Probability and Random Variables

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Lecture - 38
Autoregressive Modeling and Linear Prediction

So, yesterday we are considering this autoregressive modeling this is small mistake I made which I want to correct actually. So, let me redo it again, because I do not remember exactly where it was.

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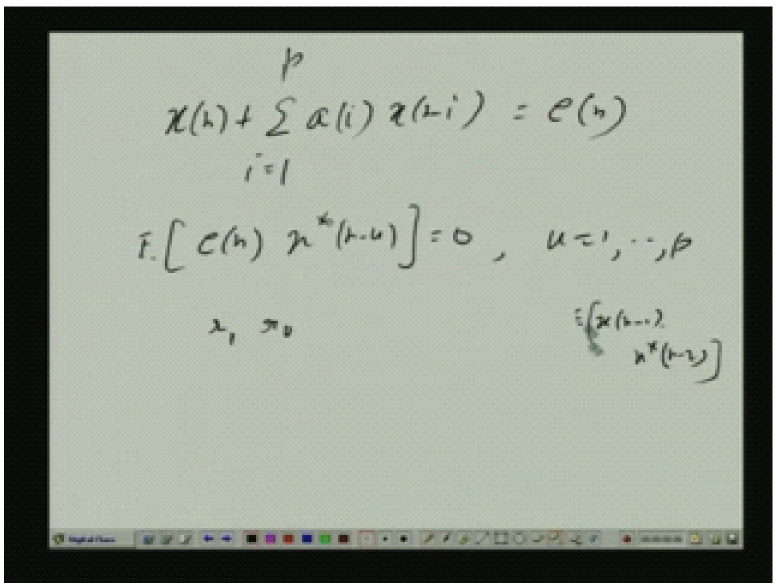

So, let us just redo it we had a model like this xn plus summation ai xn minus i, i equal to say 1 to p say p-th order model you say en. In the case of autoregressive modeling en is of a white sequence may be 0 mean white sequence. And it is p th order model it is all pole if you do the z transform find out the transform function it is only poles. Alternately you have also shown that even if, there is no such model you want to just project, you want to just predict xn linearly from its past p values xn minus 1 to xn minus p.

Then xn in the let the prediction coefficients be minus ai, then the error will be what is what you see here on the left hand side. And that error, then also you have got a similar equation. So, both linear prediction and AR modeling gives rise to same equation further. If it is linear prediction we know en is orthogonal there is uncorrelated with the past p samples. Because, what are the doing is we are projecting x of n orthogonally on the space spread by the past p samples xn minus 1 up to xn minus p.

The error, then because it is orthogonal projection the error will be orthogonal to the total plane or total space spend by xn minus 1 up to xn minus p. So, en is orthogonal to the past p values. Similarly, in the case of AR modeling we have seen the yesterday that input en which is white sequence, because of model is causal and all that. N is orthogonal to all the past samples of xn and therefore, the past p sample also.

Since, in the modeling problem where we will be estimating or finding out ai is unknown ai's from the given data we will be using only the orthogonality between en and the past p samples. It does not matter whether we are solving the linear prediction pole or AR modeling pole you get the same set of equations and same solutions. Now, there we said that how to find out these equations. We know this orthogonality E of this is what we did yesterday and then I made a small mistake. So, I am redoing this step.

Say x star n minus k that is equal to 0 for k equal to 1 2 p. These are orthogonality if you now substitute this left hand side for n here. And do the multiplication, then what you get out of this equation is you can verify easily for k equal to 1 say. We start with k equal to 1 then go to k 2 equal to k equal to 2 and then k equal to p progressively for k equal to 1 you get. Firstly xn into x star n minus 1.

So, that gives you r 1 I am putting this like in the subscript then r 0. Then what we have next term is xn minus 1 times x star n minus 2 expected value. This is equal to what this is equal to the correlation for like what n minus 1 minus within bracket n minus 2. I I think. So, what is the first 1 let me may be just the take the first 1.

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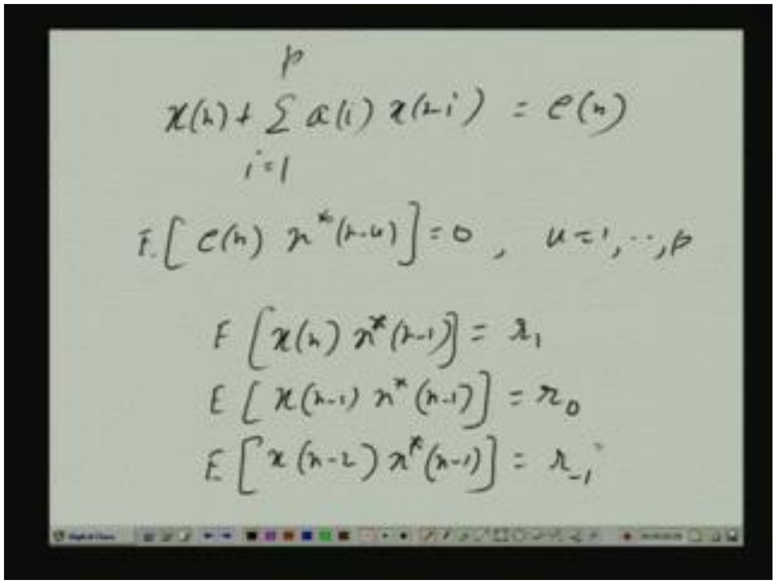

First 1 is if you put this expression for en here take the first term xn. So, xn x star n minus 1 if you take E of that you; obviously, get r 1. Next term is E of I am not taking the coefficients they will be handled separately. This this let me now elaborate a little. You know, because the skipping some steps is making mistake or may be just resulting in confusion.

Next term is n minus 2.

So, what is this, this index n minus 2 minus within bracket n minus 1? So, correlation at lag minus 1, so this is r minus 1 yesterday I put it put a star also along with it. Instead of writing as r minus 1 I made it r minus 1 star which was wrong. So, let me erase this part you now very well understand what I say.

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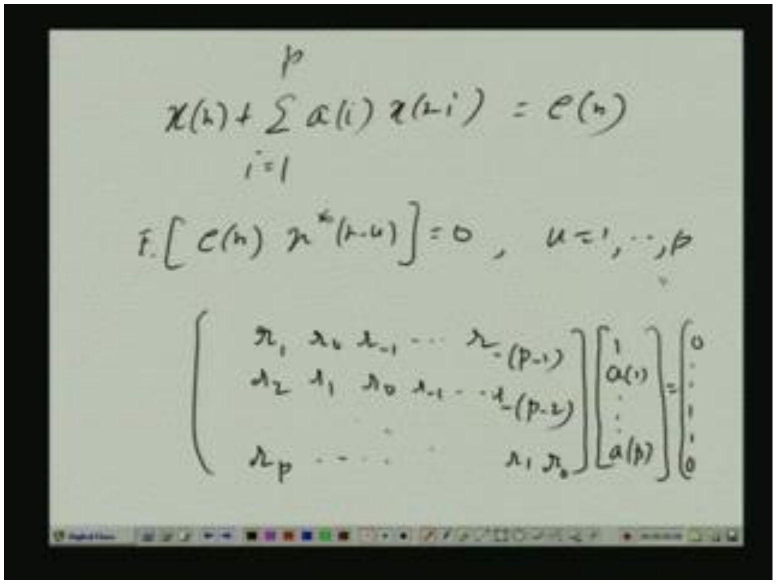

I am going to do r 1 r 0 r minus 1 dot dot dot last 1 will be what n minus p. Xn minus p x star n minus 1. So, r within bracket minus p minus 1 for the matrix 1 a 1 dot dot dot ap. This is equal to 0 dot dot dot dot 0. Again I say it is not ax equal to 0 kind of equation, because you have got a 1 here which is known quantity. So, 1 times r 1 that has to be taken out to the right hand side if they from the, if you do the first row times this column.

