Basic Tools of Microwave Engineering
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Lecture - 15
Tutorial 3: Problem Solving on Equivalent Voltage and Current in Waveguide and on Scattering Parameters

Welcome to the 15th lecture, that is tutorial 3. We will see some problem solving, on equivalent voltage and current concepts in waveguide and on scattering parameters. The first problem is determined the equivalent voltage and current for TE m0 mode fields.

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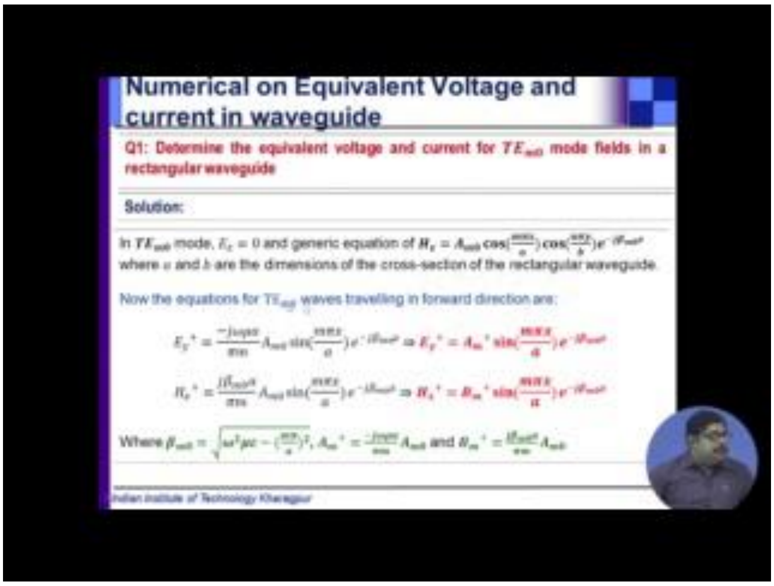

In a rectangular waveguide, we have seen how to do it now just look at the TE m0 mode. Transverse electric means, ez is equal to and hz this is the generic form a and b are the dimensions of the cross section of the rectangular waveguide. So, if you write TE m0 mode waves appeals, the transverse fields will be ey and hx plus incident fields; obviously, they are also the reflective fields.

Now, ey plus can be written in these form. So, that we are saying, that there is an amplitude part then this is the Eigen function part this is the Z propagation part, hx plus also like this where this propagation constant or phase constant since we are assuming lossless line gamma is j beta. So, beta for m 0 mode is like this. Now this m plus is this, this is the constant and bm plus is like this.

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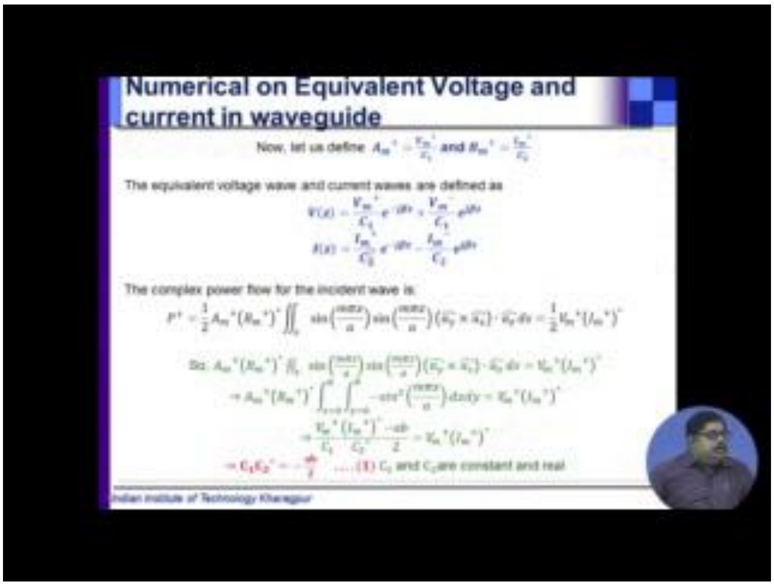

Now let us define as we said that the electric field and voltage should be proportional. So, that proportionality constant let us say c1. So, this is a voltage side quantity that we are making proportional to m plus the magnitude of the voltage wave and b m plus was the magnetic field part. So, that we are making proportional to the current, this is the proportionality constant is c two. So, this c1 c2 is a value finally, we have to find now equivalent voltage and currents are like this now enforce the power equivalence. So, first find out from the field 1 with these definitions. What is the power incident power? So, all the steps are done for you. Pointing vector then integration over the guide S means the transverse cross section of the guide; that means, for a rectangular wave guide it is from the x is from 0 to a, y goes from 0 to b.

So, if you do that now that we have done here, that these are the thing and this should be equal to the average power that was the power equivalence part half vm plus I m plus star. So, if you equate that to you get these relation c1 c2 star is equal to minus a b by 2 Now c1 and c2 are constant and real. So, c2 star will equal to c two; that means, c1 c2 product is minus a b by 2 this is1 equation. So, we are calling it equation number 1.

