2 gamma S; S 2 1 gamma L, S 1 2 gamma S. You see, no another loops are there, first order loops are there. So, there are 3 first order loops; that means, when we will sum in the denominator, you will see, sigma L 1, that will be a sum of 3 loops; so, that will be S 1 1 gamma S, plus S 2 1 this, plus gamma L S 2 2.

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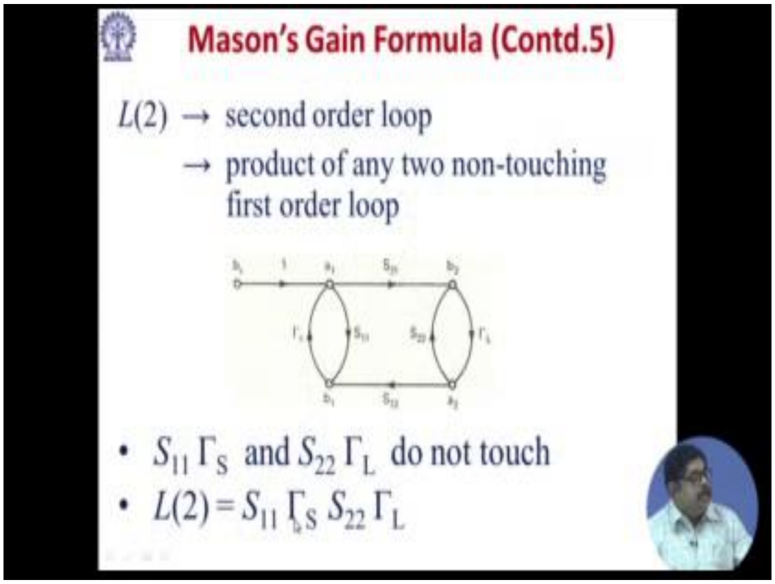

Now, the same thing, is there any second order loop? That means, are there two loops which are not touching? You see, this is one loop, but, that is touching with this; touching with this loop. So, this one and this one, they are not non – touching; but, this one, and this one, is non-touching; this one, and this one. So, there are, there is one second order loop; so, that is called S 11 gamma S and S 2 2 gamma L, they do not touch. So, we will write, L 2 is, there is one second order loop, and that loop value is product of these two; so, S 1 1 gamma S into S 2 2 gamma L.

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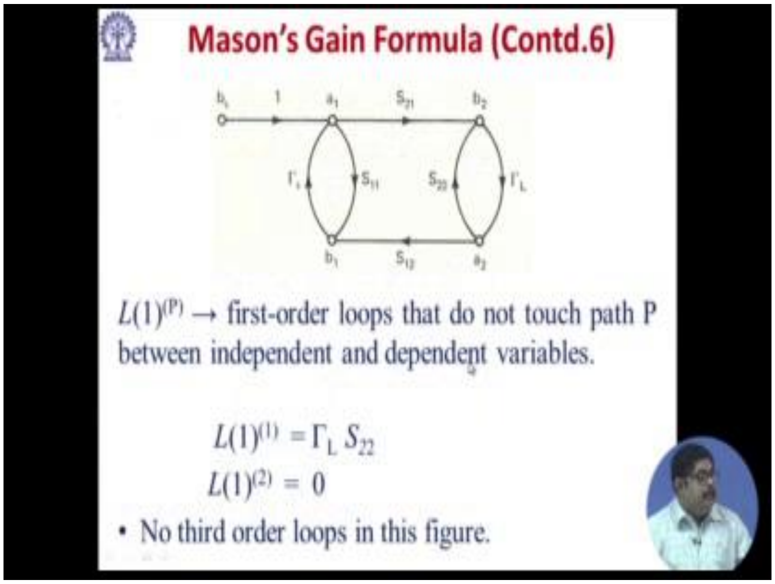

Now, we have seen, there are two paths. Now, L 1 P; so, L 1 1, L 1 2; now, L 1 1; now, there are three L 1s, as we have seen that, this is one L 1; this is another L 1, and this is another L 1. There are three first order loops, but which one is non-touching to path 1? What is path 1? This path; this was our path 1. Now, first loop, and this loop, they are non-touching means that they do not share any common node. So, first two loops, they do share common nodes with this path; but this one does not share any common node with this one. So, this one is a non -touching loop; so L 1 1, that is why, we are writing gamma L S 2 2.

And, is there any first order loop which is non-touching with the second path? What was the second path? This, to this; now, you see all the loops are touching this path. So, L 1 2 is equal to 0. And, among these three loops, is there any triplet of non- touching loops? No. So, no third order loops in this figure.

