Basic Tools of Microwave Engineering
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Lecture – 13
Properties of Scattering Parameter

In these lecture, we will see properties of scattering parameters the second tool that we have introduced.

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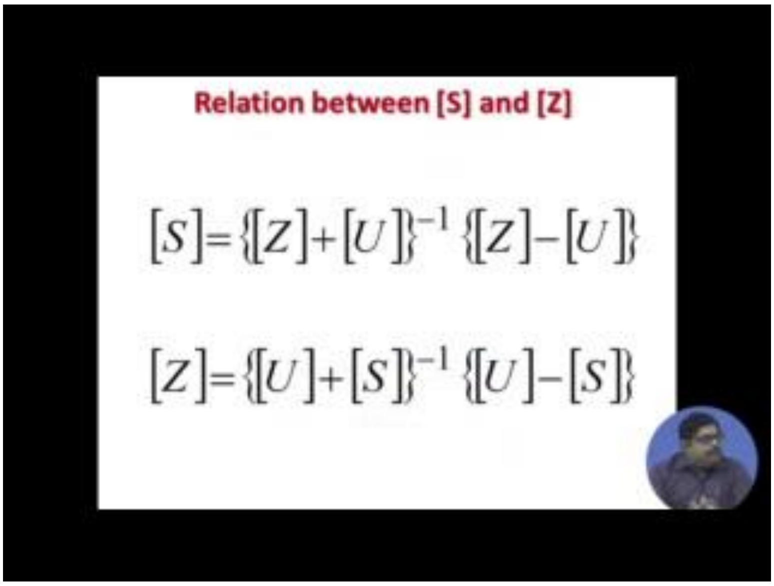

Now, this is the first that obviously, Z matrix incidence matrix is a popular tool. So, this S parameter is related to Z by these, if you know Z matrix you can find S parameter so that means, any lumped network if you know Z parameter you can always find this S parameter. We will see in one problem will give a lumped network. So, if you want to find its S parameter directly that may be a problem because for lump parameters the separating incident reflected transmitted voltages that become a problem. So, their Z parameter finding is easier you can do that and then come to S parameter.

On the other hand, this we have say that the second equation that is if you know S parameter, you can find Z. Now, for microwave networks this is helpful because in microwave networks you see that it is easier to find out the reflected transmitted and incident waves. So, their ratio is easier to find, you find S matrix, then find the Z matrix. So, suppose A transmission 9 is given, now its S matrix is easier to find, but if you want to find Z parameter of S transmission line, first find S parameter come to Z parameter by using this equation. So, these 2 relation help and once you know Z you know there are relation between Z and Y. So, if I know Z I can go to Y also there are relations between Z and H.

So, if I know Z I can go to H also. There are relations between Z and a, b, c, d. So, if I know Z I can go to a, b, c, d. So, with these 2 you can go from 1 to others. So, our advice is if a lumped element is there not a distributed 1 try to find its Z or Y whichever is convenient then finally, find Z and if you want to find S from Z, use this formula to find S. On the other hand, if there is a distributed line then try to find its S parameter and from S parameter you can come to Z or Y as required.

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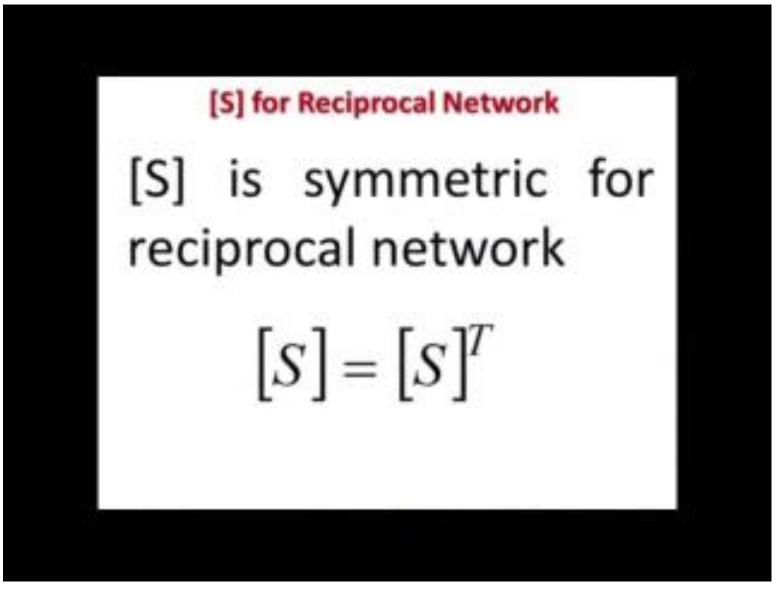

Now, various times we see that a network is reciprocal then if we do not use any non reciprocal device, 1 of the very popular non reciprocal device is ferrite. So, if ferrite is not used or other such non reciprocal special materials not used, in most of the passive devices they are symmetric or their reciprocal network.

