Basic Tools of Microwave Engineering

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Lecture – 17
Scattering Parameters of Coupler and Magic Tee

Welcome to this 17th lecture. Here we will see the scattering parameter of 2 very important microwave device - one is Coupler another is Magic Tee.

Now what is importance of coupler? If you want to couple power from one part to another part we will use coupler, but there are also other uses that we will come at the last part of this. Basically the modern microwave measurement which depends on network analyzer, there this coupler is very fundamental device similarly magic tee is propertive, why it is magic we will understand there is nothing magical about it, it is well understood. But the thing is magic tee is very important part, in another very important modern day microwave instrument that is radar.

So, that is why we have selected these 2 that network analyzer which is measurement device radar which is a useful device for you know detecting any object it is a microwave sensor. So, any object you can detect by radar at a large distance which I cannot see still radar can see. So, it is one of the fundamental parts of that radar. So, that is why we will see the scattering parameter based description of coupler and magic tee

Now, first see that in circuits we need to either divide the power. Suppose I am having a main power line from that I want distribution of power lines as you see in your day to day activities that low frequency power, the electrical power that gets divided into various branches. Similarly at high frequency also I have suppose a source of power then from that I want to distribute power, so that is called power divider.

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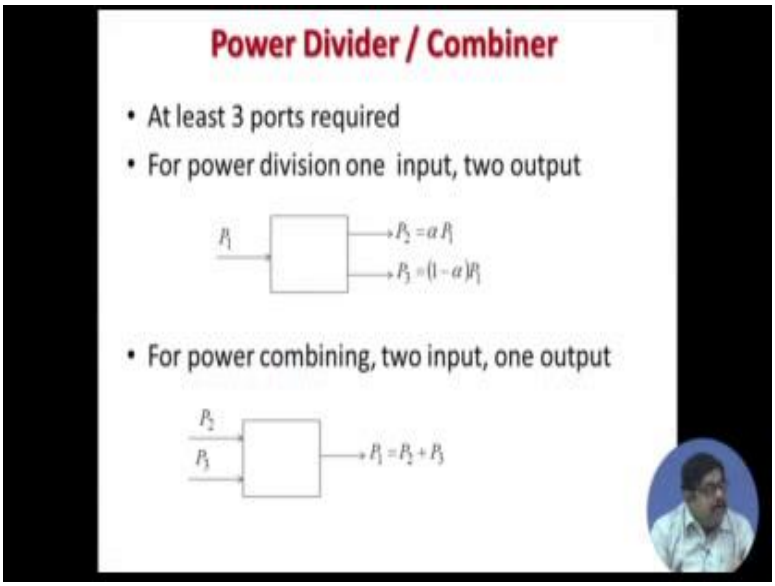

Similarly, if there are several powers are coming I need to sometimes combine the power. So, Power Divider or Combiner whatever the thing - the point is you require at least 3 port in this device. So, these are no more 2 port devices as we have seen till now. So, there will be at least 3 ports - 4 power division, you can see you require at least one port where the input will come and that will get divided at least in 2 if not more. So, at least 3 ports are required.

So, in case of power division let us say P 1 is coming here and then at output port some P 2 and another P 3 is coming. Now if it is a lossless one, I will require that P 2 plus P 3 is equal to P 1 that has been done here. But always this division may not be lossless in that case there will be some loss inside the device also. So, this is the lossless power divider. Similarly for power combining we require 2 inputs 2 ports in the input side and that will give us output. Again this is the lossless case, but if you do not have this then this relationship will not hold there will be also some additional power that is lost here.

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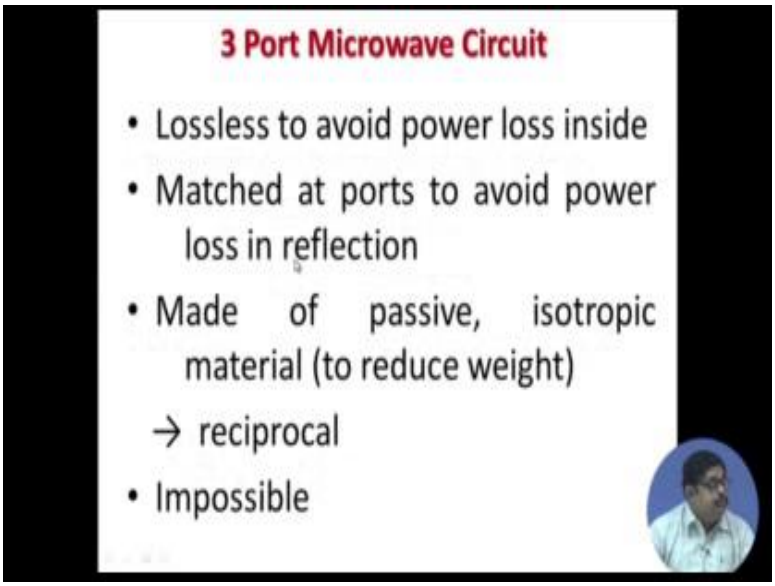

So, 3 port microwave circuit that is now we will see. Now obviously, we will want that, it is always desired to have a lossless power divider or power combiner. So, we want the microwave circuit to be lossless so that no power loss inside the device. Similarly we also want that the impedance at the ports should be matched, we have discussed in detail the need of impedance matching otherwise there is power is lost in the mismatch. So, we want that at all the ports if the device should be matched to avoid power loss in reflection also there is a need that these devices if they are made of a passive an isotropic material then their weight is less. Because in many of these modern days application these microwave devices they need to be you know microwave electronic circuitry they are day by day they are becoming miniaturized they are becoming light weight.

So, for that it is better that the non isotropic material like ferrites etcetera, they are not used. It is passive isotropic device if it is then it is very easy to go for planar technology. So, for going to planar technology if you have non isotropic material like ferrite it is not amenable to planar technology. So, that is why we also want that the device should be passive isotropic and that is why the demand is in terms of the device terminology we want that it should be a reciprocal device.

That means, we demand from a 3 port microwave device, our wish list that it should be lossless, it should be matched that all the ports, and it should be reciprocal. Now unfortunately can be shown mathematically I am not going into that mathematics, but

unfortunately that is not possible. It is impossible it can be shown that you cannot have all your 3 wishes from a 3 port microwave network. You can relax any of them you get suppose you can relax a lossless thing. So, you have lossy device; that means, you can use resistors etcetera then there will be some high amount of loss inside the device, but that can give you power division power or combining that you can have.

Also you can have that there will be some mismatch and in that case you can have a lossless and reciprocal device. Similarly you can also have non reciprocal device, but lossless and match. One example of that is circulator which is heavily used for in case of any microwave transmitter, high power transmitter and if the receiver is also at the same place one example is radar or another example is a satellite transmitter and receiver.

In that case, when the transmitter is sending power for radar, suppose a transmitter is sending high power now receivers they are generally will detect that signal which will be going coming from a long distance reflected. So, that is a weak signal that is why receivers are very sensitive, various weak signal they can detect. But when the transmitter is transmitting if receiver gets that power then receiver will be destroyed because transmitter is sending high power, receiver is detecting very low power.

Now that time you need to isolate the transmitter and receiver that isolator is generally 3 port because in one of the port the transmitter is put another port receiver is put, another the device through which power goes or comes that is antenna that is put. So that 3 ports there should be isolation between 2 of the ports when it is transmitting or receiving that device is called circulator. That is made generally of this non isotropic material called ferrite, but that is very bulky that is why nowadays people are trying to find out how to make in the low weight those things.