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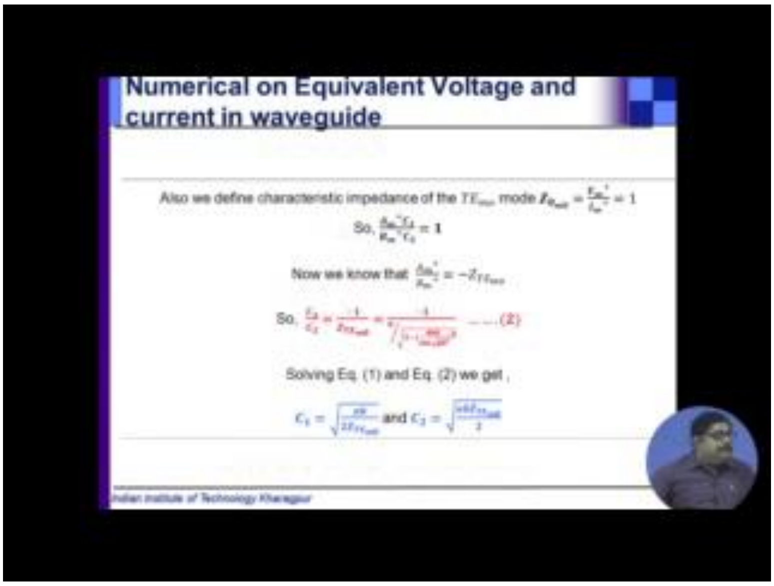

Then the next part is impedance equivalence. So, impedance is vm plus by I m plus, we said we had 2 choices either we make it equal to TE m0 mode wave impedance or equal to 1, we are taking the second 1 let us say that this is equal to 1. So, that makes our this thing 1 now am plus and b m plus their relationship is by Z TE m0 note this minus because of the power flow that always e y and h x from there standard one, 1 should be opposite to other. So, that the power can flow because y cross x that itself is negative, another negative is required that is why. So, that incident wave part we are assuming flowing in the positive Z direction, this minus. So, that if you put then c1 by c2, will be these 1 minus1 by eta. Eta is the intrinsic impedance divided by this quantity.

So, you have now 2 equations; for c1 and c2, 2 unknowns; c1 c2, 2 equations, 2 unknowns. So, you can solve if you solve you get c1 is this and c2 is this. So, depending on TE m0 you have these are the solution. So, this is simple that once we get c1 and c2 we can write as we written o. So, once you know c1 c2 you can describe v Z at for TE m0 mode v m plus by c1 you know this value this plus this. So, this is the description c1 c2 will come from there, that is all now proceed to the second problem.

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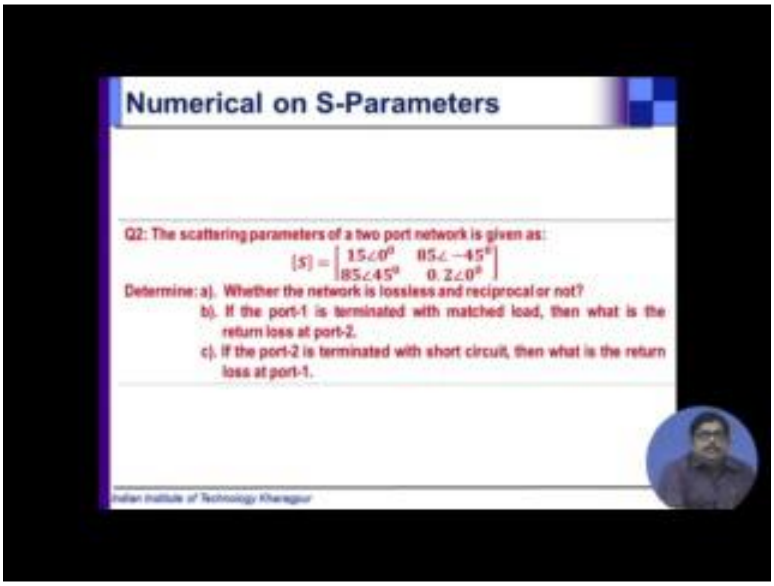

So, we have seen how to define voltage and current now scattering parameters.

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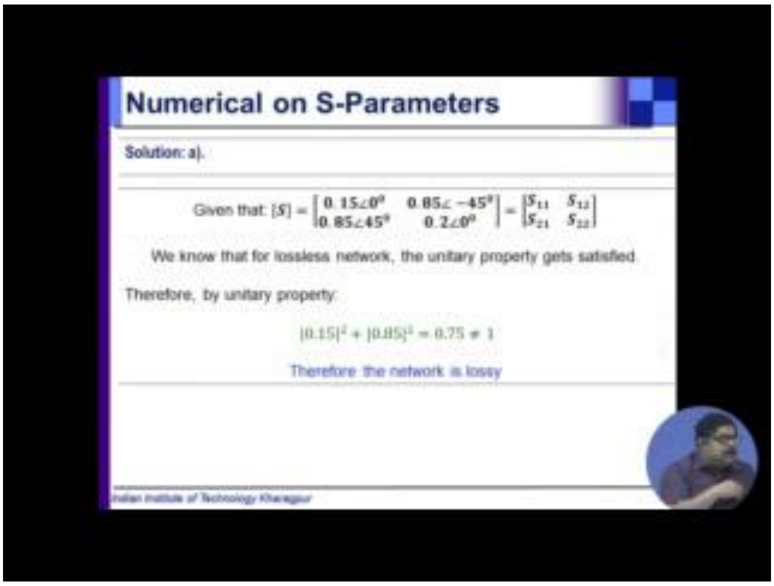