If the reciprocal the scattering matrix that becomes symmetric, now symmetric means what that S and S Transpose they are equal you see a matrix I can take a transpose of the matrix. So, for a reciprocal network is and its transport is same. So, we can easily take whether A, if we know S matrix of any network particularly with network analyzer, we can find out S matrix, we can easily check whether it is a reciprocal network or not.

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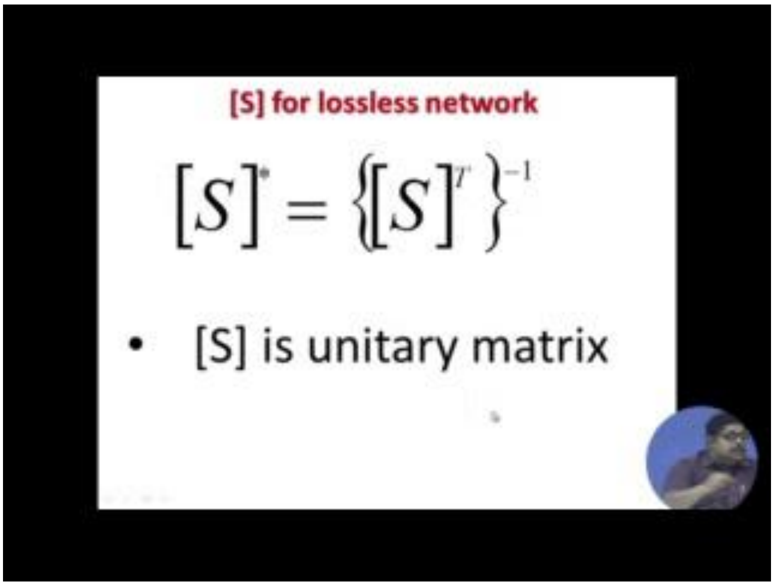

Now, if a network is lossless, lossless means there is no dissipation of power, there what about is in studying inside, there is no loss that is called the lossless network. So, for a lossless network, the S matrix is this property you see S conjugate is equal to S transpose inverse that, now, if you remember any network whose conjugate is equal to inverse of transpose that is called a unitary matrix. So, for a lossless network, S is unitary matrix.

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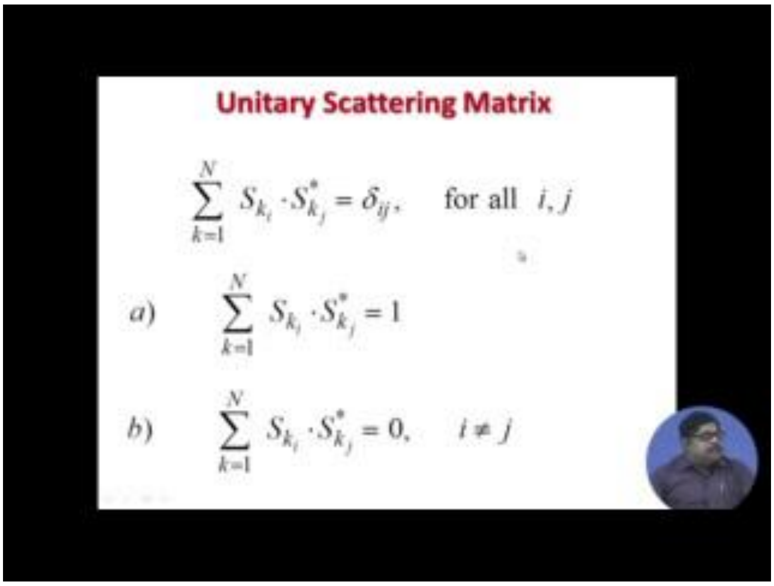

Now, if for any unitary matrix the elements are related like these is S k i dot S k j star is delta i j this delta is that means, when i j equal to 1, this delta becomes 1. If i j not equal to 1 delta becomes 0. Now, these can be broken into 2 parts; 1 is you see S k i dot S k j is equal to 1 so that means, any column of S k i means is equal to 1 means same column is S matrix any particular column EPT is dotted with the conjugate of the same column will get 1 any column conjugated with the same column will get 1 any column conjugated with the other columns that will give us 0.