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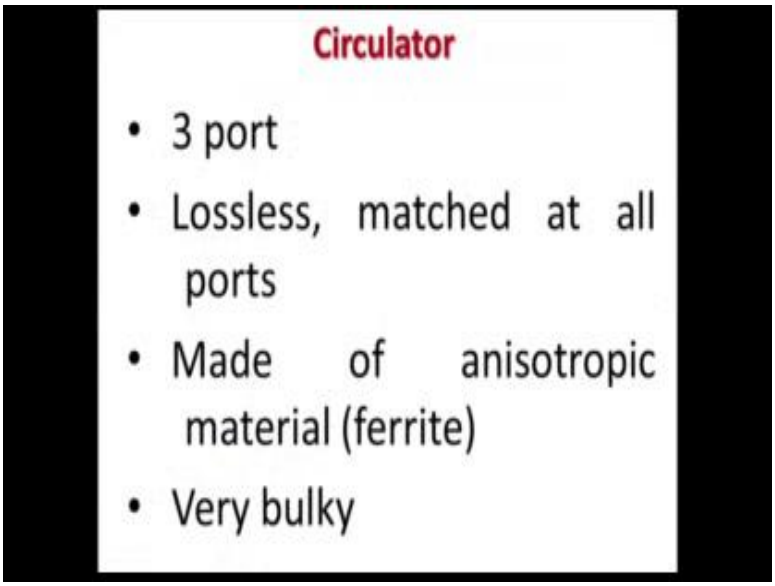

But in 3 port this is one impossibility that is why already a slide for circulator that it is a 3 port, it is lossless, matched at all ports, made of anisotropic material like ferrite, but the problem is its very bulky. So, it increases the weight of the whole system.

So, also there is resistive divider as I said that in that case you are suffering loss.

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But it will match at all ports made of isotropic material the disadvantage is we have high insertion loss. Instead of 3 port if we have 4 port that thing will show that it is called

directional coupler. It has 4 port, it is lossless, it is matched at all ports, made of passive isotropic materials, it is reciprocal.

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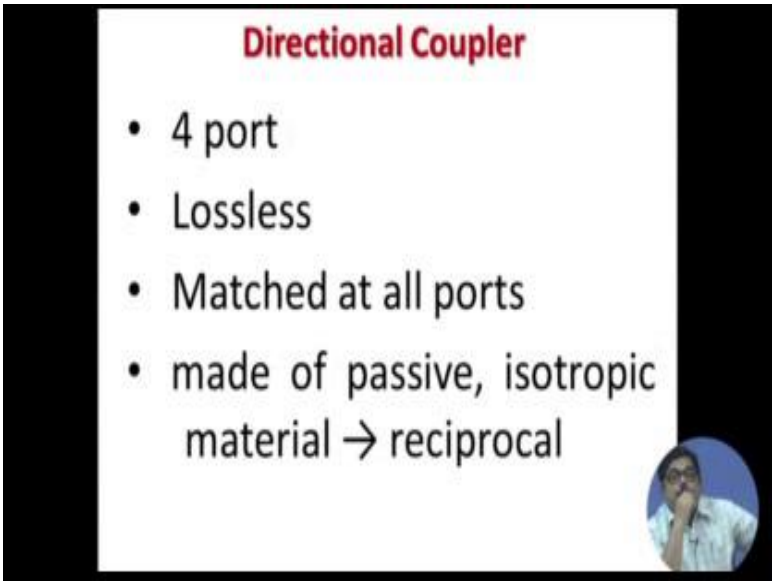

So, instead of 3 port if we open up another port obviously; that means, we are having more electronics circuitry. So, 4 port, but if we have 4 port we have our wish list completed that we have those 3 properties that we demanded from a microwave network that is lossless, matched at all ports, no mismatch and reciprocal all are satisfied. So, that is why directional coupler or couplers are very good that they are since they satisfy all our wish list they are used heavily in microwave circuit.

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This is the one, in our lab we have this directional coupler this is wave guide based directional coupler. You can also have planar directional couplers or coaxial directional couplers, but this thing is heavily used if you want to have any antenna radiating and you want to find out how much power it is sending. This is a device without which you cannot find that. So, this directional coupler you see it has 4 ports. Now out of that you can see here that the main power is add in here port 1, this is port 2 from which that power goes. So, there is a main line here the lower; you see there are 2 wave guides one is top another is here. So, port 1 and 2 is in the main line, we call it main line thing or main wave guide, this is an auxiliary line you have port 3.

Now actually port 4 is there, but port 4 generally in many applications this is inaccessible to the user. So, sometimes it is used sometimes not so this is actually 3 port directional coupler it has 4 port, but we should not call it a port because unless and until outsiders can access the electronics of that port you cannot call it a port. But to show you we have called it a port, actually is not a port because no power can come out or can be given from here. Though power is coming at this end, but it is not accessible outside that is why it is not a port, but to show you that there you can remove this and if you put this you can make it a port here.

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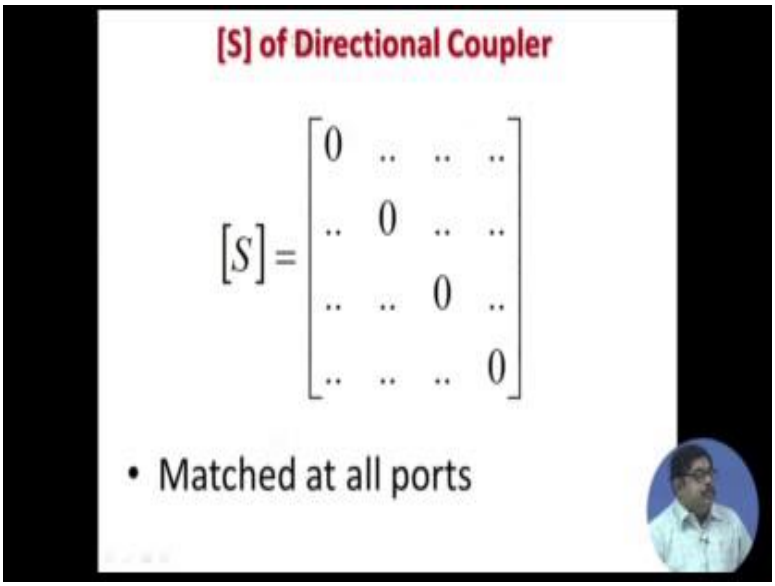

Now, let us find what is the scattering parameter of this directional coupler. You see first we said we matched at all ports; that means, the S 11, S 22, S 33, S 44, they should be 0 because match means there is no mismatch, no reflections at the ports. So, that is why in the 4 ports we have put the 4 0's other we still do not know. So, matched at all ports give us 4 elements of the S matrix they go to 0.

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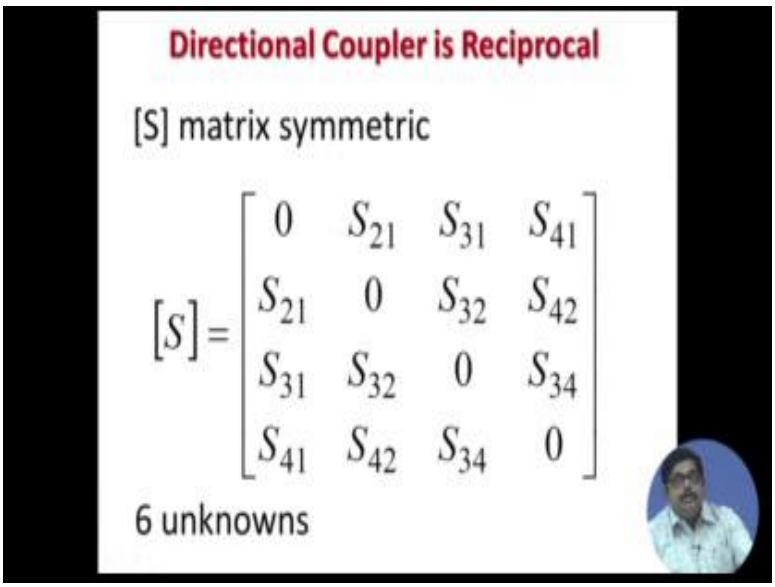

Next, we know directional coupler is reciprocal. Reciprocal means we have already found that if the network is reciprocal its S matrix should be symmetric. So, we have

taken that symmetry we have enforced; that means, this parameter S 12 and S 21 they are same. Now we could have named them S 12, both we have taken S 21 both because that gives us better insight that because S 21 means when power is given to port 1 how much it is going to port 2. So, instead of S 12 we have taken S 21. Similarly S 31, S 31, S 32, S 32, S 42, S 4 2, then S 41, S 41, S 34, is S 34.

