

Basic Tools of Microwave Engineering
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Lecture - 15
Tutorial 3: Problem Solving on Equivalent Voltage
and Current in Waveguide and on Scattering Parameters

Welcome to the 15th lecture, that is tutorial 3. We will see some problem solving, on equivalent voltage and current concepts in waveguide and on scattering parameters. The first problem is determined the equivalent voltage and current for TE m0 mode fields.

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Numerical on Equivalent Voltage and current in waveguide

Q1: Determine the equivalent voltage and current for TE_{m0} mode fields in a rectangular waveguide

Solution:

In TE_{m0} mode, $E_z = 0$ and generic equation of $H_z = A_{m0} \cos(\frac{m\pi x}{a}) \cos(\frac{n\pi y}{b}) e^{-j\beta_{m0} z}$ where a and b are the dimensions of the cross-section of the rectangular waveguide.

Now the equations for TE_{m0} waves travelling in forward direction are:

$$E_y^+ = \frac{-j\omega\mu_0}{\pi b} A_{m0} \sin(\frac{m\pi x}{a}) e^{-j\beta_{m0} z} \Rightarrow E_y^+ = A_m^+ \sin(\frac{m\pi x}{a}) e^{-j\beta_{m0} z}$$

$$H_x^+ = \frac{j\beta_{m0} b}{\pi a} A_{m0} \sin(\frac{m\pi x}{a}) e^{-j\beta_{m0} z} \Rightarrow H_x^+ = B_m^+ \sin(\frac{m\pi x}{a}) e^{-j\beta_{m0} z}$$

Where $\beta_{m0} = \sqrt{k^2 - (\frac{m\pi}{a})^2}$, $A_m^+ = \frac{j\omega\mu_0}{\pi b} A_{m0}$ and $B_m^+ = \frac{j\beta_{m0} b}{\pi a} A_{m0}$

In a rectangular waveguide, we have seen how to do it now just look at the TE m0 mode. Transverse electric means, E_z is equal to 0 and H_z this is the generic form a and b are the dimensions of the cross section of the rectangular waveguide. So, if you write TE m0 mode waves appears, the transverse fields will be E_y and H_x plus incident fields; obviously, they are also the reflective fields.

Now, E_y plus can be written in these form. So, that we are saying, that there is an amplitude part then this is the Eigen function part this is the Z propagation part, H_x plus also like this where this propagation constant or phase constant since we are assuming lossless line γ is $j\beta$. So, β for m 0 mode is like this. Now this m plus is this, this is the constant and B_m plus is like this.

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Numerical on Equivalent Voltage and current in waveguide

Now, let us define $A_m^+ = \frac{V_m^+}{C_1}$ and $B_m^+ = \frac{I_m^+}{C_2}$

The equivalent voltage wave and current waves are defined as

$$V(x) = \frac{V_m^+}{C_1} e^{-j\beta_2 x} + \frac{V_m^-}{C_1} e^{j\beta_2 x}$$

$$I(x) = \frac{I_m^+}{C_2} e^{-j\beta_2 x} - \frac{I_m^-}{C_2} e^{j\beta_2 x}$$

The complex power flow for the incident wave is:

$$P^+ = \frac{1}{2} A_m^+ (B_m^+)^* \iint_S \sin\left(\frac{j\pi x}{a}\right) \sin\left(\frac{j\pi y}{a}\right) (\vec{E}_y \times \vec{H}_z^* - \vec{E}_z \times \vec{H}_y^*) \cdot \vec{a}_x \, dy \, dz = \frac{1}{2} V_m^+ (I_m^+)^*$$

$$\text{So: } A_m^+ (B_m^+)^* \iint_S \sin\left(\frac{j\pi x}{a}\right) \sin\left(\frac{j\pi y}{a}\right) (\vec{E}_y \times \vec{H}_z^* - \vec{E}_z \times \vec{H}_y^*) \cdot \vec{a}_x \, dy \, dz = V_m^+ (I_m^+)^*$$

$$\Rightarrow A_m^+ (B_m^+)^* \int_{y=0}^b \int_{z=0}^b -\cos^2\left(\frac{j\pi y}{a}\right) dy \, dz = V_m^+ (I_m^+)^*$$

$$\Rightarrow \frac{V_m^+ (I_m^+)^*}{C_1 C_2^*} = -\frac{ab}{2} = V_m^+ (I_m^+)^*$$

$$\Rightarrow C_1 C_2^* = -\frac{ab}{2} \dots (1) \quad C_1 \text{ and } C_2 \text{ are constant and real}$$

