Pattern Recognition and Image Understanding

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Lecture - 37
Temporal Pattern Recognition

Hello, so till now what we have discussed is the different classification and recognition techniques of patterns, as well as different clustering techniques of patterns. So, the patterns that we have discussed so far, they are actually static patterns, that is the patterns, which do not have any team temporality in it. So, if I have a set of feature vectors which are static over time, then I can either classify those feature vectors or I can cluster those feature vectors. Today, what we are going to discuss is the classification of Temporal Patterns or patterns, which actually unfolds in time. So, what do you mean by the patterns, which have temporality or the patterns, which unfolds in time.

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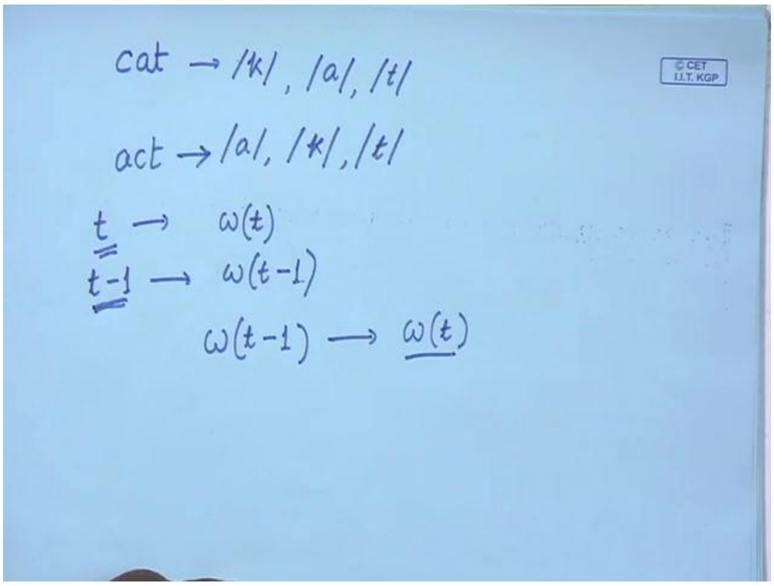

Let us take for example as speech signal in, suppose I want to recognise a word or spoken word say cat. So, if I simply write cat c a t, so if I want to recognise this word cat as it is spoken, you find that this word cat it can be broken into three different phenomenon. So, the first phoneme say ka second phonemena is and the third phonemena is ta. So, I can represent this as the first phoneme which is ka, then a phoneme and then a phoneme ta. So, if I utter this phonemes as given in this sequence ka, ta I pronoun the word cat or as if I simply altered the temple sequence of these three defined phonemes on the suppose I first, I utter aa then I utters the phoneme ka.

Then I utters ta its simply becomes act, so the word that I pronounce in this case is act. So, the phony though the phonemes are same, but they are uttered in a different sequence in time. So, as the same phonemes are utters in different sequence in time you find the word that is being pronounced, the word itself changes. So, this kind of patterned, which is a temporal patterned. So, when we talk about the problem of speech recognition, when we talk about the problem of speaker recognisor any signal, which varies with time say for example, some system identification task. There we get a signal or the signal varies with kind it is a temporal signal.

So, these are the examples of temporal patterns so the patterned which actually unfold in time. Similarly, coming to our vision context of vision applications we can have the different applications of say gesture recognition. So, we want to identify with the a person is running or the person is walking the person is scrolling. So, that these different kinds of act that is running, walking, scrolling. So, we want to classify or we want to recognise these different kind of tasks or possibly all of your ours of sign language.

In case of sign language you find that it is also the sequence of hand gestures the different hand positions, which actually convey the meaning of the information. This convey the information to the others, so that particular sign language this is also an example of temporal patterned. So, today what we are going to discuss is, how to recognise some temporal patterns or how to classify such temporal patterns? So, hope the kind of tools that we will use for such temporal patterned recognition is something, which is similar to sequential machine or finite state machine, which all of you might have done.

I hope all of you have done during your digital circuit course, so what is a sequential machine or finite state machine in case of sequential machine or finite state machine? We actually define a finite set of steps and a machine can be in any of the states that any given time instant. That same time we also have a set of out puts so if any state when it makes the transition from one state to other it outputs a symbol. So, we have a set of states and a set of output symbols, so we can say that act time instant t the machine can be at state set of mega t. The time instant the previous time instant the machine can be at state, so at time instant t minus 1 the machine can be at state of mega t minus 1.

