

Basic Tools of Microwave Engineering
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Lecture – 10
Tutorial 2: Impedance Matching Network Design by Smith Chart

Welcome to the second tutorial of this course, where we will be attempting to solve some problems that will clear our concepts that how to design Impedance Matching Network. And we will be using obviously, the tools that we have learnt in our earlier module that is the Smith Chart. So, with smith chart how to design.

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L-Section Matching Network Design

Q1: Design an L-section matching network to match a series RC load with impedance $Z_L = 200 - j100 \Omega$ to a 100Ω line at a frequency of 500 MHz. Determine the frequency response of the matching network.

Solution:

Step-1:

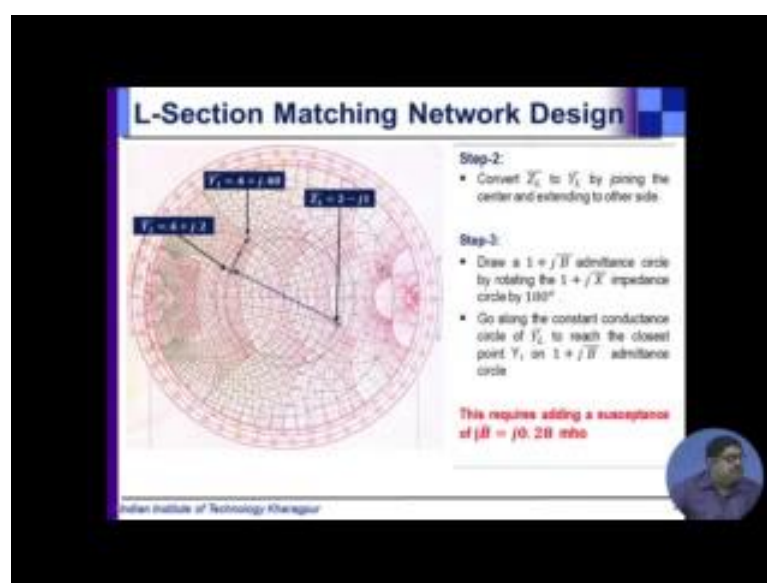
- Determine the normalized load impedance, $Z_L = \frac{Z_L}{Z_0} = 2 - j1$ and plot it on smith chart.
- Determine the region (Region 2)
- Choose any of the two generic networks ($C_1 - L_1$) or ($L_2 - C_2$)
- Let, we choose the network $C_1 - L_1$

The circuit diagram shows a series combination of a capacitor jX and a load Z_L connected to a source Z_0 . The matching network consists of a series capacitor jX and a shunt inductor jB .

The first problem is this that design obviously, that L-Section matching network to match a series RC load with impedance Z_L is equal to 200 minus j 100 ohm to a 100 ohm line at a frequency of 500 mega hertz. Also determine the frequency response of the matching network.

Now, here you know, we will have to do a lumped element matching L-Section matching. So, the first part is we will have to find out where we are in this region because based on that we will be having the generic designs. So, the first thing is plot the normalized impedance that comes to 2 minus Z 1. So, 2 minus Z 1 plot in the smith chart.

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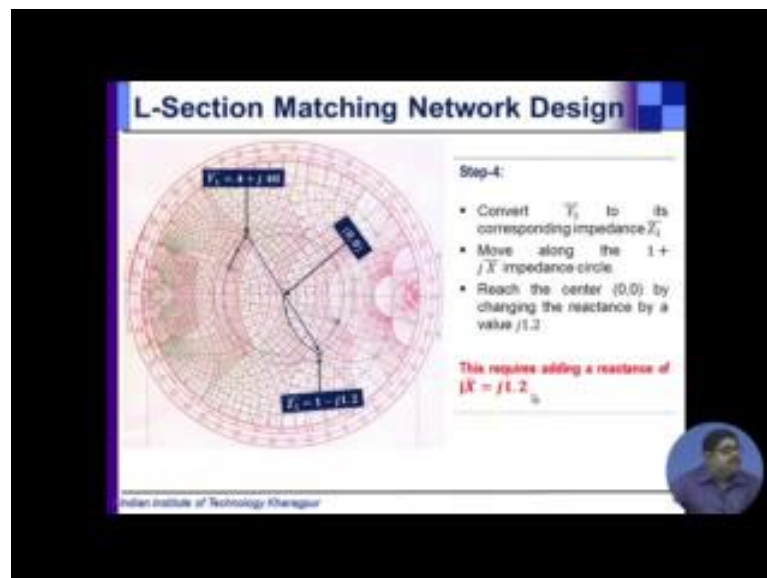
You see that we are plotting the smith chart, it is here. Now this is which part, if you remember that this part is region 2 that is why determine the region, it is region 2. Now in region 2, if you now go back and see that what are your choices you can have a C p L s or L p C s; that means, firstly you can have a first (Refer Time: 02:36) capacitor in parallel with the load then a series L s or a inductor parallel with the load and then is a (Refer Time: 02:48).