So, for unitary both these condition should be simultaneously satisfied, if any of them is highlighted it is not an unitary matrix. So, for a unitary matrix S k i dot S k j star is equal to 1 for i is equal to j. So, this condition there should be.

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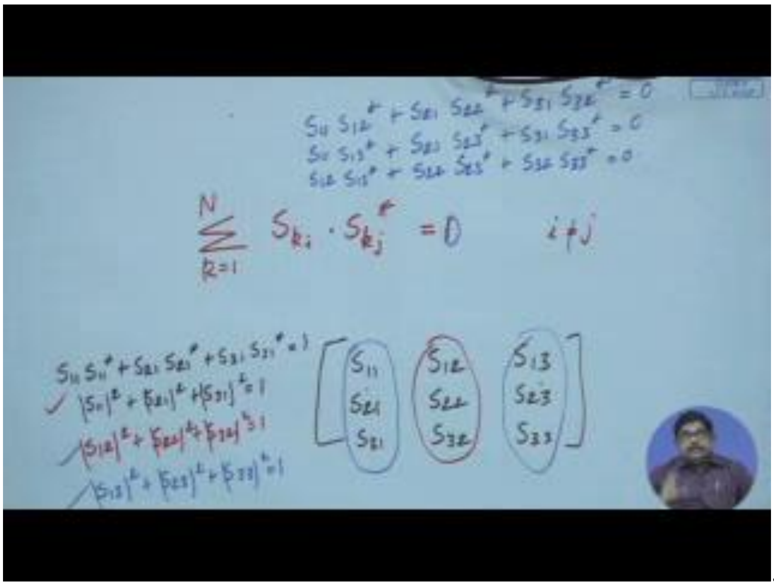

An additional qualification necessary here S k i dot S k j star is equal to 1, i is equal to j that means, the same column you dot with conjugate of the same column you will get 1 and any column conjugate with other column. So, suppose I have S 1, S 12, S 13, S 21, S 22, S 23, S 31, S 32, S 33. Now, the first 1 says that this is a column these column. So, if the matrix is unitary I will get S 11, S 11 star plus S 21 S 22 star plus S 31, S 31 star is equal to S 11, S 11 star means what S 11 square plus S 21 square plus S 31 is equal to 1 also I will have for these that means, S 12 square.

So, this is 1 then I should have S 12 square plus S 22 square plus S 32 square is equal to 1 then S 13 square plus S 23 square plus S 33 square is equal to 1 also this is the first property; that means, for unitary I will have these 3 equations. Also I will have 3 more equations that 1 with the other. So, when this will be 0 this will be 0; that means, S 11, S 12 star plus S 21, S 22 star plus S 31 S 32 star this will be 0 these 2 column 1 column conjugate with other column these are all different columns. So, that is 0 then this 1 with conjugate of these S 11, S 13 star plus S 21 S 23 star plus S 31 S 33 star this is 0.

Also this 1 conjugate with this one; that means, S 12 S 13 star plus S 22 S 23 star plus S 32 S 33 star is equal to 0. So, to be unitary these three and these three the six equations needs to be satisfied then will say the matrix is lossless or unitary matrix. So, in lossless network its scattering parameter matrix satisfies all these equations

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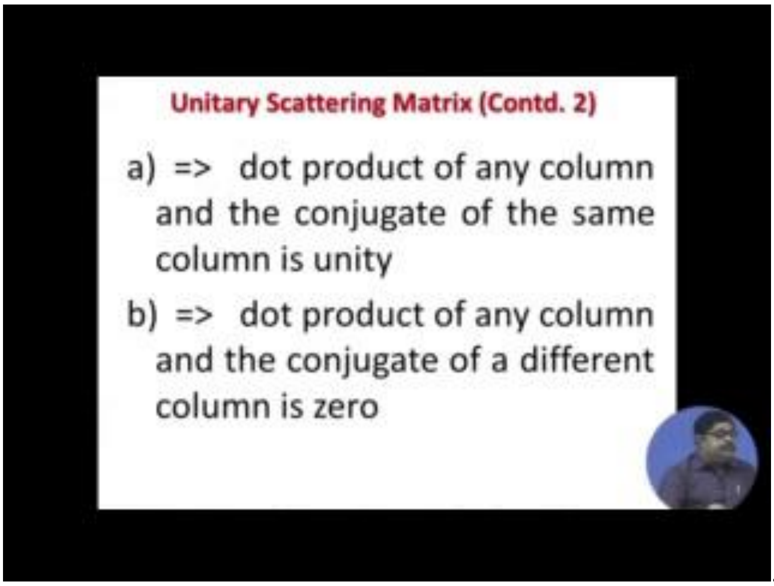

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So, that we are saying the dot product of any column and the conjugate of the same column is unity dot product of any column and the conjugate of a different column is 0. So, both the things should be simultaneously satisfied, if any of them is highlighted it is not unitary scattering matrix that means, the network is not lossless.

