Basic Tools of Microwave Engineering

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Lecture – 16
Relation between S-parameters and transmission parameters

Welcome to the sixteenth lecture. We have seen scattering parameters; we have seen relationship with Z parameters. Particularly today we will see the relationship of S parameter with ABCD parameter or transmission parameters as you know that in a complex network we divide the network into several parts. So, each sub block we characterise by S parameters. Now, while connecting instead of connecting them in cascade in terms of S parameter, if we know the ABCD parameter we can connect them better because ABCD parameters are transmission parameters are designed for cascading of various sub blocks of a network.

Now, as you know that ABCD parameter depends on total voltage and current at ports whereas, scattering parameters they reflect the true wave nature of the voltage and current waves. So, if we convert from scattering parameter to ABCD parameter then we can find for each block first the scattering parameter, then we can convert to ABCD parameter then find the overall networks ABCD parameter by cascading those sub blocks and then we can reconvert back from ABCD parameter to S parameter, so that the overall S parameter of the whole network will be able to find. So, that is why this relationship between S parameters and transmitter parameters 1 should know, 1 microwave engineer should know and that is why we start our discussion.

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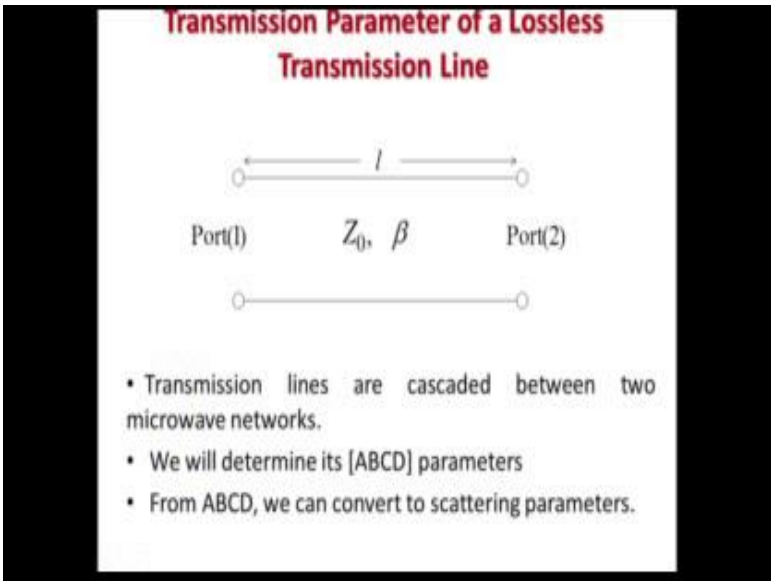

Now, first let us find the transmission parameter of a lossless transmission line. You are given a simple transmission line, find the ABCD parameter? Now, obviously this is the moment we say transmission line, transmission line means there it will be the wave nature that will come into play. So, we will have to find the incident voltage wave, incident current wave, reflected voltage wave, etcetera. Find the S parameters and also we can find the ABCD parameters and from ABCD we can convert back to scattering parameters. So, this strategy we will follow.

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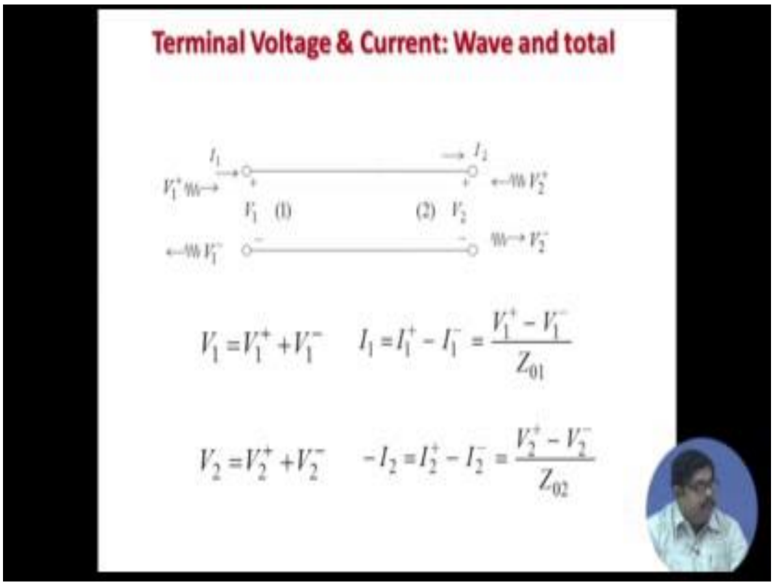

So, you see we have a piece of transmission line of length l characteristic impedance of the transmission line is Z naught phase constant is beta that means, we are assuming lossless transmission line. So, propagation constant gamma that does not have any real part, it has the imaginary part j b that is why the face constant is only shown.

Now, we are considering 1 port there are 2 ports. So, on the left side you have port 1 on the right side you have port 2. So, let us find its ABCD parameter? Now, for that we need to break the voltage wave V 1 the total voltage V 1 that should be broken into the incident voltage wave and reflected voltage wave. So, V 1 is equal to V 1 plus V 1 minus also the total port current I 1 that should be broken into the incident current wave I 1 plus and incident reflected current wave I 1 minus and you know that their relationship is I 1 plus minus I 1 minus I 1 is equal to and that we can again relate to the voltage waves by bringing the impedance reference impedance at port 1.

Let us say that is Z naught 1 in most of the cases this Z naught 1 is taken as Z naught of the transmission line. Similarly, you can do it at the port 2 that you can write the total voltage V 2 in terms of V 2 plus and V 2 minus you can also write I 2 in terms of I 2 plus and I 2 minus. Here 1 point I want to mention that in a transmission matrix ABCD parameters when we find generally the port current I 2 is taken as it is going out from port 2 because in a cascade that helps now in other 2 port parameters Z y etcetera.