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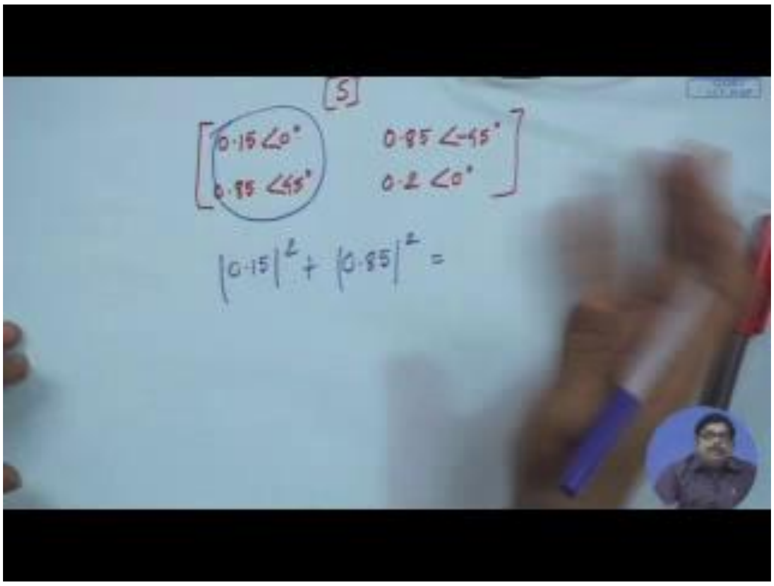

Suppose, that this parameter there are this in the slight the values are a bit wrong. So, correct it in a this slide it is given. So, a 2 port network scattering parameter is given, now determine whether the network is lossless and reciprocal or not. So, you know this is S parameter. So, we know that reciprocal it will be S is symmetric; that means, S is equal to S transpose and lossless it will be if S is unitary. So, you will have to change that to then, it is said if the port 1 is terminated with match load what is the return loss at port 2 and third part if the port 2 is terminated with short circuit what is the return loss at port 1.

So, let us see the first part that here it is correctly written the S matrix. So, for lossless network unitary property, now unitary property is the any 1 row conjugated with itself will be equal to 1. So, row means let us say this row. So, 0.15 square this conjugated a dotted with conjugate of these; that means, that will be 0.5 square plus0.85 square, now these value turns out to be 0.75 it is not 1. So, 1 - not satisfied immediately we can say loss the network is not lossless; that means it is a lossy network.

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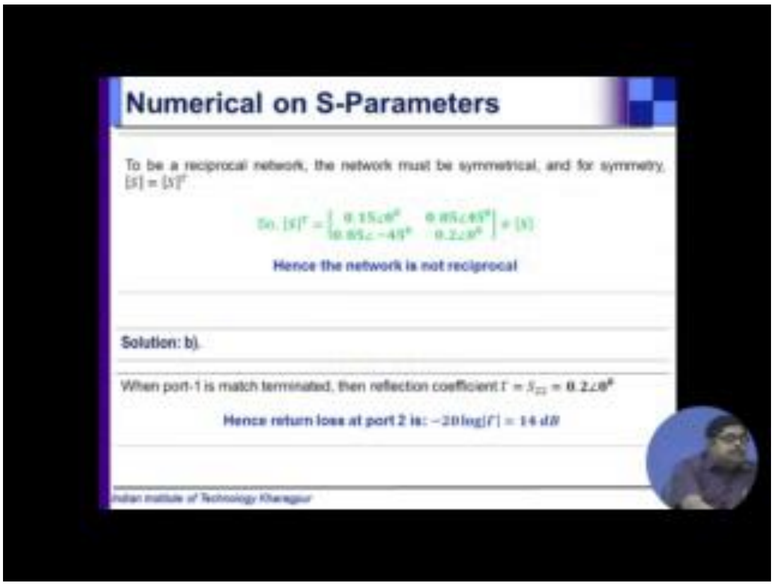

But whether it is reciprocal, for that you see that is it symmetry; that means, if I transpose this1 what I will get?