Let we choose the first one C p l s. So, you see with the load we choose that there is a capacitor in parallel and this is a series inductance that is why we are calling J X and j b. So, you see the follow the steps then convert since our choice is a parallel something. So, instead of Z L we will go to Y L, so that these 2 can be added. So, this Z L is converted to Y L that you now know from a thing that this is your Y L this point; now this 1 plus J X circle you changed by 180 degree rotating. So, this is your 1 plus j beta circle, this one you see 1 plus j beta circle if you look at your smith chart you are here. So, staying on the same constant resistance; that means, 0.4 you now move on this same constant 0.4 circle g bar is equal to 0.4 circle and go towards generator and you see you are cutting this 1 plus J B circle here. You see this was 1 plus J B circle you are cutting it here. So, that we are calling that Y 1.

Now, you can say that instead of that I can further go on and here also I will cut this circle somewhere here. So, I can go on and this also; that will be another solution, let us

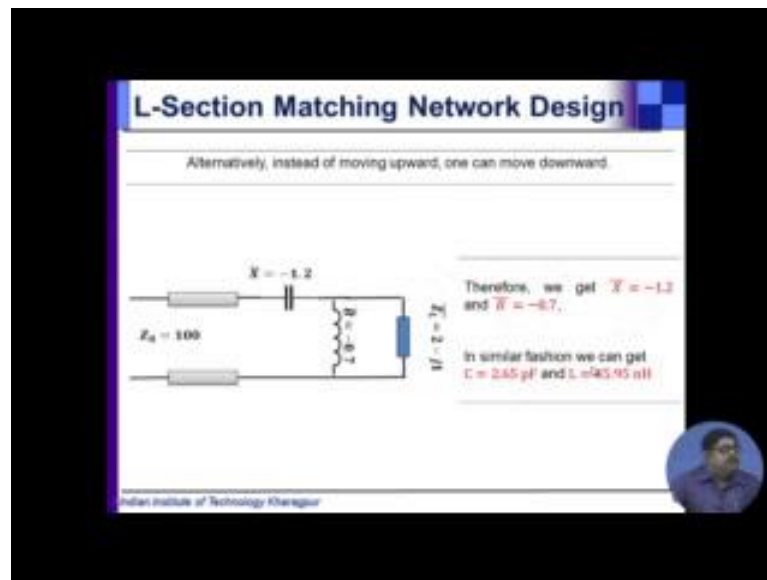
first see that this one. So, here if we go along the constant conductance circle of Y_L to reach the closest point Y_1 this requires adding, so how much susceptance I need to add to change from here to here? Constant conductance, so I am basically changing my susceptance, that susceptance you can read out that what is the susceptance value here, what is the susceptance value here, the difference is you need to put. Who has put that? That is put by that C_p , this fellow has put that thing. So, that he has put you here now this fellow. Now you see this is another series arm. So, here we have come up to here, from here now we need to change to impedance. So, we will impedance that Y_1 its impedance is here Z_1 bar.

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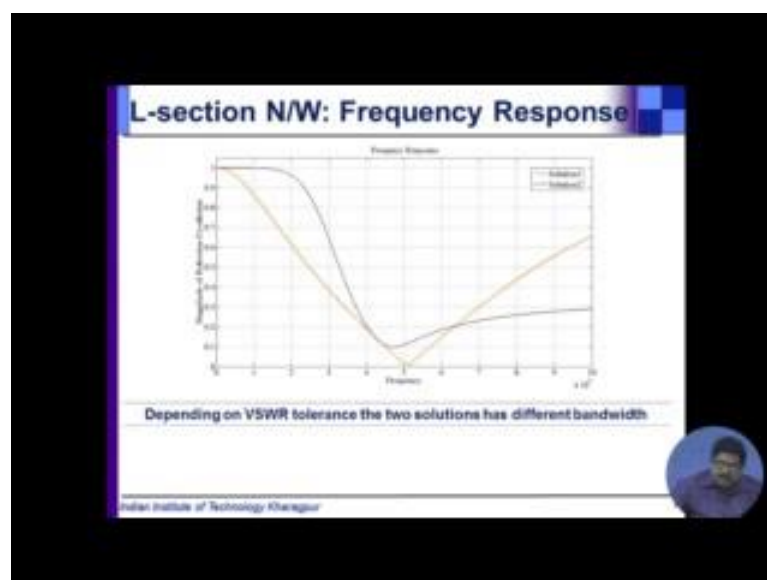
Now, we are already on the $1 + jX$ circle from here we will have to go here, how that can be done. So, this requires again a change of reactance that is equal to this. So, you can find this value.