So, in case of a sequential machine or a finite state machine the state of the machine, that time step t the time step t is directly influenced by the step of the machine, at time step t minus 1. So, there is a transition from state or mega t minus 1 to the state of mega t. So, the state of the machine at time step of mega t is influenced by what is the state of the machine, at the time step t minus 1. So, let us just try to briefly review what we had in case of a sequential machine or a finite state machine. So, in case of a sequential machine it has a finite set of state say S, which is actually set of states and O, which is the set of outputs.

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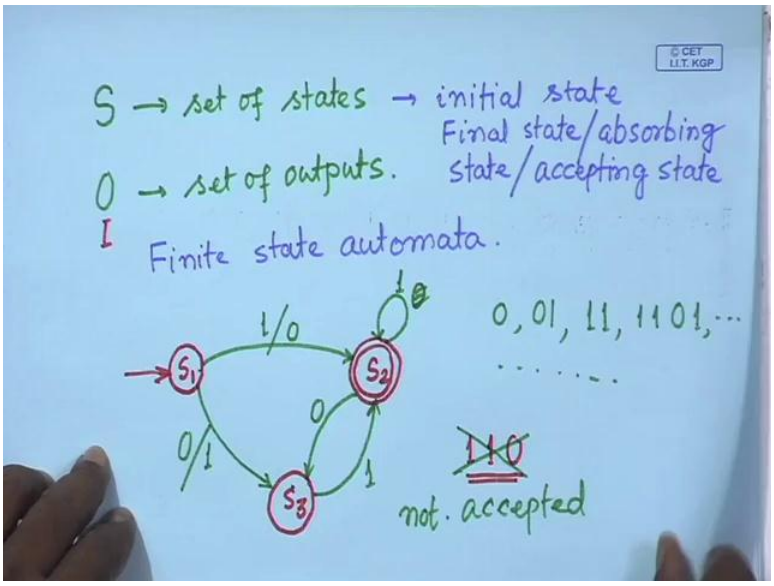

So, when a we have this set of finite set of states one of the state is called an initial state and one of the states is a final state all and absorbing state. So, we can have out of these sets one state is an initial state and one state is final state or it may also be called absorbing state or accepting state.

The set of outputs is actually the set of symbols, which the machine outputs or visible symbols, which can be measured or it can be observed, when the machine makes the transition from one step one state to another. If you remember, if these set of outputs symbols is limited to only two symbols that is either 0 or 1. Then this finite state machine is called finite state automata. So, a finite state of automata is nothing but a sequential machine or a finite state machine, which outputs only one of the two symbols one of them is 0 and the other one is 1. So, let us just take one such finite state machine, suppose I have a finite state machine, which have got three states one is S 1, one state is S 2 and the other state is S 3.

Suppose, out of these three is two is my final state or observing state and S 1 is the initial state. Now, along with the set of states is and the set of output O there is another set, which is I that is the set of inputs. So, the machine makes the transition from one state to another state with some input, which is actually the trigger. So, in this sequential machine having three different states I have assume the state S 1 is my initial state, S 2 is my final state and S 3 is any other state. So, the machine can make a transition from state one to is two and give him output. Let us say 1, 0 it can make a transition to S 1 to S 3 and give him output, which is say one it can make a transition from S 3 to S 2.

It can give an output 1 it can also make a transition from S 2 to S 3. For this transition it can output symbol, which is 0 it is also possible that a machine will have a safe transition that there will be self lope. In this case I can have something like this like S 2, I can have transition to S 2 itself and for that it will output a symbol let us say 1. So, given such a sequential machine and the outputs, which are emitted by this machine whenever it makes a transition from one state to another state. Over here, as I have assumed that S 1 is my initial state and S 2 is final state.

So, you find the kind of the outputs, which are actually accepted by this machine or the kind of or the sequence of outputs, which are generated by this machine. That can be 0, 1 it can be 1, 1, 1 it can be 0 then 1, then 0, then 1, then 1 and all this sort of things. So, the kind of sequences, which is accepted by this machine, will be say 0. As with output 0 the machine is making a transition from S 1 to S 2 and S 2 is the accepting state it can be 0 followed by 1. Because, with 0 we are making a transition from S 1 to S 2, then S 2 it can make transition with in S 2 to S 2 with an output equal to 1.