This multiplication you get 1 equation out of which r 1 into 1 is a known continue that, can be taken to the right hand side same for other rules.You can set of linear equations solve them. And by solving whatever values you get you plug in in this model. You get a transfer function. So, you know the mod square of the transform function times of variants some constant will be the power spectral density.

So, basically you have to solve these equations what are the other equations here. Now, put k equal to 2 if you put k equal to 2 what you get absolutely similar steps. If you put k equal to 2 you have here r 2, then r 1 r 0 r minus 1 dot dot dot r mi this e minus 2 so on and so forth. Where, we have got k equal to p then you have got r 0 here. And what you have here a equal to p means r p dot dot dot this kind of thing.

Now, I also now put k equal to 0 here suppose. Why leave out k equal to 0 case you may put k equal to 0. What you get here is just n there are k equal to 0 case. What is this in the case of by that is linear prediction or AR modeling in in case in each case.

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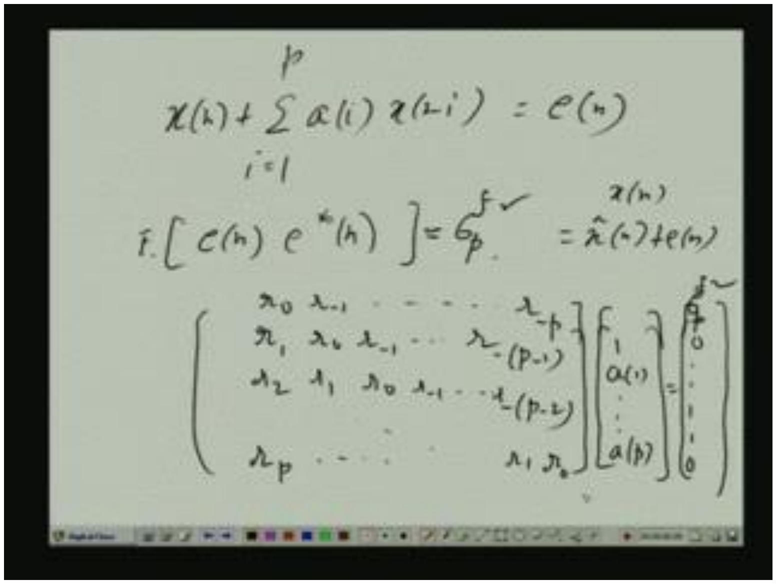

In a case of linear prediction say what is xn? Xn is x cap n plus en.What is x cap n? It is the orthogonal projection rather than prediction of xn from the past p values. This is x cap n is nothing but,a linear combination of xn minus 1 up to xn minus p. And the error is en a summation of the 2 is xn. If you put that back here en is orthogonal to x cap n, because x cap n consists of the past p samples xn minus 1 to xn minus p en is orthogonal to them.

So, en times that, with a conjugate on this x cap n. hat becomes 0 because; they are orthogonal therefore, uncorrelated. So, are you left with is e of en into e star n that is e of mod en square which the variance of en. Alternately, in the case of AR modeling problem we know that, I mean you view x cap n as what x cap n as the prediction or orthogonal projection of xn not on the space, spend by the just past p samples, but by all past samples.

There is x cap n and the error is en en is orthogonal to this x cap n by the same logic and you put them back here. You once again get e of mod n square which is a variance. So; that means, if you put r equal to 0 here in the equation what you get you substitute the left hand side for en as as before you will see you will get this equation. And this is equal to the variance the variance we denote as this is same as en now. This is not 0.

This is called forward prediction because, from the past sample. So, we are predicting few results. So, it is a forward direction prediction. So, that is why, is a forward prediction error variance. Sigma square is usually this use for variance. So, sigma F square forward F for forward. You also put a subscript p to denote that is a p th order linear prediction or p th order modeling problems. Mind you, this is unknown, because input en or the prediction error en is not known to us and therefore these are unknown, but still we put like this.

So, how to solve the equation? First forget out the first row just take the second row to the p-th row to the last row. Solve them get a 1 to ap. Now, put them back in this vector now, take the first row times this column whatever you get that is sigma pf square.That is the input variance. So, what is the output power spectral density input variance times was input is white.

So, input variance times Mod Square of the transfer function. Transfer function you know, because you what it az. That is 1 by 1 plus a 1 z inverse plus a 2 z inverse 2 plus dot dot dot plus apz inverse p. So, all the a 1 to ap are not are now known, because of, because of you solve this equations. So, you know the transfer function you know the mod square of that times these variance, that will be the power spectral density.

That is how you have to go about it. This equation is a famous equation in the context of linear prediction or AR modeling and this is called a Yule Walker equation.

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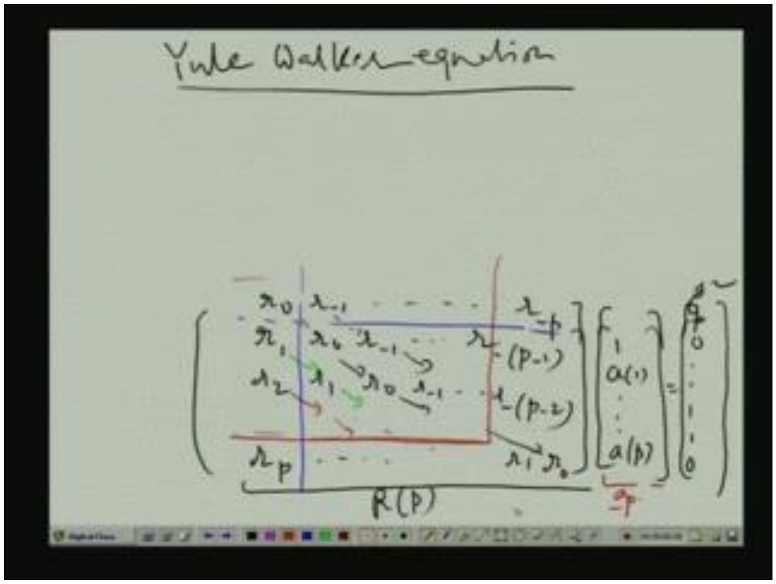

As far as power spectral density is constant you just have to solve it that is all once you a 1 to ap. You are through. Of course, you need the values for r 0 r 1 and all that, but in the previous case only while discussing PO diagram I told you given the sign data record for xn of sufficiently long data record. That you can you can do sample estimate you know I just sample estimate. I am giving the formula also sample estimate for r 0 r 1 r minus 1 and all that.