They are the I 2 flows into the second port, but in transmission parameter it goes out that is why we have shown it as I two, but when we are writing the I 2 plus and I 2 minus I 2 plus is inside the port I 2 minus is out of the port that is why I 2 plus minus I 2 minus that direction is minus I 2 note this because this is 1 point of confusion generally people have that. Now, this I 2 plus and I 2 minus they can related to V 2 plus and V 2 minus with the help of the second port characteristic impedance Z naught 2 again most of the cases Z naught 2 I assume to be Z naught the transmission lines characteristic impedance..

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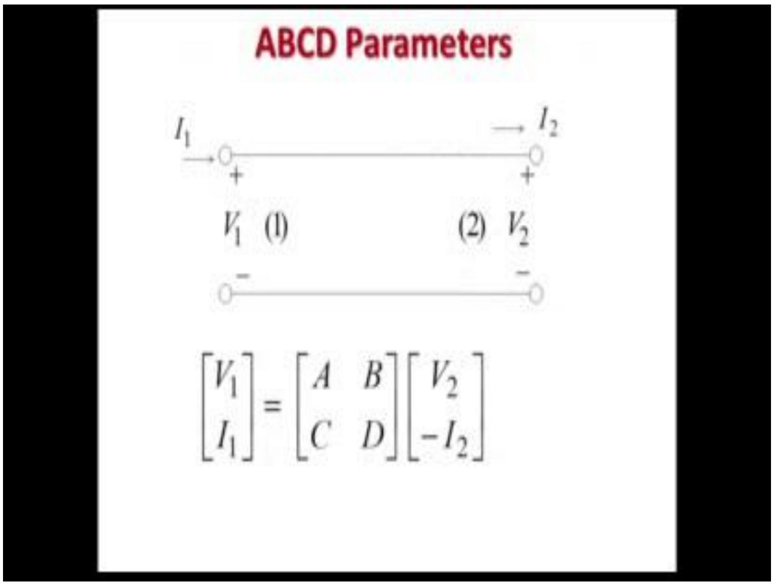

Now, you know that this is your earlier knowledge that ABCD parameters are defined as the port 1 quantities port 1 voltage and current V 1 and I 1 total that is related to V 2 and I 2. Again note that minus I 2 because of the changed direction of I 2 for ABCD parameters, but in terms of our usual I 2 which goes into the port 2 that minus has been introduced.

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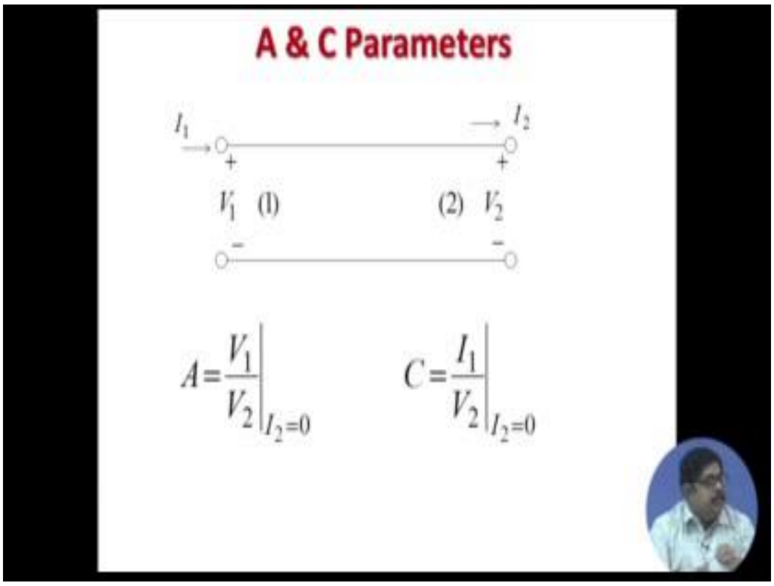

So, this is the definition now to find in A, B, A and C you see in both these cases we require I 2 should be made equal to 0. Now, I two; that means, if we make in the port 2 I

2 is equal to 0 means it is an open circuit. So, we will have to make that port condition port 2 should be open circuited and then the ratio of V 1 by V 2 will give us A and ratio of I 1 by V 2 will give us C. So, let us see that we have made open circuit port 2 is open circuit I 2 is equal to 0.

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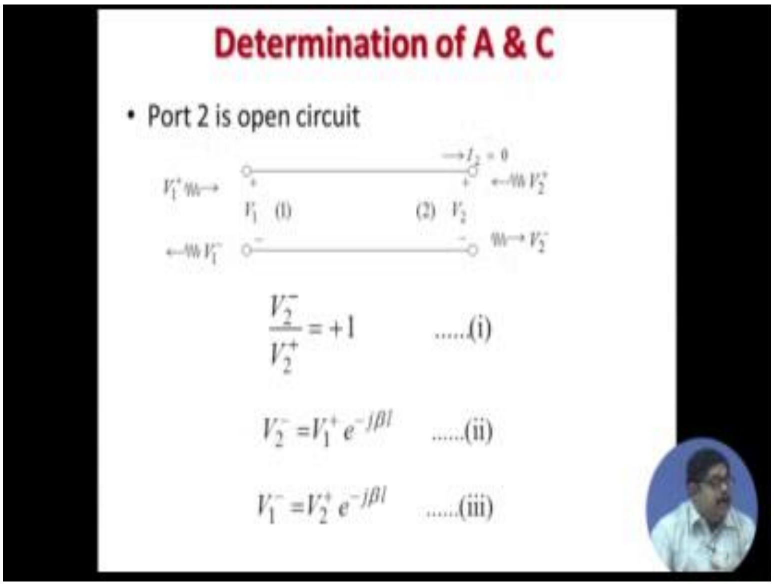

Now, in an open circuit you know what happens relation between the reflected voltage and the incident voltage. So, in an open circuit the reflection coefficient is plus 1. So, V 2 minus by V 2 plus then you will be plus 1 then V 2 and you know that V 2 minus what is V 2 minus this is this wave how that wave came this V 1 plus wave has transmitted through this transmission line by a distance l. So, V 2 minus is related to V 1 plus by this relation this propagation relation that e to power minus j beta l because any wave when it moves by a distance l it acquires an added phase of minus j beta l. So, that is why this second relation similarly the V 2 plus that is falling here on port 2 that is troubling through this transmission line by again at distance of l and that is coming out as V 1 minus that is why we are writing V 1 minus is equal to V 2 plus e to the power minus j beta l

Now, with this three relations you can solve for these quantities V 1 plus V 1 minus V 2 plus V 1 V 2 minus.