So, that is still in accepting state, so I can also accept a stream 0, 1 I can accept a stream one when the machine makes a transition from S 2 to S 3, then it makes a transition from S 3 to S 2 with an output 1. So, 1, 1 there is a stream, which will be generated by this sequential machine. It will also be accepted by the sequential machine I can also have 1 1, 0 1, this is also an accepted sequence. So, I can have a number of sequences of such output symbols, which are accepted by this machine, but you find that if I have a symbol say 1 1 0. So, from the initial state the machine comes to state S 3 with an output one from S 3 it goes to S 2 with output one from S 2 it comes to S 3 with output zero, but S 3 is not an accepting state.

So, this symbol or this sequence of symbol does not terminate the machine in an accepting state. So, a result this symbol will not be accepted by this machine or this sequence of symbols will not be accepted by this sequential machine. So, because for acceptance of a sequence of symbols, by a sequential machine or a finite state machine with the sequence the sequence must terminate in 1, in un accepting state. So, this sequence 1 1 0, this is not accepted by this sequential machine, this is not accepted, so this is what I have a sequential machine. In case of a sequential machine the probability of transition from one state to another it depends upon an input.

Say for example, here if I given input say 1 it moves to the state S 2 with an output one, but if my input i zero it moves to state S 3 with an output one. So, with one as input the machine moves from state one to S 2 with an output 0. If the input is 0 in state S 1 the machine moves from S 2 to S 3 with an output one. So, this is what a sequential machine or a finite state of automata, if the set of output symbols is limited to 0 and 1. So, a tool which is similar to this sequential machine or very similar to this finite state machine, which is used for temporal pattern recognition, is called a Hidden Markov model.

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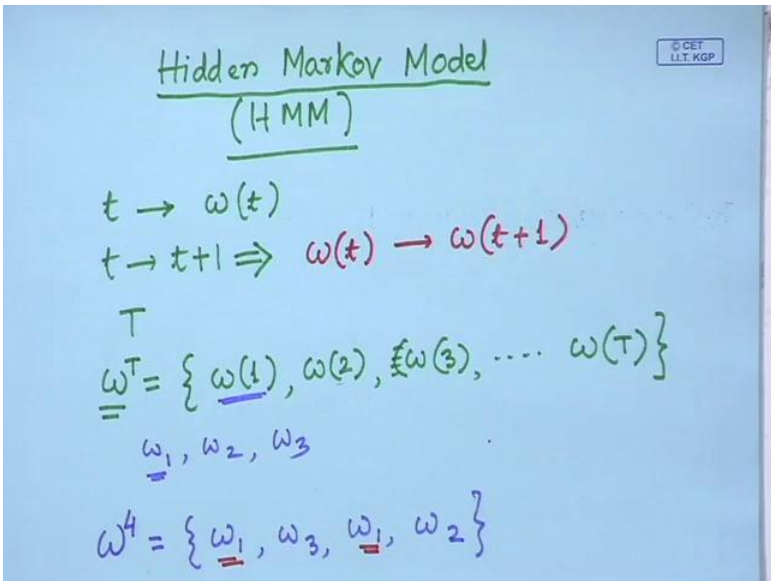

So, what we will discuss today is, what is called a hidden Markov model? or in short it is written as H M M. So, as we said that this hidden Markov model, which is used for temporal pattern recognition is very similar to the sequential machine or the finite state machine. That is this hidden Markov model will have a set of states and the hidden Markov model, the machine can be in any of the states at the time instant t. So, suppose this at time instant t the machine is in state say omega T. What I can have is as the machine moves a transition from one state to another that is from time instant t to time instant t plus 1.

So, from t to t plus 1the machine makes a transition or H M M makes a transition from state omega T to state omega T plus 1. So at time step t H M M was in state omega T at time step t plus 1at the next time step the machine was is in state omega T plus 1. So, that is a transition from omega T to omega t plus 1. Because, the machine makes transitions from one state to another from in different times steps. So, the machine generates a sequence of states and a sequence of states are sequence of length, say capital T is usually represented as omega capital T, which is nothing but a sequence of states omega 1, omega 2, omega 3 continues like this up to omega T.

So, it simply says that at equal to one the machine was in state to omega 1, so this omega 1 may be any of the permissible states of the machine. Similarly, at times step two the machine makes a transition from omega 1 to omega 2. So, this omega 2 again may be any of the permissible states of the hidden Markov model. So, this way over total duration of capital T the machine moves from one state to another. The sequence of steps or the sequence of states through, which the machine moves this sequence is represented by this omega T. So, it is a sequence of states of capital T number of states through, which the machine moves and this sequence is nothing but omega 1, omega 2, omega 3 up to omega T.