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Now let us define as we said that the electric field and voltage should be proportional. So, that proportionality constant let us say c_1 . So, this is a voltage side quantity that we are making proportional to m plus the magnitude of the voltage wave and b_m plus was the magnetic field part. So, that we are making proportional to the current, this is the proportionality constant is c_2 . So, this $c_1 c_2^*$ is a value finally, we have to find now equivalent voltage and currents are like this now enforce the power equivalence. So, first find out from the field 1 with these definitions. What is the power incident power? So, all the steps are done for you. Pointing vector then integration over the guide S means the transverse cross section of the guide; that means, for a rectangular wave guide it is from the x is from 0 to a , y goes from 0 to b .

So, if you do that now that we have done here, that these are the thing and this should be equal to the average power that was the power equivalence part half v_m plus I_m plus star. So, if you equate that to you get these relation $c_1 c_2^*$ star is equal to minus $a b$ by 2. Now c_1 and c_2 are constant and real. So, c_2^* will equal to c_2 ; that means, $c_1 c_2$ product is minus $a b$ by 2 this is 1 equation. So, we are calling it equation number 1.

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Numerical on Equivalent Voltage and current in waveguide

Also we define characteristic impedance of the TE_{m0} mode $Z_{0_{TE_{m0}}} = \frac{E_{m0}}{H_{m0}} = 1$

So, $\frac{E_{m0}}{H_{m0}} = 1$

Now we know that $\frac{E_{m0}}{H_{m0}} = -Z_{TE_{m0}}$

So, $\frac{E_{m0}}{H_{m0}} = \frac{1}{-Z_{TE_{m0}}} = \frac{1}{\sqrt{\frac{1 - \beta^2}{1 - \beta^2}}} \dots (2)$

Solving Eq. (1) and Eq. (2) we get,

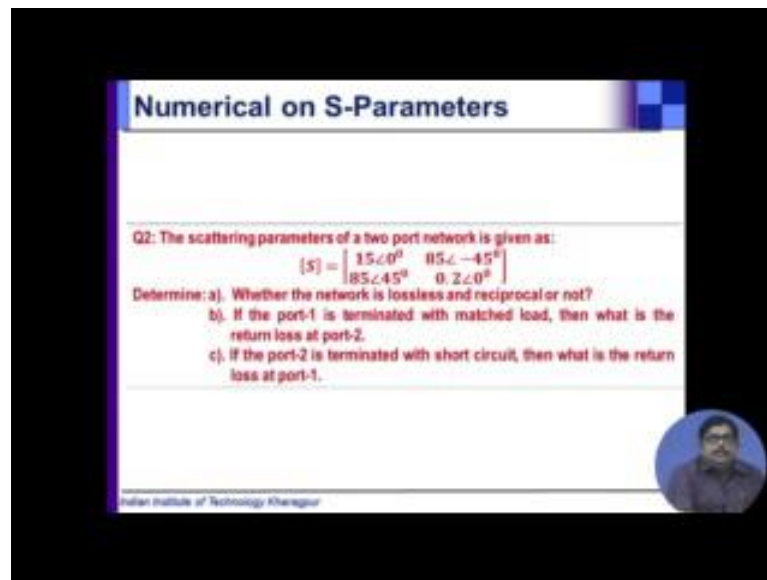
$c_1 = \sqrt{\frac{\epsilon_0}{2Z_{TE_{m0}}}}$ and $c_2 = \sqrt{\frac{j\omega\epsilon_0}{2}}$