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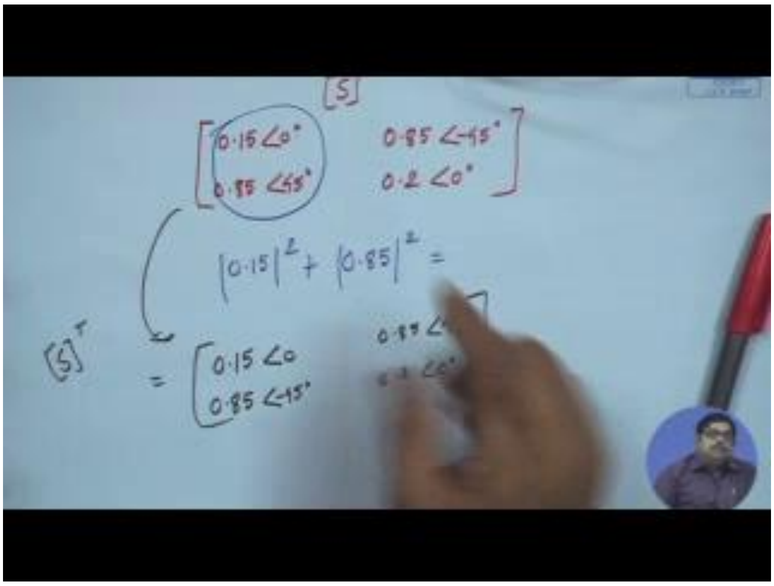

Transpose of this S of t that will be how much? Transpose means the row becomes column.

So, this will be the new 1 at the same no you see here minus 45, 45. So, the network is not reciprocal also. So, part a is solved that it is not unit lossless not reciprocal. Then port 1 is match terminated. So, we know that what is the return loss at port 2 that is same as S

22, s22 is point to these. So, return loss it is a scalar quantity. So, that is minus twenty log the reflection coefficient magnitude then that turns to be 14 db. So, port 2 return loss will be a 14 db when port 1 is match terminated, that is solution b pretty simple.

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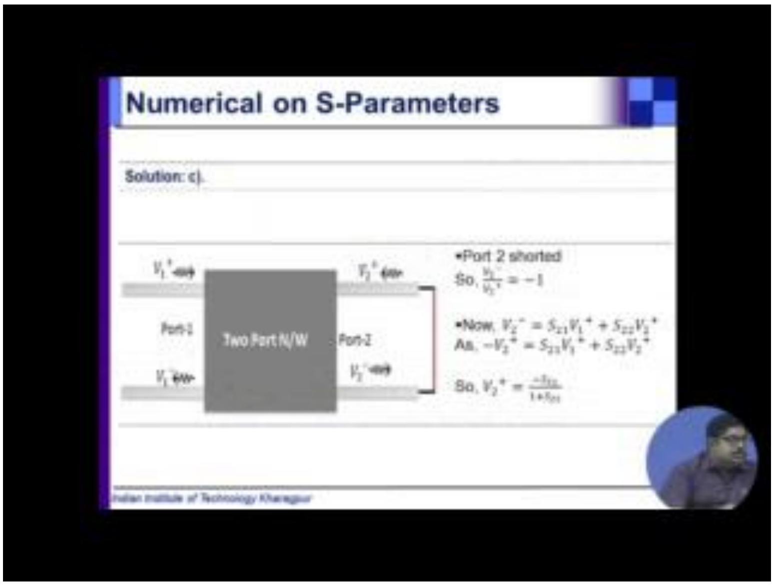

Now, solution c it says that port 2 is shorted, what is the return loss at port 1? So, port 2 is shorted, shorted means v2 minus by v2 plus reflection coefficient is minus1 then, v2 minus from the general S matrix definition we can write v2 minus is S 21 v1 plus S 22 v2 plus, now here we put these value that v2 minus is nothing, but minus v2 plus that these. So, from here I can solve for what is v2 plus.

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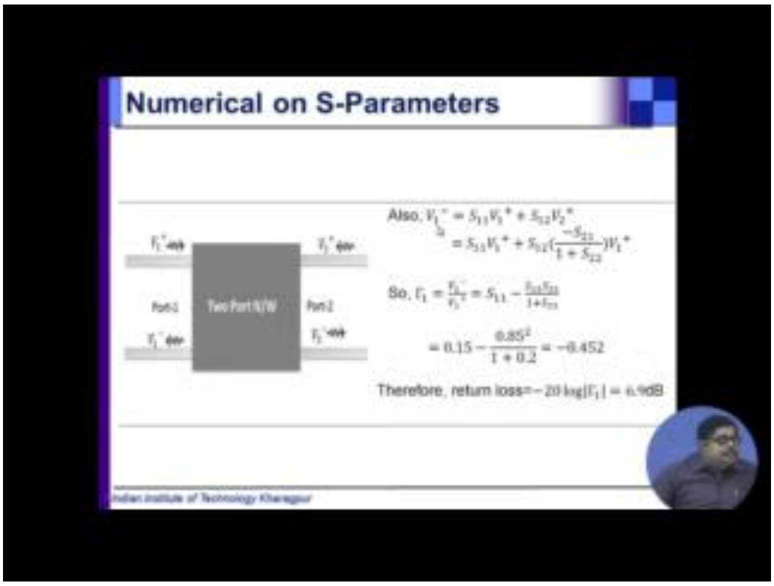

Now, go to the other equation of S matrix v1 minus is this, we are put the value of v2 plus. So, everything is now in terms of v1 minus and v1 plus. So, I can take the ratio that becomes the so, in general this is and at the reflection coefficient is equal to reflection coefficient on port 1 is equal to s11 minus s12, S 21 by 1 plus S 22. You see that reflection coefficient gamma 1 is not equal to s11 it is equal to s11. When these portions become 0, when those positions become 0? When I make that port 2 matched.