So, this is the state in which the machine was at time step one this is the state in which the machine was at time step two and so on. So, this is I have total capital T number of states, so for example if I have a machine with say states omega 1, omega 2, omega 3, so these are the states of the machine. Now, you find the difference between this omega 1 where 1 is put with in pan thesis and this to omega 1 this is a particular state of the machine omega 1 is one of the states. As the machine is having three states omega 1 omega 2 and omega 3 where as I when I write in this form it is one of these states in which the machine was at time step one. So, please note the difference between these two notations.

Suppose, I can have a sequence of states of links four this may be say omega 1, omega 3 omega 1, omega 2, so this may be my sequence of four states. So, a machine moves from one state to another in this particular sequence. So, at t equal to 1 or omega 1 was these omegas 1 at t equal to 2 the omega 2 the state of the machine might have been omega 3 at t equal to 3. In the third step the machine might have been in state of omega 1, in the four step the machine is in state omega 2. So, the four states through, which the hidden Markov model makes it a machine, so as I have four different states. So, this is the sequence of length four, which is usually represented as omega 4

Here, again you find that it is not necessary that empty state a single state has to appear only once in a sequence. A state can appear more than once in different orders. So, here you find that omega 1 and omega 1, these two states appear twice. So, it means that from omega 1 the machine may has made a transition to omega 3 from omega 3, again it has made a transition to omega 1, from omega again it has made a transition to omega 2. Now, this production of state sequence in case of a hidden Markov model is described by a transition probability. So, the transition probability is something like this the probability P that the machine makes a transition two states, omega t plus 1to state omega j at time step t plus 1 given the machine was in state omega i at time state t.

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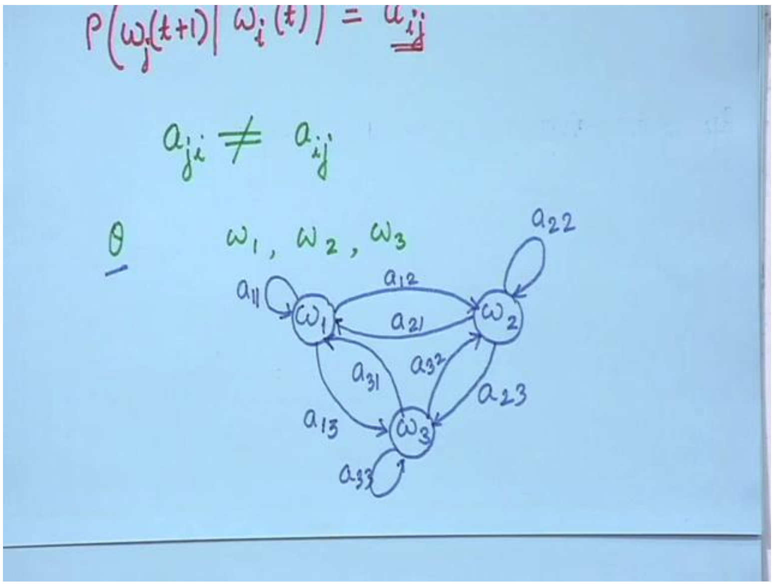

So, what is the probability that the hidden Markov model will make a transition from omega i, which is the i state to state omega j? If the machine was in state omega i at time step t at the machine moves to state omega j at time step t plus 1,that is at the next time state. So, the probability of this transition that the machine makes a transition from state omega i at time state t to state omega j at time state t plus 1, this is what is given by the transition probability a i j, compare with this or sequential machine. In case of sequential machine, we said that depending upon input the machine moves makes a transition from one step to another. So, if my input is 1 the transition probability from S 1 to S 2 is equal to 1 where as if my input is 0 the transition probability from S 1 to S 3 is 1.

So, this probability it will make a transition from S 1 to S 3, but if my input is 0 the machine cannot make a transition from S 1 to S 2. Similarly, if the input is one this sequential machine cannot make a transition from S 1 to S 3. So, that is the kind of restriction we have in case of a sequential machine. Where as in case of hidden Markov model from state omega 2 the machine make a transition to state omega 1, it may make a transition to state omega 3, it may make a transition to omega 5.