Suppose, if we have a resistive network obviously, you know resistance creates dissipation of power. So, that matrix cannot be lossless.

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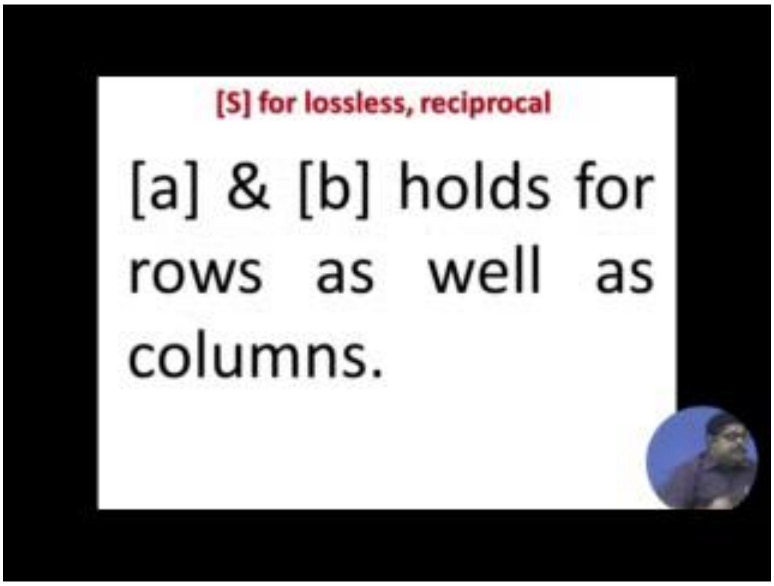

So, if any you have resistance used in the network you can see that is S matrix cannot be lossless a and b holds. So, now, if lossless as well as reciprocal material network that case the property holds for rows as well as column because for reciprocal the network and its transpose is same you know columns after transposition becomes rows.

So, here this property holds for columns, now for reciprocal rows also will have these rows means these conjugated with this row will be 1 these conjugated with this row will be 1 these conjugated with this row will be 1 these conjugated with another row will be zero these conjugated with another row will be 0 like that. So, all these are required when we try to solve various particular networks these properties helps us to find out these things this very useful properties.

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Now, we come to another thing that let us see in a 2 port network.

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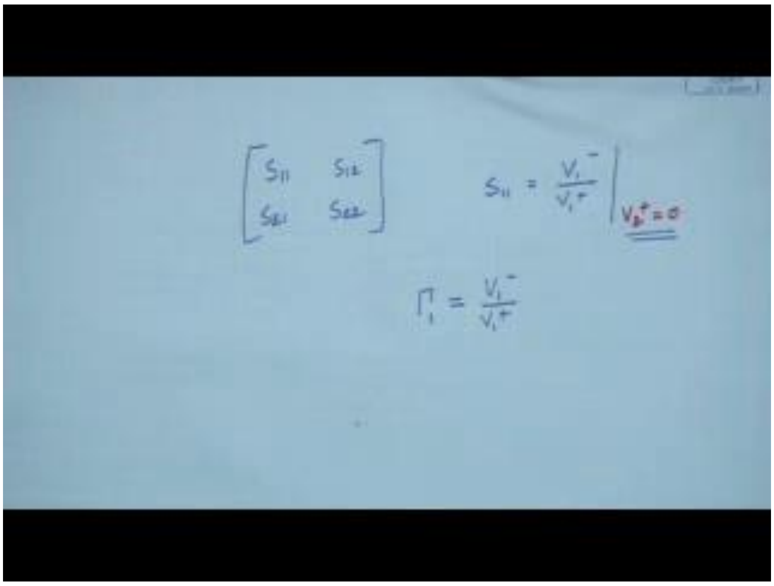

We have S 11, S 12, S 21, S 22 and what is S 11, S 11 is V 1 minus by V 1 plus when V 2 plus is made 0 match load, now is S 11 equal to deflection coefficient in general what is deflection coefficient of port 1 deflection coefficient of port 1 is V 1 minus V 1 plus, but S 11 is V 1 minus by V 1 plus with the added condition of these.