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Then the next part is impedance equivalence. So, impedance is v_m plus by I_m plus, we said we had 2 choices either we make it equal to TE_{m0} mode wave impedance or equal to 1, we are taking the second 1 let us say that this is equal to 1. So, that makes our this thing 1 now a_m plus and b_m plus their relationship is by $Z_{TE_{m0}}$ note this minus because of the power flow that always e_y and h_x from there standard one, 1 should be opposite to other. So, that the power can flow because y cross x that itself is negative, another negative is required that is why. So, that incident wave part we are assuming flowing in the positive Z direction, this minus. So, that if you put then c_1 by c_2 , will be these 1 minus 1 by η . η is the intrinsic impedance divided by this quantity.

So, you have now 2 equations; for c_1 and c_2 , 2 unknowns; c_1 c_2 , 2 equations, 2 unknowns. So, you can solve if you solve you get c_1 is this and c_2 is this. So, depending on TE_{m0} you have these are the solution. So, this is simple that once we get c_1 and c_2 we can write as we written o. So, once you know c_1 c_2 you can describe v_Z at for TE_{m0} mode v_m plus by c_1 you know this value this plus this. So, this is the description c_1 c_2 will come from there, that is all now proceed to the second problem.

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Numerical on S-Parameters

Q2: The scattering parameters of a two port network is given as:

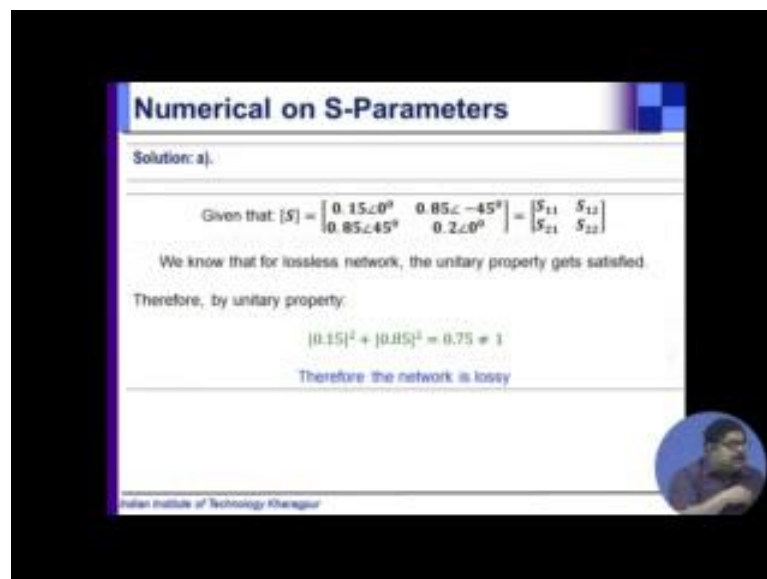
$$[S] = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle -45^\circ \\ 0.85 \angle 45^\circ & 0.2 \angle 0^\circ \end{bmatrix}$$

Determine: a). Whether the network is lossless and reciprocal or not?
b). If the port-1 is terminated with matched load, then what is the return loss at port-2.
c). If the port-2 is terminated with short circuit, then what is the return loss at port-1.

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So, we have seen how to define voltage and current now scattering parameters.

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Numerical on S-Parameters

Solution: a).

Given that: $[S] = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle -45^\circ \\ 0.85 \angle 45^\circ & 0.2 \angle 0^\circ \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

We know that for lossless network, the unitary property gets satisfied.