So, all these transition are possible and there is a transition probability that it makes a transition omega 1 to omega 3. There is a probability that it makes a transition from omega 1 to omega 5, there is a probability that it makes a transition from omega 1 to omega 10 and all this. So, from any state at any instant of time it can make a transition to any of the other states or to the state itself with a finite probability of transition. So, that is what we are saying over here that, if the machine was in state omega i at time instant t. Then the probability that it will make a transition to state omega j at time instant t plus 1is given by a i j. So, this is the probability of transition from state omega i to omega j it is also possible, that the machine will make a transition from omega j to omega i.

So, at the probability of transition is actually given by a j i this is the probability transition from omega j to state omega i. The probability transition from state i to j is given by a i j. In general, this a i and j and a j i there are not equal, that is the transition probability from state omega i to omega j may not be same as the transition probability, from omega j to state omega i. So, these defined transition probabilities will actually define the different hidden Markov models. So, when we design a hidden Markov model or when we go for training of the hidden Markov model. The major task of training of the hidden Markov model learning these transition probabilities a i j, which we will see later on.

Now, suppose you are given a particular hidden Markov model, which can recognise a particular temporal pattern. So, we use a represent such a model by a symbolic theta. So, this theta represents a specific hidden Markov model and we have. So, for example a machine this particular hidden Markov model theta has, say three different state one of the state omega 1, one of the state is omega 2 and the other state is omega 3.

Now, when we have such a kind of hidden Markov model having a number of different states. One of the state is an absorbing state or an accepting state the concept is, once the model moves to that particular state the model cannot come out of that state either from any of the other states. The model can move a make a transition to that absorbing state that once in absorbing state, the model cannot come out of it.

We will come to the utility of such a specific state, which is an absorbing state or an accepting state or a final state later on. So, suppose this machine ahs this three different states given by omega 1, omega 2, omega 3. Let us represent this as say I have state omega 1 I have state of omega 2 and I have state of omega 3. So, this machine can make a transition from omega 1 to omega 2 with a probability of transition, which is given by a 1, 2 it can make a transition from omega 2, omega 3, sorry omega 2 to omega 1 with a probability of transition given by a 2 ,1.

It can make a transition from omega 1 to omega 3 here I will have a probability of which is 1, 3. a 1, 3 it can make a transition from omega 3 to omega 1 with probability of transition a 3, 1. It can make a transition from omega 3 to omega 2 with probability of transition a 3 2 a 2 3. It is also possible that it will have a transition to the same state. So, over here omega 2 to omega 2, here the probability transition will be a 2 2. Similarly, here the transition from omega 1 to omega 1 I can have two different states consequently I am in same state appearing success simply I can also have similar such situation.

So, this transition from omega 1 to omega 1, which will be represented by a 1 1. Similarly, transition from omega 3 to omega 3 itself, which is over here, which will be written as a 3 3. So, suppose I have hidden Markov model specified by theta, this hidden Markov model let us say as four different states omega 1, omega 2, omega 3, and omega 4.

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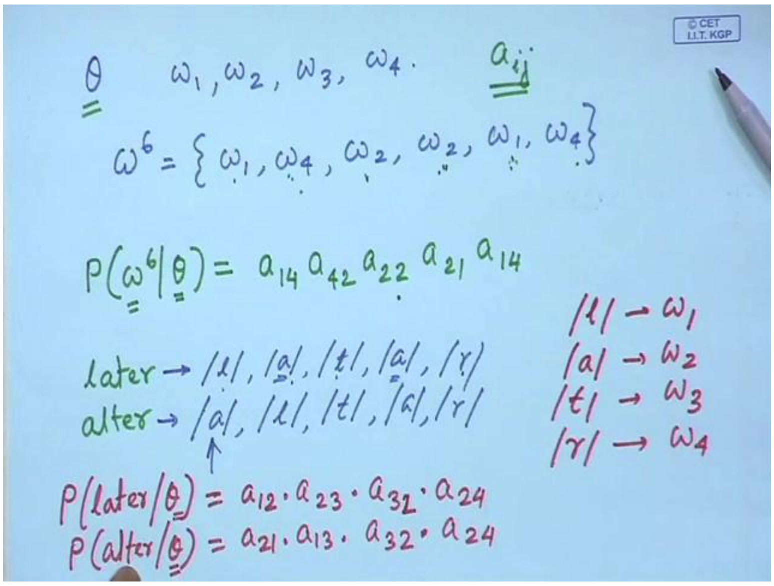

So, I have four different stated, so I have a model theta that is the hidden Markov model having four states omega 1, omega 2, omega 3 and omega 4. Suppose, we have been given a sequence of states, so sequence of six states omega c, where this sequence is say omega 1, omega 4, omega 2. I can have omega 2 again I have again omega 1 omega 4. So, this is the sequence of state to reach the machine makes a transition, now given this I can find out the probability that machine, theta has made this sequence of transition. How I can do it? So, what I have to find out is I have to find out what is the probability P omega 6 to give him theta.