So, if you count that you have 6 unknowns here, after invoking the symmetric property you are having how many unknowns - S 21 is an unknown, S 31 is an unknown, S 41 is an unknown, S 32 is an unknown, S 42 is an unknown, and S 34 is an unknown. So, 6 unknowns are there in the S matrix. So, for directional coupler we have invoked 2 properties, one is matched at all ports another is symmetric that gives us a 16 element matrix - 4 by 4 matrix means 16 elements out of that 10 we have eliminated. So, it has now 6 unknowns.

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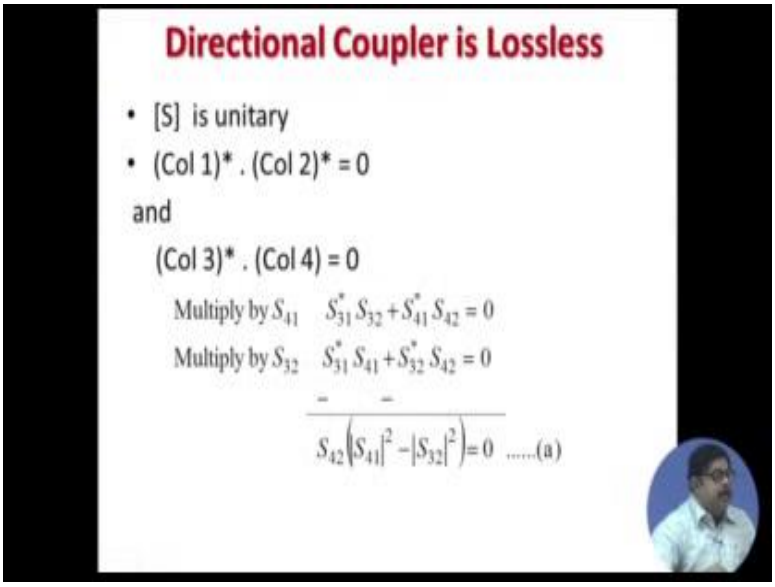

Let us see another property that directional coupler is lossless that is the third wish list we have. So, S should be unitary and let us further simplify the S matrix. So, S unitary means we can have the one column conjugate with another column that will be 0. Now this choice is arbitrary. So, let us start that column 1 conjugate dot column 2 sorry; column 1 conjugate dot column 2 that will be 0 not this one. So, you correct there is a extra conjugate came, the correct one will be column 1, this extra star would not be there please correct it.

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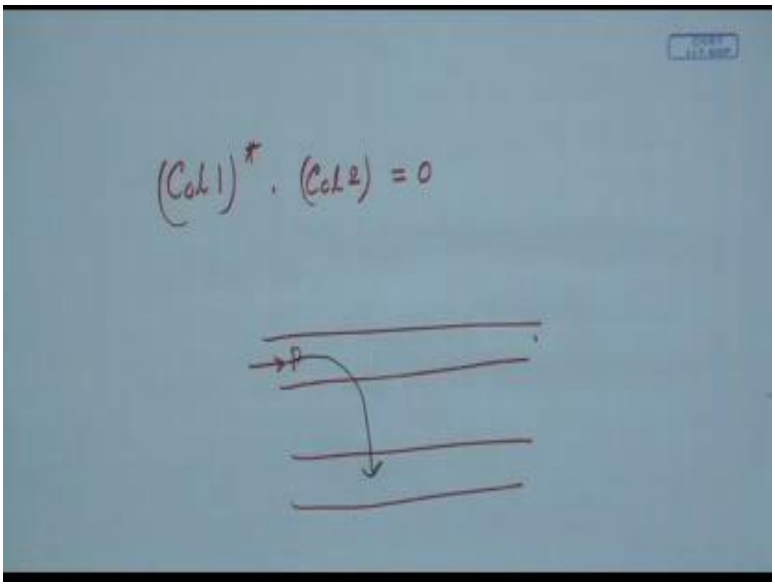

So, and column 3 dot column 3 star that means, column 3 conjugate dot column 4 is 0. That is why we have given it, you can take any other various choices we just do illustration sake.

Now, we have 2 equations. Now you see that, if I want to simplify these equations. So, these first 2 terms if I want to eliminate you see that I need to multiply by S 41 the first equation and multiply by S 32 the second equation and then subtraction, by that I get this formula that I am calling a, this is one formula. Unitary property means there will be various such conditions, various columns all there with other column if it is conjugate it is 0 with itself conjugation gives you 1. Various equations will form, this is the first one.

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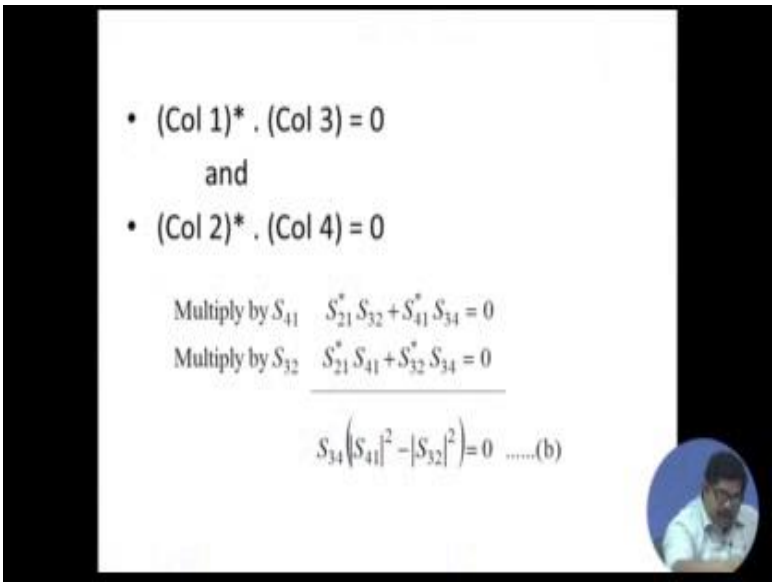

Earlier we have column 1 with column 2. Now, column 1 with column 3 - column 1 conjugate with column 3, 0 and column 2 conjugate with column 4, 0; again we have shown what to do. So, you get this formula it is we are calling equation b.