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So, we have found that this is 0.28 and this is 1.2 therefore, you can now unnormalize that and find out what is your l c value. So, basically you have a (Refer Time: 06:33) so and so and C of so and so at 500 mega hertz. Now as I already said that instead of moving upward you can further go and cut the point downward, these 2 values will get. So, these are the 2 other values.

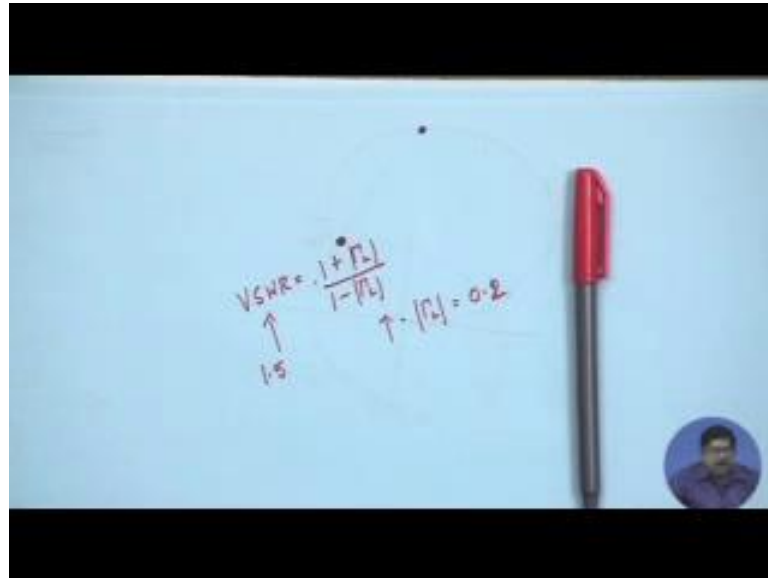
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Now, which one is preferable? You plot as we have seen that, you can plot the frequency response. So, if you plot the frequency response you see 1 and 2 are coming like this.

Now, depending on the application that which value of VSWR you are able to tolerate. If suppose VSWR of 1.5 is tolerable from. So, 1.5 VSWR means what? That VSWR is what $1 + \Gamma_L$ by $1 - \Gamma_L$.

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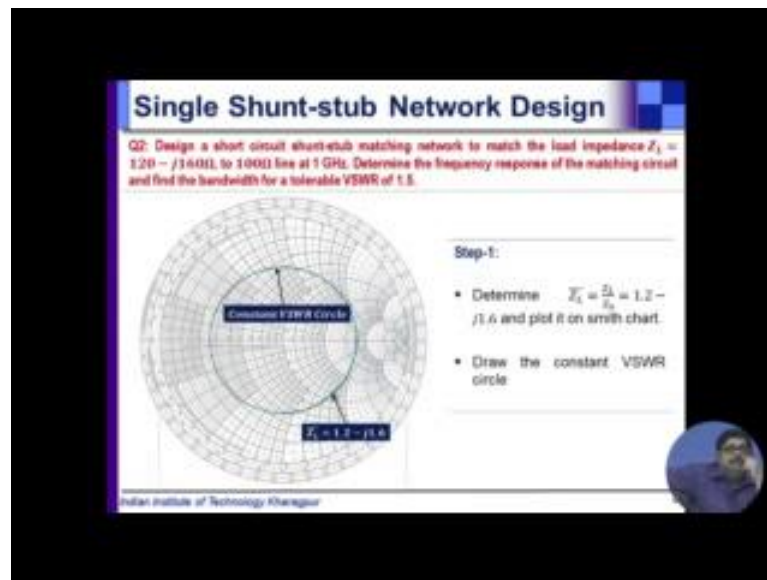
Handwritten notes on a blue surface showing the VSWR formula and a calculation. The formula is $VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$. Below the formula, an arrow points from the value 1.5 to the VSWR term. To the right, an arrow points from the value 0.2 to the $|\Gamma_L|$ term in the formula.