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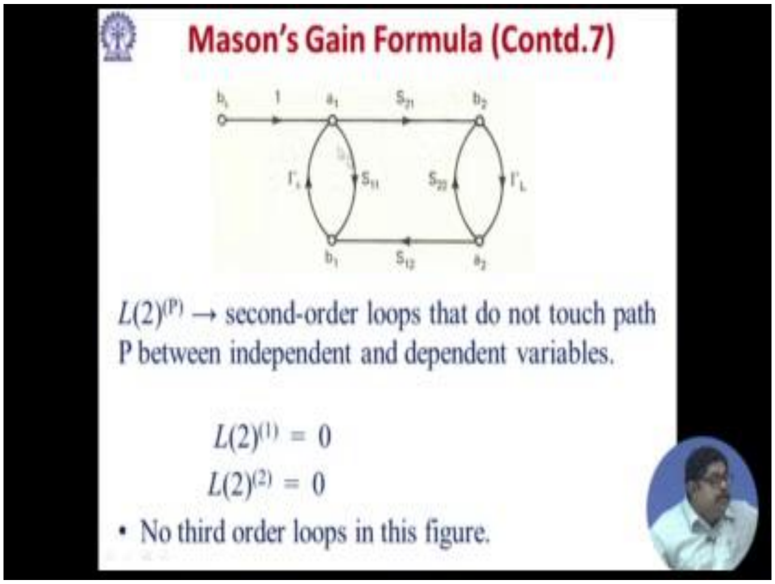

Then, second order loop, we have already seen, but is there any second order loop? What is the second order loop? This one, and this one; not touching path 1, but, it touches path 1. So, that is why, we have written L 2 1 is 0. Similarly, the second order loop that touches path 2 also; this is path 2; it touches that; that is why, L 2 1 is equal to 0; L 2 2 is equal to 0; and, no third order loops in this figure.

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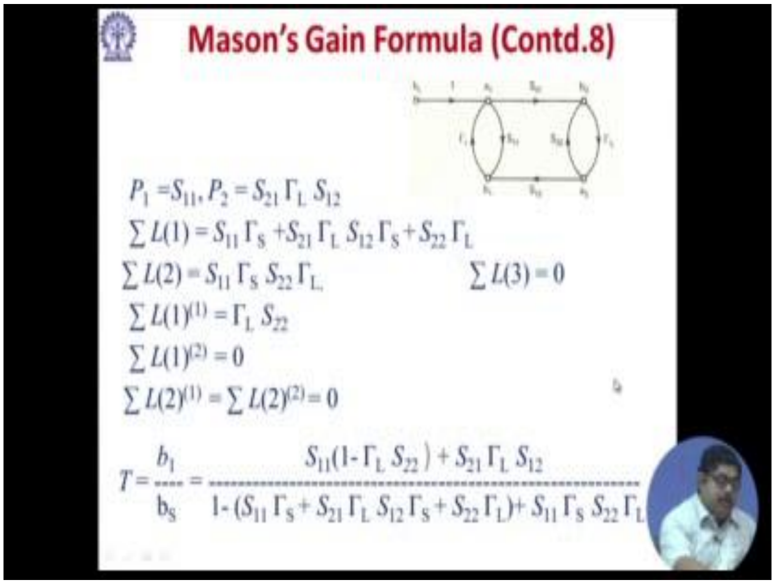

So, in Mason's gain formula, we can put all that. We have written here, P 1 value; we have written P 2 value; we have written L 1; we have written the three loops, three first order loops; second order loop, one second order loop; no third order loop; L 1 1 is only existing; L 1 2 is not there; L 2 1, L 2 2 is not there. So, you can write, this will be that transfer function b 1 by b s, for this one, if you want to find b 1 by b s, this will be the value.

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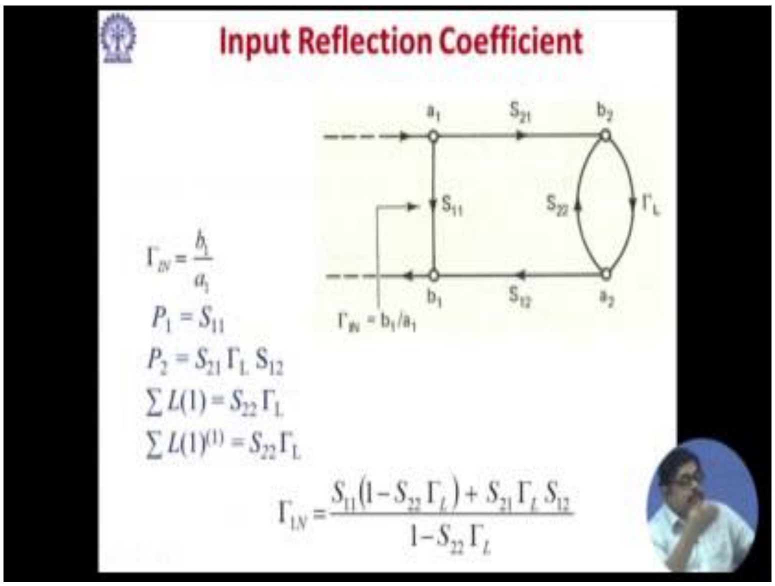

