Basic Tools of Microwave Engineering
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Lecture – 11
Voltage and Current at Microwave Frequency

Welcome to the eleventh lecture of this course. In earlier lectures we have seen the first tool smith chart, which microwave engineers use very often and with that tool we have also seen how to measure impedance and we have solved various problems for impedance matching. We have seen the design of impedance matching network. Now, in this week, in this 5 lectures will try to see the second tool that microwave engineers use which is called S parameter or scattering parameter. Now to kick start that discussion, the first lecture will see voltage and current concept at microwave frequency, now at lower frequency than microwave frequency.

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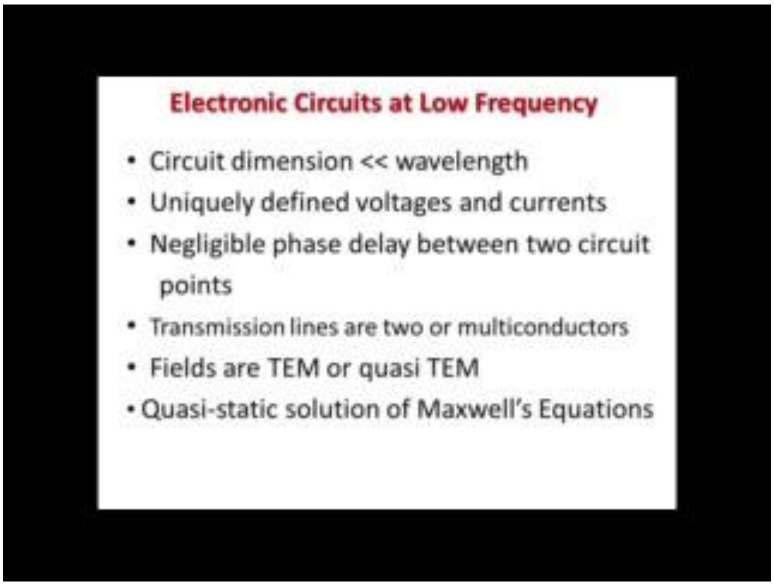

We know that the circuit dimensions or components that are used in electronic circuit their dimensions are quite less than wave length in low frequency because low frequency wave length is quite large. So, typically in kilohertz up to megahertz frequencies the circuit dimension that we use for various registers, transistors, diodes and various amplifiers, etcetera that we built from them their dimension is much, much less than wave length that is why the phase difference between 2 points in the line for a voltage or current wave that is not much and that is why we can uniquely define the voltage and currents at low frequency and we assume that there is no wave type of thing there though inherently there is wave, but at low frequency wave.

So, voltage and currents obey the form of (Refer Time: 02:50) voltage law and current law. So, that unless and until a component is encountered the voltage and current do not change, but actually for a wave definitely the voltage and current change as we move on. So, we have negligible phase delay between 2 circuit points at lower frequencies and also we can use to conduct a transmission lines like Maxwell lines in low frequency and into conduct a line it is very easy to define the voltage and currents, so uniquely.

Also in low frequency the fields that are transmitted to those 2 conductor transmission lines they are mainly transverse electromagnetic type or even if not transverse electromagnetic type they are quasi TEM we call that means, almost like TEM for all practical purposes like TEM that is why the solution of Maxwell's equation at lower frequency, they also for electrodynamic cases, also they are quasi-static that means, something like electro static or magneto static type of solution. So, at lower frequency the whole thing though dynamic though field based though wave based, but their solution appears to be like the electro static or magneto static cases. So, that voltage and current definitions suffice.

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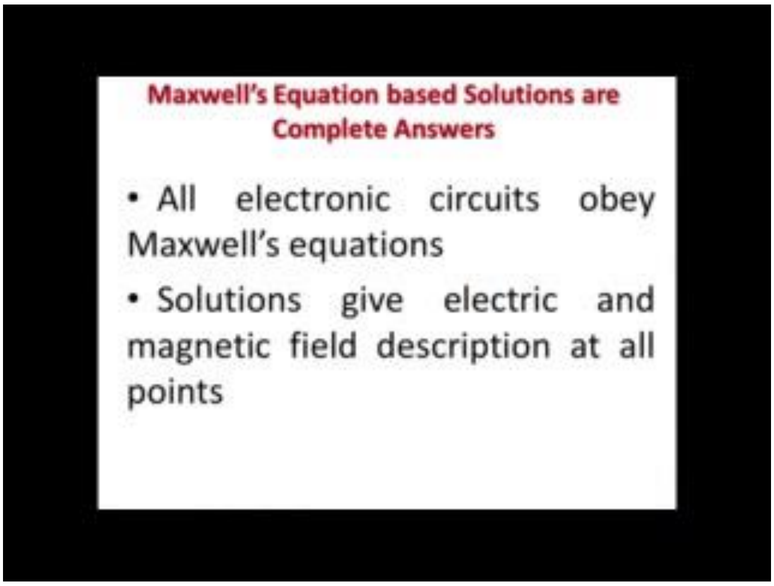