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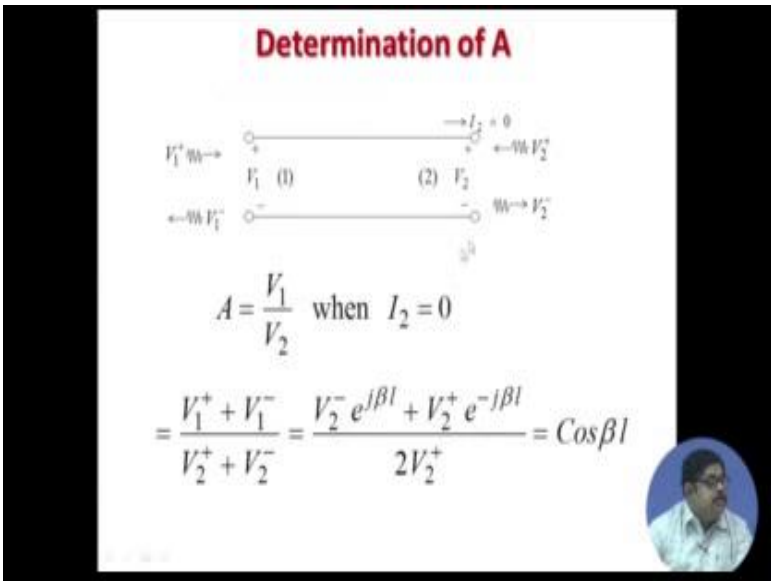

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So, that is done determination of a. So, a is V 1 by V 2 when I 2 is equal to 0; that means, that condition. So, simply you can V 1 you write as V 1 plus plus V 1 minus and V 2 also V 2 plus plus V 2 minus then you put those equations from equation 2 and three and the first equation which was open circuit at condition that shows that V 2 minus and V 2 plus both are equal. So, we are calling it 2 V 2 plus then this V 2 minus will also be replaced by V 2 plus. So, all V 2 plus they gets cancelled you get e to the power j beta l plus e to the power minus j beta l by 2 which you know is cos beta l. So, as value is becoming cos beta l. So, for a pure lossless transmission line the a parameter is cos beta l

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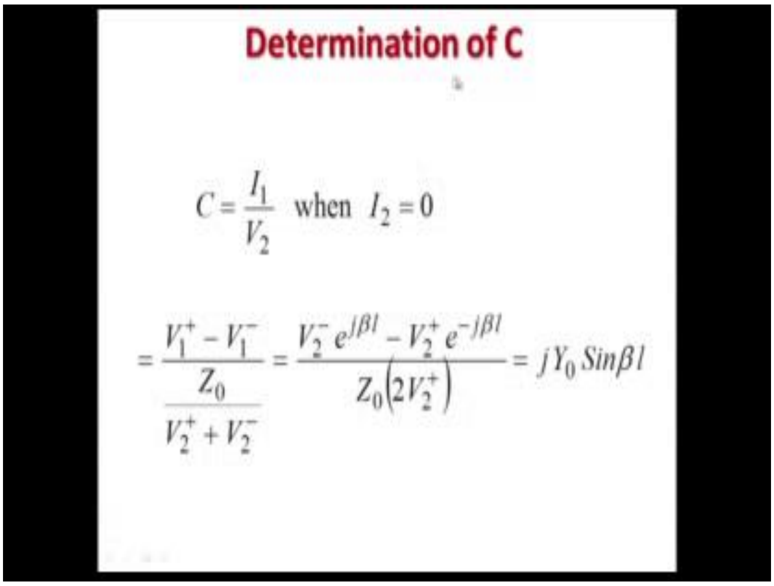

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Now, also you can determine c c is I 1 by V 2 I 1 we have already said V 1 plus minus V 1 minus by Z naught and V 2 as it is V 2 plus plus V 2 minus again you do manipulation you see that it comes out to be j y naught y naught is nothing, but 1 by Z naught the characteristic admittance of the transmission line into in this case sin beta l. So, this is an imaginary quantity A was pure real C is imaginary.

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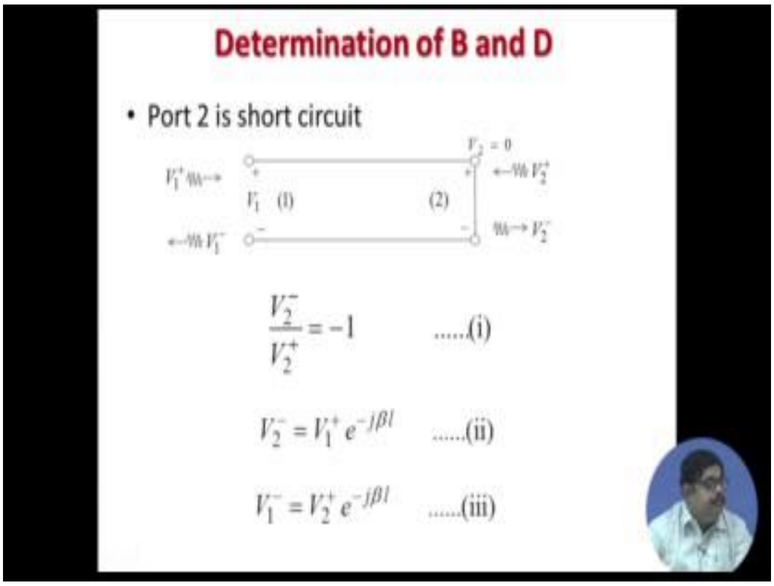