If I matched port 2 nothing comes from port 2 to port 1 that time s1 to become 0 and this whole thing become 0 that time it becomes equal. But not now at least not here because it is not match terminated the port 2 here is shorted. So, tau 1 is this. Now, you know these values from the S matrix given you can find these values and this. So, return loss you can once you know reflection coefficient, you can find the return loss reflection coefficient is the complex quantity, return loss is the scalar quantity. Please remember that that is why it is a magnitude of tau 1 log of that.

So, we have solved the 3 part, given the S parameter we can find out wave what is the property of the network? Whether it was lossless, whether it was reciprocal then given various port conditions we are asked to find reflection coefficients at various ports that we have done in b it was that 1 port is match terminated the other port return loss.

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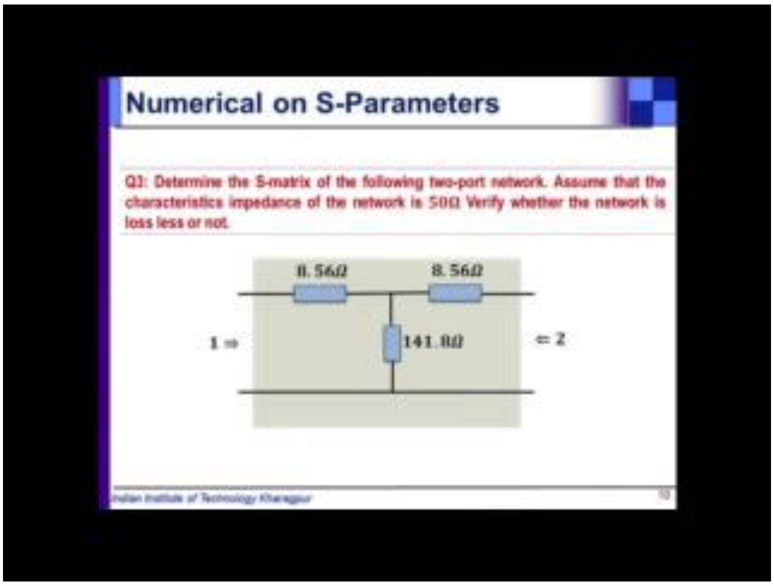

In c1 port is shorted, other port return loss. Now see the third problem, that S matrix of the following 2 port network. Again verify whether the network is lossless or not, that you see this is a purely lumped element case.

So, it is easier to go to Z matrix find the Z matrix of this is a simple symmetrical t network, you can very easily find Z matrix find the Z matrix from Z matrix; use the formula to find S matrix because otherwise if you try to find S matrix you will have to find out what is the reflection etcetera at various ones. You can do that, but that is a bit tricky, but you can just come to Z matrix.

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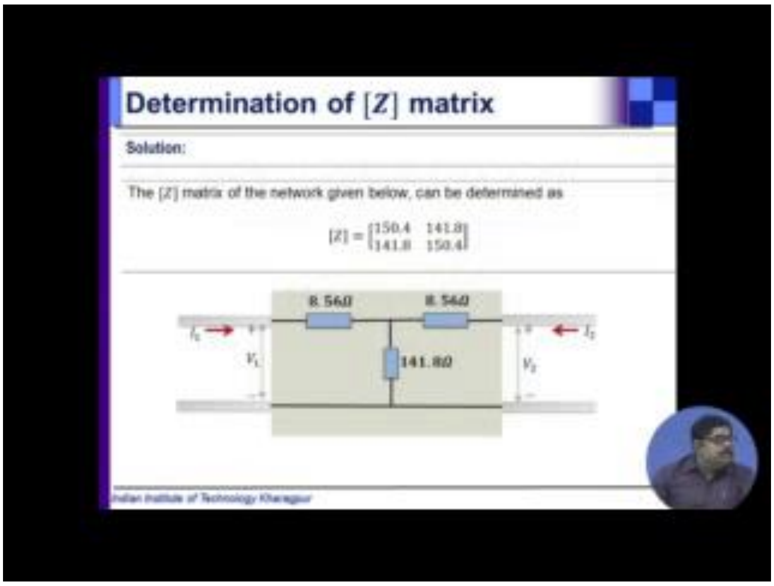

If you do the Z matrix will be like this. This I think all of you know from your earlier knowledge how to find Z matrix of a t network. So, this will be the Z matrix.