But at higher frequency obviously, that is not the case there we need to consider the non TEM type of propagation Maxwell's laws gets solved there by various modes that is why power gets transported by various modes and we will have to solve those problems with the help of Maxwell's equations and boundary conditions, but in reality though Maxwell's equation and its solution gives us all the information about the electric and magnetic field throughout the whole universe you can say and we are very exact.

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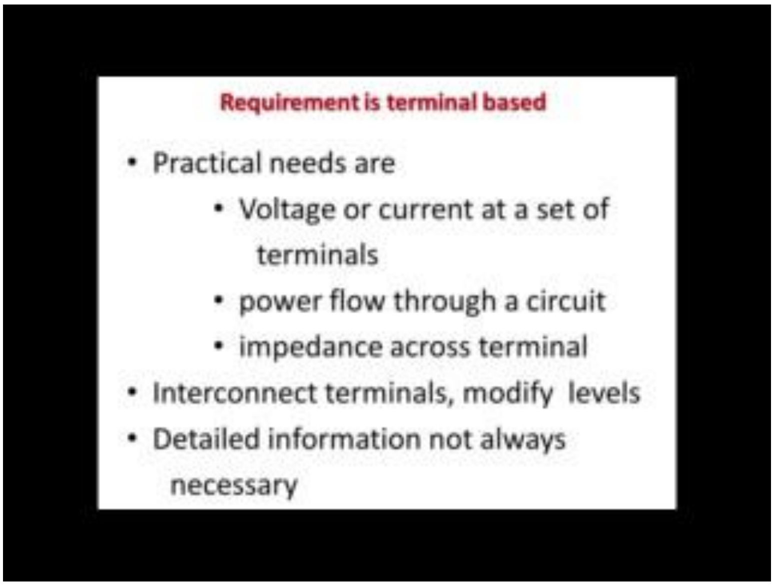

But always in even in high frequency design microwave frequency circuit designs we do not require. So, many information if we look at our practical need is always terminal based you know a complex design problem. Generally, we solve by considering or breaking the whole problem into several parts and that is why we try to resort to block diagrams and where a sub block its characterization, first we do then we design that block, then we inter connect that block and hence in practical cases we want to know at a set of terminals which are generally the ports of the sub blocks there.

What is the voltage or what is the current going into 1 port or coming out of 1 port of the block? What is the power flow through that block? What is the impedance across those port terminals? And then we interconnect those terminals to connect several such blocks, if required we modify their impedance levels etcetera and exact electric field and magnetic field description and all these points inside the block is not always necessary.

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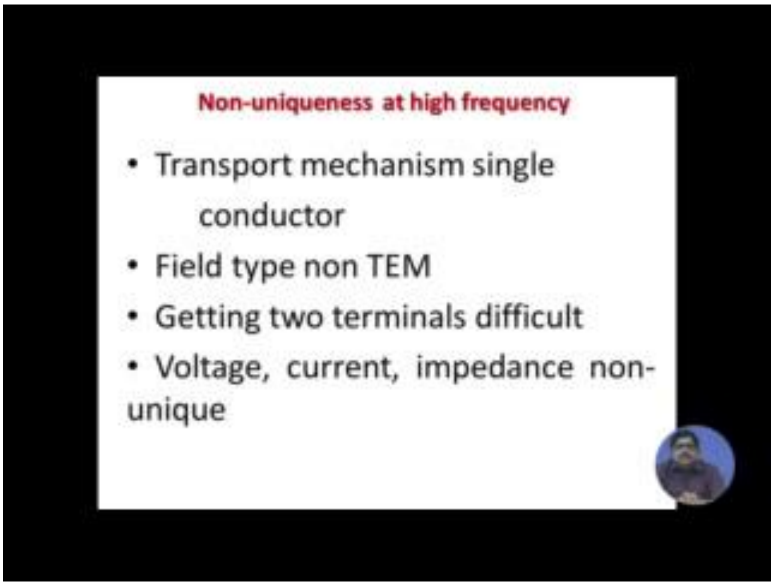

So, engineers try to find that can we have some voltage and current concept defined at microwave frequency, but 1 problem of this high frequency thing is here the transport mechanism or the power that gets transferred through the medium that is generally in the form of waveguides which are a single conductor which are not 2 conductors because 2 conductors becomes losse higher microwave frequencies. So, single conductor does not support a traverse electromagnetic or quasi TEM type of waves. So, field here is non TEM mainly in the form of various combinations of transverse electric and transverse magnetic modes.