So, when I have this hidden Markov model theta, that means I have the sequence of states. I also have the complete table or probability of state transition table, that is I have all the a i j is for all values of i and j. That is what specifies a hidden Markov model. So, suppose this theta is specified by these are the states and i also has a i j, which is to be learnt, but once the model is specified, i have the complete set of a i j for all values of i and j. So, given such a machine theta and given this sequence of states, that is machine makes a transition from omega 1 to omega 4, omega 4 to omega 2, omega 2 to omega 2 again omega 2 to omega 1, and then omega 1 to omega 4.

So, what is the probability that this model of theta has made, such a sequence has transited through such a sequence of states. So, effectively what I want to find out is what is P of this omega 6 give him theta. You find that this P of omega 6 of even theta is nothing but the probability of transition from omega 1 to omega 4, which is nothing but a 1 4, then probability of transition from omega 4 to omega 2, so this multiplied by a 4 2, which is the probability of transition from state omega 4 to omega 2. Then the probability of transition from omega 2 to omega 2 itself. So, I have a 2 2, which is the probability of transition from state omega 2 to omega 2. Then from omega 2 to omega 1, which is nothing but a 2 1, then the probability of transition from omega 1, omega 4 which is a 1 2.

So, the probability that this model theta has made a transition through a sequence of states is nothing but the probability of transitions, between successive states. So, it is the product of the probability of transition between successive states. So, the machine this theta will have these transitions the probability of that. That is P omega 6 given, theta is nothing but Pomega a omega 1 4, which is the probability of transitions from state omega 1 from state omega 4, multiplied by the probability of transitions from omega 4 to omega 2. That is a 4 2 multiplied by probability of transitions from omega 2 to omega 2 itself, which is a 2 2 multiplied by the probability of transitions from omega 1 2 to omega 1, which is a 2 1 multiplied by the probability of transitions from omega 1 to omega 4, which is a 1 4.

So, this is the probability that, this machine theta has made a transition through this sequence of six states as specified by the omega 6. So, that that is fine I mean I have a model the model has a number of states. The machine makes a transition through a sequence of states over time and what is the probability that the machine has generated? Or has transited through a given sequence of states? I can compute that probability, now what are this state’s actually.

Let us say that I have again in with an example to my speech recognition. I have two different words one is the later other one is alter. So, you find that they use the same phonemes one is later other one is alters. If I break them into phonemes this consists of a sequence of phonemes, la, ta, aa again and ra, where as in case of alters this is aa, this is ra, this is ta, this is aa, again this is ta.

So, you find that it is the same set of phonemes, but occurring in different order in time sequence. That is what makes the difference between later and alter. So, I can say that in my hidden Markov model every state corresponds to one such phoneme, every state will correspond to one such phoneme. So, as here I have one two three and four, four different phonemes, because this phoneme and this phoneme is same. So, I have four different phonemes, so i can have four different states to model this particular spoken word. Similarly, the same different states, but occurring in the different sequence will generate this spoken word alter.

So, this may be the different states and if I say the first phoneme will la corresponds to states omega 1, aa corresponds to state omega 2, ta corresponds to state omega 3 and ra corresponds to state omega 4. In that case the probability Plater that is the sequence given theta. That is my model it will simply be here you are making a transition from la to aa, so omega 1 to omega 2. So, it will be a 1 2 multiplied by a 2 3 multiplied by again ta to aa. So, 3 to 1 a 3 1 multiplied by aa to ra, right? Sorry, ta to aa. So, 3 to 2 multiplied by aa to ra, so 2 to 4. So, this is the probability that hidden Markov model will generate the spoken word later to it can recognise the spoken word later is this.