So, once you have a sufficiently long data record for exchange just by sample estimate technique. Sample an average technique say you can find out the correlation values. Now, this Yule Walker equation is fine, but you know these equations lead to something else. There are firstly, there will be some very fast algorithm to solved this 1. That algorithm gives to a structure a kind of filter called Lattice filter which has some interesting properties and very useful in linear prediction context. I will see if I can give some introduction to Lattice.

So, this matrix I call R within bracket p you see this is a this is called a Tripoli'smatrix. In a Tripoli's matrix if you 1 diagonal entry that entire diagonal is is repeating you see. Similarly, r minus 1 it repeats along these r 1 repeats along this r 2 repeat along repeats like that. And r minus 1 r 1 these elements are actually conjugate of each other you know r minus 1 r 1 star or r minus 1 star is r 1. We know r minus 2 star this is actually a it is actually conjugate symmetric matrix.

Any I-th element and ji th element they are conjugate of each other and also Tripoli's. Any conjugates symmetric matrix in short called Hermitian matrix. So, it is Tripoli and Hermitian both see if you know 1 row it is enough you can construct the entire matrix. First the diagonal element will be same. So, you can construct you can slide along the diagonals or sub diagonals and construct 1 half the upper triangle. And, then using the conjugate symmetry, if this is r minus 1 you know this would be r 1 so and so forth. And you can construct the lower half also.

This vector I call say ap ap vector sorry this p need not be here there is a p-th order modeling or p-th order linear prediction problem that has been considered. Remember 1 thing is Rp and if I if can if I partition this matrix like this, take out first row and first column. What you get here is, Rp minus 1 that is instead of Rp put Rp minus 1 whatever I would have got like here I went up to 2 r minus p. For p minus 1 th for Rp minus I would have gone up to r minus within bracket p minus 1 that is where I am getting here.

So, this part is Rp minus 1 and then 1 row and 1 column alternatively you can also take this matrix like this. Where this part is Rp minus 1?

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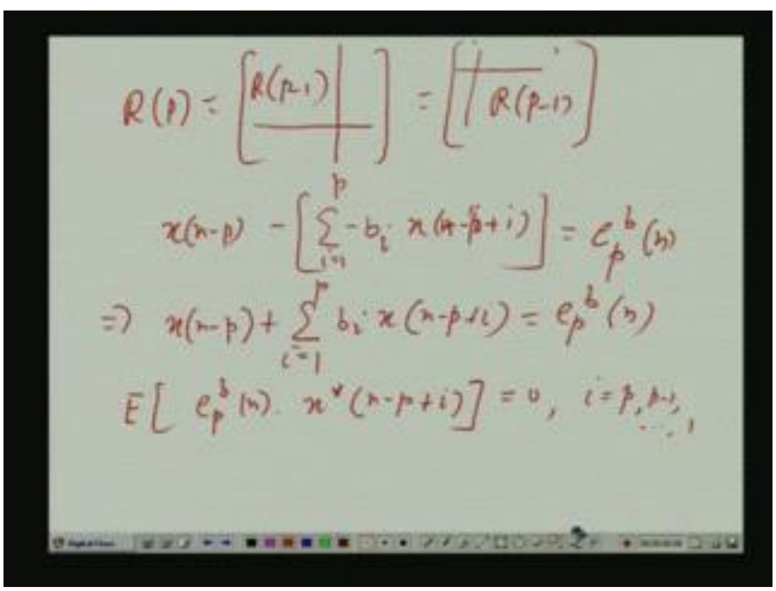

So, we write like this 1 row 1 column leave it this is Rp minus 1 alternatively. This called matrix partitioning. This we know keep this result as somewhere. Now, along with the forward prediction there is another problem that is also you know tackle simultaneously. There is called problem of backward prediction. Backward prediction means if you were to predict x the past sample say xn minus p from its p future values. What are the p future values x of n x of n minus 1 dot dot dot x of n minus p plus 1.

So, from the p future values if you want to predict xn minus as a linear combination and the let the coefficients be minus bi. Minus bi you write like this. P plus I i equal to 1 to p. So, this is the error this error you call e previous error earlier. So, long I denoted en that will now be written previously I whatever was en that will now be written as ef. Because, the rest it will be forward prediction ef p-th order n.

So, whatever we we did for en whenever you had en earlier wherever you replace that n by this epfn. P-th order forward prediction error. Using that terminology I mean analogously this will be p-th order backward prediction error epbn. It is a then not n minus p, because this is the convention we look at the latest sample in this left hand side. Latest sample in when I equal to p you put p here you still get xn. That is xn minus p is predicted from p future values.

So, p future values are xn minus p plus 1 xn minus p plus 2 xn minus p plus 3 dot dot dot up to xn. Since n is the characteristic the convention is to put n here. And these are the coefficients. So, what you get actually minus minus become plus. So, you get; obviously, this is orthogonal to what orthogonal to the space spend by those p future elements. And, then it was p past elements now p future elements. So, if pbn is orthogonal to xn xn minus 1 xn minus 2 dot dot dot up to xn minus p plus 1. So, the p future values starting I mean for this sample xn minute p.

That is if you go p steps in future. So, those p future values samples, they are orthogonal to epbn. That means, if I now use that orthogonality in this way that. Multiply the left hand side by say; that means, if I put it like this. E of into x star this will be equal to 0 you start at I equal to p go backward p p minus 1 dot dot dot up to 1. If I do that you can see 1 thing for suppose put I equal to p. You put I equal to p it is x star n.

If you put x star n, what you have? If you put x star n and this epa epbn is replaced by the left hand side. In the left hand side you started the in this summation for I equal to p. Take I equal to p case. Then p minus 1 p minus 2 dot dot dot I equal to 1. Then n minus p this is have you go about. There is I i repeat my steps epbn is replaced by the left hand side here of this equation.

Then you have to do term wise multiplication, but we start first with whom in the left hand side. You have a term involving xn minus p with that. Then next time I consider xn minus.First we consider the term that is xn that is when you put I equal to p pn minus be cancels xn. First we start with the xn then xn minus 1 then xn minus 2 dot dot dot and then finally, xn minus p.

This is how I order this thing actually what are you doing was we are just putting epb and replacing epbn by the expression. And doing the term wise multiplication only we are doing it in a particular order then that order is (( )). So, first I equal to p and once you have I equal to p case this is x star n and this is x star n. And then you are starting with xn here So, what we are getting is I need space here.