$$VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

↑ 1.5 ↑ $|\Gamma_L| = 0.2$

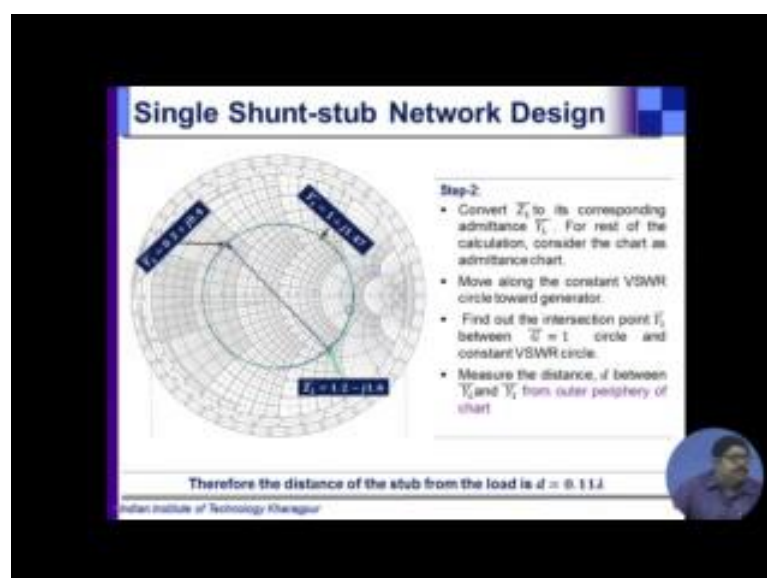
So, if this is 1.5 you find out what is Γ_L it may be something like how much it will come, 0.1 more than 0.1 it will come. So, let us say 0.2 let us arbitrarily take that this is 0.2. So, you take this 0.2 and then you see that which one obviously, more or less for 0.2 both of them is having the same match. So, if you ask me in that sense it is same, but still I will prefer this, what is that black one why because you see the red one that is very sharp. Now near the design frequency of 500 mega hertz it is giving a very sharp whereas, this is not giving near design frequency it is not giving match very good match but it is less sharp. So, I will choose that, but if you say that 0.1 then I will definitely go for red one not this one. So, depending on your application you need to choose.

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Now, that was L-Section designs. So, like that if you are in other regions you can always find that. Now this one you see design a short circuit shunt stub matching network to match the load impedance, a complex load 200 ohm line; that means, characteristic ohm 100 ohm, 1 Giga hertz and here it is given that find also that VSWR of 1.5, this from this I could have said tolerable VSWR 1.5. So, let us see that again determine single step matching if you remember single step matching that Z_L bar you first plot then draw the constant VSWR circle as we have done that this is the Z_L bar. So, constant VSWR circle.

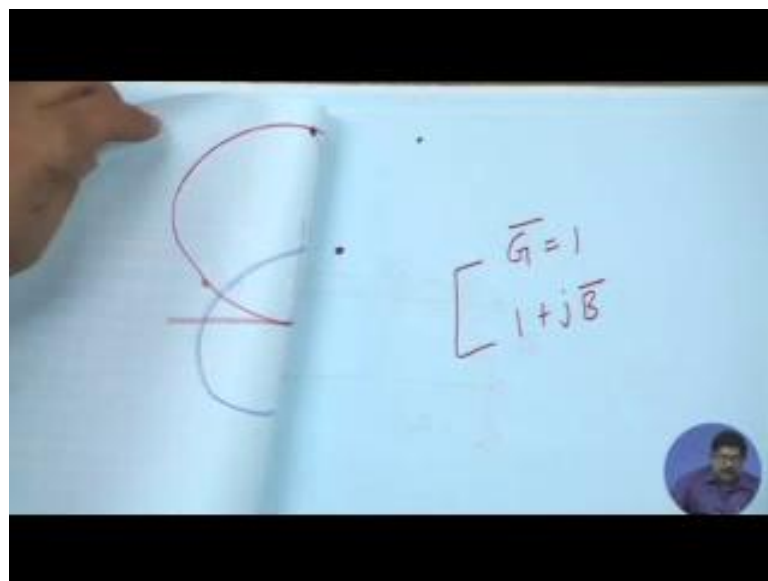
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Then there will be a movement. So, since it is shunt design the specification is shunt stub. So, Z_L we need to convert to Y_L because admittance plane it will be easier. So, we are converting Y_L and consider this as admittance chart. If you use $Y Z$ charts then you need not consider anything you know that what is the conversion. So, I prefer $Y Z$ chart, but if that is not available. So, the solution is given in terms of impedance chart, but if you have $Y Z$ chart it is always you need not on your mental frame change it to admittance or it is there as a different colour one. So, we have come to this Y_1 .

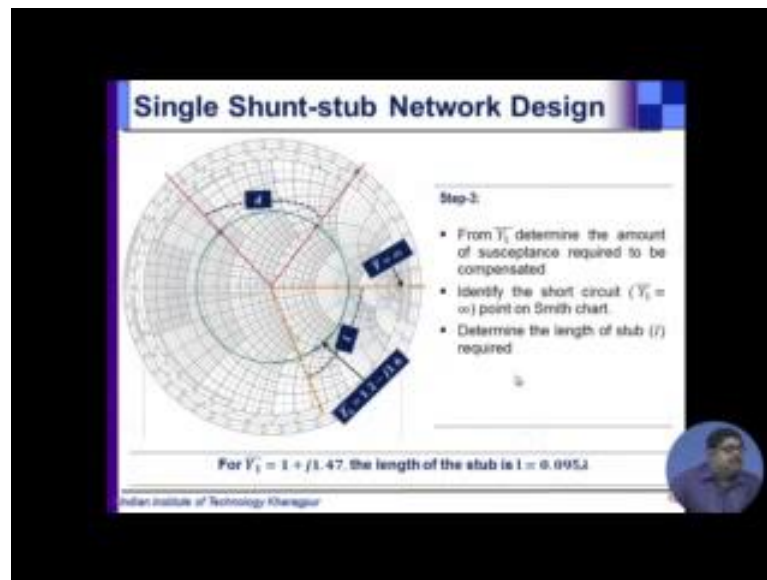
Now, you will have to move along the transmission line; that means, move along the constant VSWR circle. So, this is the constant VSWR circle and moving towards generator means this clock wise direction towards generator now find out where this g is equal to one circle or $1 + jB$ circle both we call, either we call it. Sometimes we call it g is equal to one circle, sometimes we call $1 + jB$ circle.