Similarly, the probability that it has generated alter give him theta this will be simply aa to la. So, omega 2 to omega 1 so a 2 1 multiplied by la to ta this was ta so a omega a 1 3 multiplied by ta to aa again ta to aa. So, a 3 2 multiplied by aa to ra, so that is a 2 4. So, you find the these two probabilities same a 3 2, a 3 2, a 2 4, a 2 4, but these two a 1 2 as we say that may not be same as a 2 1. Similarly, a 2 3 may not be is not usually not same as a 3 1. So, because of this the probability that this theta generates later and the theta generates alter they will be different for a given hidden Markov model theta.

So, if I actually train a hidden Markov model to recognise the word later the spoken word later. Then when I have the same word later, which is spoken with these different phonemes. Then the probability that the theta will return for the word later will be more, then the same theta which will return the probability for the spoken word later. So, once you train a hidden Markov model with a particular sequence, the model actually learnt the probabilities of transition from one states to another for another word. Another spoken word this probability of transition may not match though the states might be same, so that we will generate a lower probability of generation or lower probability of performance.

So, this is how given a model and given a sequence of states I can find out what is the probability that the machine has transited or the hidden Markov model has transited, through that given sequence of state. Now, as I said that in this hidden Markov model, there is a specific state, which is an accepting state or an absorbing state and this specific state is usually represented by the symbol omega 0.

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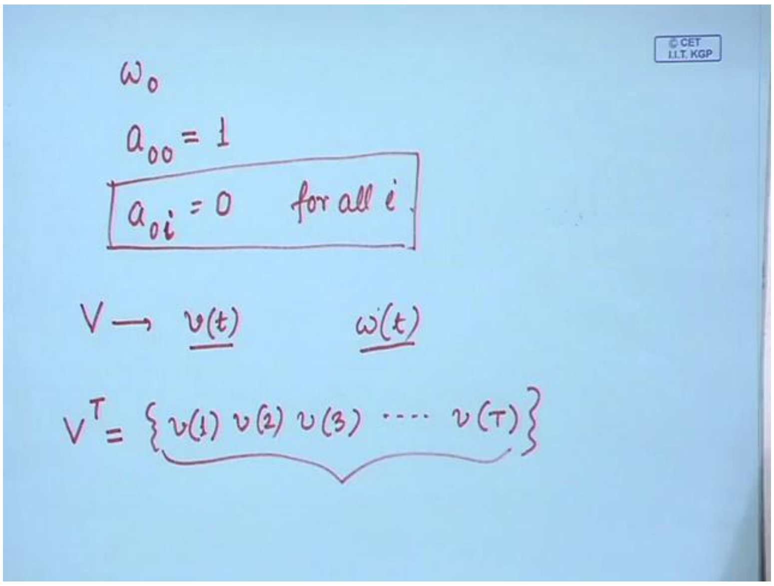

As I said that once this hidden Markov model enters the state omega 0 it cannot come out of that state. However, it can make infinite number of transition with in that state. So, as a result I will always have a 0 0 that is the probability of transition from state omega 0 to itself will always be equal to 1 whereas, because I cannot have a transition from omega 0 to any other state.

So, I will always have a 0 i is equal to 0, because from omega 0 i cannot make a transition to any other state omega i. So, a 0 i will always be 0 for all i, so this is what is very important. I can have, I will have a 0 0 equal to 1, because from the final state, it can make infinite number of transitions within the same state, but from the final, state it cannot make transition to any other state.

So, as a result a 0 i for all states i will be equal to 0 where as a 0 0 will equal to 1. Now, you find that we have, so far discussed this hidden Markov mode with respect to steps with respect to states. Now, this states are the once, which are not really visible or it cannot be absorb by the preserver what is visible or what is absorbable to the preserver is certain set of symbol, which can be absorbed or which can be measured. Right? Whereas, this states are not really visible I am in the same thing applies to our sequential this state B S 1 S 2 S 3. They are not really absorbable I cannot absorb them, but what I can absorb is the output, which is emitted by the machine. So, I can absorb this output 1 i can absorb this output 0.

So, this is the outputs which are really absorbable the state is not really absorbable. So, in the same manner, in case of this hidden Markov model the states that we have talked about. These states are not really absorbable, but what is absorbable? Or what can be measured based on which? We can take a decision whether a given word or spoken word or a sequence of actions belongs to a particular cars or belongs to a particular category. This classification has to be done based on absorbable symbols I cannot absorb the state through, which the machine will transit. So, along with the states I also have to have a set of absorbable symbols.