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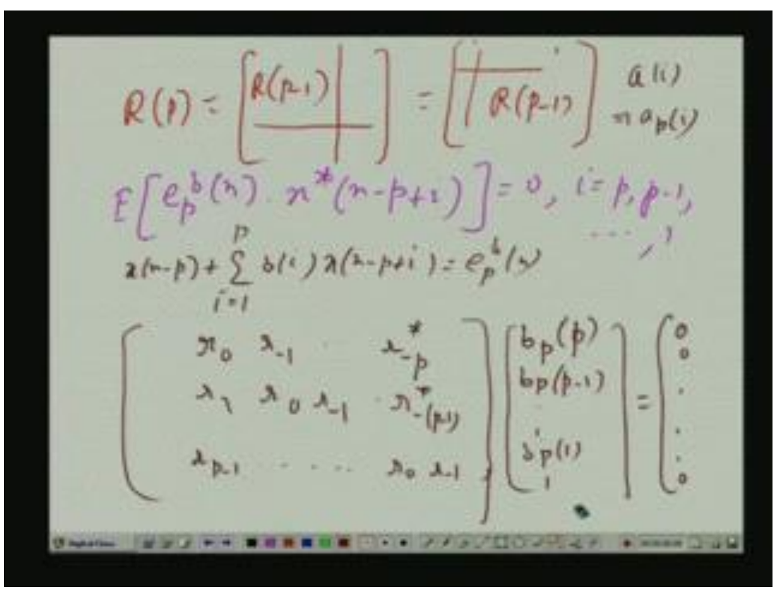

So, may be what we do I rewrite this here and generate some space. The same equation I am rewriting in the, I will erase this bottom 1. So, space will be generated I will use it.

And what was epbn epbn was just for our reference what epbn xn minus p plus bi xn minus p plus i. I was 1 to p that was your epbn. Now, this epbn is replaced by this left hand side in this equation and we do term wise multiplication in the particular order.

So, first I equal to p I equal to p meanswhat you have? I equal to p means x star n add in that summation. You start at xn. So, essentially product will give rise to r 0. Then xn minus 1 into x star n product will give rise to r minus 1. Then xn minus 2 you understand. First it was xn xn into x star n it gave you r 0. Then xn minus 1 into x star n it gave you r minus 1. Then r minus 2 dot dot dot and finally, xn minus p xn minus p into x star n. That will give you r minus p star.

Then what you have? So, I repeat again what I did I took p equal to 1 p i equal to p. And that is orthogonal epbn is orthogonal to that. So, epbn is this and I do term wise multiplication xn into x star n. That will give rise to r 0 xn minus 1 to x star n that will gives rise to r minus 1 dot dot dot. And finally, xn minus p into x star n that will gives rise to this.

Then p I equal to p minus 1 if you do that you will get I am sure r 1 here, because xn into x star n minus 1 that is r 1 then r 0. Then r minus 1 dot dot dot r minus within bracket p minus 1 star so on and so forth. When I equal to 1 n minus p plus 1 n minus p plus 1 x star n minus p plus 1, so xn into that. So, it will be r e minus 1 dot dot dot r 0, then n minus p n minus p plus 1 star till r minus 1 you will get that. Times bp we are writing it like this earlier I just wrote ap if a 1 a 2 a 3. But I am now putting subscript p just to indicate that these are related to p th order backward prediction.

Similarly, previously if we had ai that will be now api. Earlier I did 1 there confusions I did bring the subscript. All these are zero’s again you can solve them it is not again ax equal to 0 equation, because there is a 1 here. So, 1 for the first equation 1 times r minus p star that is an known quantity you can take it to the right hand side same for other rows. And you solve you get these values. So, you know what is the backward prediction error and this coefficients and all that.

Now, look at this matrix and now I want to do the case for I want to also include a particular case.

What is that case? I went from say I equal to 0 if I take I equal to 0 then what happens. If I take I equal to 0 that is this goes and this not 0 surely, because its xn minus p. And what is xn minus p the by now you should be able to do this what is xn minus p xn minus p has got 2 part.

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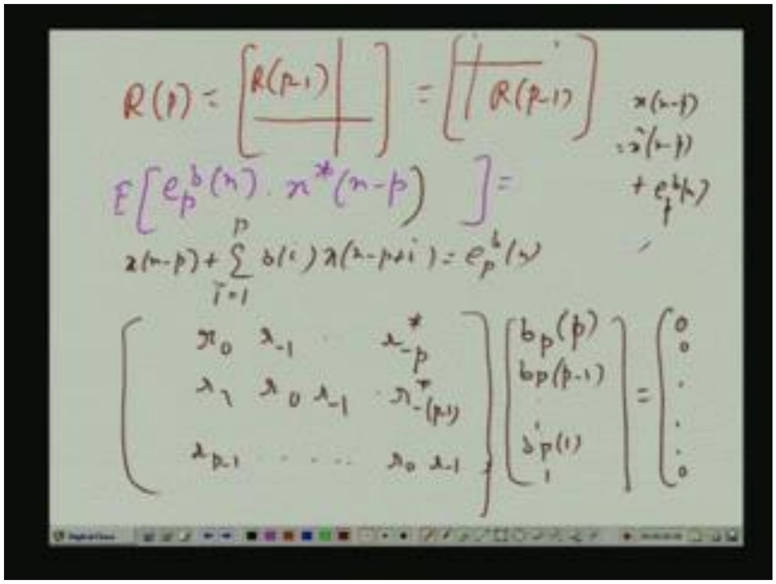

1 is xn minus p you can write separately here. As its prediction which is a linear combination of the p future elements and the corresponding error epbn. This x cap n minus p is what its prediction is a linear combination of the p future values. That is orthogonal to epbn. So, that product will go as 0. So, what you get is not nothing but e of mod epbn square. That is the variants of the backward prediction error for p th order.

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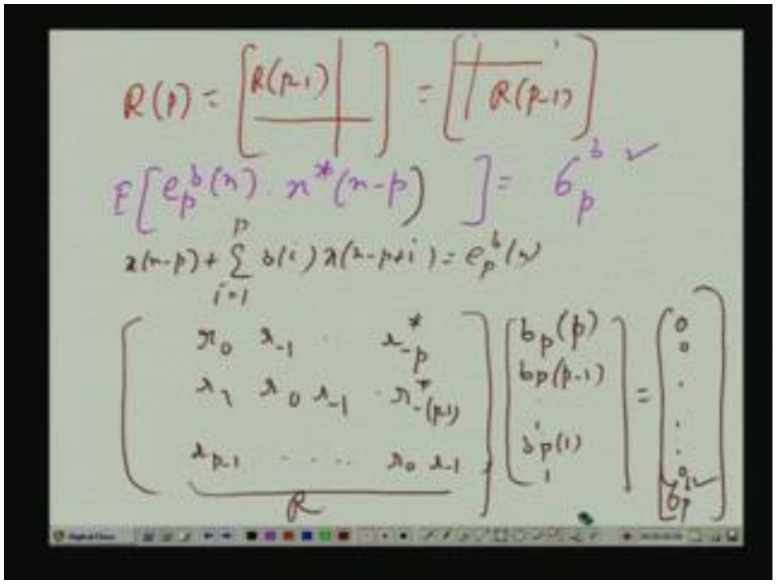

So, that means exactly similar step as I did for the forward prediction case this is nothing but sigma p, but this term not f, but b. B for backward square. I am showing here you have this quantity sigma p b square this is unknown. So, first you solve for the b values the scope is here. So, also know them. The last row times this column will give you the unknown quantity.