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These have same meaning. So we need to now find out that where they are cutting. So, from this Y_L we will have to move along the generator and you see at this point it is cutting now, also I can say no I will continue I will not stop here and go here, here also it is cutting this is the $1 + jB$ circle, here also, here also. So, these are the 2 solutions. So, you can take any of that find out the intersection point, measure the distance d between Y_1 from outer periphery of chart. So, here in this solution that d is coming to be 0.11λ .

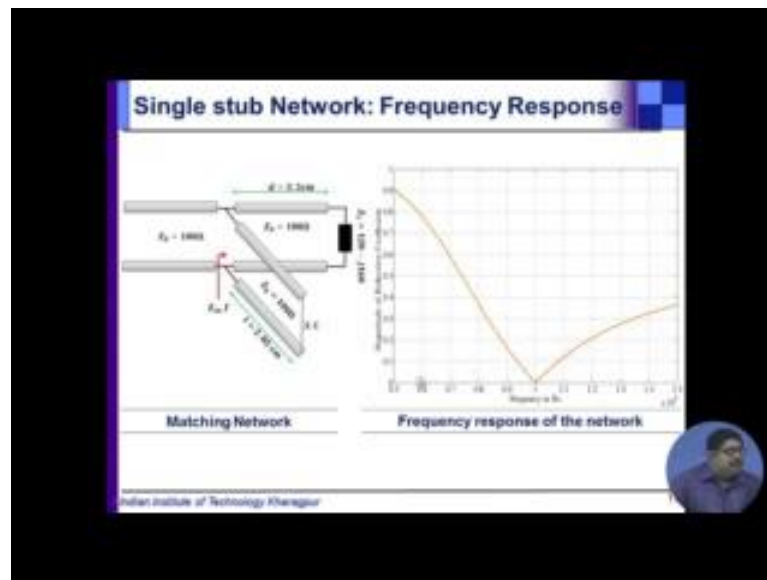
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Then determine the amount of susceptance so you read this value and find out, suppose here Y_1 is equal to $1 + j1.47$; that means, now I will have to have the shunt stub and I want that stub should then balance this class $j1.47$; that means, shunt susceptance will be minus $j1.47$.

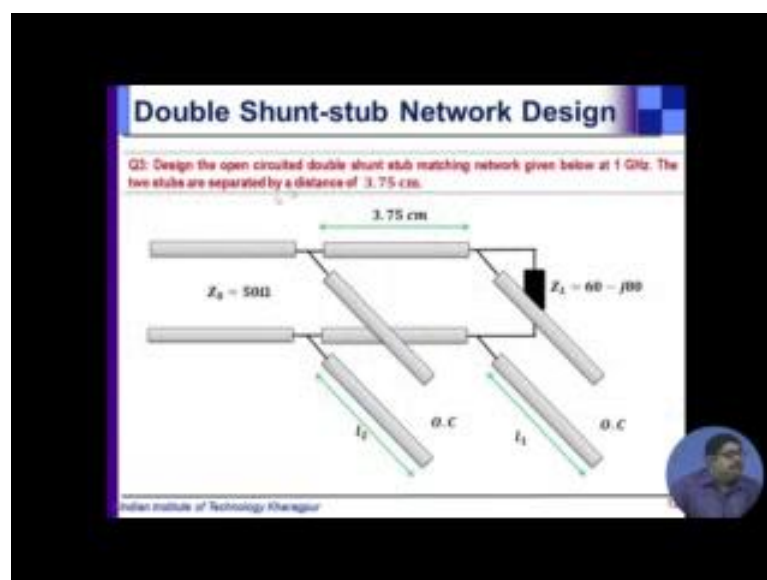
Now if it is a shorted stub short circuit stub. So, this is the in the admittance chart this is the short circuit, in the impedance chart this is the short circuit. Now this is admittance chart short circuit, from here we will have to go towards the generator, I will have to see that where is minus $j1.47$ it will be from here I will have to come somewhere and probably here that minus 1.47 and that will be the length of the stub. So, we have determined this is 0.095λ . So, based on your λ value this problem I think get how much 1 Giga hertz. So, 1 Giga hertz mean 30 centimeter λ , you can find out L is 0.095 into 30 centimeter, so your design is complete.