Then you need to determine B and D the condition for B and D determination will come if we short circuit port one; that means, they require that V 1 should be equal to 0 not V 2 is equal to 0 port sorry port 2 is short circuit. So, you know that V 2 minus by V 2 plus is minus 1 for a short circuit the reflection coefficient is minus 1. So, V 2 minus by V 2 plus is shorted and you can find out that V 2 minus again the propagation condition that V 2 minus that has come because of V 1 plus with this transmission. So, that is given by e to the power minus j beta l similarly V 1 minus which is coming out here due to V 2 plus that is again e to the power minus j beta l. So, again from this 3 equations you can solve for those V 2 plus and V 2 minus.

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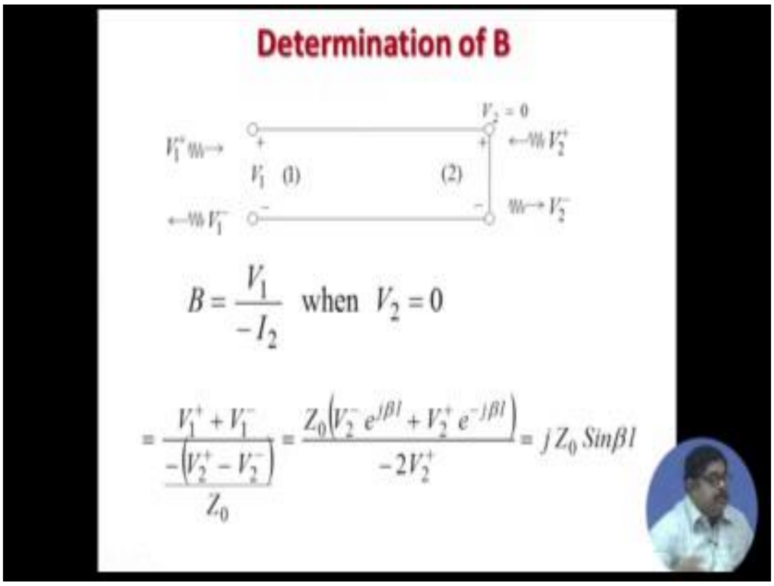

So, first solve for B parameter is V 1 minus I 2 when V 2 is equal to 0. So, that if you put those values you get j Z naught sine beta l again an imaginary quantity.

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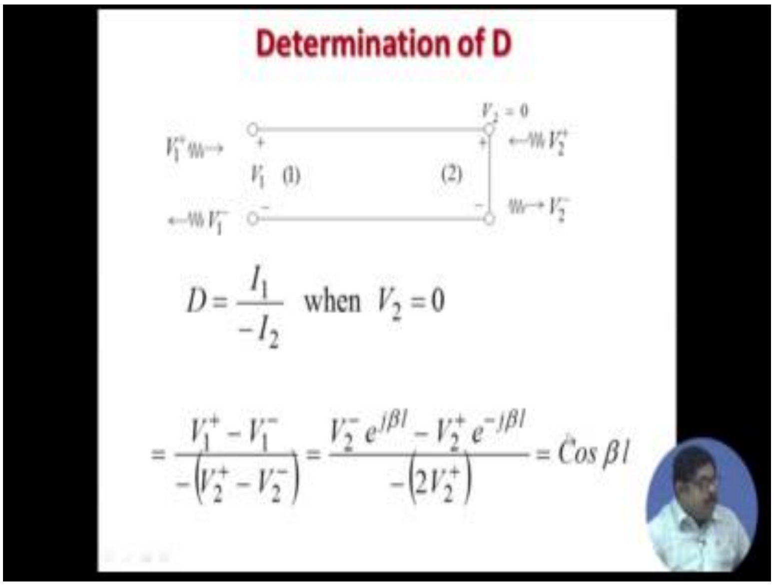

And determination of D that is ratio of 2 currents this, so that will give you this D is equal to cos beta l and again a real quantity.

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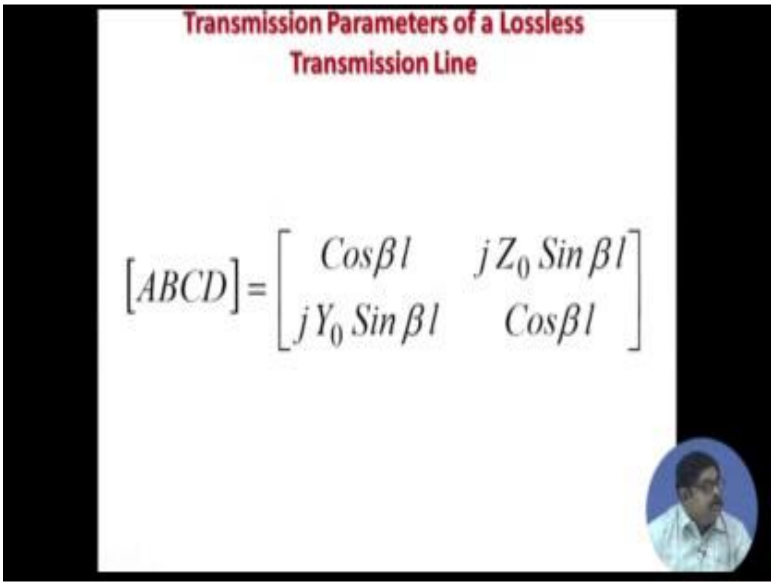

So, finally, you get the transmission parameters of a lossless transmission line to be cos beta l j Z naught sine beta l etcetera.