So this set of absorbable symbols is actually, this given by a set say v this is the set of absorbable. Accordingly, i can have a visible state, which is given by v t. So, for hidden Markov model I have two types of states one type set of states, which cannot be absorbed which are not visible or hidden those states are hidden, which are represented by omega T and a set of states, which have absorb it, which can be absorbed, which can be measured. That is the visible state, so at a time step t the machine can be state omega T, which is hidden state and it can emit and absorbable state, which is given by v t. As you have done in case of the states of the hidden states, that the machine can transit through a sequence of hidden states.

Given a model I can find out what is the probability that the model has transited through such sequence of hidden states, a similar manner for the visible states. I can have a sequence of visible states, so a sequence of visible states of length capital T, which is given by v t in the same manner. It is represented by v 1 that is visible state, which is emitted by the machine at time step t equal to 2, similarly v 2 similarly, v 3 and it continuous up to v t. So, this is the sequence of visible states or visible symbols, which are emitted by the machine at different steps in time.

This is actually such sequence of absorbable symbols they actually constitute your pattern or temporal pattern. I have to recognise such temporal pattern using hidden Markov models, so that is the task that we have. So, as we have two different sets of states one is the set of hidden states given by omega T. Another is the set of visible states or visible symbols given v t. We have said that, we have a transition probability a i j. That is the probability of transition from state omega i at time step t to state to omega j at time step t plus 1.

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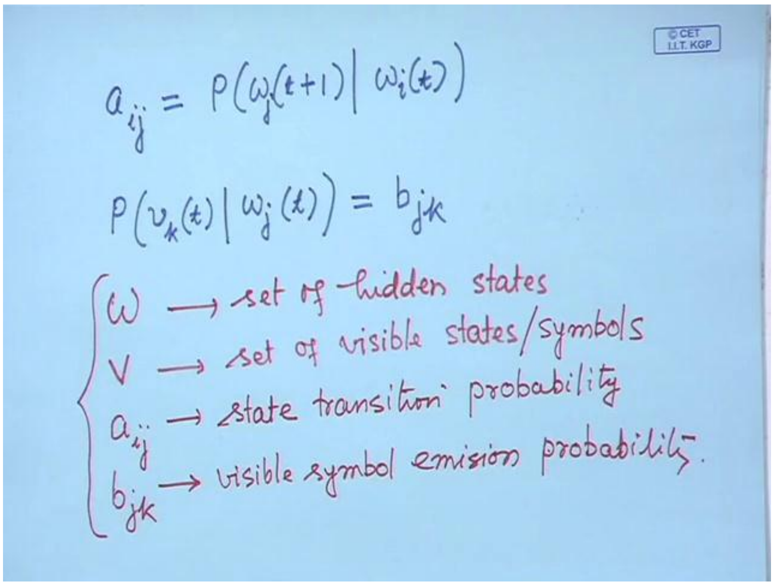

So, we have said that this a i j is equal to probability that the machine will be in state omega j at step t plus 1, given the machine was in state omega i at time step t. So, this was the probability of transition in the same manner there is also a finite probability. That in the heights state or in state omega i state omega j the machine will output ab visible or ab visible state, which is v k t.

So, I also have the probability P that the machine will output the visible symbol v k t. At time step t gives him the machine is in state omega j at the same time state. So, while in the hidden state omega j at time state at time step t, the machine out puts are visible symbol v k t. At the same time step t with a probability given by P v k t, given omega j t and this probability is usually represented by b j k.

So, this is the probability that the machine emits a visible symbol v k at visible state v k from state at state omega j. So, this is what is b j k? So, whenever I have a hidden Markov model I have to have, whenever I specify hidden Markov model I have to have three things. One is the set of omegas which of the hidden states a set of v. So, I have to have omega, which is nothing but set of hidden states. I have to have a set v, which is the set of visible states or visible symbols. I have to have a i j, which is the state transition probability.

I have to have b j k, which is this also I can term as state transition probability. Because, both v and t both omega and t we are coming them as cats, omega is the set of hidden state and v is the set of visible state. So, this b j k, I can also term as state transition probability. This state transition probability is from a hidden state to a visible state, but actually this is the probability that my hidden Markov model emits a visible symbol v k, when it is in the hidden state of omega j.

So, this is actually visible symbol emission probability. So, these are the four different items which must be specified for specification of a hidden Markov model. So, I will stop this discussion today here. In next class I will elaborate on this hidden Markov model. We will see that how the temporal pattern recognition can be achieved through hidden Markov models.

Thank you.