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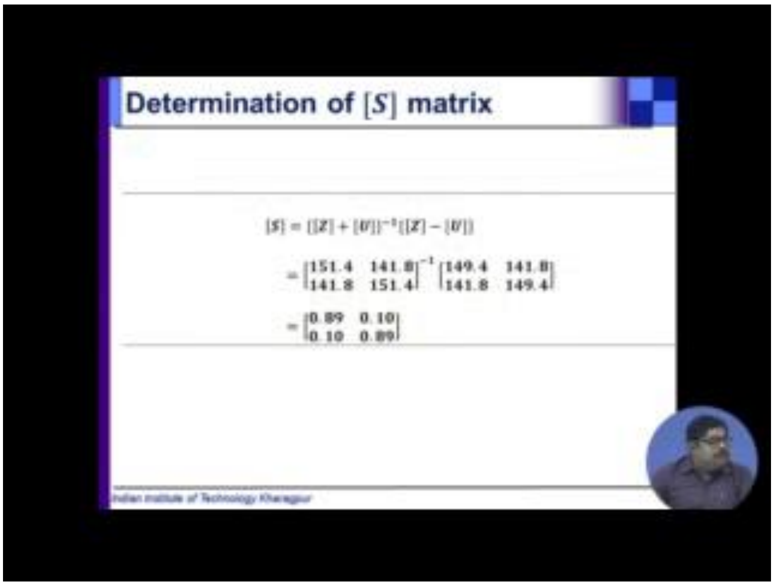

Now, once you have found Z matrix you can use this formula S is equal to Z plus u inverse Z minus u. So, do that it is simple matrix algebra and then, it will become like this. This is you can see in some books these are wrongly given, but these are correct answer. Then the checking of whether network is lossless. So, you again apply unitary property and see it is not unitary.

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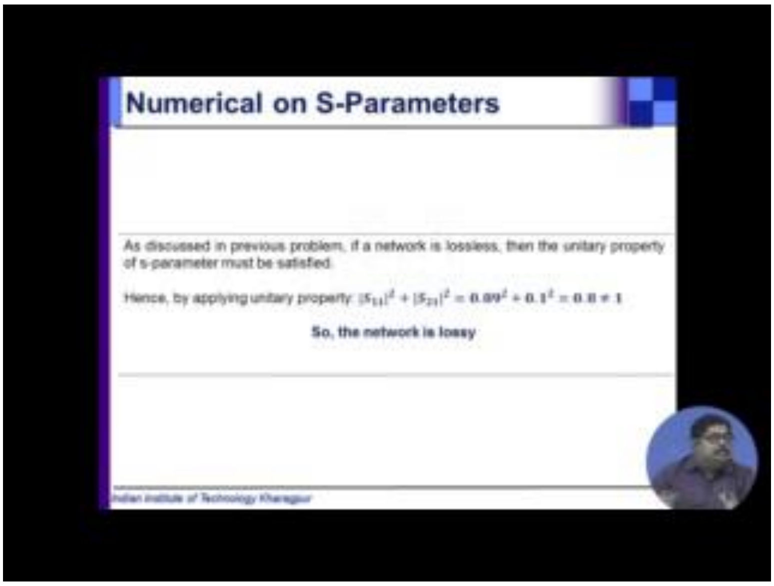

So, it is a lossy network as; obviously, it should be because the network was very. So, from that also you could have said that it should be lossy, but mathematically you will have to prove it that is why these thing.

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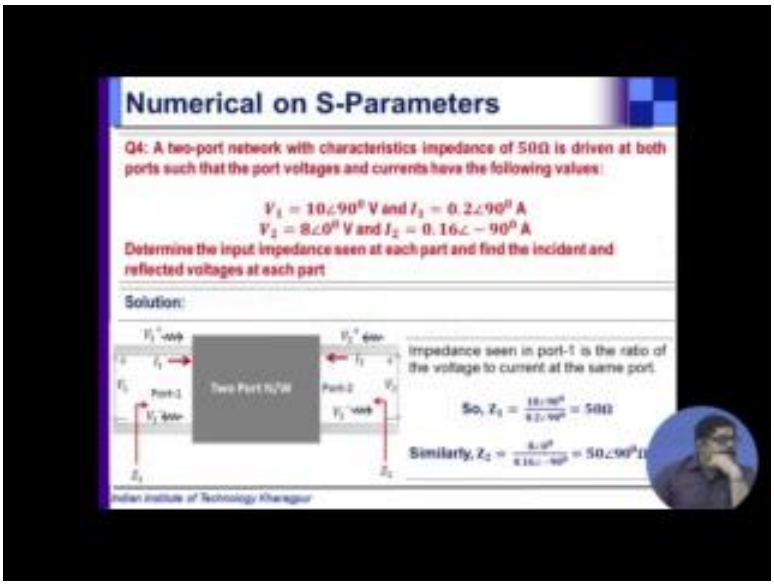

Now, it 2 port network with characteristic impedance 50 ohm is given at both ports, such that these are the various port voltage and current values. So, determine the input impedance seen at each port and find the incident and reflected voltages at each port. So, it is given at both ports. So, impedance seen in port 1, self impedance that will be a ratio of voltage currents you can find Z 1, also if you look from port 2 Z 2 will be self impedance Z 2 will be these.