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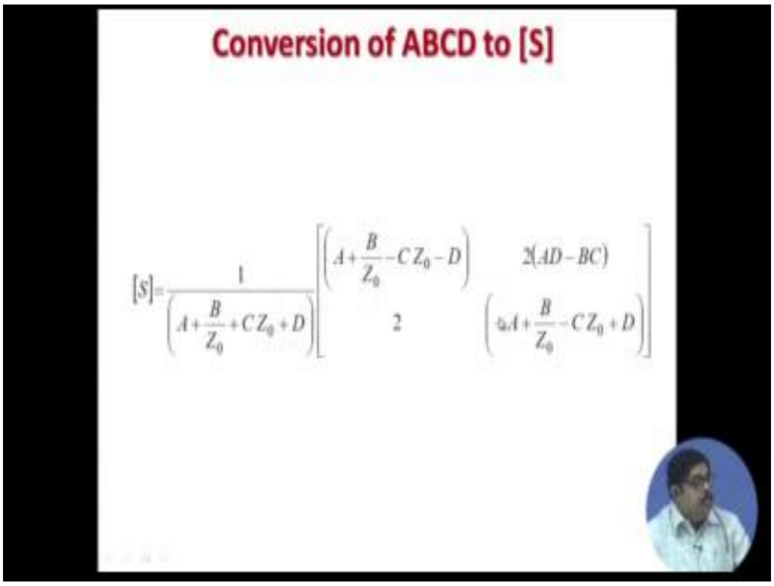

So, now this conversion of ABCD, ABCD to S this formula is there for inter conversion. So, S is this is common for the whole matrix and then these formulas.

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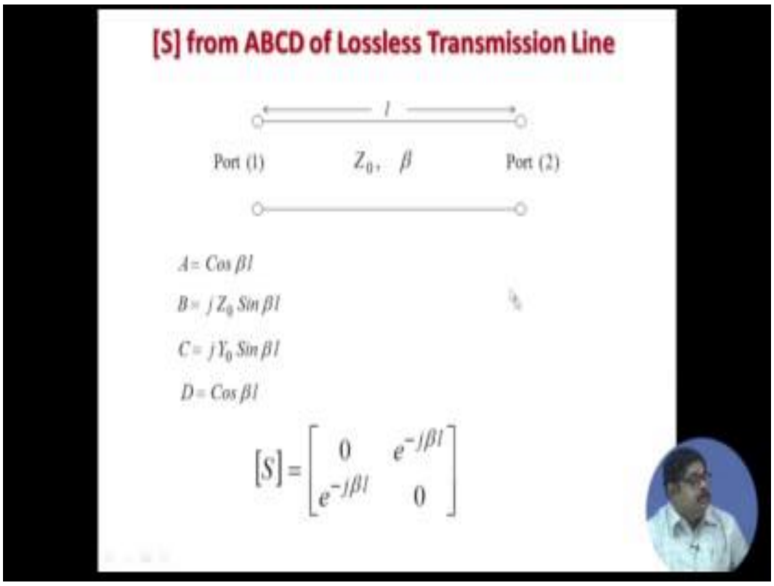

So, if we put that then we have ABCD we have written A once again here, if you do those conversion formula you get that S parameter of the lossless transmission line is its S 11 is 0 its S 22 is also 0 its S 12 and S 21 are e to the power minus j beta l. So, as you see that this S parameter from the S matrix we can see it is a reciprocal network because there are no active devices etcetera is used. So, its S parameter is reciprocal; that means, if we take for S I j is equal to S j I we can say that or we can also say that if we take the transition of this matrix that will be same as this matrix.

So, this is a reciprocal matrix also you see this satisfies unitary property because here if we take that this column if we take conjugate with each it will be just the; that means, it will be e to the power minus j beta l into e to the power plus j beta l which will be equal to 1. So, that 2. Similarly, this and conjugate of this that will be 0. So, it is a pure unitary matrix and that is why we get that this is the lossless network. So, this is the S matrix you remember transmission line.

Obviously, it is on the transmission parameters S 12 and S 21 they are present it does not have because it is a match line we have assumed that port 1 reference impedance is equal to Z naught port 2 is also Z naught that that is why it is called a matched transmission line. So, for a matched transmission line there is no reflected wave there is no reflected wave in port 1 as well as in port 2 and we get S 11 and S 22 both of them at 0. So, this we got from converting from that ABCD parameter we can get this now.

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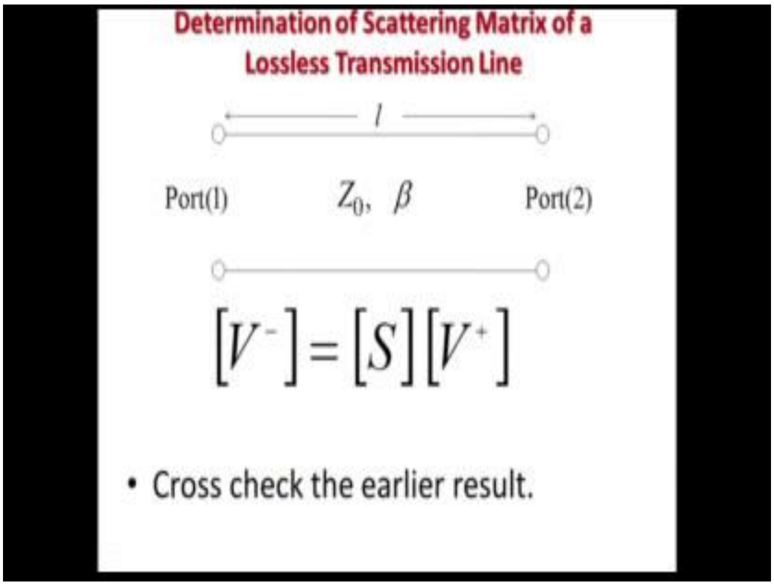

So, we can write that V minus is equal to S V plus you can cross check the earlier result that it will be there. Now, for let us do that the previous formula we derived for the S parameter that was from conversion from ABCD parameter. Now, let us see how the scattering matrix of a lossless transmission line how we derive that then if this 2 S parameters comes to be same we can cross check the result. So, here now we will do the same thing find the scattering parameter by directly finding scattering parameter we know how to do it for scattering parameter determination you know that if you want to determine S 11 and S 21 then you need to match terminate the port 2.