Now, look at this matrix again you can see that this is again same R. Same R matrix this is the beauty same R times of backward prediction error vector of this type. Backward prediction coefficient vector of this set will given equation of this kind where, the top ones are 0 and last 1 is unknown, but positive quantity. Previous case R into those are another vector where starting 1 was 1 1 ap 1 ap 2 dot dot app.

Then on the right hand side first first term was an unknown positive quantity all other terms are 0. These 2 equations are to be solved simultaneously. And there I will be cleverly using this partitioning of Rp. And there an order requires a solution comes which gives rise to Lattice filter. We can just easily derive it now. So, I do not have to bother about the structure of R now. R you remember it is an order records a solutions.

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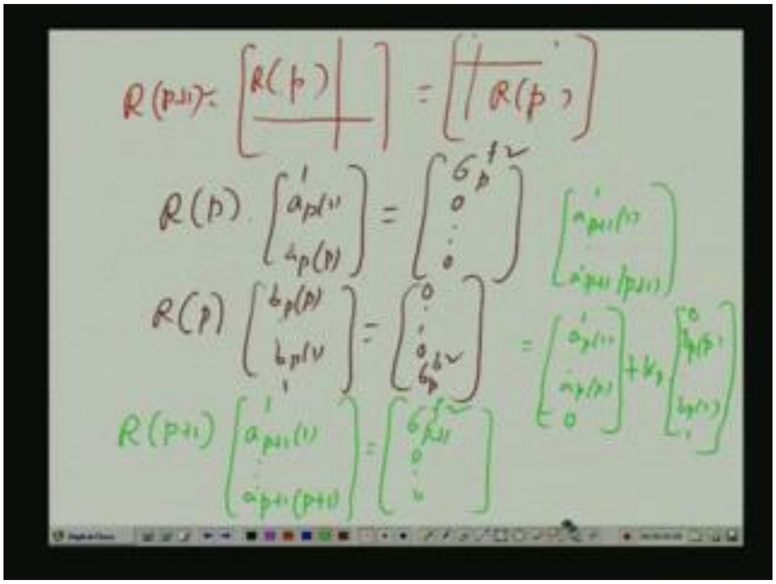

So, suppose I known the p th order solution. That is suppose, Rp into 1 ap 1 dot dot dot ap p this equation is known. Sigma pf square 0 dot dot dot 0. This is solved. So, I know this is unknown coefficients I known sigma pf square. And also Rp times this also is known. What will be the corresponding solution for pa p plus 1-th order?

So, we will develop a order requires solution. So, you can see if I have Rp plus 1 the equation that I am trying to solve here is this. See length has gone up by 1 earlier I had up to p now this p plus 1. And this will be again the, that unknown quantity sigma p plus 1 f square 0 dot dot dot 0. Suppose I want to solve this. You can see 1 thing I can if I propose a solution like this that look, but before that I know this matrix partitioning.

So, if it is instead of p if I make it p plus 1 then this becomes this p this p is replaced by p plus 1. Now, suppose I first propose a solution like this that can the solution be of this type 1 ap plus 1 1 dot dot dot ap plus 1 p. This is the unknown vector in this equation. Can it be a linear combination of the 2 vectors of the previous order? That is 1 ap 1 dot dot dot app this is the a vector for the previous order p-th order i. We are assuming that, we have already solved this for the p th order.

So, we know these coefficients. So, take them and put a 0 and some constant may be kp you can call times again 0 here and then this previous solution for backward prediction bpp dot dot dot bp 1 1. By point is can I am just proposing and can let us verify that is possible by the particular choice of kp. Can we write the p plus 1-th order forward predictor? If you call it forward predictor coefficients are called predictor coefficients. So, forward predictor.

As a linear combination of the p-th order forward predictor and the 0 and b th order p-th backward predictor is 0 on top can you do like this. Suppose, we do what is happening you have put then this on the left hand side can it solve. And can it satisfy equation like this where on the right hand side unknown is a positive quantity, but unknown quantity and all the co terms are 0.

Now, you see first if you have Rp plus 1 times this vector you do this first partition of the matrix Rp and then column column. So, Rp plus 1 into this vector 1 ap 1 up to app and then 0. What does it amount to actually it amounts to nothing but Rp times this first part and 0 times whatever be the last element of each row here and 0 takes 0 makes everything 0. So, actually if you put that back here what you get. I hope you can remember this part, because there is a shortage of space.

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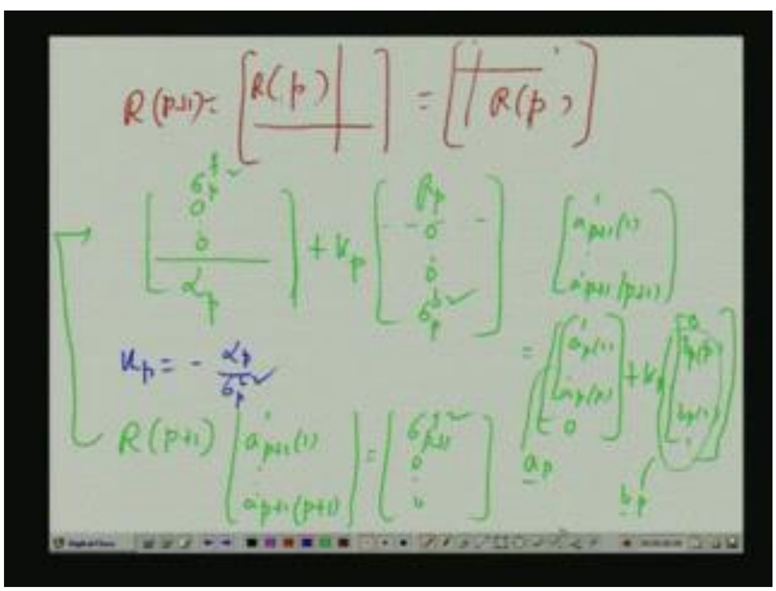

If you put there put this back here what you get using the first partition of the matrix? Rp times may be you can call it ap what is ap this part this part you are calling ap. This part says you are calling say bp. So, Rp times ap plus whatever be the singular element left in this column. That times 0. So, 0 takes Rp time ap and we already have solved this equation why I know what is Rp times ap. That is sigma tf square and then all zeros. And then there is a last row this last row times this.