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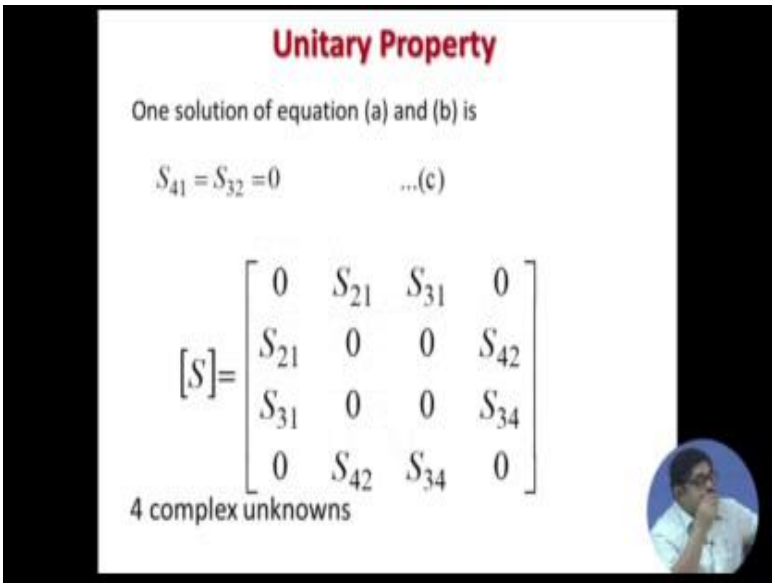

Then those 2 formulas that we got a and b, if you note that there are various solutions of that possible, out of that we are taking one solution that S 41 and S 32 are 0, S 41 and S 32 are 0. What is S 41? S 41 means I have given power at input port 1. How much is coming out of port 4? So, that is 0 - that means, if I give power at port 1 is that port 4 no

power will come. Similarly if I give power at port 2 at third port no power will come. So, those are 0's. So, now, we put it into the S parameter. So, you see 2 of those unknowns due to unitary property they have gone. So, we are remaining with 4 complex unknowns.

But we have not exhausted unitary property we will try to further simplify, but out of 6 unknowns invocation of some of the unitary properties are 4 unknowns, but remember there are actually 4 complex numbers because S parameters are complex number. So, basically we have 8 unknowns because each complex number means a plus j b, a b both are real. So, one complex number means 2 real numbers. So, if I have 4 complex unknowns basically I need to find 8 real numbers to find or determine the S matrix. So, till now we are here.

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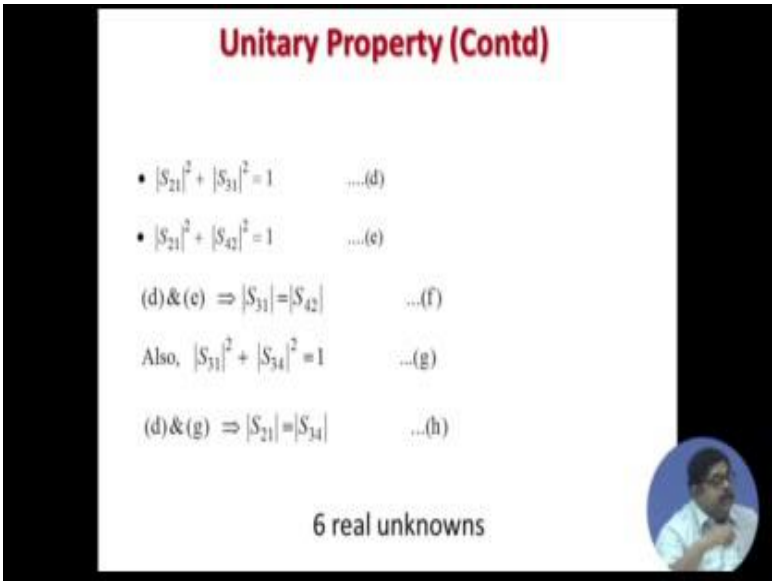

Then we invoke further those earlier 2 (Refer Time: 19:26) different columns they are conjugate one of their conjugate that was 0. Now another property is self thing, by that you can whatever matrix you got from here I can say S 21 square plus S 31 square is equal to 1, S 21 square plus S 42 square magnitude is equal to 1 this plus this, this square plus this square 1, this magnitude square plus this magnitude is magnitude square 1. So, 4 such equations, out of that we have written only 2 and that has done that these 2 says S 31 and S 42 they should be equal.

Now, we invoke another property; that means, the third rows that gives us that S 21 and S 34 they are. So, from this we see the guidelines about the magnitude part of the thing.

So, a complex number can also be written instead of a plus j b form which is called rectangular form, it can also be expressed as a magnitude phase form. So, you see that here about magnitude something I said. So, finally, from here we have got 6 real unknowns what are they that S 31, S 42, S 21, S 2 4 you see here that from this 4 complex unknowns one we are removing and finally, we are having 6 real unknowns. Out of 8 what I said you have now 6 real unknowns you need to determine.

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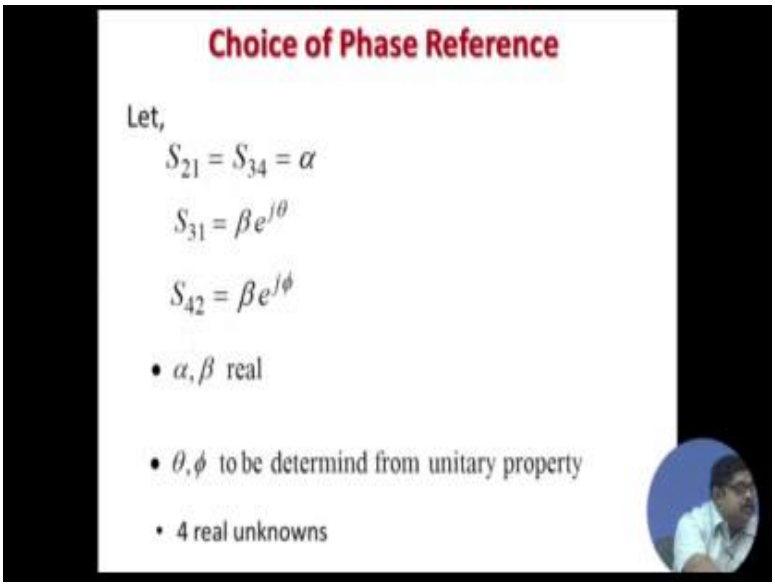

Now, this is your choice. So, generally this choice is made that S 21 and S 34 they though they are complex we take the reference that they are their phase are 0. So, S 21 and S 34 you see that S 21 means I have given power at port 1 I am taking it at port 2. So, that one we are taking as reference to alpha - alpha is a real number. S 34 means I am giving at power 4 - power at port 4 taking at 3 that is totally unconnected with S 21 because S 21 is giving at power is given at port 1, S 34 is power is given at port 4. So, that is why these 2 we have made alpha.

But S 31 I cannot choose arbitrarily because I have taken S 21 as alpha then S 31 should have some phase and also S 31 is different from S 21. So, 31 I am taking the magnitude is beta, a real quantity and phase is theta another real quantity in angle. So, phase and S 42 will be beta because S 31 magnitude and S 42 magnitude, S 31 magnitude and S 42 magnitude they are same that is why we are taking the both of them are having beta, but their phase is different if you refer j 5.

So, now how many unknowns we are remaining, alpha beta 2 real unknowns and theta phi 2 real phase thing. So, 4 real unknowns remain for finding any S parameter of a directional coupler.