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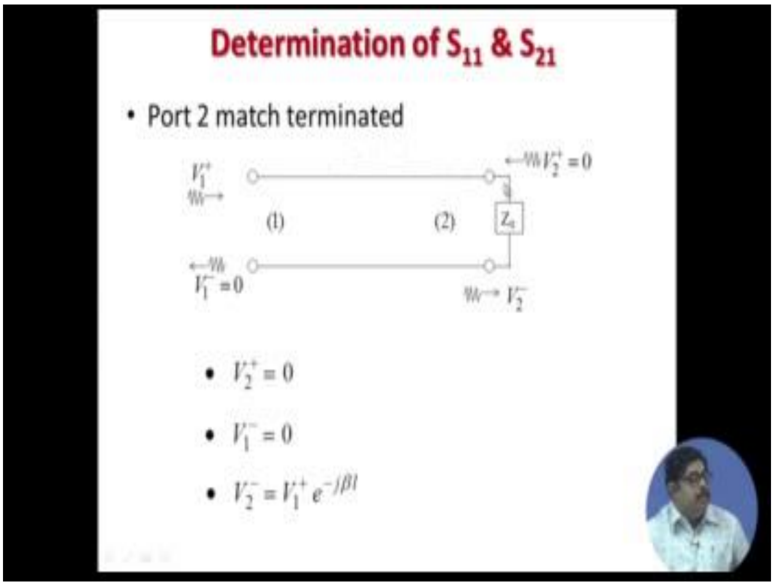

So, you see we have match terminated match terminated means the termination should be an impedance whose value is equal to the characteristic impedance. So, Z naught we have connected under this condition we will have to find for S 11 it will be V 1 minus by V 1 plus and for S 21 it will be V 2 minus by V 1 plus. So, you see we have written that V 2 just because this is Z naught. So, there is no incident wave V 2 plus will be 0 and the moment V 2 plus is 0, V 2 plus after travelling here come out here as V 1 minus. So, V 1 minus also will be 0 and V 2 plus we know that it is 0 V 1 minus 0 and V 2 minus; that means, we have V 1 plus. So, that after transmission will come at V 2 minus and their relationship is like a wave travelling through a length l in a transmission line e to the power minus j beta l is the relationship

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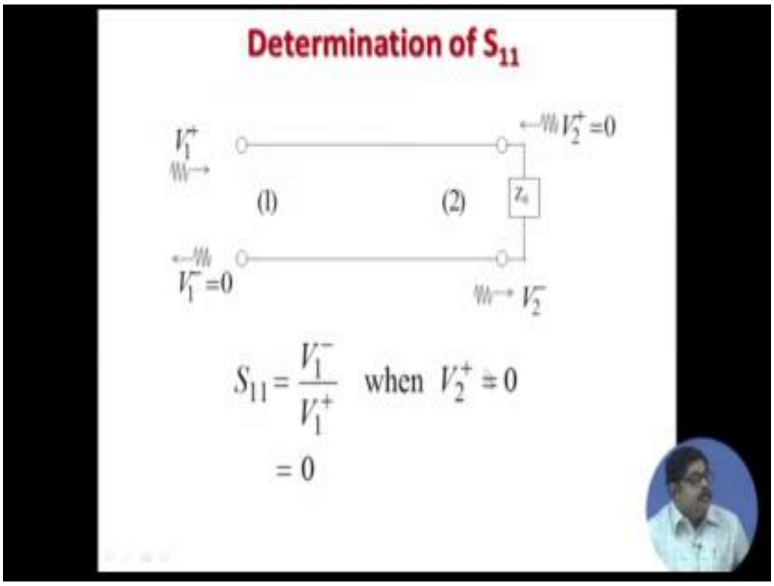

Now, let us determine S 11 which is V 1 minus by V 1 plus as we said that V 1 minus is becoming 0 under match condition. So, S 11 is becoming 0.

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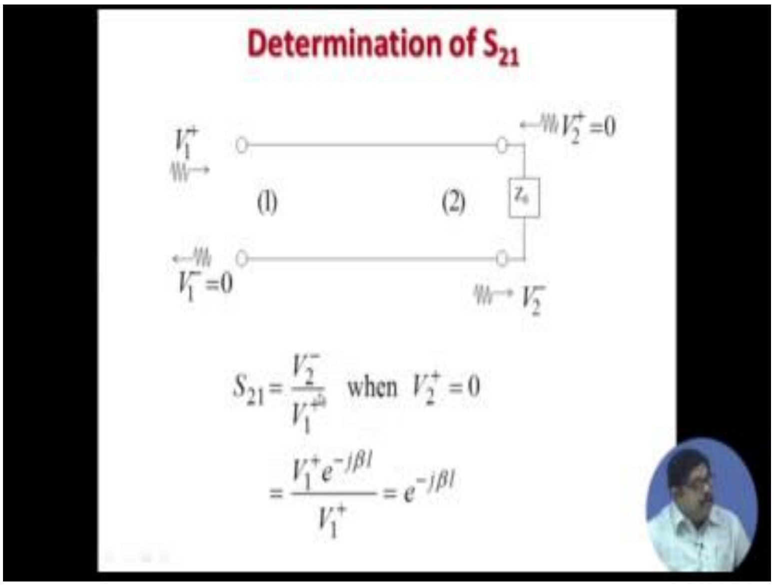

Now, under the same condition see what is happening to V 2 minus plus V 1 minus we can put that relationship and it becomes e to the power minus j beta l.