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Now, you find out that they said that write these expressions and find the frequency response it is like this you see that at 1 Giga hertz it is good match, but then it is changing. So, we will have to find out at which points that is you do find the bandwidth response, find the bandwidth for a tolerable VSWR of 1.5. So, 1.5 means you will have to see how much is the 1.5 there you will have to make that.

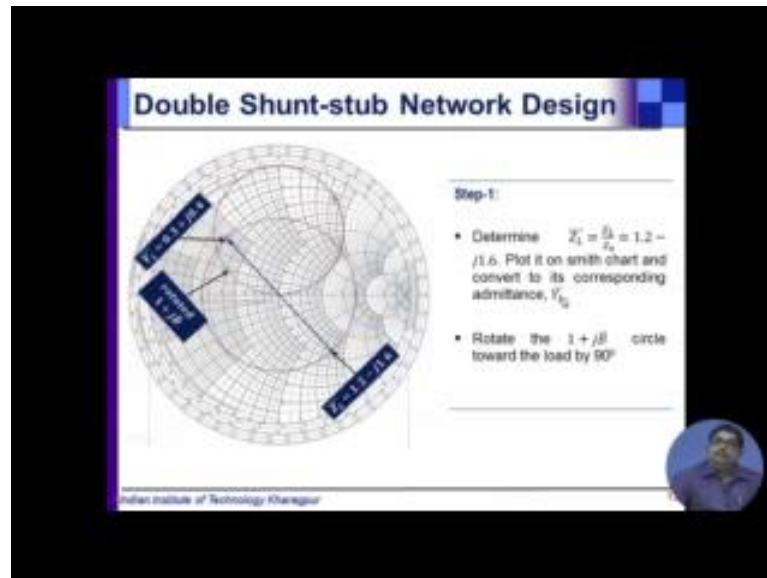
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Similarly, double shunt stub matching network, you see this single stub that has a frequency response and you can plot it like this.

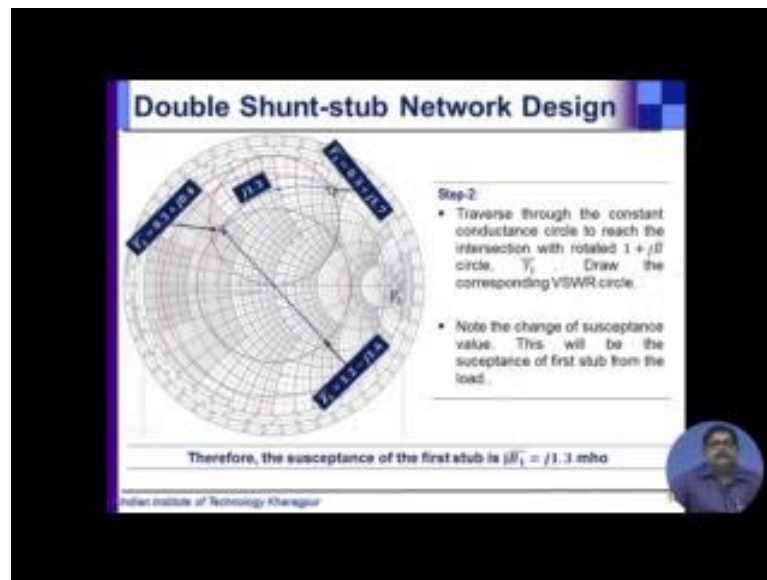
Now, let us come to double shunt. So, open circuited double shunt stub matching network, we have specified that this thing as we have seen in our laboratory photo that there is a fixed distance. So, let us say that fixed distance is 3.75 centimeter, Z_L is $60 - j80$ ohm - this is 50 ohm. So, we have 2 open circuited stub connected in shunt, now design this L_1 and L_2 , find this L_1 , L_2 value.

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So, again determine first the load impedance convert it to the admittance. Now since 3.75 at 1 Giga hertz; that means, 30 centimeter. So, this is $\lambda/8$, $\lambda/8$ separation means 90 degree rotation that is why you rotate the $1 + jB$ circle, $1 + jB$ circle is this, rotate it by 90 degree.