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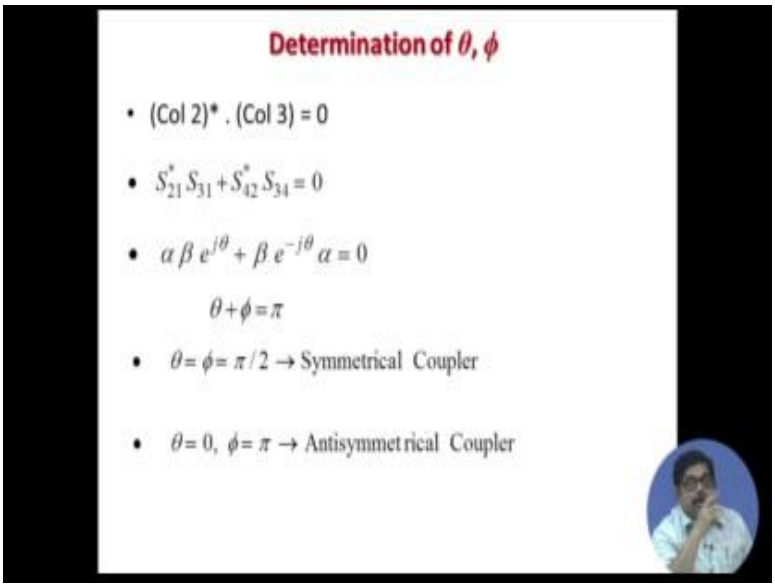

Now, for that again we invoke the property column 2, this we have not invoked till now we have taken 1 2 3 4 etcetera - column 2 star column 2 conjugate dot column 3 is equal to 0, if we invoke that then we get a relationship between the 2 phases theta plus phi. It turns out that theta plus phi are not independent their sum should be phi.

Now, from here there are 2 choices - one is called symmetrical coupler another is called antisymmetrical coupler. In a symmetrical coupler this equation theta plus phi is equal to pi that is solved. So, both of them are taken as same phase, theta and phi both are taken half of this phase that is pi by 2 that is called symmetrical coupler, if we take antisymmetrical coupler then theta is taken 0 and phi is taken phi.

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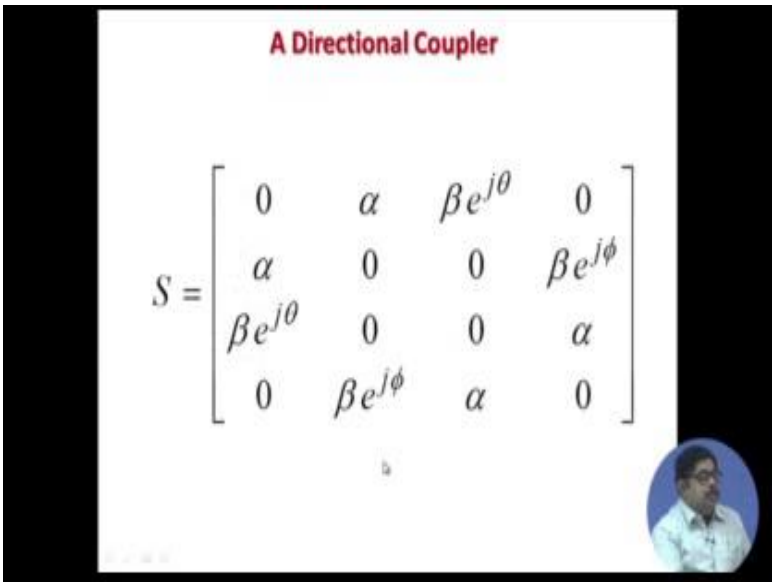

So, this becomes a directional coupler. You see that whatever we have said, directional couplers should have this scattering matrix you see 4 unknowns alpha, beta, theta, phi.

Now, it can be symmetric, it can be antisymmetric.

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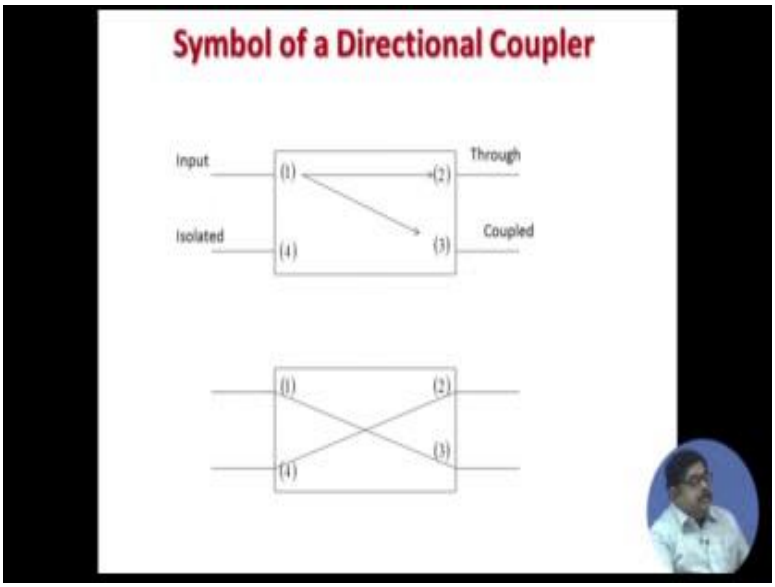

Now, this is the symbol of directional coupler. Directional coupler input is given; you see that input goes to port 2; whatever is given at port 1 it goes to port 2 it also goes to port 3. Let us see here you see that this column shows you what is given at port 1. Port 1 something is given, nothing is coming out at port 1 because its matched then some power

goes to port 2, some goes to port 3, nothing goes to port 4 - that is why port 4 of any directional coupler is called isolated port. That, even if you give power at port 1 - port 2 and 3 get powered, but port 4 do not get powered.

Similarly, what happens if we give a port 2? Power at port 2, you see nothing is coming out from this port 2. Power is going to port 1 power is going to port 4 nothing is also coming to 3. So, if you give power at port 2, port 3 is an isolated port. Similarly if you give power at port 3 then port 2 is isolated other ports gets something then, port 4 if you give power then one is isolated. But the nomenclature of the ports is in terms of the port 1 thing because you will have to standardize, so nomenclature is genuinely we give power at port 1. That is why port 4 is called isolated port.

This second port is the main line if you remember in the network analyzer, in the directional coupler diagram. So, this is called through port 2 and this third port is called coupled port, that is why the name coupler. So, you see in the symbol also we give that thing that this is coupled port this is through port, sometimes this spelling through also is written t h r u as a short, but actually it is through.

Also, this is the diagram of network analyzer, directional coupler that this symbol also is sometimes used that you are giving at port 1 you see port 2 it is going, but main thing is it is getting coupled. That means, coupling means I had a line, wave guide or transmission line now another line is here. I am giving power obviously, power will go here, but this is not coupled power this is through power because this is the main line this is main for taking that power.

But the power if that some how goes here then that is called coupled power. You can say how it is good, there is no radiation mechanism here actually the 2 lines they are kept one over other and there are some holes at some selected well designed places by that the power comes. That coupling is shown here. That one is getting coupled to 3, similarly if you give power at 2 it is getting coupled at 4.

Now, you can say why I will give power at 2 I am giving at port 1 - true, but that power is going to 2 and then if there is a mismatch inside because see ports we have (Refer Time: 28:41) but sometimes if the load etcetera they are some other thing which is not in our hand because up to this port we have designed, but after that sometimes if some instead of match loader or something if they is some other load then it will come back.

Now, that will go to port 4 that will not disturb and come to other ports, like port 3 etcetera.