So, that is some unknown quantity there is some not unknown this is the know quantity. You can call it say alpha some alpha p. What is alpha p last row of Rp plus 1 times this vector apn 0 that will be here plus Kp times. This time value is the second partition if you view the second partition. What you have first row times this vector 0 and then bp these vector 0 and bp what will what you that will be.

That will be some some quantity you have to calculate that is bp. And all the values are known here you are call it beta p, but what happens after that first entry of each row here times the 0. So, forget out the first center, because 0 will take care of it. So, basically you get Rp times bp vector. And you see what is Rpap Rpap you have seen, because we are already solved that equation what is Rpap. That is nothing but, what is Rpap sigma pf square and then 0 0 up to 0 and what is Rpbp Rpbp is sigma pb square then 0 0.

So, these 2 should be equal to this right hand side right hand side is what sigma p plus 1 f square then 0, 0 sigma p plus f square is unknown, but positive quantity. Now, you see 1 thing there are 2 vectors on the left hand side here and both vectors have some nonzero some non 0 quantities on the top and the bottom, but in the middle there are zeroes.

So, at least I am able to get all the zeroes here 0 plus kp times zeroes are the zeroes are matching in the last row alpha p plus kp times sigma p b square that also, must be equal to 0. So, solving that I get a k so; that means, what kp what is kp for the last row I know what will be kp. So, I can choose a kp like this that alpha p plus kp times sigma pb square equal to 0 last equation is satisfied, which gives rise to what put that kp here back here. So, sigma pf square plus that kp times beta p that will be now this unknown quantity sigma p plus f square.

So; that means, just we have to find out kp and using the kp have to just linearly combine the 2 solutions ap and the 0 and 0 and bp. So, these, so that means, what is the solution then what is this equation what is the solution. Let me erase every thing now.

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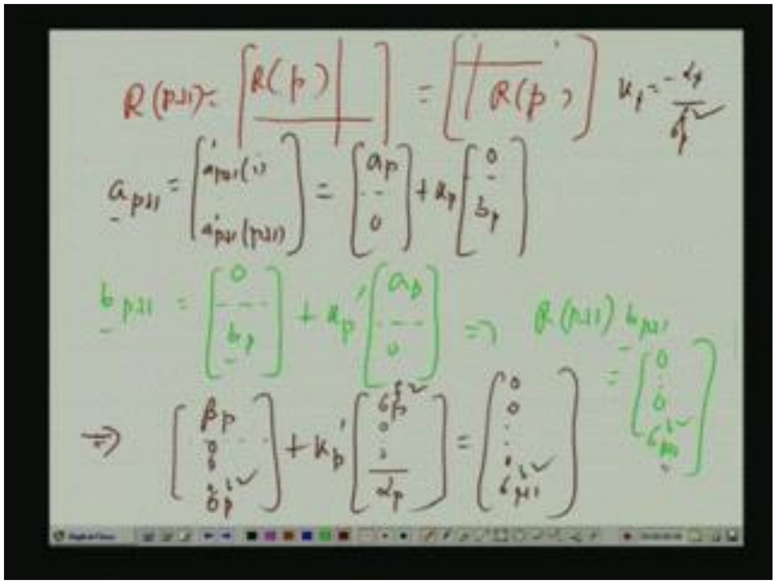

What is the solution? Ap plus 1 which by definition is of this form ap plus 1 1 up to ap plus 1 p plus 1 that is ap 0 plus kp 0 bp. You see 1 thing the top coefficients even if you linearly combine the right hand if you carry out the right hand side equation. I mean summation you still get 1 at the top, because what is the top post term in ap that is 1. 1 plus kp into 0. So, you will get back the 1. So, that is satisfied please note that.

So, this 1 solution we already know what is the; what is kp kp k about to be some minus alpha p by sigma p f square. Similarly, we can find out bp plus one.Now, it will be very simple, because it will very analogous to what we did so far. We propose bp plus 1 of this form 0 bp plus some coefficient kp prime times ap 0. Question is is it possible is it viable for this equation you have to solve. We have to solve these equations Rp plus 1 bp plus 1 is equal to something of this form 0. And then unknown positive quantity sigma p plus 1 b square.

If you substitute bp plus 1 here by this right hand side expression what do you get by similar argument what you get here is this. You will be using this matrix partitioning as before. So, if in the first terms Rp plus 1 times this 0 and bp this vector I will be using the second partitioning. And; obviously, what I get is this; I will get an unknown quantity on the top sorry a unknown quantity is a beta p on top. The first row of this matrix times this vector 0 and bp that will give some beta p. And then first element of each of this rows that will get multiplied by 0 here.

So, forget that so Rp times bp and what is Rp times bp Rp times bp is we know 0 0 dot dot 0 and then sigma p b square. And kp prime times you put that vector again this time use the first patrician Rp times ap, only the other element the last column elements get multiplied by 0. So, forget that Rp ap is what again from the previous solution we know Rp ap is sigma pf square. Then 0 0 0 and the last term is some alpha b. This must of this form 0 dot dot 0 sigma p plus 1 b square. So, all the zero's are already made 0 plus kp prime time 0 is 0 except for the first term beta p plus kp prime time sigma pf square. That means b equal to 0. So, I can find out what is kp prime here.

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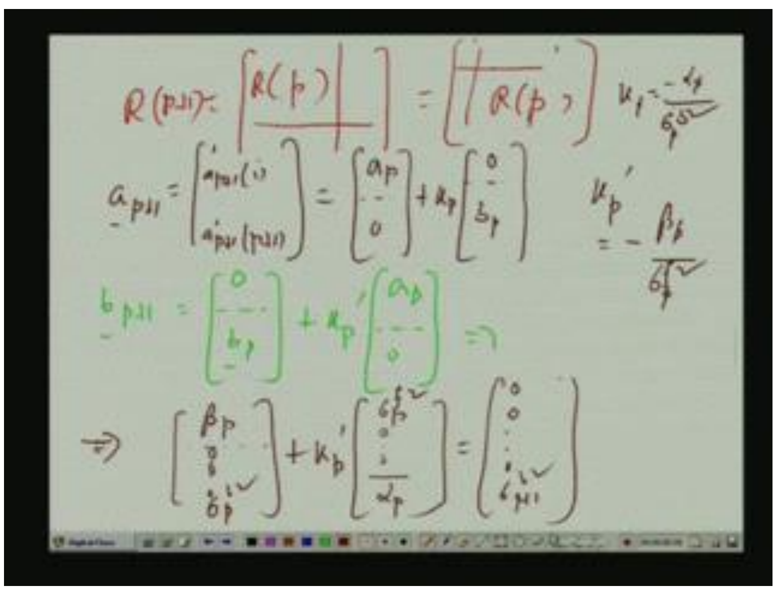

Kp prime should be equal to erase this kp prime is equal to I think this was previously it was wrong with. In fact, so you put that back here. And you now get what is sigma p plus 1 b square from the last equation. In fact, I would not waste my time to go into all those details, but kp and kp prime can be shown to be same. You know this equation can be same. In fact, 1 is the conjugate of the other to be fine. Ap prime is the conjugate kp and vice versa sigma pb square sigma pf square they are same.