So, we can say that if port 2 is terminated in match load then only deflection coefficient then S 11 are same, otherwise they are different S 11 is under this condition deflection coefficient, deflection coefficient that is why we are saying that in general they are different that is why the first answer is no and second answer is yes only when all other ports are matched then S 11 and deflection coefficients same generally is a miss conception many times people interchangeably say these, but you should remember that under only these condition their yes otherwise they are 2 different entities.

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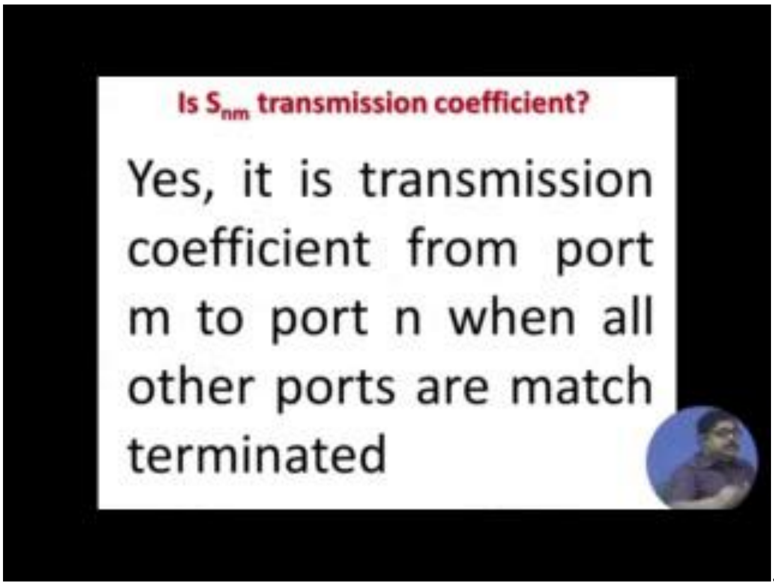

Is S n m is transmission coefficient again our answer is yes or no. So, it will be yes when it is transmission coefficient from port m to port n when all other ports are match terminated. So, this match transmission is important other ports match terminated then is parameter S and m are transmission coefficient. So, in a 2 port network say that easiest one.

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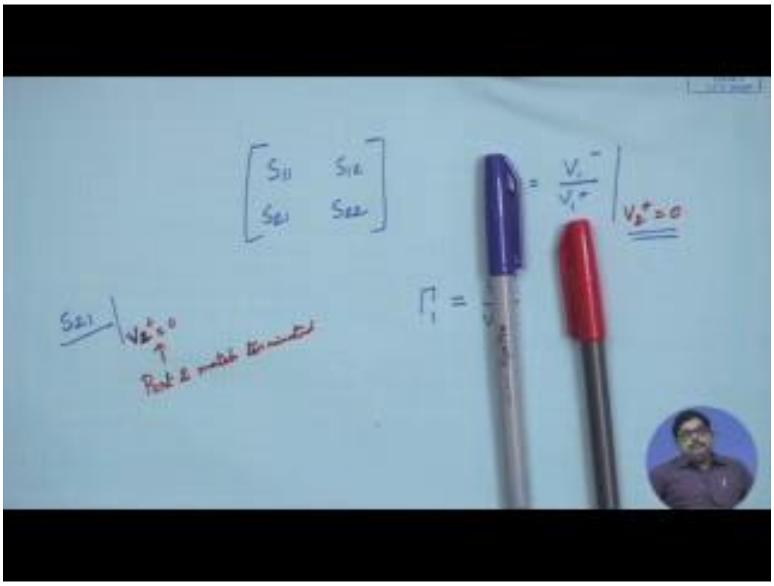

Transmission coefficient from port 1 to port two, the answer is yes if you have met the yes. The second port 2 port is match terminated that means, again V 2 plus is equal to 0, if you make V 2 plus is equal to 0 this means port 2 match terminated then S 21 is transmission coefficient. So, a network analyzer measures S 11 is 21. Now, remember that they are in actual cases when the terminations are different in port 1 or 2 based on that there can be some other deflection coefficient, only when a port 2 is matched. Then S 11 is deflection coefficient, similarly for S 21 only when port 2 is matched S 21 is transmission coefficient.