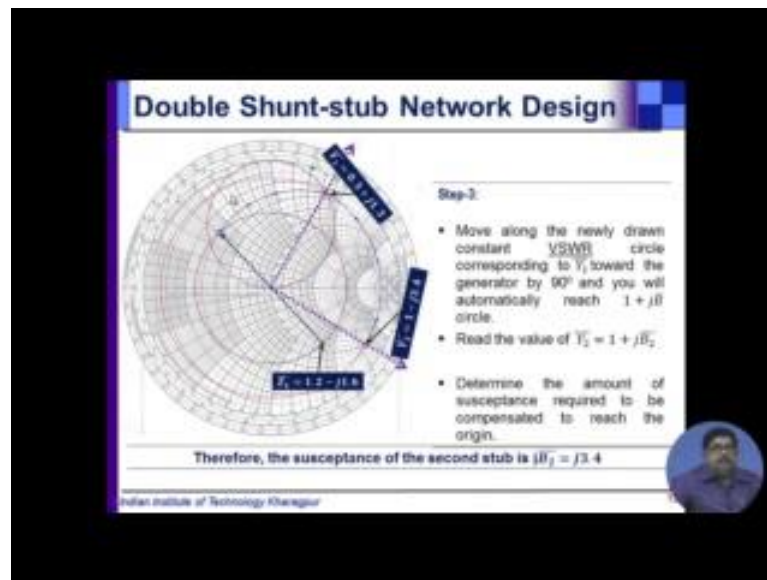
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Then, traverse to the constant conductance is the load Y_L , now traversed to the constant conductance circle to reach this circle. So, that is this part and you are reaching here. So, this is one solution you can continue and you can cut this in another point also here; that means, if you continue it here come, here another solution will be here.

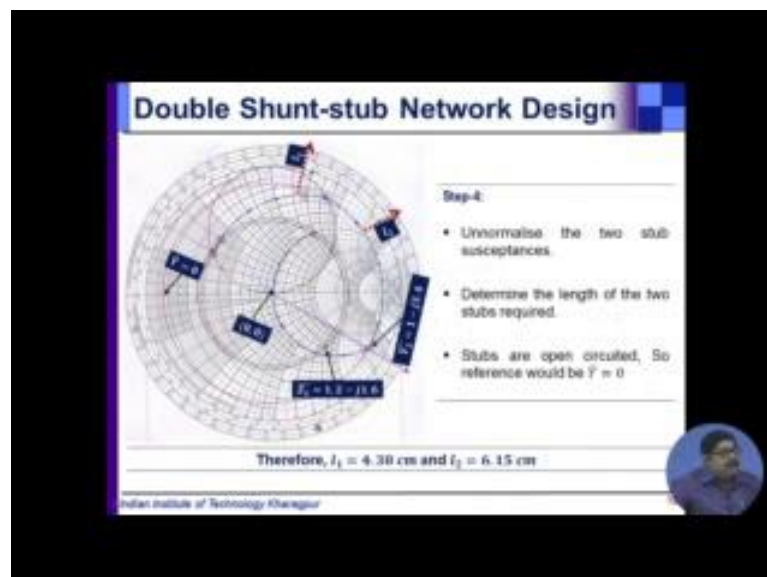
So, let us take this one then read this value. So, you are getting this Y 1 thing and then this change of susceptance is given by the first half because from coming here to here you have changed the susceptance that was given by the first half. So, from this you get the first half value.

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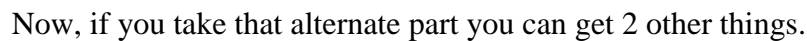
Then again from here it is a move along the newly. So, now, draw another VSWR circle because now you will have to move along the transmission line.

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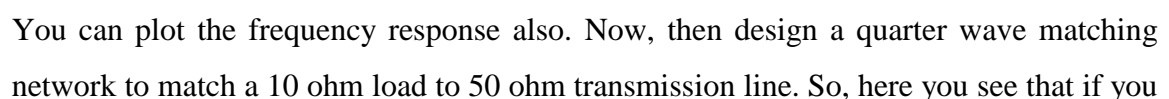


For that this new VSWR circle you move and move there then, come here are normalize the 2 stub impedances. So, corresponding to Y_1 and you will automatically reach $1 + jB$ circle that is why you have rotated earlier. So, that you can now come here, read the value of y_2 and that this $j\beta_2$ that needs to be compensated to reach the origin of the scale that value we have given here and then unnormalise the 2 stub impedances

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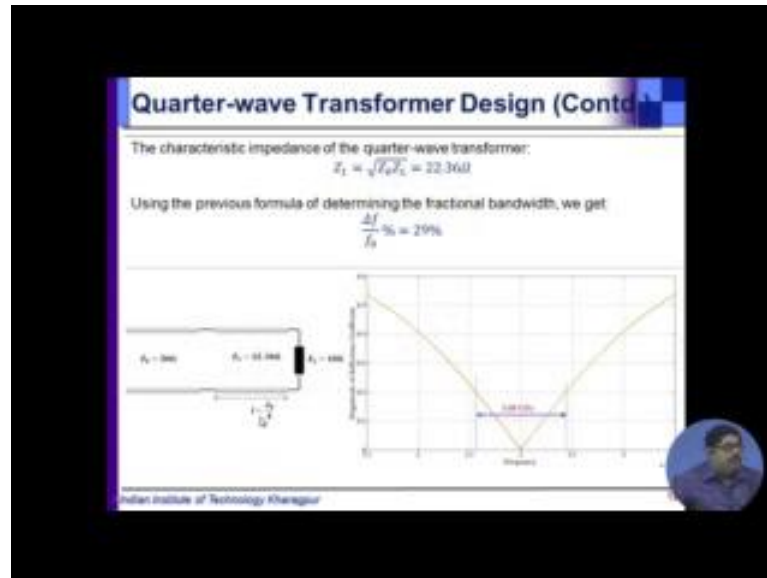