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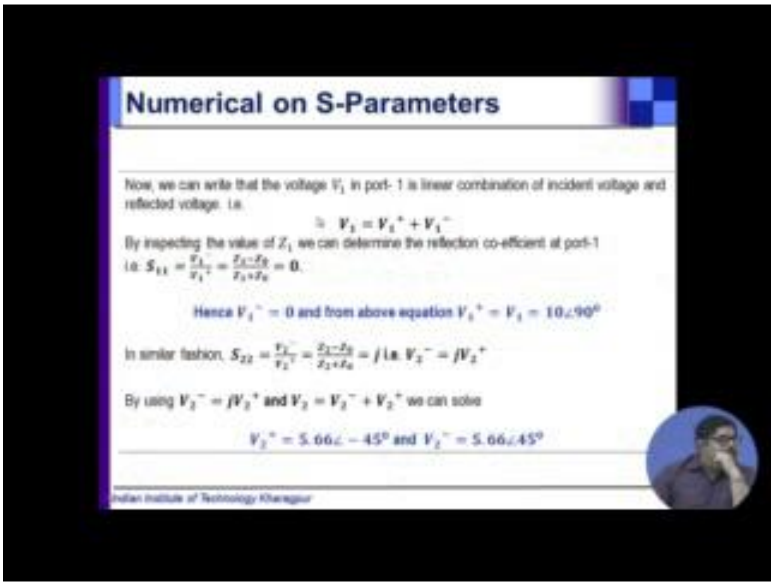

Now, voltage v1 is v1 plus plus v1 minus. So, reflection coefficient at port 1 you can find by you knows Z 1 Z 0 the characteristic impedance is given. So, z1 minus Z 0 by z1 plus Z 0 and if you look at impedances you can find s1 and s0. So, you can say v1 minus definitely should be 0 and. So, v1 plus is equal to v1.

In similar fashion you can find out that what is v2 minus S 22 that is j. So, v2 minus is this and v2 we can solve. So, v2 plus is this and v2 minus this. So, you see those problems are also solved.

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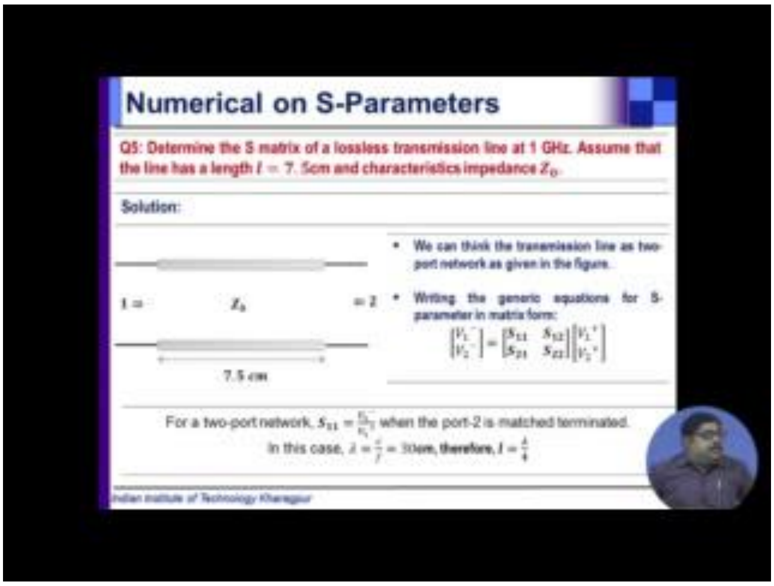

Now, the last problem that determines the S matrix of a lossless transmission line, this is a distributed line. So, it is easier to find it is S matrix at 1 Giga hertz, assume that line as a length 7.5 centimetre characteristic impedance Z 0.

So, let us see transmission line as 2 port network; obviously. So, S parameter is like this that is the definition, now for a 2 port network s1 one is this. So, n port 2 is match terminated, in this case lambda. So, 1 Giga hertz; 1 Giga hertz means lambda will be thirty centimetre and 7.5 centimetre means it is a lambda by full line. So, electrical length will be how much beta into l, beta is 2 pie by lambda phase constant. So, beta l will be pie by 2; that means, quarter wave line remember it has electrical length of pie by 2, 90 degree.