In a 2 single conductor like wave waveguide it is very difficult to get 2 terminals to define voltage or current for voltage you require 2 terminals. So, that we measure the potential difference we are not that, much about the absolute potentials the potential difference. So, for that we require 2 terminals similarly current if it enters to 1 terminal it returns to another terminal. So, it also requires 2 conductors or 2 terminals to get it defined and obviously, the characteristics of the network is the impedance. So, if voltage and current that non unique because here what happens due to wave propagation at various points we have various voltages and currents. So, impedance is also non unique at high frequency.

Now, the (Refer Time: 08:56) can we make in spite of this difficulty in the same waveguide type of cases single conductor cases, can we in an equivalent wave define unique voltage and current because if we can then instead of field theory we can talk the help of wave theory obviously, distributed theory. So, the circuit theory in the form of transmission line that theory we can take the help of and so our exact field solutions which is a vector solution three dimensional solution that will become much more easier.

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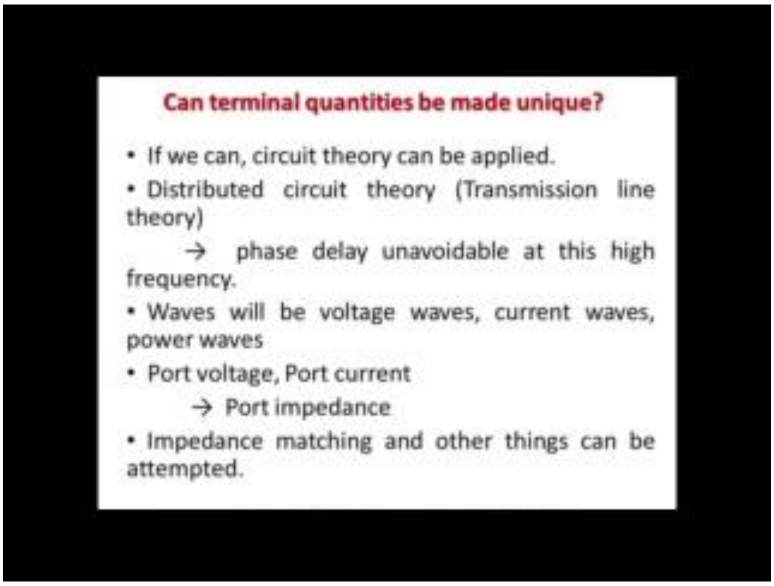

So, the question is can this terminal quantities been made unique now obviously, phase delay is unavailable at this high frequency as I said that here, since we are very high end of frequency gigahertz sort of frequency or several megahertz of frequency several hundreds of megahertz of frequency. So, even a small length that adds to quite good amount of phase. So, we cannot avoid the phase delay.

And obvious that is why here voltage will be in the form of voltage wave, we will have to find out how much it is going across a component. So, voltage in the form of voltage waves will exist here, current in the form of current waves will exist here, power also we prescribe in terms of power waves as we have seen in the case of impedance matching we have shown power waves and at the port, if we very precisely can define ports that means, on a terminal plane if we fix the terminal plane then it appears that we can have a try to define port voltage and port current and obviously, if we can do that we can get port impedance, but it can be due to the non uniqueness of the solutions it can be done in various ways. So, what is the best possible way by which we can define voltage and current if we can do that then we can impedance matching and other design things.

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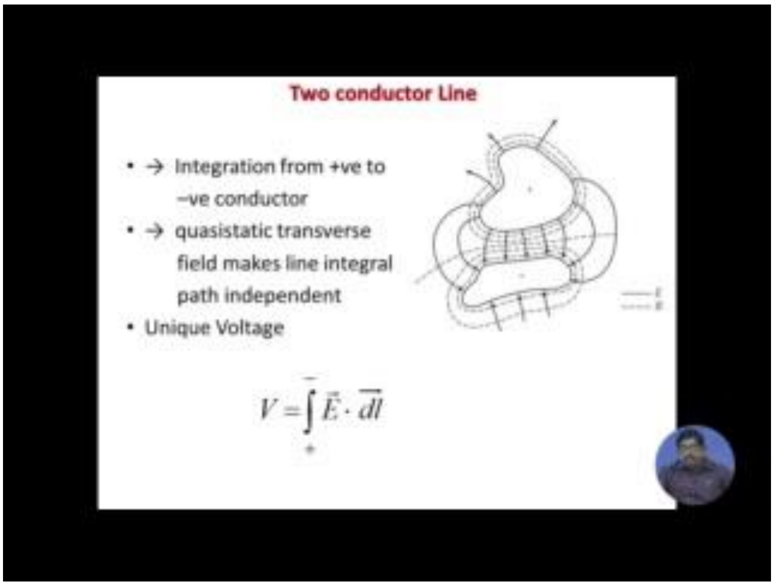