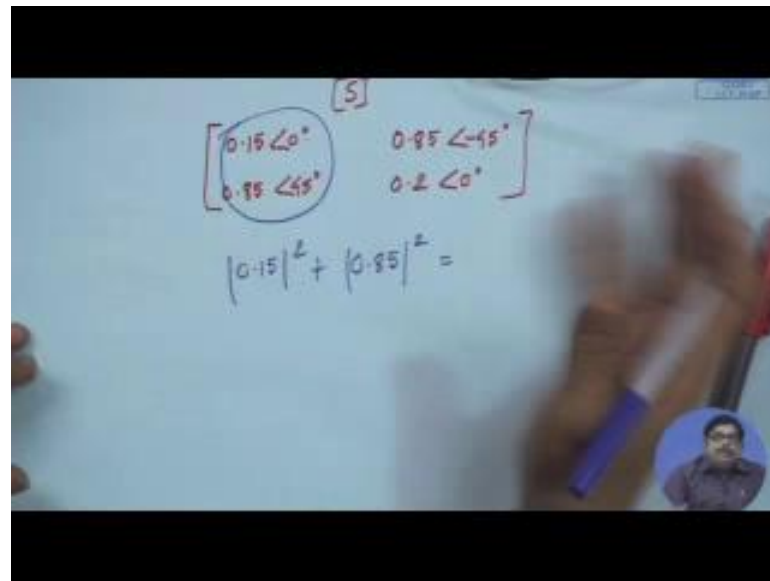
Therefore, by unitary property:

$$|0.15|^2 + |0.85|^2 = 0.75 \neq 1$$

Therefore the network is lossy

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A photograph of a whiteboard with handwritten mathematical expressions. At the top, an S-matrix is written as
$$[S] = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle -45^\circ \\ 0.85 \angle 45^\circ & 0.2 \angle 0^\circ \end{bmatrix}$$
 The first column of the matrix is circled in red. Below the matrix, the calculation $|0.15|^2 + |0.85|^2 =$ is written.

Suppose, that this parameter there are this in the slight the values are a bit wrong. So, correct it in a this slide it is given. So, a 2 port network scattering parameter is given, now determine whether the network is lossless and reciprocal or not. So, you know this is S parameter. So, we know that reciprocal it will be S is symmetric; that means, S is equal to S transpose and lossless it will be if S is unitary. So, you will have to change that to then, it is said if the port 1 is terminated with match load what is the return loss at port 2 and third part if the port 2 is terminated with short circuit what is the return loss at port 1.

So, let us see the first part that here it is correctly written the S matrix. So, for lossless network unitary property, now unitary property is the any 1 row conjugated with itself will be equal to 1. So, row means let us say this row. So, 0.15 square this conjugated a dotted with conjugate of these; that means, that will be 0.5 square plus 0.85 square, now these value turns out to be 0.75 it is not 1. So, 1 - not satisfied immediately we can say loss the network is not lossless; that means it is a lossy network.

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Numerical on S-Parameters

To be a reciprocal network, the network must be symmetrical, and for symmetry, $[S] = [S]^T$

So, $[S]^T = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle 45^\circ \\ 0.85 \angle -45^\circ & 0.2 \angle 0^\circ \end{bmatrix} \neq [S]$

Hence the network is not reciprocal

Solution: b).

When port-1 is match terminated, then reflection coefficient $\Gamma = S_{22} = 0.2 \angle 0^\circ$

Hence return loss at port 2 is: $-20 \log(\Gamma) = 14 \text{ dB}$

But whether it is reciprocal, for that you see that is it symmetry; that means, if I transpose this what I will get?

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$[S] = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle -45^\circ \\ 0.85 \angle -45^\circ & 0.2 \angle 0^\circ \end{bmatrix}$

$[S]^T = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle 45^\circ \\ 0.85 \angle 45^\circ & 0.2 \angle 0^\circ \end{bmatrix}$

$|0.15|^2 + |0.85|^2 =$

Transpose of this S of t that will be how much? Transpose means the row becomes column.

So, this will be the new 1 at the same no you see here minus 45, 45. So, the network is not reciprocal also. So, part a is solved that it is not unit lossless not reciprocal. Then port 1 is match terminated. So, we know that what is the return loss at port 2 that is same as S

22, s_{22} is point to these. So, return loss it is a scalar quantity. So, that is minus twenty log the reflection coefficient magnitude then that turns to be 14 db. So, port 2 return loss will be a 14 db when port 1 is match terminated, that is solution b pretty simple.