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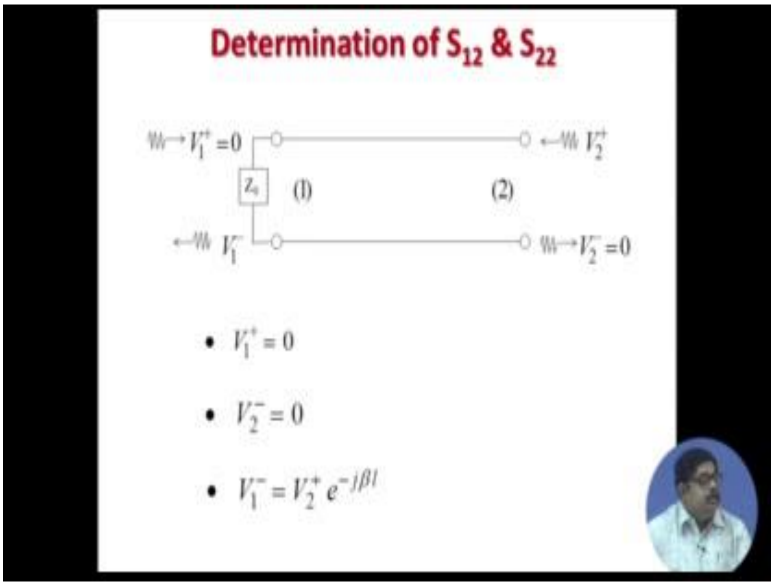

So, we have got S 21, now for S 12 and S 22 we need to match terminate port 1. So, that we can find the 2 parameters here the match terminating port 1 means V 1 plus will be 0 V 2 minus will be equal to 0 obviously, because V 1 plus becomes here V 2 minus that becomes 0 and who are then present V 2 plus and V 1 minus their interrelationship is like e to the power minus j beta l.

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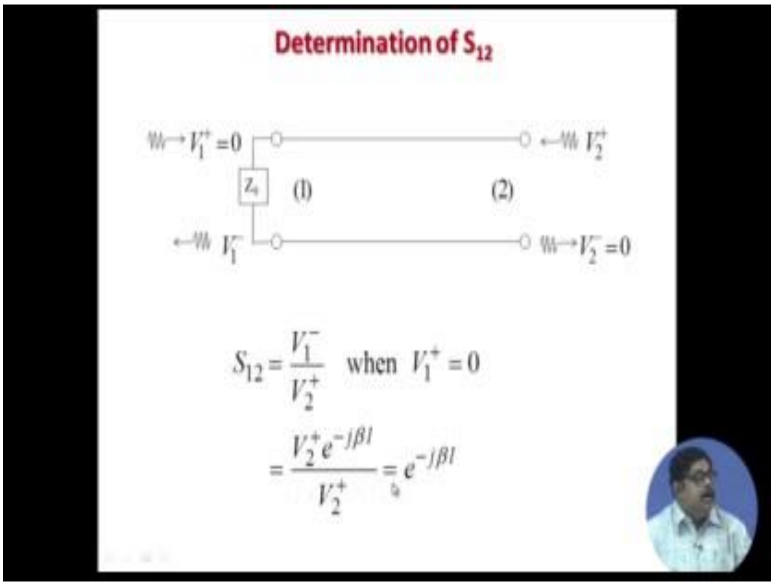

So, now we can determine S 12. So, we put the values of V 1 minus and V 2 plus and that becomes e to the power minus j beta l.

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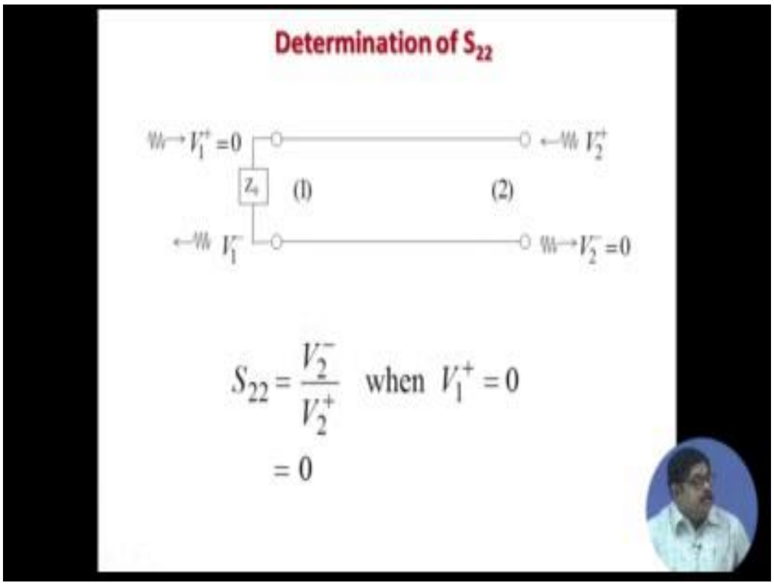

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Similarly, S 22 is V 2 minus is 0 because this is 0 V 2 plus exists that is why 0 by some value that is 0. So, S 22 is also 0. So, we get the same thing that we got from conversion from ABCD parameter that shows that our formula is correct.