So, first let us see a 2 conductor transmission line which is the quasi line when it is extended up to gigahertz in up to gigahertz, nowadays it went to 10-20 gigahertz, it is huge, it is modified a very good designed quasi line nowadays can work up to 20 gigahertz also and this in this picture you are seeing that if we have 2 conductors 1 is positively charged another is negatively charged and obviously, there will be electric fields and magnetic fields between them.

Now, quasi-static transverse field as we said that if we have quasi-static field in 2 conductors in quasi-static type of fields also other non TEM waves also propagate to them, but if we make the quasi-static approximation and then the line integral voltage we know is the line integral of electrical field and generally, the unique voltage definition is from negative to positive that is why you were saying from positive to negative the conductor if we go, if we take the line integral of the electric field that is voltage. Now, in case of 2 conductor line this voltage definition is unique.

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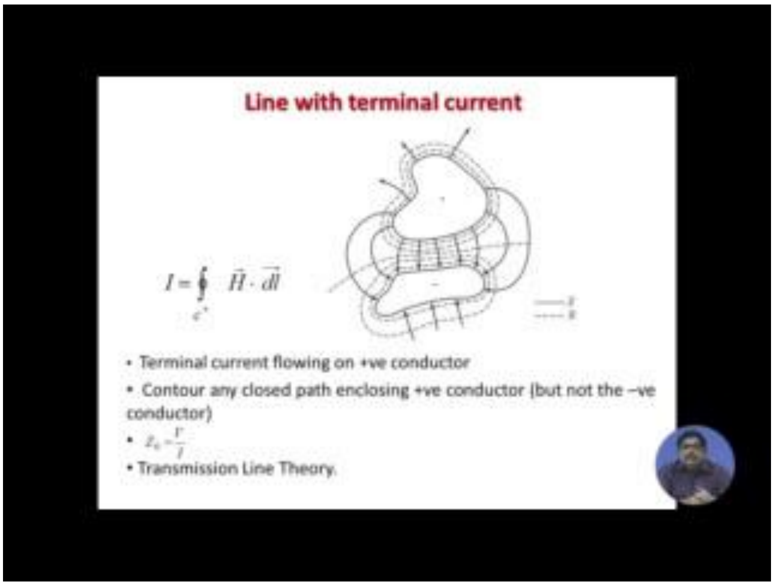

Similarly, we know that the closed line integral of magnetic field gives us current. So, here the terminal current that is flowing on positive conductor, if we can have that current uniquely defined for any contour closed path enclosing positive conductor then we can define the ratio of voltage and current. So, defined as the impedance and this is basically the realm of transmission line theory.

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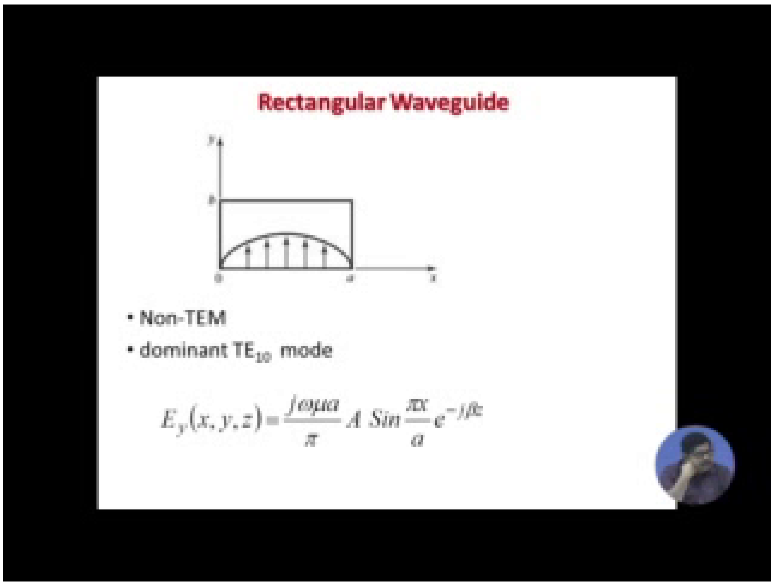

But, let us see rectangular waveguide, rectangular waveguide as dominant field distribution as TE 10 it is a non TEM mode and electric field. The transverse electric field it has only 1 transverse electric field it is along the narrow dimension of the waveguide that is generally we call y e direction the expression is given the variation is along x the variation is shown in the figure and it is a sinusoidal variation the wave is propagating in the positive Z direction.