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Numerical on S-Parameters

Solution: c).

Diagram of a Two Port N/W with Port-1 and Port-2. Port-2 is shorted.

•Port 2 shorted
So, $\frac{V_2^-}{V_2^+} = -1$

•Now, $V_2^- = S_{21}V_1^+ + S_{22}V_2^+$
As, $-V_2^+ = S_{21}V_1^+ + S_{22}V_2^+$
So, $V_2^+ = \frac{-S_{21}}{1+S_{22}}$

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Now, solution c it says that port 2 is shorted, what is the return loss at port 1? So, port 2 is shorted, shorted means v_2^- minus by v_2^+ plus reflection coefficient is minus 1 then, v_2^- minus from the general S matrix definition we can write v_2^- is $S_{21} v_1^+$ plus $S_{22} v_2^+$ plus, now here we put these value that v_2^- is nothing, but minus v_2^+ plus that these. So, from here I can solve for what is v_2^+ .

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Numerical on S-Parameters

Diagram of a Two Port Network:

- Port 1: V_1^+ (incident), V_1^- (reflected)
- Port 2: V_2^+ (incident), V_2^- (reflected)

Also, $V_1^- = S_{11}V_1^+ + S_{12}V_2^+$
 $= S_{11}V_1^+ + S_{12}\left(\frac{-S_{22}}{1 + S_{22}}\right)V_1^+$

So, $\Gamma_1 = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12}S_{22}}{1 + S_{22}}$
 $= 0.15 - \frac{0.85^2}{1 + 0.2} = -0.452$

Therefore, return loss = $-20 \log|\Gamma_1| = 6.9 \text{ dB}$

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Now, go to the other equation of S matrix v_1 minus is this, we are put the value of v_2 plus. So, everything is now in terms of v_1 minus and v_1 plus. So, I can take the ratio that becomes the so, in general this is and at the reflection coefficient is equal to reflection coefficient on port 1 is equal to s_{11} minus s_{12} , S_{21} by 1 plus S_{22} . You see that reflection coefficient γ_1 is not equal to s_{11} it is equal to s_{11} . When these portions become 0, when those positions become 0? When I make that port 2 matched.

If I matched port 2 nothing comes from port 2 to port 1 that time s_1 to become 0 and this whole thing become 0 that time it becomes equal. But not now at least not here because it is not match terminated the port 2 here is shorted. So, τ_1 is this. Now, you know these values from the S matrix given you can find these values and this. So, return loss you can once you know reflection coefficient, you can find the return loss reflection coefficient is the complex quantity, return loss is the scalar quantity. Please remember that that is why it is a magnitude of τ_1 log of that.

So, we have solved the 3 part, given the S parameter we can find out wave what is the property of the network? Whether it was lossless, whether it was reciprocal then given various port conditions we are asked to find reflection coefficients at various ports that we have done in b it was that 1 port is match terminated the other port return loss.

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Numerical on S-Parameters

Q3: Determine the S-matrix of the following two-port network. Assume that the characteristics impedance of the network is 50Ω . Verify whether the network is loss less or not.

The diagram shows a two-port network. The input port is labeled 1 and the output port is labeled 2. The network consists of two series impedances of 8.56Ω and one shunt impedance of 141.8Ω .

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In c1 port is shorted, other port return loss. Now see the third problem, that S matrix of the following 2 port network. Again verify whether the network is lossless or not, that you see this is a purely lumped element case.

So, it is easier to go to Z matrix find the Z matrix of this is a simple symmetrical t network, you can very easily find Z matrix find the Z matrix from Z matrix; use the formula to find S matrix because otherwise if you try to find S matrix you will have to find out what is the reflection etcetera at various ones. You can do that, but that is a bit tricky, but you can just come to Z matrix.