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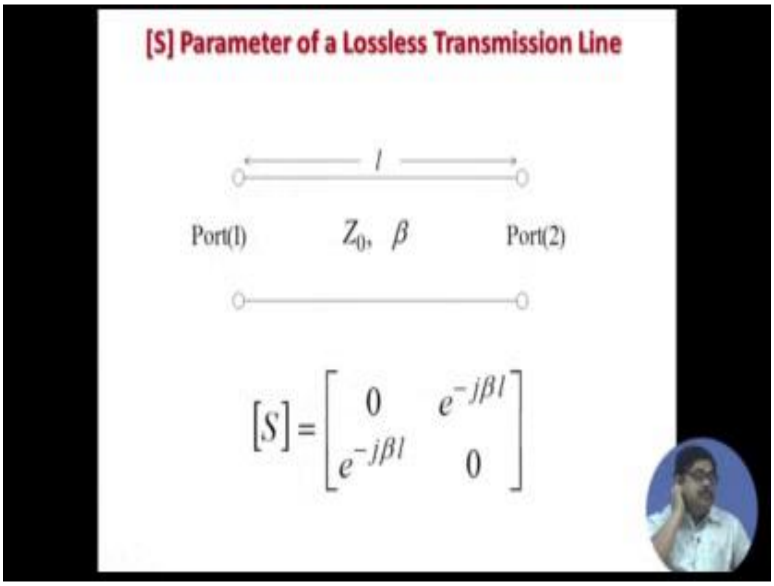

So, now you know that if you have any circuit, if it is distributed circuit it is better to find its S parameter because actual transmission is happening through waves. So, you can find S parameter then S 2 Z you can reconvert back that formula is given here.

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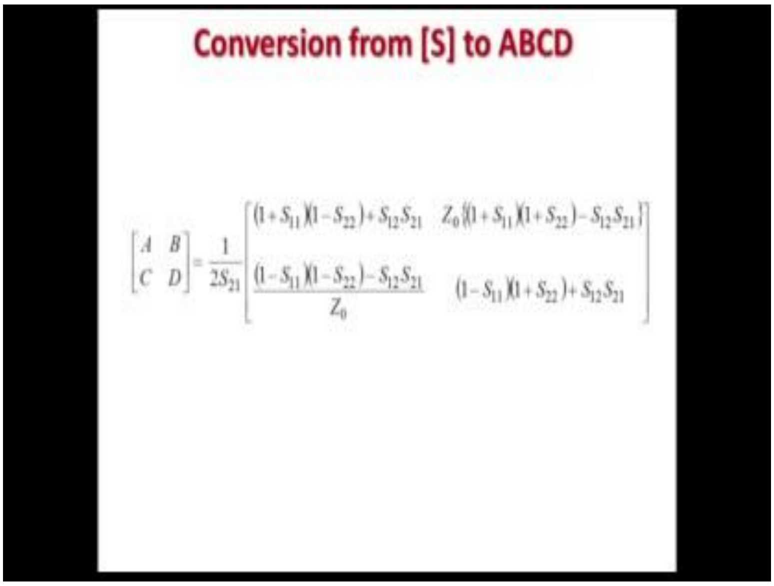

That if you know S parameter, how to find ABCD parameter you can find this.

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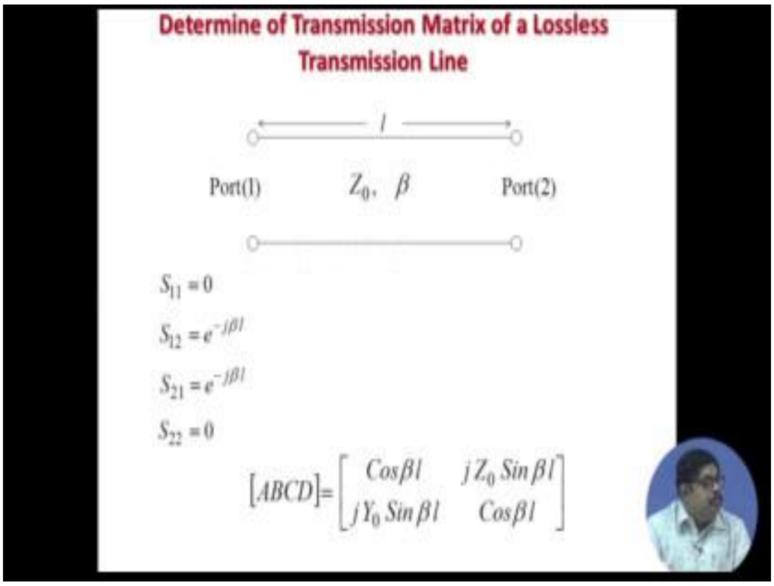

So, we have done this and if you apply these formulas you will have to note actually these are available. So, you can always refer to that in your professional life and find out then if you find you get back the same ABCD parameter that we got. So, now, you know the interconnection between the ABCD parameter and transmission parameter that was this lecture.

So, this completes that we know now Z parameter you can convert through Z parameter you can convert to y parameter etcetera. So, that is that was 1 journey another journey is S to ABCD and ABCD to s. So, now, you can measure S parameter by network analyser and from that you can find out as the case may be suppose you are 1 sub block let us 1 filter you have found its S parameter now you want to that filter will be connected to some receiver or some transmitter circuit. So, there will be other blocks

Now, for all the blocks if you find S parameter then from each you can find ABCD parameter let us say they are being added in cascade. So, a filter an amplifier all they are in cascade. So, you know each ones S parameter you can convert each ones S parameter to its ABCD parameter then you can find out overall S parameter of the whole cascaded network and then again you can come back to S parameter if that is required or if is ABCD parameter is sufficient you stay there. So, by this the second tool S parameter you have seen that how microwave engineers they use scattering parameter to characterise network

Now, in the next lecture we will see another 2 very important devices, microwave device passive device that S parameter will see that how S parameter helps to understand what those devices do that is the next seventeenth lecture there you will see.

Thank you.