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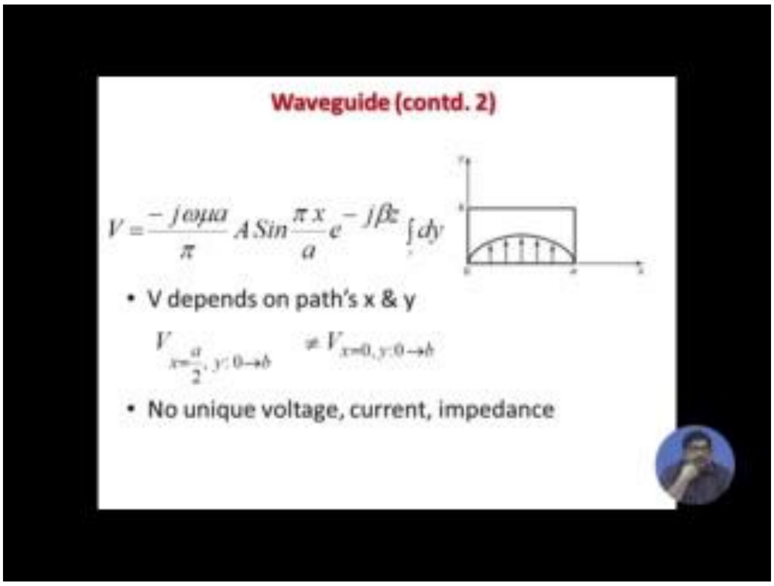

Now, from that if we have that voltage is equal to the line integral of this, we come across the expression shown here obviously, this V, the voltage depends on which path we are taking to integrate and also the values are x and y that we are traversing on the way. So, that is why we are saying, suppose we take the middle path that is x is equal to a by 2 at the middle if we take that integration of y from y is equal to 0 to the top wall y is equal to b we will get some value of that line integral and that obviously, is not equal to the thing if we do that let us say x is equal to 0; that means, on the narrow wall.

So, you need voltage current impedance these are not defined in case of a single transmission line. So, what is the wave out fortunately microwave researchers have come up with waves to tackle this problem, they say you can still define voltage and current, but obviously, for a particular non TEM mode that means, for TE 10 mode there will be a voltage current definition for t 20 mode.

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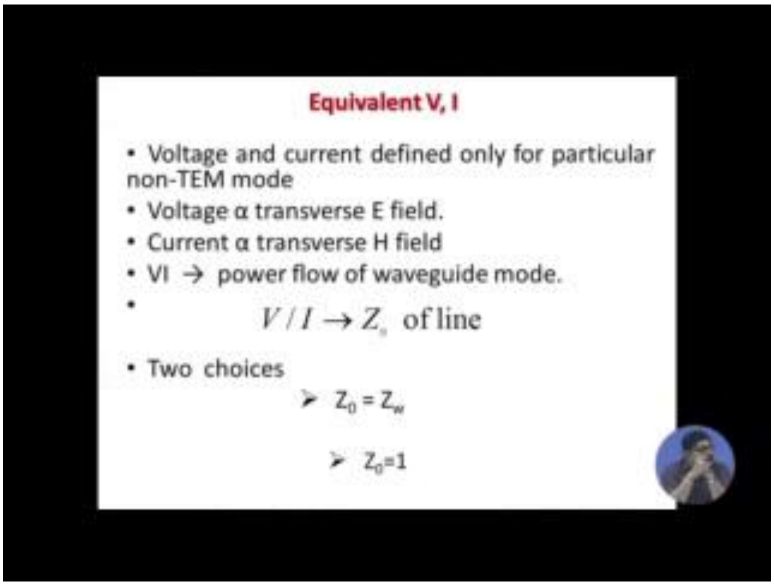

There will another voltage current definition for TEM 10 mode there is another current voltage and current definition etcetera. Now, there are 3-4 guidelines, the first is you know every mode, you know, what is the transverse field? So, voltage should be defined proportional to the transverse electric field is the first and the second one is the current also should be made proportional to transverse magnetic field.