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Determination of [Z] matrix

Solution:

The [Z] matrix of the network given below, can be determined as

$$[Z] = \begin{bmatrix} 150.4 & 141.8 \\ 141.8 & 150.4 \end{bmatrix}$$

The circuit diagram shows a two-port network. The input port has voltage V_1 and current I_1 entering. The output port has voltage V_2 and current I_2 entering. The network consists of two 56Ω resistors in series on the top line, with a 141.8Ω resistor connected in shunt from the node between them to the common ground line.

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If you do the Z matrix will be like this. This I think all of you know from your earlier knowledge how to find Z matrix of a T network. So, this will be the Z matrix.

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Determination of [S] matrix

$$[S] = ([Z] + [U])^{-1}([Z] - [U])$$
$$= \begin{bmatrix} 151.4 & 141.8 \\ 141.8 & 151.4 \end{bmatrix}^{-1} \begin{bmatrix} 149.4 & 141.8 \\ 141.8 & 149.4 \end{bmatrix}$$
$$= \begin{bmatrix} 0.89 & 0.10 \\ 0.10 & 0.89 \end{bmatrix}$$

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Now, once you have found Z matrix you can use this formula S is equal to Z plus u inverse Z minus u. So, do that it is simple matrix algebra and then, it will become like this. This is you can see in some books these are wrongly given, but these are correct answer. Then the checking of whether network is lossless. So, you again apply unitary property and see it is not unitary.

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Numerical on S-Parameters

As discussed in previous problem, if a network is lossless, then the unitary property of s-parameter must be satisfied.

Hence, by applying unitary property: $|S_{11}|^2 + |S_{21}|^2 = 0.89^2 + 0.1^2 = 0.8 \neq 1$

So, the network is lossy

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So, it is a lossy network as; obviously, it should be because the network was very. So, from that also you could have said that it should be lossy, but mathematically you will have to prove it that is why these thing.

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Numerical on S-Parameters

Q4: A two-port network with characteristics impedance of 50Ω is driven at both ports such that the port voltages and currents have the following values:

$V_1 = 10\angle 90^\circ \text{ V}$ and $I_1 = 0.2\angle 90^\circ \text{ A}$
 $V_2 = 8\angle 0^\circ \text{ V}$ and $I_2 = 0.16\angle -90^\circ \text{ A}$

Determine the input impedance seen at each port and find the incident and reflected voltages at each port

Solution:

Impedance seen in port-1 is the ratio of the voltage to current at the same port.

So, $Z_1 = \frac{10\angle 90^\circ}{0.2\angle 90^\circ} = 50\Omega$

Similarly, $Z_2 = \frac{8\angle 0^\circ}{0.16\angle -90^\circ} = 50\angle 90^\circ \Omega$

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Now, it 2 port network with characteristic impedance 50 ohm is given at both ports, such that these are the various port voltage and current values. So, determine the input impedance seen at each port and find the incident and reflected voltages at each port. So, it is given at both ports. So, impedance seen in port 1, self impedance that will be a ratio

of voltage currents you can find Z_1 , also if you look from port 2 Z_2 will be self impedance Z_2 will be these.

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Numerical on S-Parameters

Now, we can write that the voltage V_1 in port- 1 is linear combination of incident voltage and reflected voltage. i.e.

$$V_1 = V_1^+ + V_1^-$$

By inspecting the value of Z_1 we can determine the reflection co-efficient at port-1
i.e. $S_{11} = \frac{V_1^-}{V_1^+} = \frac{Z_1 - Z_0}{Z_1 + Z_0} = 0$.

Hence $V_1^- = 0$ and from above equation $V_1^+ = V_1 = 10 \angle 90^\circ$

In similar fashion, $S_{22} = \frac{V_2^-}{V_2^+} = \frac{Z_2 - Z_0}{Z_2 + Z_0} = j$ i.e. $V_2^- = jV_2^+$

By using $V_2^- = jV_2^+$ and $V_2 = V_2^- + V_2^+$ we can solve

$$V_2^+ = 5.66 \angle -45^\circ \text{ and } V_2^- = 5.66 \angle 45^\circ$$

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Now, voltage v_1 is v_1 plus plus v_1 minus. So, reflection coefficient at port 1 you can find by you knows Z_1 Z_0 the characteristic impedance is given. So, z_1 minus Z_0 by z_1 plus Z_0 and if you look at impedances you can find s_1 and s_0 . So, you can say v_1 minus definitely should be 0 and. So, v_1 plus is equal to v_1 .