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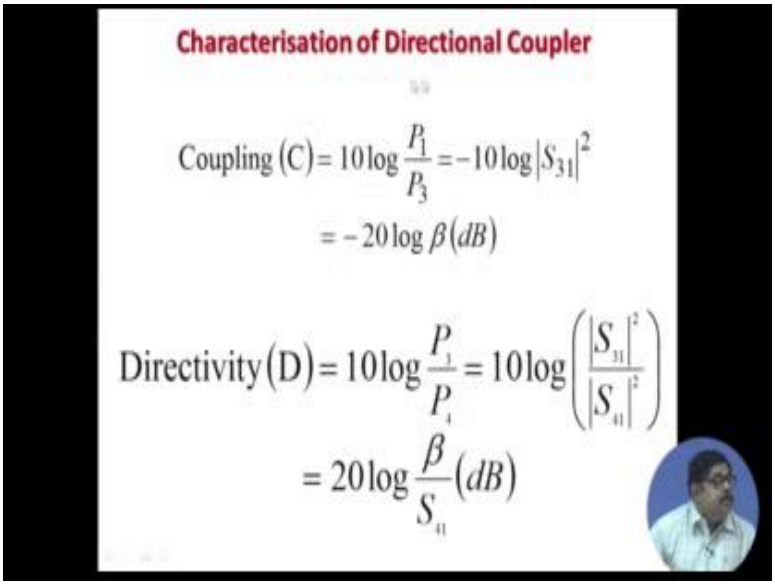

So, these directional couplers they get characterized by 4 quantities - 2 are here. Coupling, it is how much power is from whatever power you are giving at port 1 how much is going to port 3. Its definition is 10 log P 1 by P 3 you see that remember it is not P 3 by P 1 because then the log will be negative, but coupling we want to say suppose 10 dB coupler that means, that out of whatever power I am giving one-tenth is going through coupled port that we write as 10 log P 1 by P 3, and in terms of S 31 you can write it at we have chosen S 31 to be beta that is why I have written it as 10-20 log beta in dB.

Similarly, there is another quantity - directivity. Directivity means that same thing that in forward direction I am trying to couple power to port 3 you see this diagram. In forward direction I am trying to couple my port 1 power to port 3, if the power is mismatched it will come from port 2 it will come to port 4. Now I want to find that this should be I whatever coupling I am making here and this should be low because even if there is a mismatch that is not desirable. So, that is why that physically is directivity that how much power is going to 3 divided by 4. So, ideally this should be very high, in terms of S parameters I have written that - that they depends on S 31 and S 41 and in our chosen ones it is beta etcetera. S 41 we have assumed here to be 0, you see S 41 is 0, but in real

life S 41 is not 0, that is why that S 41 needs to be measured and for finding directivity you need to know that.

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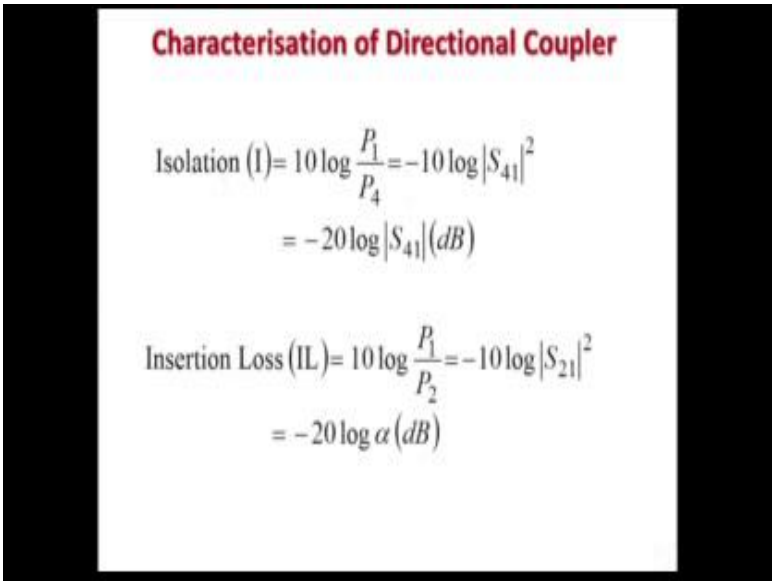

Also there is another thing – isolation, as I said that how good is your isolation; that means, S 41 though we are saying to be 0, but in real life it is not 0, that is isolation in log P 1 by p 4 and that you needs to measure. Similarly, insertion laws that we are saying that the there is no power going here, but insertion loss means the directional coupler we are assuming to be lossless, but the thing is when we are giving P 1 some power is going to other ports.

The full power I am not getting at P 2 that is because we are having coupled ports so some power we are deliberately doing, this is not loss but you can say that if the forward line transmission is going on there how much power I am not getting. So, you see sometimes as in a radiation we say spreading loss, though there is nothing loss there, similarly here also insertion loss is not loss inside but as a transport device I am not getting that because some power I am deliberately coupling. So, to get an idea that how much I am not getting, how much I am using for coupling and measurement etcetera that is called insertion loss.

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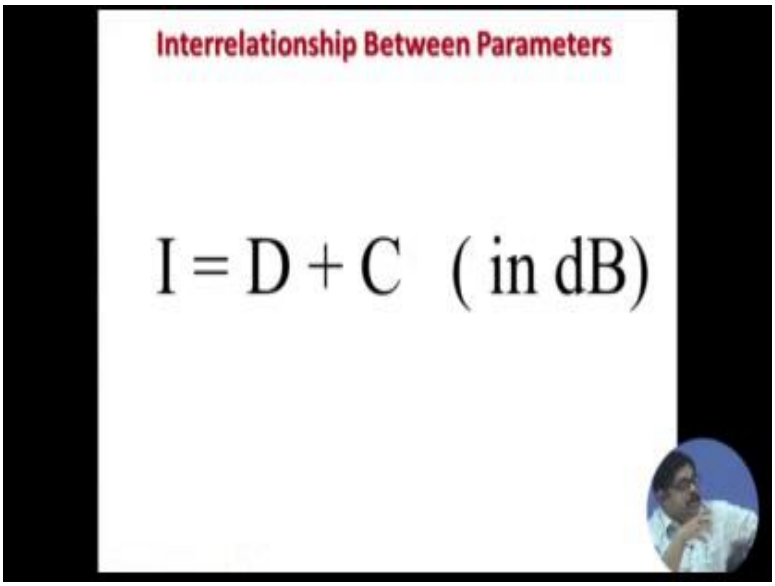

This is the definition and you can do that at interrelationship between them is the isolation is equal to D plus C, but the relationship is in dB, you can easily exercise that.

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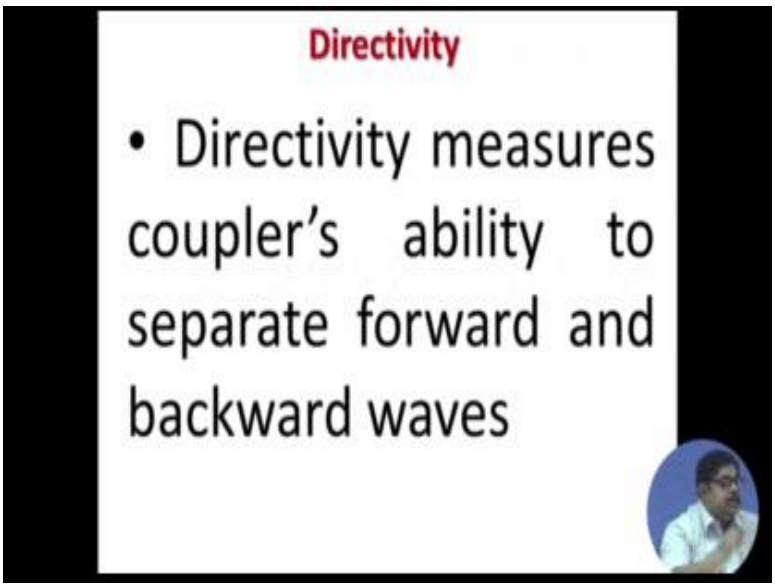

Now, directivity measures couplers ability to separate forward and backward waves. As I said that if you go back to this diagram that you see in this second one, it is showing that directivity means how much I am going here and how much here. Now this is incident wave, incident wave is going to port 3 and whatever is reflected at port 2 that is coming here as port 4 and they are getting separated. Till now we have not mentioned this point,

but now you see that in actually any wave guide there is some incident power and some reflected power. But with this 4 port device I can separate them, because in the main line whatever incident they are going here this one is not at all coming here they are isolated. So, from reflected wave is not coming here, incident is coming here - incident is not coming here. So, port 4 does not have any incident wave.