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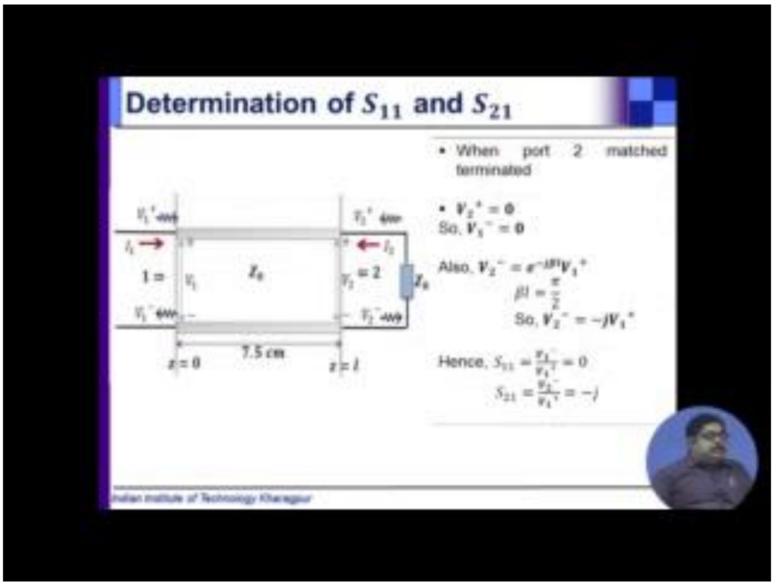

Now, port how to determine for finding s11 and S 21? You will have to match terminate port 2 port 2 match terminated as we have seen you connect Z 0 here. So, v2 plus the moment you put Z 0 here v2 plus; that means, whatever signal is coming from here nothing is going. So, v2 plus is0 if v2 plus does not go v1 minus also become 0. So, now, what is v2 minus? v2 minus is what is coming here, that is because you are exciting from here. So, this wave is coming here. So, in a transmission line of length l a lossless transmission line of length l a wave started from here v1 plus after going distance l it will add a phase of e to the power minus j beta l. So, that is what we have written v2 minus is e to the power minus j beta l into v1 plus beta l as I say pie by 2. So, v2 minus become minus j v1 plus e to the power minus j pie by 2 is minus j, minus here.

So, the moment you got that you can find out s1 one s11 is v1 minus by v1 plus under this condition already we are here. So, we are not written it explicitly. So, v1 minus by v1 plus that is 0 because v1 minus is 0 here, and S 21, S 21 is v2 minus by v1 plus that is minus j.

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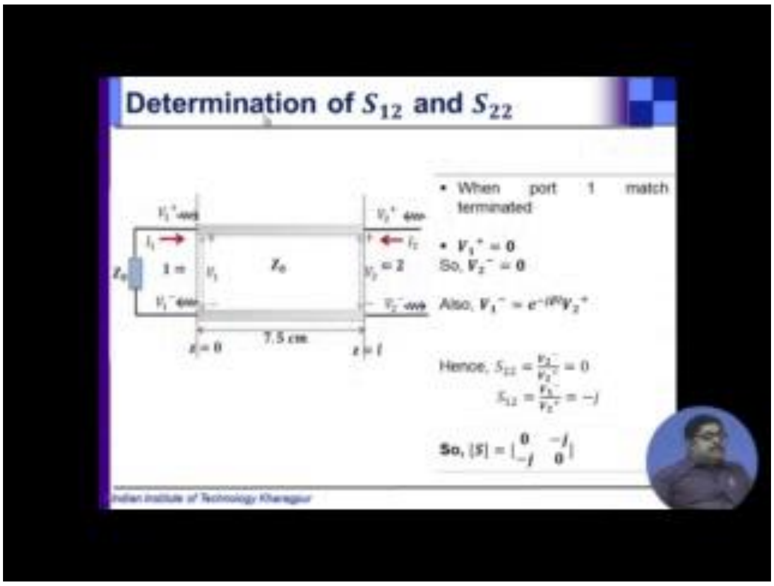

So, you got the first row first column of S parameter. Let us see the second for second column determination you need to match terminate port 1 port 1 match terminated. Now port 1 match terminated means v1 plus is equal to 0 the moment you see v1 plus is equal to 0 v2 minus also is to be equal to 0.

Now, in this case you are exciting from here. So, how much this v2 plus will be when it is coming here that will be v1 minus. So, what is relationship between v1 minus and v2 plus? v1 minus is equal to e to the power minus j beta l v2 plus, transmission line wave moving from 1 end to another. So, once you have that find out what is S 22, S 22 is v2 minus by v2 plus v2 minus is 0. So, S 22 will also be 0 and s12 is v1 minus by v2 plus v1 minus by v2 plus that is e to the power minus j pie by 2 that is minus j.

So, now you put 0 0 minus j minus j. So, a transmission line as S matrix that if the transmission line with characteristic impedance Z 0 your reference impedance is also Z 0, then there is no reflection at port1 also at port2 there is no reflection, but there is a transmission that is why it is transmission line, but for a quarter wave line you are getting both in the forward wave and or incident wave and reflected wave, they are getting a phase change of minus j. If you want to avoid that minus j, you will have to change the line. So, that you can get a non-quadrature thing, but this is the term scattering parameter.

Now, if it was as to you find out the Z parameter of that, then you can use that formula we have given from S parameter you can go to Z parameter, but directly evaluating S parameter is easier, because this is a distributed line here wave picture is more prominent or more explicit you can say. That is all these type of problems as a microwave engineer you need to solve. So, that to find S parameters of various circuits networks etcetera and for measurement your help is the network analyser.