In similar fashion you can find out that what is v_2 minus S_{22} that is j . So, v_2 minus is this and v_2 we can solve. So, v_2 plus is this and v_2 minus this. So, you see those problems are also solved.

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Numerical on S-Parameters

Q5: Determine the S matrix of a lossless transmission line at 1 GHz. Assume that the line has a length $l = 7.5 \text{ cm}$ and characteristic impedance Z_0 .

Solution:

$l = \quad Z_0 \quad = 2$

7.5 cm

- We can think the transmission line as two-port network as given in the figure.
- Writing the generic equations for S-parameter in matrix form:
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

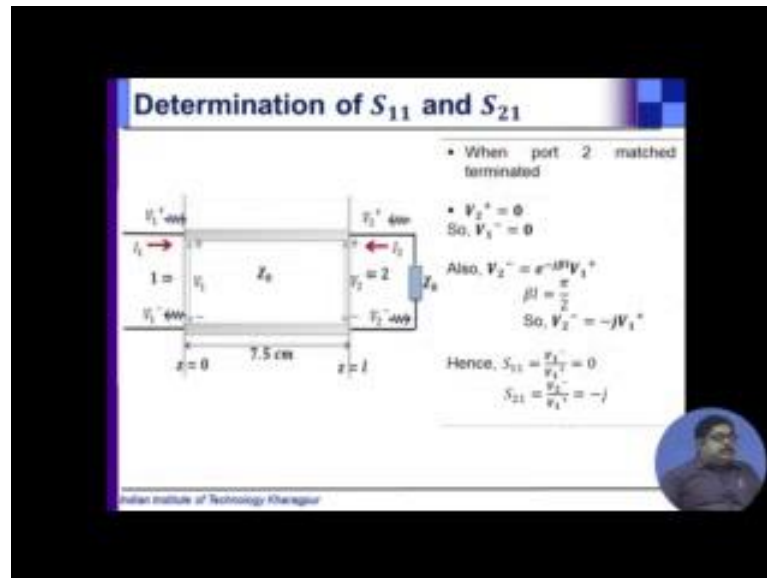
For a two-port network, $S_{11} = \frac{b_1^-}{a_1^+}$ when the port-2 is matched terminated.
In this case, $\lambda = \frac{c}{f} = 30 \text{ cm}$, therefore, $l = \frac{\lambda}{4}$

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Now, the last problem that determines the S matrix of a lossless transmission line, this is a distributed line. So, it is easier to find it is S matrix at 1 Giga hertz, assume that line as a length 7.5 centimetre characteristic impedance Z_0 .

So, let us see transmission line as 2 port network; obviously. So, S parameter is like this that is the definition, now for a 2 port network s1 one is this. So, n port 2 is match terminated, in this case lambda. So, 1 Giga hertz; 1 Giga hertz means lambda will be thirty centimetre and 7.5 centimetre means it is a lambda by full line. So, electrical length will be how much beta into l, beta is 2π by lambda phase constant. So, beta l will be π by 2; that means, quarter wave line remember it has electrical length of π by 2, 90 degree.