That comes from (( )) these results I am not showing here. So, from to past solution you can find out the future solution. Only thing is how to start the recursion. Now, we erase this. We know the solution we know what is kp and kp prime. The solutions are known to us. And how to start the recursion as I assume that ap is known bp is known and then found out ap plus 1 bp plus 1.

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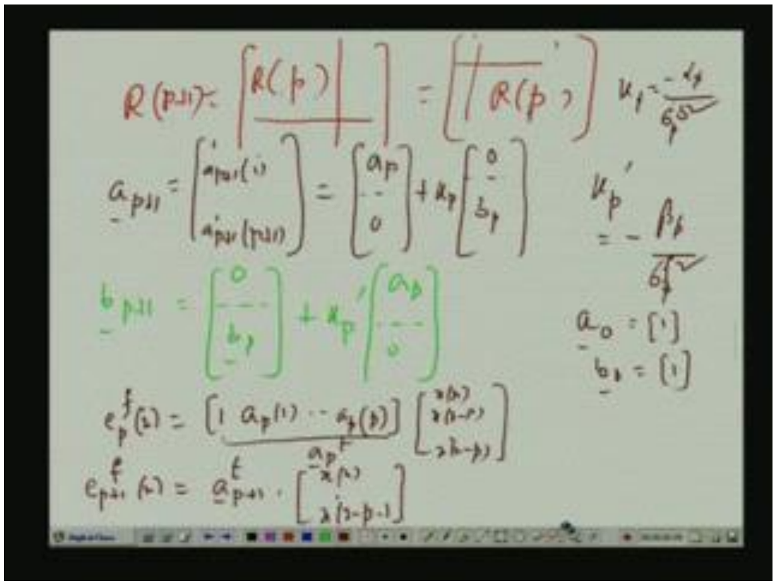

A 0 is; obviously, 1 b 0 is 1, because there is a top most coefficient. So, you can find out what is a 0 a 0 vector is actually is a vector I mean element 1 b 0 vector is 1 because, the top most element is 1. So, when you consider the 0-th order you are left with only that, so 1 1. So, this is how you start 1 thing you note that is important is that what was epfn that was 1 times. Now; that means, this is what ap transpose ap vector transpose. So, what is ep plus 1 fn that will be by the same logic times what xn dot dot dot of 2 1 more term minus 1.

Now, we know what is ap plus 1 if you substitute ap plus 1 by this thing ap 0 and then kp into 0 kp into the vector with 0 bp. Put that back here what you get. You can put that back here you see these are the simple steps. And there is no point in wasting time on that. First term is ap 0. So, forget out the last entry of this vector. So, you get ap time ap transpose xn. Ap transpose xn last entry of this vector xn minus p minus 1 that is not that has to be discarded here, because that gets multiplied by 0.

So, what you get is ap transpose xn. And what is ap transpose xn there is again epfn. And kp kp is what beta p minus this inverse of the kp. Kp times now what is this you you have 0 bp this vector may transpose from times this matrix, So, 0 into first entry xn forget that. So, you started n minus 1. So, bp transpose which vector you get a term like bp transpose xn minus 1 dot dot dot up to x. N minus p minus 1 that you can write as n minus 1 minus p. Suppose n minus 1 is n prime. So, it is xn prime dot dot dot up to x n prime minus p. So, that will be again what epb at n prime n prime is n minus 1. And what is the ap transpose xn we have already seen this is from previously you have seen.

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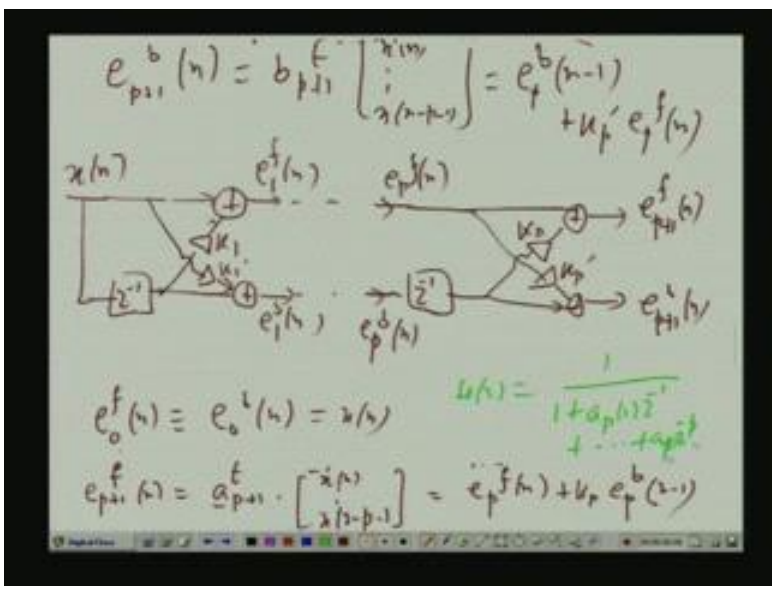

This is epfn similarly, so ep plus 1. This is very important equation ep plus 1 fn is epfn plus kp times epbn minus 1. Then the question is what is ep plus 1 bn. That is again bp plus 1 transpose this vector. Bp plus 1 transpose this vector xn up to xn minus p minus 1 up to xn minus p minus 1. Now, you replace bp plus 1 by what you have on the right hand side.

So, in the first case you have got a vector that has 0 and bp transpose. 0 cancel the first term. So, what you get is what you have seen already first is epbn minus 1. And second term that is kp prime. Another vector is ap 0 transpose we have already done it in the previous case ap 0 apn 0 transpose. So, the 0 is the last term forget the last term. Ap transpose a vector that starts at xn and goes up to xn minus p. So, if ap transpose that vector is once again epfn.

So, we just consider these equation these 2 equations now forget the entire thing. You get a beautiful structure. What is the structure? If you have this is kp multiplied this is kp prime. These 2 are you know conjugate of each other you are not I am not proving it. It can be proved easily you can try it is given in books. So, this is order recursive. And the same structure I can use again again again. So, I can use I can just cascade this structure after this you know I mean just connect it the same structure after this. Just coefficients will be kp plus 1 and kp plus 1 prime.

So, for ep plus 1 fn and ep plus 1 bn I can generate the same errors for p plus 2 th order and then p plus straight order so and so. So, these order recursive very first stage you know that for 0-th order case p, 0-th order case forward and backward backward prediction errors are same and there is xn. So, xn is that is e 0 fn there 0 th order is same as e 0 bn is same as xn. So, you can generate this first order things in the same way with k 1 this k 1 prime this called a Lattice filter.

This is an 1 of the most celebrated things in statistical signal processing this is used for AR modeling and is particularly used in speech context. Speech context, because you know I mean our speak generation mechanism in our vocal cord has been modeled as some kind of model by an AR model, which is given by some white sequence. Only thing the AR model parameters get changed from time to time and you produce various sounds.