Now, let us come to our one point that, suppose, I want to find the input reflection coefficient of a loaded circuit. This, this part is the S parameter part, up to this. Now, there is a load. So, there is a load reflection coefficient. Now, from here, if I want to look, what will be the value? So, you see that, what is my P 1? P 1 is, that means, I am coming from, you see, b 1 by a 1; that means, this is independent variable, this is dependent variable. So, how I can come from a 1 to b 1? One path is this; we are calling it P 1; that is, S 1 1.

Another path is, I can come here; I can come here; I can come here; so, S P 2 is S 2 1 gamma L, S 1 2. Now, is there any loop? Yes, one loop is, this one is a loop. You see, there are no other loops possible; so, that means, only one first order loop. And so, obviously, L 2 and L 3, they are all 0. But, this L 1 1, that means, first order loop, that means, this loop, is it non -touching with path 1? Yes. So, L 1 1 will be this loop S 2 2 gamma L. L 1 2, does it, is it non -touching with this path? No. So, L 1 2, 2 superscript, that will be 0.

So, now you put, gamma IN will be this.

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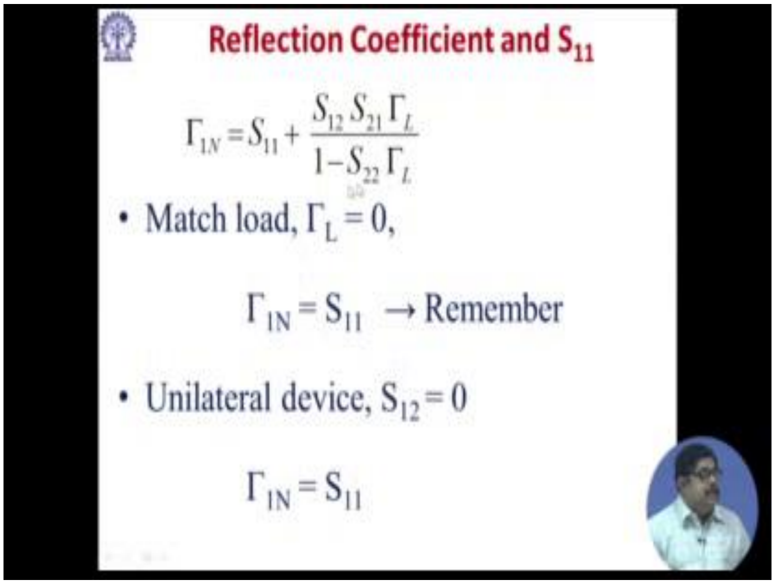

So that you can put now the values, and this is the well known value; you see, how simply we got it in actual S parameter, things we have derived that, but, that requires some manipulation. But here, there is, directly we have got this value, and when that matched load, when gamma for, if the load is a matched load then gamma L will be 0; and, you will get that, gamma IN is nothing but S 1 1; that we know; that we have discussed in scattering parameter also; that, generally, reflection coefficient that the input port is not S 1 1, but, under second port matched, you get this. Also, unilateral device; that means, generally, a device, if there is, from 2 to 1 if there is no path, then you can also get gamma IN.

Basically, gamma L is equal to 0; matched load makes this path 0; but, there are also some devices whose S parameter is such that, S 1 2 is 0. In that case also, you can get that, gamma IN is S 1 1.