Now obviously, this proportionality constant can be many things. So, this is not uniquely said, but then 2 more conditions are imposed on these 2 to make the whole thing very unique that one thing is a instantaneous power flow should be equal to the product of voltage and current and the ratio of this voltage and current that should be defined as an impedance and there are 2 choices that impedance. So, defined can be made equal to the wave impedance because every mode has a modal impedance which is the ratio of the transverse electric field to transverse magnetic field, so that the power can propagate to Z direction that is called Z dome w or wave impedance.

So, you can make that impedance. So, defined form voltage by current either as wave impedance or you can normalize that this 2 choices are existing. So, if this is done then at a terminal plane we can uniquely define voltage and current, obviously, voltage and current will propagate in the Z direction in the form of a wave, but the terminal voltage and current they will be unique.

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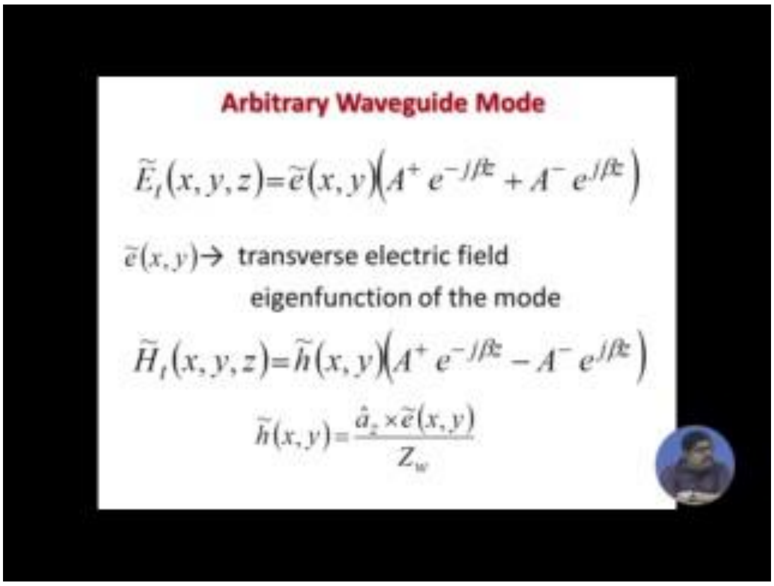

So, that is why let us say that an arbitrary waveguide mode its transverse component of electric field will in general be a function of x, y, z. So, that can be written as eigenfunction e x, y into 1 positive going wave e to the power minus j beta Z and other negative going wave e to the power minus e to the power plus j beta Z we are calling j beta that means, we are assuming it is lossless, if it is not lossless it should be gamma Z and minus gamma Z a propagation constant similarly the transverse magnetic field also can be written like this this is the breaking of any web guide any electric field into various web guide modes.

Now, we know that electric field eigenfunction and magnetic field eigenfunction they are related through the modal impedance j omega, this is the field theoretic point now to make equivalent voltage current.

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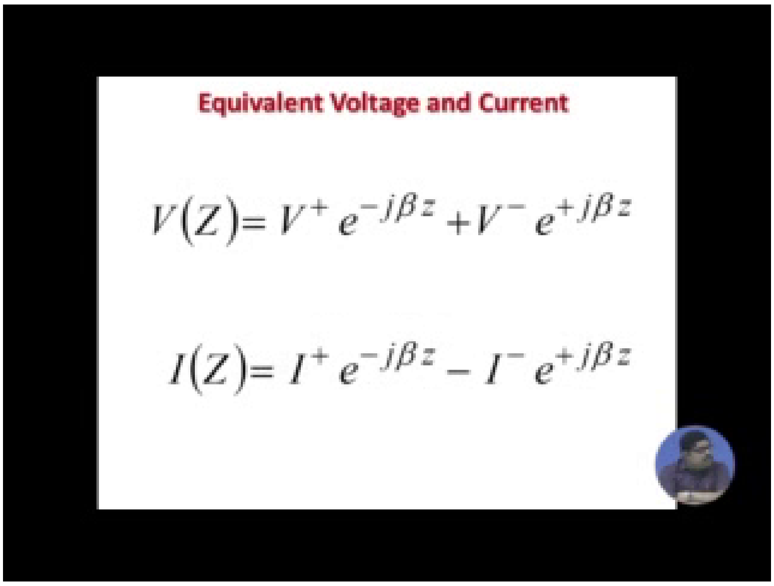

What we do we write V Z as a positive going voltage wave V plus e to the power minus j beta Z and obviously, also a reflected voltage wave V to the power minus e to the power j beta Z. Similarly, the current wave also you break into 2 parts 1 is incident wave another is reflected wave remember that incident and reflected wave there is a negative sign between them because of power propagation direction.