So, this this AR model is exactly predicted I mean you know I mean model by this I mean from this you can find out the model parameters and all that. That is why it is very useful for speech speech processing you can I will not go into to that, because this is not a course on speech process. So, it just shows how modeling AR modeling problem is is related with linear prediction and Lattice filter and all that.

Only thing that I have so far not shown and I should show in may be 5 or 10 minutes in the next class is that when you solve the Yule Walker equations you get a set of coefficients a 1 ap 1 ap 2 dot dot dot app you put them back in the transfer function for az.

So, you get a transfer function suppose like this az as these at AR model transfer function. Now, this is that all pole transfer function. The poles for this model to be viable that is stable and causal the model the poles must lie within unit circle. And we will show that by solving Yule Walker equations. Whatever you get there is those coefficients indeed ensure that the roots here there is the poles here.

That is the coefficients ap 1 ap 2 ap p whatever you obtained app. They are such that the corresponding poles are indeed within unit circle. Such a polynomial is called minimum page it is looks like within unit circle this shall prove in the next class. And then I will go for another application of the stochastic process theory which is general mean-square distribution and Wiener filter and adaptive filter. And some examples of the that will take us to the end of this course. Thank you very much.

Preview of the Next Lecture

Probability and Random Variables
Prof. M.Chakraborty
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 39
Linear Mean Square Estimation – Wiener (FIR) Filter

Last class we have been discussing autoregressive modeling. And then we give you the equations which solution gives rise gives rise the AR parameters also we derived what is call that linear prediction lattice. Just few things little more things about this AR modeling will be consider today.

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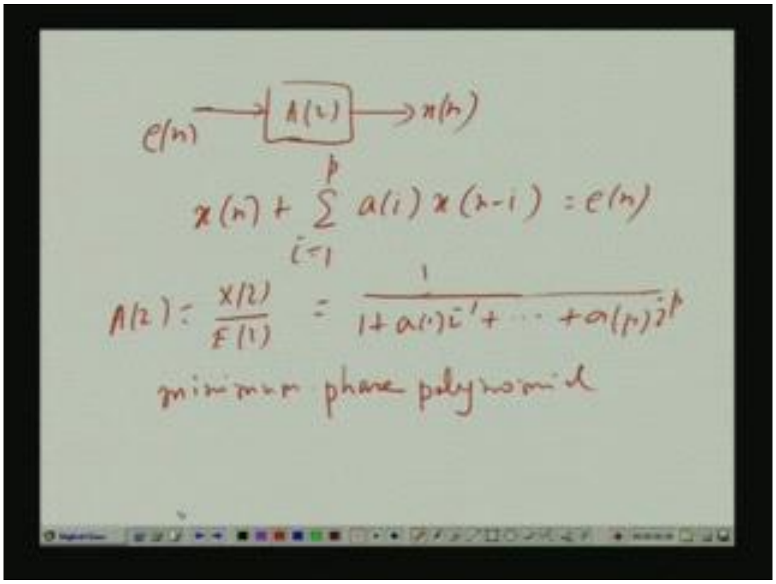

First that is suppose you have got a white process un I called I called it en is last table be I change it un, so en. This is xn you are considering either AR modeling or linear prediction problem in any case we have got this equation. So, p-th order problem into en if it is a linear prediction problem en is orthogonal to the first p samples that is xn minus 1 xn minus 2 up to xn minus p. If we decide AR modeling problem en is a white sequence and is orthogonal to all the past of x.

In that case, of linear prediction n is not a white sequence n is orthogonal only to xn minus 1 up to xn minus p. In general, it is not a orthogonal sequence in the any case n will be sequence. So, you can model this even in the case of linear prediction you can model this by a filter like this that suppose, there is a linear prediction errors sequence called en may not be white. And white in the case of AR process it is passing through a model AZ what is AZ that is transfer function of this AR system gives raise to xn. What is AZ AZ is xn by EZ? If you take Z transfer left hand side and right on side; obviously, we get that all pole transfer function and that is your az.

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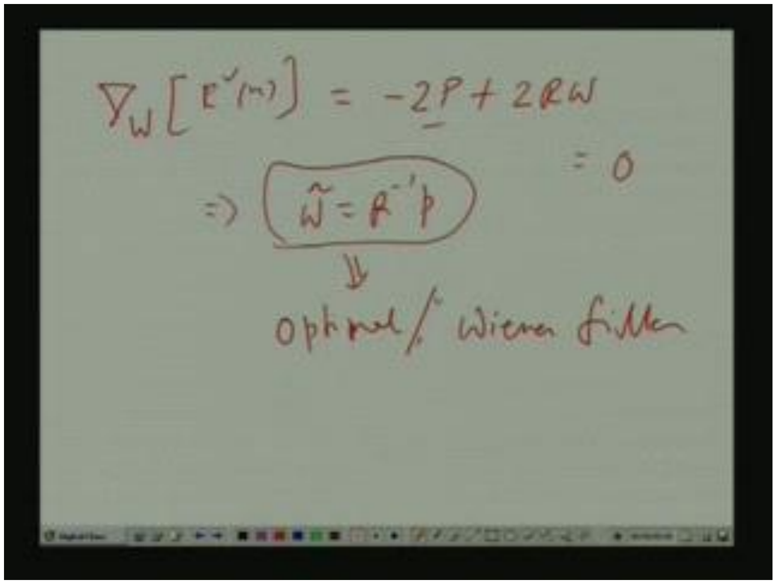

Here, I got twice RW remember this is a gradient expression. I leaves it it the adaptive filter this gradient should be 0 when I have got the minimum. That; obviously, will gives rise to the same Wiener filter you can see w cap equal to R inverse p.This is the optimal filter. In practice, if you know rnp prime you can calculate this R inverse p, but problems is rnpn not always known.

In that case, if to supply information these information externally, what we do? This actually gives rise to the interesting case of adaptive filter. When rnpn not known or when the input statistics or the joint statistics between dn and xn they change from time to time you cannot have 1 filter design once for all. So, you what you should have you should try to have some adaptive mechanism by which your filter learns from the data. That is coming in and tries to adjust itself by iterative method.

So, that finally, it converges on what should be the corresponding Wiener filter for a given rn given p. Then again after a while if rnp changes then again that adaptive mechanism will be used it will further read from the data. And readjust itself, so that, you know. You get a new Rn new p and you get another corresponding Wiener filter.

So, that will be continuous adaptive some mechanism if input R and p changes it will track. And it will have an iterative self adjustment procedure and it will finally, converge every time that iterative procedures with converge that the corresponding Wiener filter. There are plane q of adaptive filter algorithms, but most celebrated is that least mean square a logarithm which will be as considering in the next class. And I will give 1 or 2 applications. You know lecture on correlation and common equation and all that. That we will do in this course.

Thank you very much.