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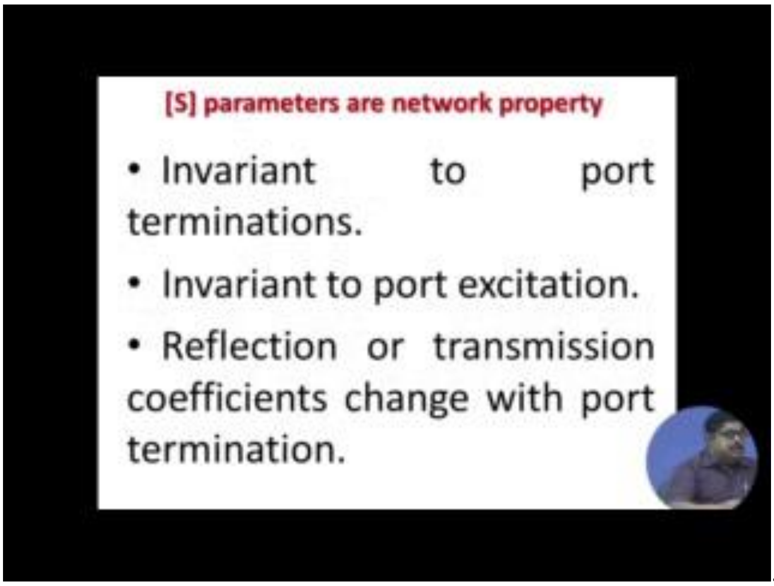

Now, S parameters are properties of network their characters of network. So, S parameters do not depend on port terminations it will change the port terminations S parameter do not change as we said that reflection coefficient may change transmission coefficient may change they depend on port termination, but S parameters are network property they are not depend on excitation of any one. So, they are invariant to termination port excitation does not do think, similar concept that when we say Z parameter it does not depend on what voltage I apply, at which port it is a property of the network it does not depend on excitation or response at the ports similar thing here.

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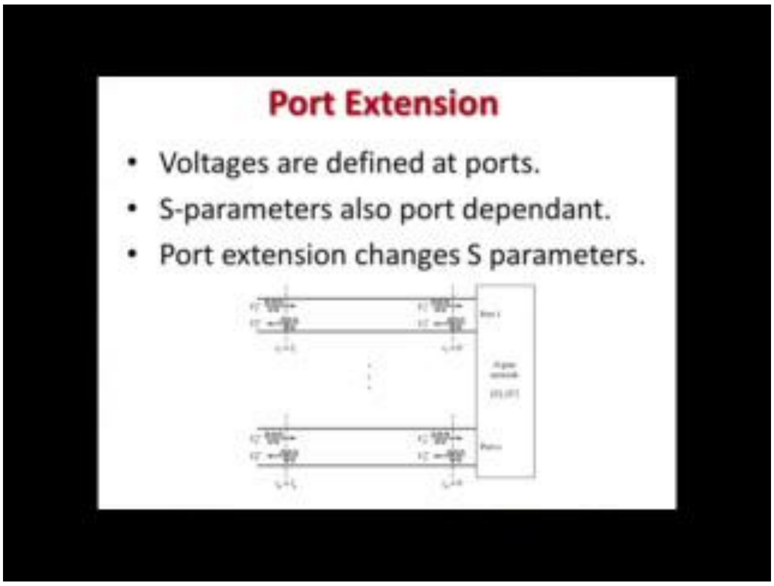

Now, 1 thing that we have seen in the first lecture today that in general for at microwave frequency with web guide type of things, we cannot define voltage uniquely, but under certain restriction you have define, but that definition we that times say is only at the particular terminal plain at a particular port I can define uniquely voltage or current.

So, voltages are defined at ports S parameters are also port dependent. Now, if I change that port that means, if I extend that port that terminal plain I can extend, I can then need to redefine the voltages and the S parameters will also get change so that means, see in this case that I have a network, I had the ports here port one, port two, port nine, etcetera. It is an input network the S parameter is S, now suppose I extend the port and put that instead of this plain I say these dash plain here that means, I have extended the port. So, definitely is this V 1 plus has been changed here V 1 plus dash etcetera.

So, S parameter of the whole thing will be changed and that new thing we are calling this S dashed instead of S with respect to this new ports is S dash. Now, what is the relation between the 2 S? Suppose, I these extension is every port I extend by an electrical length of theta that basically it is a transmission line. So, they are for a lossless transmission line of electrical length theta that means, if physical length l electrical length is theta the relation is theta is equal to beta into l.