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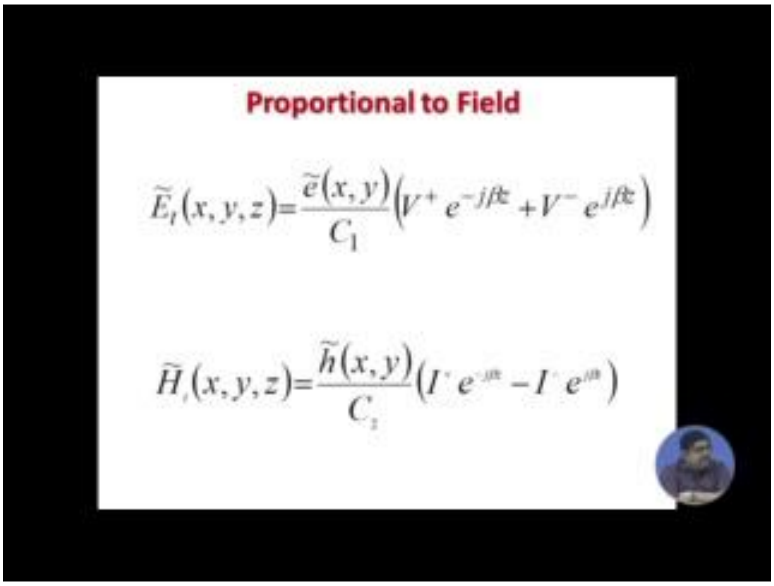

Now, see that the guidelines for defining equivalent voltage current the 2 first points are the equivalent voltage is proportional to the electric transverse electric field that is why we are writing e t x, y, z that should be written as V plus y c 1, c 1 is constant obviously, the eigenfunction will be there, but other parts we are writing as V plus by c 1. Similarly, the magnetic field is meant made proportional to the current the constant of proportionality in this case is c 2.

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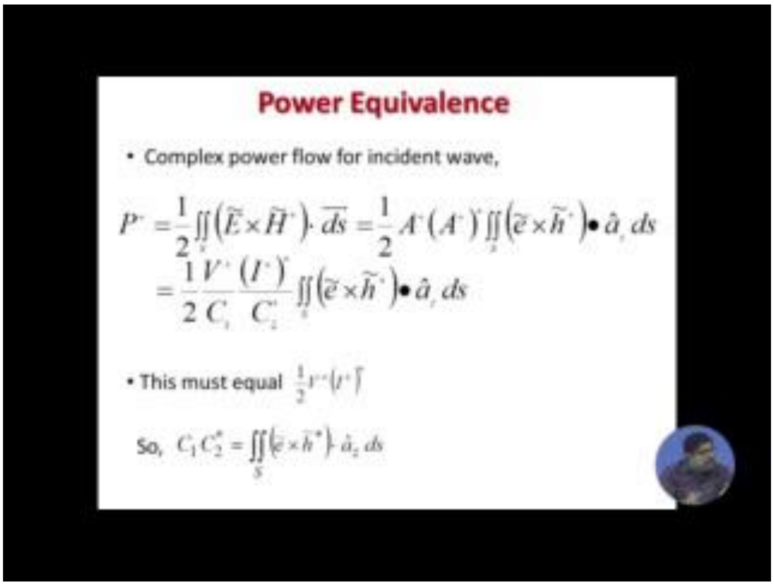

Now, we are forcing the power flow condition that instantaneous power flow should be the product of voltage and current. So, if we assume time harmonic fields, that is sinusoidal fields are one form of time harmonic field e to the power j omega t, then we can also define an average power wave power flow.

Now, the incident power flow can be written like we know, it can be expressed as the pointing vector, surface integration of pointing vector. So, half comes from that e to the power j omega it is a sinusoidal variation, any sinusoidal quantity its average is half. So, half e cross h star that is the pointing vector it is a power intensity power flow per unit area that dotted with the e s vector. So, that expression we wrote finally, we also wrote it as we know already we have made those a plus thing as proportional to voltage and current.

So, in terms of voltage and current it is written, now the demand is instantaneous power flow should be equal to V i. So, in terms of complex things the average power flow should equal to half p plus half i plus star. So, the first equation top equation we have written in terms of field and the second equation that it should be equal to this half V plus i plus star. So, this 2 if we made equal then we get 1 condition on c 1, c 2 as shown that their product is certain things depending on the eigenfunction. So, depending on the waves mode modal variation and the geometry of the guide this can be evaluated right inside also impedance should be equivalent that means, the Z naught that is equal to e y plus by h x plus which turns out to be e by h that should be also equal to V plus by i plus or ratio of V minus by i minus.