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have a VSWR specified as 1.5. So, maximum value of the gamma that will be 0.2, 0.2, you can put in that formula this gamma max value z_0 and Z_L value are given z_0 is z_0 is 50 or 500 ohm and 500 ohm and this is 10 ohm. So, you can get what is this then do it.

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And the characteristic impedance of the transformer is GM I said geometric mean, 22.360 ohm, this thing is 22.360 it is L is $\lambda/4$, $\lambda/4$ what is λ ? λ is 3 Giga hertz, 3 Giga hertz means 10 centimeter. So, it will be 2.5 centimeter length and its percentage bandwidth if you see from here that it is giving you a bandwidth is 880 mega hertz you see by IEEE definition 500 mega hertz more than anything is called ultra wide band. So, this is giving you that, percentage wise it is 29 percent. So, a quarter wave transformer though you are calling it not a very high band because it is a single transformer, single section, but still it is giving you 29 percent then you see we could achieve 880 mega hertz bandwidth, huge bandwidth.

There are very few signals who have this type of bandwidth.

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Quarter-wave Transformer Design

Q5: Design a three section matching network to match a 100Ω load to 50Ω line at 2 GHz. The network is desired to show a maximally flat type frequency response around 2 GHz. Determine the values of characteristic impedances required for each section of line and also determine the fractional bandwidth if the maximum allowable VSWR is 1.22. Also plot the frequency response of the matching network.

Solution: Given $N=3$, $Z_L = 100\Omega$ and $Z_0 = 50\Omega$

Recalling the generic equation of maximally-flat type frequency response:


$$T(\theta) = A(1 + e^{-j2\theta})^N \text{ with the value of } T(0) = A2^N$$

When, $\theta = 0$, then $T(0) = \frac{Z_L - Z_0}{Z_L + Z_0} = A2^N$. Therefore $A=0.0456$

Again we know the recursive relation $\ln(Z_{n+1}) = \ln(Z_n) + 2T_n$ and

$$2T_n = 2^{-N} N_{C,n} \ln\left(\frac{Z_L}{Z_0}\right)$$

Therefore, value of the impedances are:
 $Z_1 = 64.88\Omega$, $Z_2 = 84.69\Omega$, $Z_3 = 92.27\Omega$



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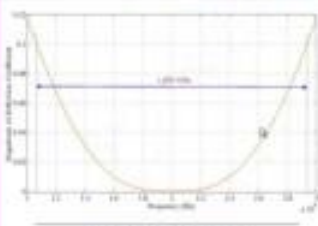
Now, extend that to a 3 section matching network to have a maximally flat response. So, it is said you take 3 sections, you take 3; Z_L is 100, Z_0 is 50 you see if you directly ask them to match there will be huge mismatch, but by this we have those formulas. So, gamma naught is equal to a 2 to the power n from that you can find a's values, again that recursive relation you can use and one by one you can find out these Z_1 , Z_2 and Z_3 values. So, you see that from 100 ohm you are first having 92.27, 84, 64 then that is matching to 50 ohm.

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Quarter-wave Transformer Design

It is mentioned that the maximum allowable VSWR is 1.22 i.e. $\Gamma_{max} = 0.1$


Therefore, $\frac{Z_L}{Z_0} = 2 - \frac{1}{0.1} \cos^{-1} \left[\frac{1}{2} \left(\frac{0.1}{0.0456} \right) \right] = 93.1\%$



Frequency Response



Matching Network



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Let us see, for VSWR of 1.22 which is a good VSWR value, γ_{\max} is 0.1 you see the bandwidth it is 1.869 Giga hertz you are getting the bandwidth percentage wise. You see 3 section that could give you 93.1 you see the response also it is maximally flat. So, this is the drawing, this is the 50 ohm section, this is the 100 ohm load by you are progressively making it by progressive making you get a good matching network such a good bandwidth percentage wise see it is a very wide band design. So, now, you understood how to do a wide band design, wide band impedance matching.

With that we conclude this section. So, now you have learnt how to do impedance matching both for a narrow band design, but if you require a wide band design that idea also you have learnt. So, you will be able to do that in your professional career.