Now, you see port 4 is only having the reflected wave and port 3 is only having the incident wave it is not having any reflected wave. So, 3 and 4 they are separating the incident and reflected wave a very good thing because though the whole thing is total, but actually in 3 and 4 we are getting separated incident and reflected wave. So, that ability of this to separate this that is called directivity.

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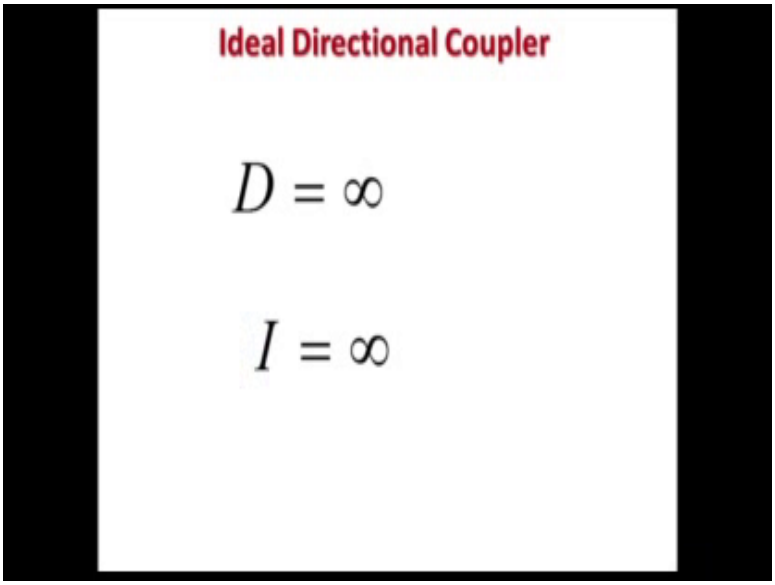

Ideal directional coupler this now this is instrument you see, this is the directional coupler.

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You see, I have seen in the coupled port power is getting measured you see the sensor is connected here power is being fed from this side; power is given at port 1. 4 port in this one in laboratory we do not use it, but in actual life the directional coupler port is also used. In network analyzer inside there are various directional couplers they use port 4.

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Same thing here, the opposite thing here the coupled port is matched, is a match termination and I am sending the power here.

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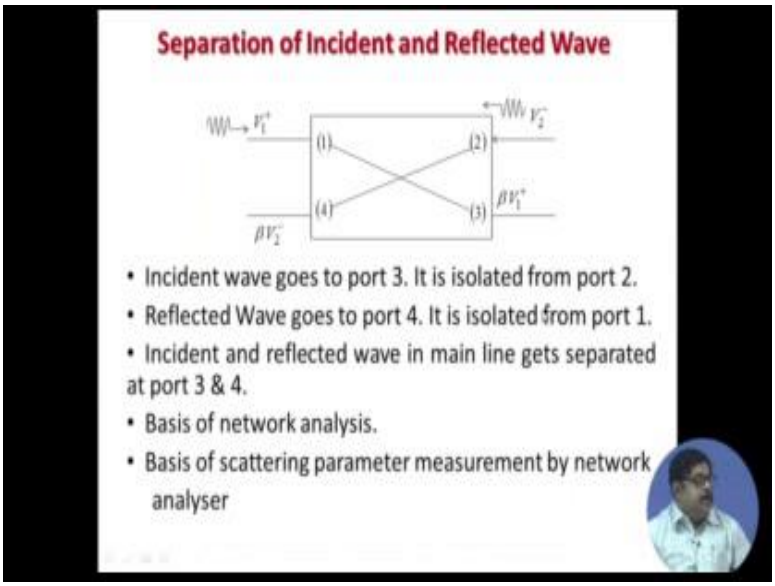

Now, separation of incident and reflected wave, as I said incident wave goes to port 3 it is isolated from port 2. Reflected wave goes to port 4, it is isolated from port 1 incident and reflected wave in main line gets separated at port 3 and 4. This is the basis of network analysis, basis of scattering parameter measurement by network analyzer. So, that is why it is so important to understand directional coupler.

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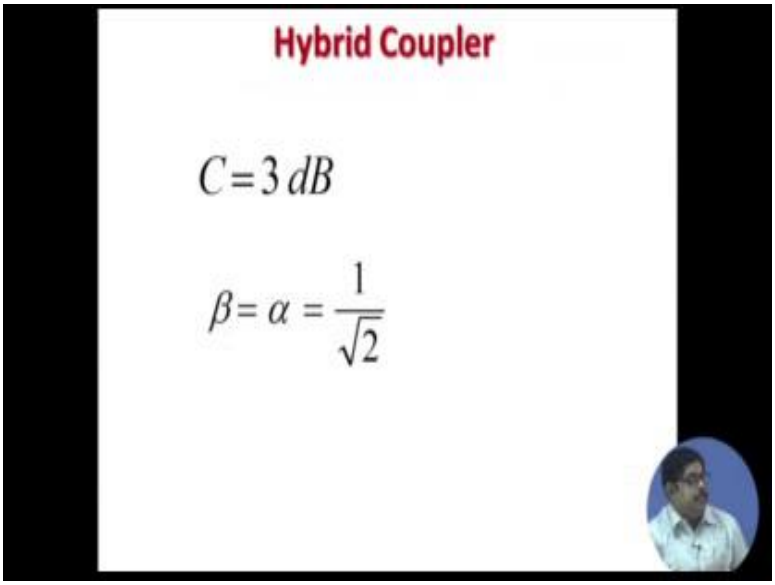

Now, a particular form of directional coupler is called hybrid coupler where this coupling factor is 3 dB. Coupling factor 3 dB means, whatever power I am feeding at

port 1 half of that power is going to the coupled port, if you do that then the alpha and beta they turns out to be 1 by root 2.

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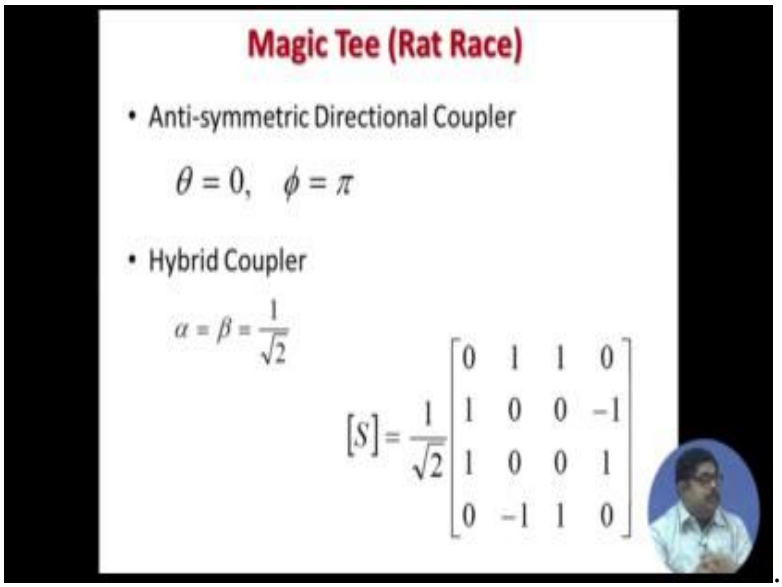

Now this magic tee what we mentioned initially that also is called Rat Race. That is basically this hybrid coupler. So, its alpha and beta both are 1 by root 2, also it is an antisymmetric directional coupler. So, if theta is equal to 0 and phi is equal to phi. So, its S matrix turns out to be this. You can easily find out that from whatever we left for a general directional coupler if you put the values of alpha beta and theta and phi, these are the 4 unknowns. So, if you make this choice then you get directional coupler like this.