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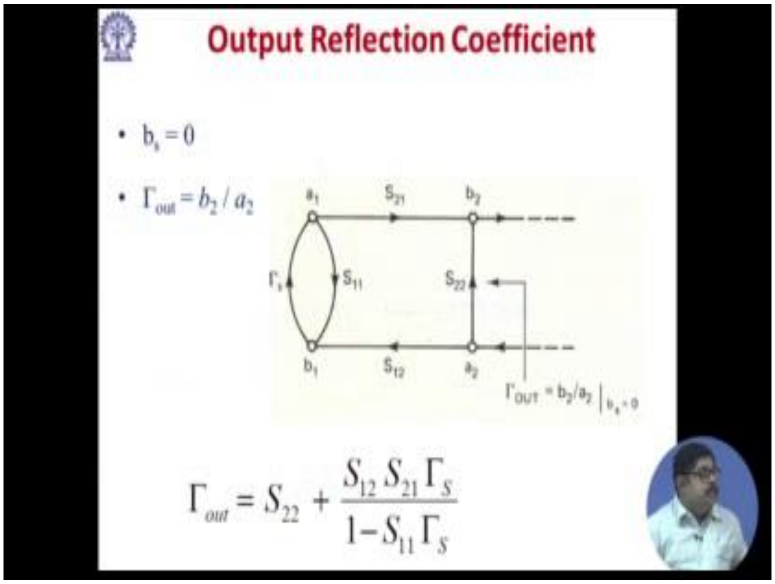

Similarly, you can find it out from output reflection coefficient; that output reflection coefficient will be this b 2 by a 2. That means how you can come from a 2 to b 2? So, there are, again, if you put that, that one path is this; another path is, go here, come here; so, then find out, what is the loop; there are, one loop is here; another loop is, you can say that, this is another loop. So, these two loops, you do that, and you can get this.

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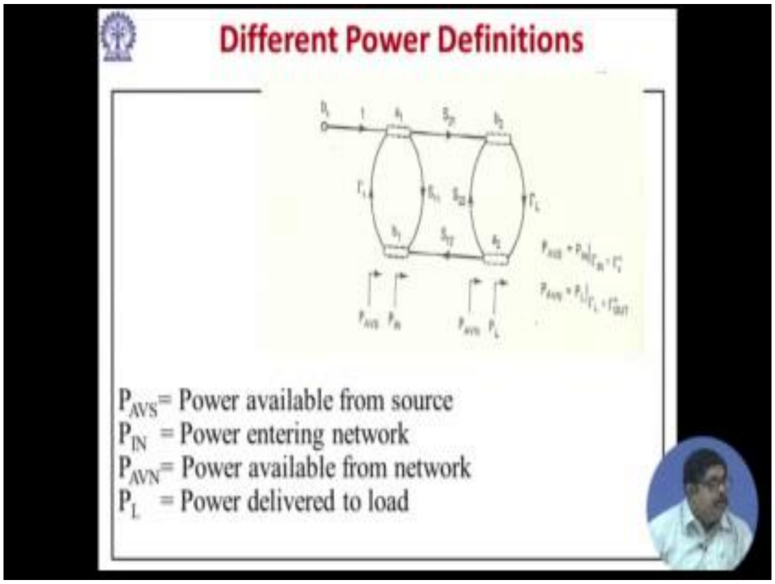

Similarly, there are various power definitions; we are not going there. When you study amplifiers, you require that.

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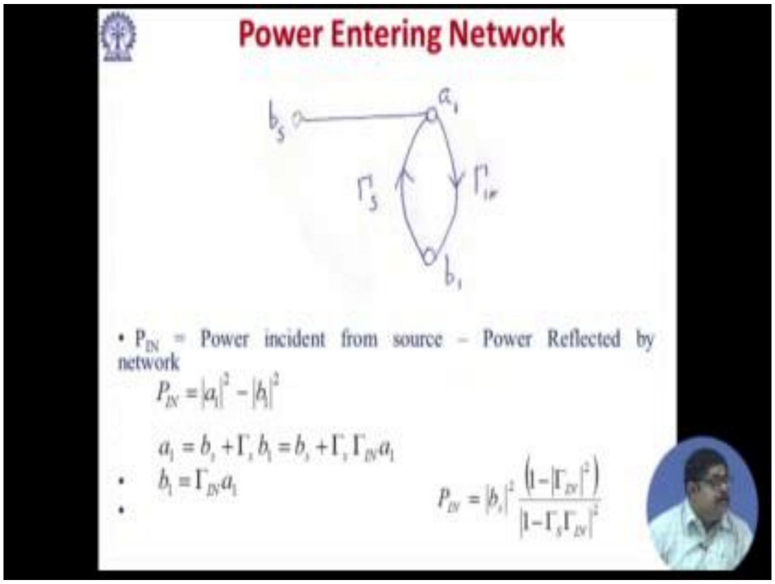

So, I think, we have completed that, signal flow graph, we have seen the introduction, how to manipulate that, and later, in the next lecture, we will try to find out its application. Already, I have shown you some application; a very important application, that we have introduced you to network analyzer. So, its calibration comes easier, if you do with signal flow graph; that we will see in the next lecture.

Thank you.