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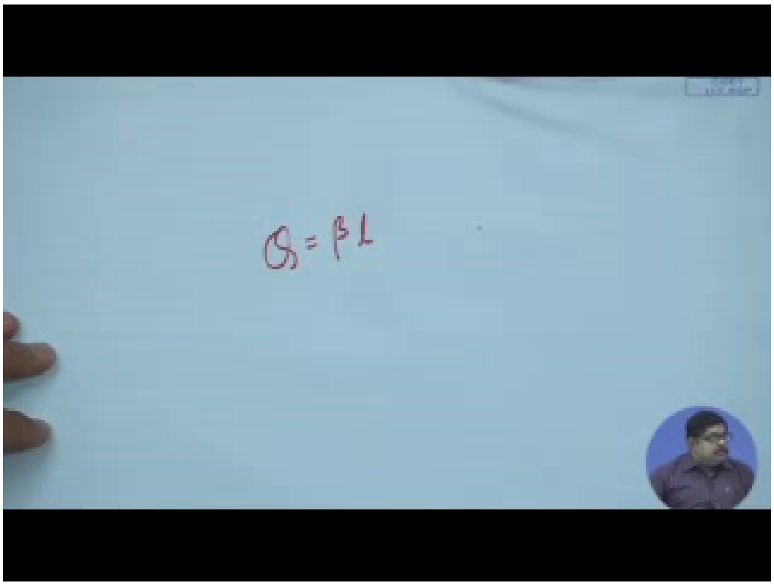

So, this electrical length of this 1 is theta. So, the S dash the new.

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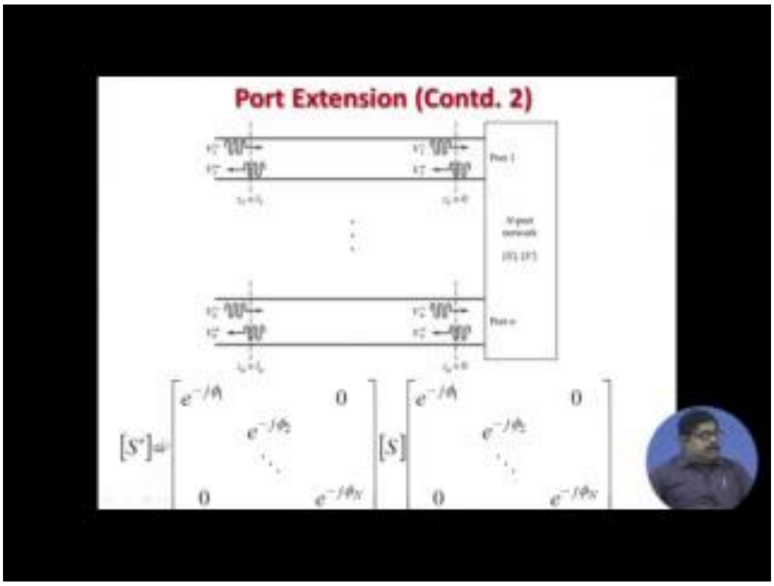

S parameter that will be even by this that port 1 is extended by let us say or theta. Now, we are calling phi, phi 1 then port 2 by phi two, etcetera. So, you write the matrix like this, then S. So, this will be your new port definition. So, S dash and S is related by these called port extension.

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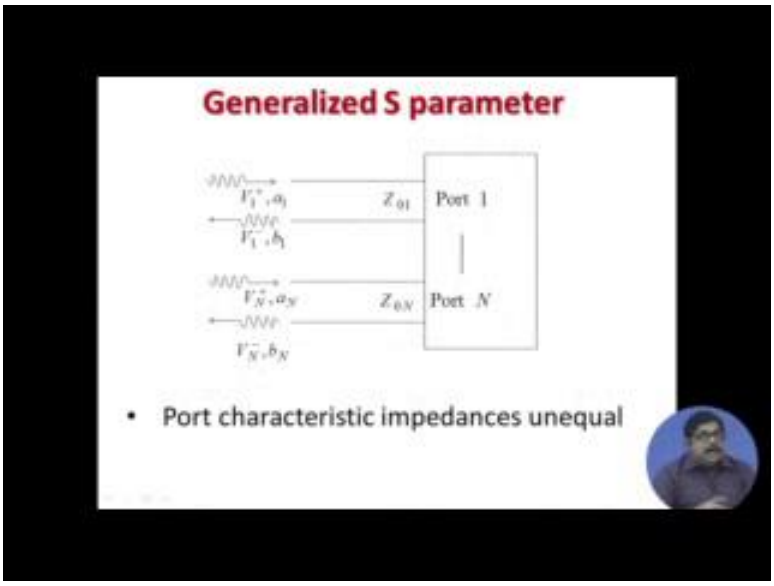

And another thing is that various ports, now we define parameters. Now, let us say that characteristics impedance of various ports are different. This thing happens suppose many times we write suppose in a web guide there are 5 numbers of modes going. So, we say that it is though there may be a single port there, but we say electrically it is equivalent different ports and each mode may have different port impedances. So, this is the how to write generalize parameter when the various ports has various impedances 1 is Z naught 1, Z naught 2, Z naught 3, etcetera. So, port characteristics impedance is unequal.