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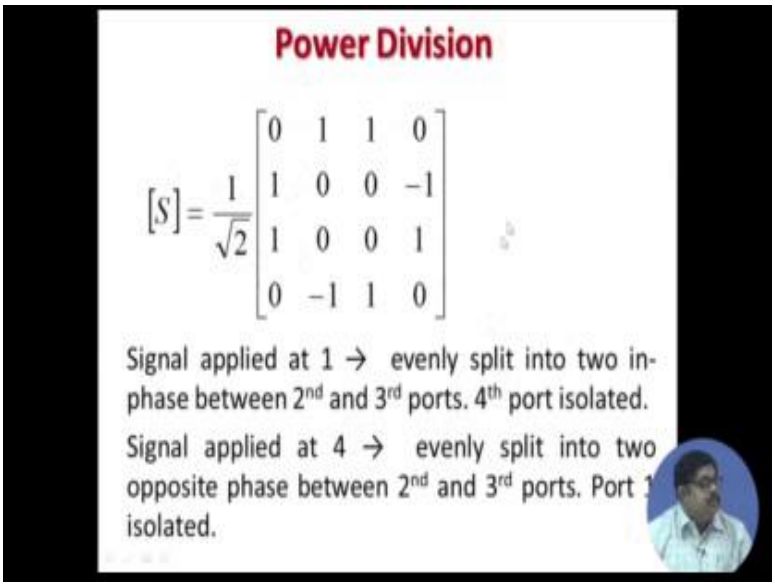

Now, what is the beauty of this? You see that, if I apply signal at port 1 that means, look at this row, what happens? That signal gets divided between port 2 and 3, their phase is also same. So, we have due to an evenly split into 2 in-phase signals between 2nd and 3rd ports, 4th port is isolated.

Similarly if signal is applied at port 4, look at this 4th column. The signal is evenly split between two, 2 and 3 port, but there is their opposite phase and no signal is going to port 1. So, port 1 is isolated that means I can use it as a power divider. If I give signal at either port 1 or port 4. In both the cases equal power is getting into the 2 ports, it is getting divided into 2 ports.

Now, depending on your wish if you give it at port 1 you can get that the 2 dB divided powers they are at equal phase. If you give the power at port 4 they are at opposite phase. So, power division, but 2 types of power division you can get from here.

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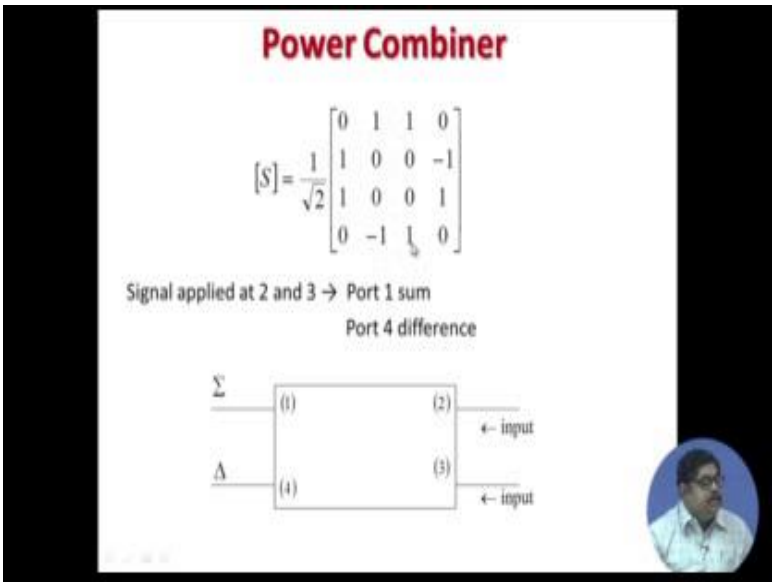

You can also get it as a power combiner. You see the same scattering matrix now I apply signal at port 2 and 3. Now 2 means this 3 means this, so this is a linear circuit, you super impose that - that means, if I give power at 2 and 3 then signal is coming only at port 1 and port 4, now sum this- this is 2, this is 0. That means, at port 1 whatever power I am giving at 2 and 3 their sum I am getting and at port 4 I am getting 0; that is why you see that I am giving input at 2 and 3, at 1 I am getting sum, at 4 I am getting difference.

So, this is very important because in radars etcetera, you want to have that to determine the angle precisely I need to find out that whatever signal I am getting I need to sum and I need to divide and that if I divide this sum and divide signal, we get precise angle measurement that is called mono pulse technique there this rat race is used.

But in our case why it is called now magic, you see this is the picture of a magic tee that I can use this port 4 device either as a power divider or as a power combiner.

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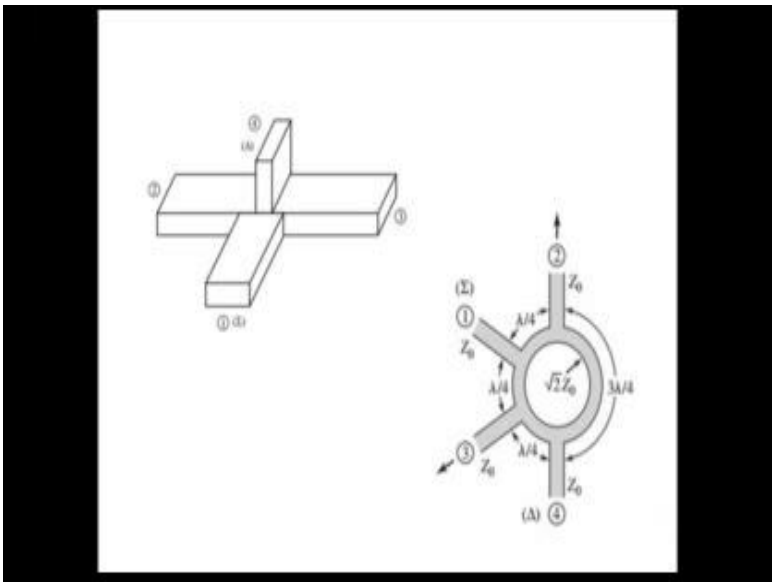

Remember what we started this lecture with that we need to sometimes make power divisions sometimes make power combiner, generally they are different device. But rat race one device can give you either a power divider or a power combiner that also whether you have one sum up power or difference of power, whether you have splitting in equal phase or splitting in opposite phase. This is the wave guide version.

Left side of a directional coupler it is the 2 planes actually 2 tee junctions in 2 orthogonal planes that makes it the fourth port is here, this is the difference port, this is the sum port and this is the main line. So, you see there are 2 auxiliary lines here; similarly this is a planar version of the same thing. It is like a rat racing that you are giving power here that is coming here etcetera that is why it is called Rat Race.

In our lab we have this power heating, you see these students they have retained because if they get sometimes confused, which power port is which one.

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So, you see these 2 are - 2 and 3 port. This one will be 1 port and this is port 4, this is the difference port, this is the sum port these two are your 2 (Refer Time: 41:28). So, I think that we have seen, very 2 important devices rat race or magic tee and directional coupler they is, S parameter. So, S parameter gives us to understand what is happening in the physics behind it.

Now in terms of total thing we could not have done that, total voltage, total current does not give us that inside picture, but the S parameter which is how the wave is flowing that gives us that intricate picture.