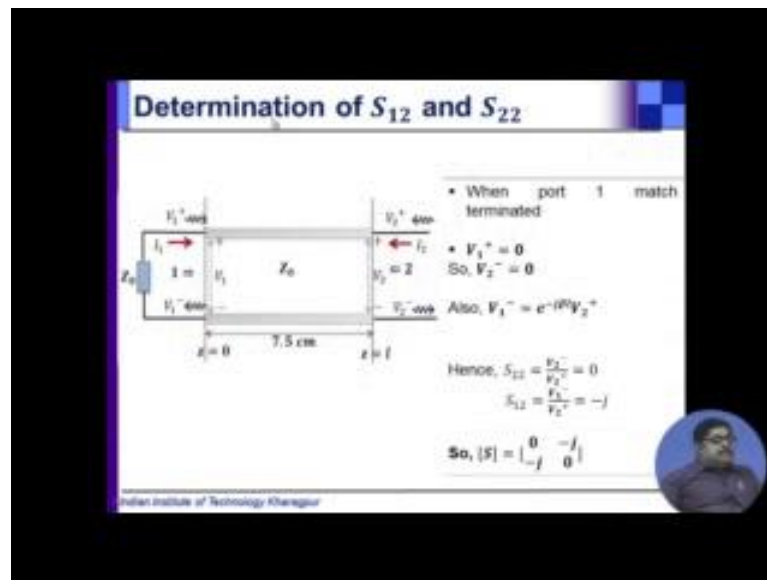
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Now, port how to determine for finding s_{11} and S_{21} ? You will have to match terminate port 2 port 2 match terminated as we have seen you connect Z_0 here. So, v_2 plus the moment you put Z_0 here v_2 plus; that means, whatever signal is coming from here nothing is going. So, v_2 plus is 0 if v_2 plus does not go v_1 minus also become 0. So, now, what is v_2 minus? v_2 minus is what is coming here, that is because you are exciting from here. So, this wave is coming here. So, in a transmission line of length l a lossless transmission line of length l a wave started from here v_1 plus after going distance l it will add a phase of e to the power minus $j\beta l$. So, that is what we have written v_2 minus is e to the power minus $j\beta l$ into v_1 plus βl as I say π by 2. So, v_2 minus become minus j v_1 plus e to the power minus $j\pi$ by 2 is minus j , minus here.

So, the moment you got that you can find out s_{11} one s_{11} is v_1 minus by v_1 plus under this condition already we are here. So, we are not written it explicitly. So, v_1 minus by v_1 plus that is 0 because v_1 minus is 0 here, and S_{21} , S_{21} is v_2 minus by v_1 plus that is minus j .

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So, you got the first row first column of S parameter. Let us see the second for second column determination you need to match terminate port 1 port 1 match terminated. Now port 1 match terminated means v_1 plus is equal to 0 the moment you see v_1 plus is equal to 0 v_2 minus also is to be equal to 0.

Now, in this case you are exciting from here. So, how much this v_2 plus will be when it is coming here that will be v_1 minus. So, what is relationship between v_1 minus and v_2 plus? v_1 minus is equal to $e^{-j\beta l}$ v_2 plus, transmission line wave moving from 1 end to another. So, once you have that find out what is S_{22} , S_{22} is v_2 minus by v_2 plus v_2 minus is 0. So, S_{22} will also be 0 and s_{12} is v_1 minus by v_2 plus v_1 minus by v_2 plus that is $e^{-j\beta l}$ v_2 plus by v_2 plus that is $e^{-j\beta l}$ that is $-j$.

So, now you put 0 0 minus j minus j . So, a transmission line as S matrix that if the transmission line with characteristic impedance Z_0 your reference impedance is also Z_0 , then there is no reflection at port1 also at port2 there is no reflection, but there is a transmission that is why it is transmission line, but for a quarter wave line you are getting both in the forward wave and or incident wave and reflected wave, they are getting a phase change of minus j . If you want to avoid that minus j , you will have to change the line. So, that you can get a non-quadrature thing, but this is the term scattering parameter.

Now, if it was as to you find out the Z parameter of that, then you can use that formula we have given from S parameter you can go to Z parameter, but directly evaluating S parameter is easier, because this is a distributed line here wave picture is more prominent or more explicit you can say. That is all these type of problems as a microwave engineer you need to solve. So, that to find S parameters of various circuits networks etcetera and for measurement your help is the network analyser.