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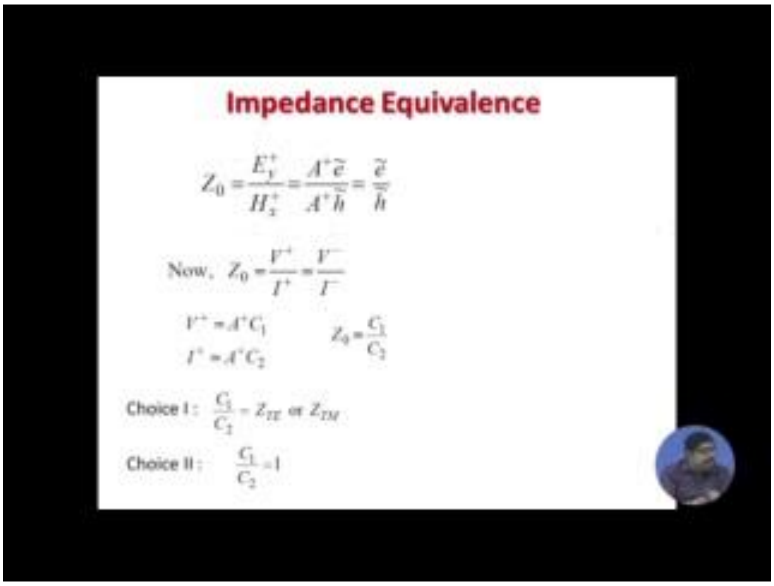

Now, if we meant that again we say there are 2 choices c 1, c 2 Z t in this case it is Z t because we are talking of TE 10 or you can make c 1 by c 2 is equal to 1, we will solve problems latter in the tutorial, we will take the second choice and show you can also take the first choice.

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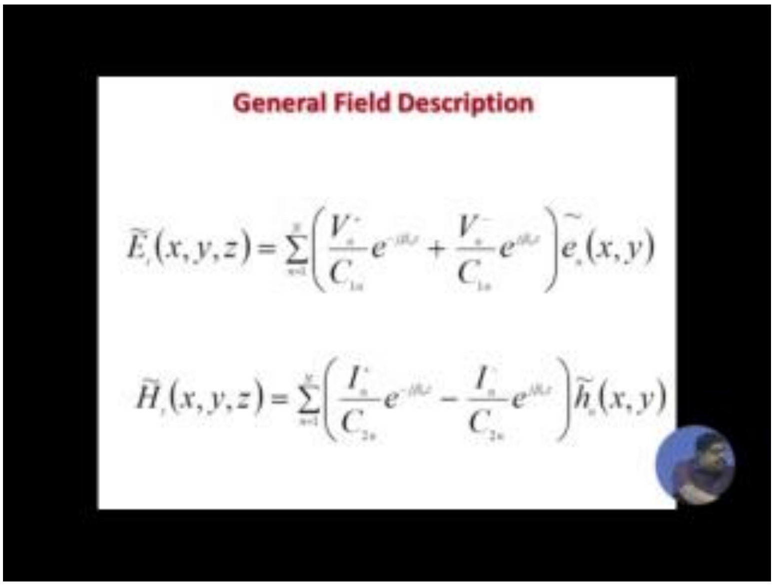

So, generally the field can be described like this that transverse field e t. Now, can be described in terms of those equivalent voltage and currents as V n plus by c 1 n as you see the sum of n plus and n minus, similarly the transverse magnetic field also can be written in terms of equivalent currents i n plus and i n minus c 2 c n are there you can solve for the values of c 1 and c 2 by enforcing the power equivalence and impedance equivalence those 2 will 2 unknowns 2 equations.

So, you can find out their values and those values you can put. So, various model things as there can be various modes possible particularly near the discontinuity etcetera or near the excitation, when we have very near to the source or very near to the load or very near to any discontinuity the instead of a single mode various higher order modes also are present that time, we can write that in form of various modal voltage and currents this is the description.

And this completes that how you can define the equivalent voltage and current in microwave frequency. Obviously, this is a dynamic problem, electro dynamic problem at higher frequency. It is not as simple as the low frequency voltage and current and (Refer Time: 26:50) voltage and current flow type but it can be done. Now, every mode has its own counterpart of voltage and current. So, that is a good step forward where now we can attempt using transmission line theory to these. So, that instead of solving Maxwell’s equation we can get away with lesser information, but as an practical engineer we always not go for all exact information things, but the practical useful things. So, this will be shown to leaders to useful concept, which is a second tool which is focus of our, this series that scattering parameter in the next lecture.