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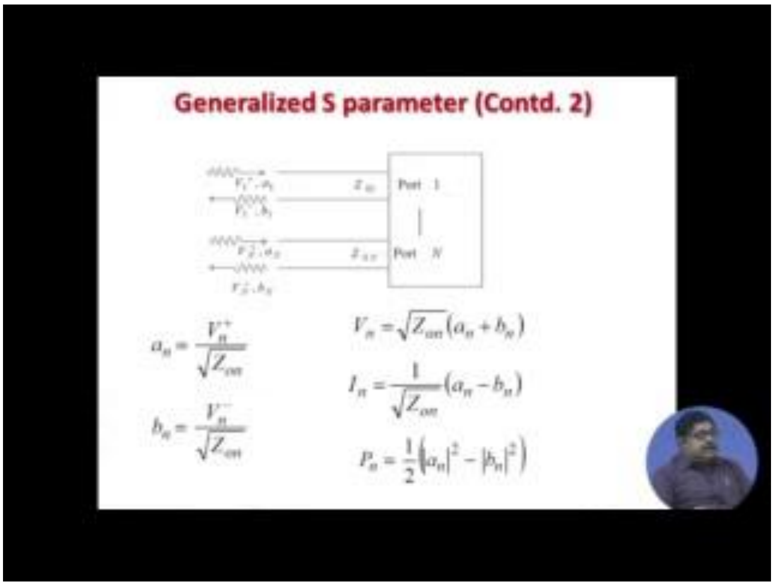

In that case, what we suggest that you redefine the voltage, the positive going voltage and reflected voltage that incident voltage V n plus you divide it by root over impedance of the port and that let us call a n. Similarly, that reflected voltage from the port nth port that you divide by the square root of impedance and call it b n and instead of b 1 plus b 1 minus, let us say that a 1, b 1 are the two voltages, they are hypothetical ones this a n b n, a 1 b 1, a 2 b 2 etcetera.

So, beauty is here if you write now V n that the voltage here that will be dependent on root over Z 01 that Z 0n I will be also dependent. On Z 0 n p n also dependent on you see that in a p n expression there is no impedance coming. So, voltage current they are in this new generalized thing they are impedance dependent, but power is not if you write it terms of a n, but if you write it in terms of b it will be power will be dependent on impedance as well.

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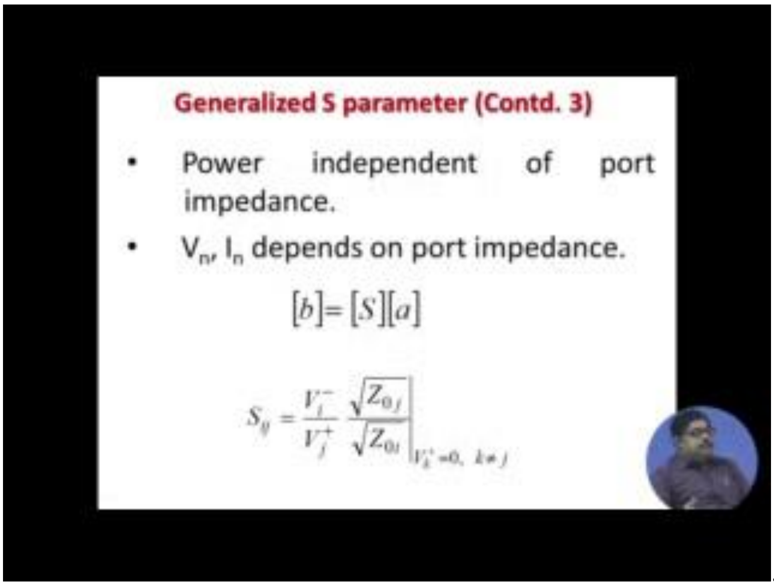

So, power independent of port impedance generally, this power is the thing that is required in many cases. So, if you want to make power expression independent of port impedance, this generalized is parameter a b can be used. So, V n i n depend on port impedance. So, you see that S i j depends on port impedances, but in terms of b a there are no impedance coming this is 1 advance concept just we are mentioning because we are introducing S parameter that, if you have various port getting you can write it in terms of this a b, but this generally in simple cases you do not require this is when in in a particular guide various higher order modes are traveling you can use these generalized scattering parameters a b instead of V plus V minus.

All we had in these properties of S parameter. So, we have seen properties in the next lecture will see the instrument network analyzer that what is asked and how it helps to find out S parameter measurements.

Thank you.