Probability and Random Variables
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Lecture - 36
Spectrum Estimation-Non Parametric Methods

So, in the last class we are doing one thing in a hurry. So, maybe I just start from that and do it again.

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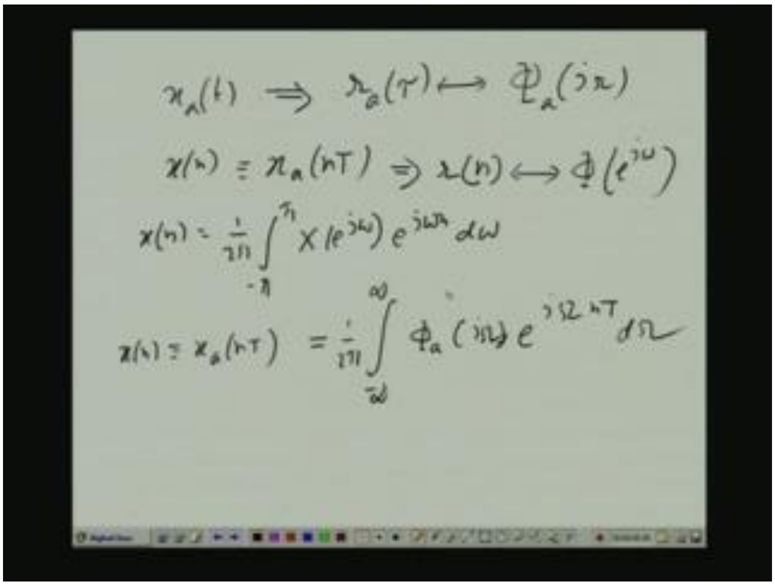

That is suppose there is a random continuous value that is analog random process xa t. And this is sampled at a time period sampling period of T to generate a sequence xn. So obviously, xn also is a random sequence. So, you have xn equal to the n th sample that is if you sample at T equal to 0, then t equal to T, then twice t tl so on and so forth. N th sample I call it as xn, then n th sample of the sequence xn that is equal to x nt.

So; obviously, xn also is a random sequence xa t has got this auto correlation there the auto correlation r a tau. If you take the Fourier transform of it you will get the analog power spectral density. Analog power spectral density that we will denote as phi a for analog j Omega Omega is radiant per second which is analog frequency. And xn this has got the the discreet correlation function r maybe you can say rk or rn say rn or rk does not matter there must be an integer.

This has got the discreet time, if you take discreet time fourier transform, then you have got the corresponding discreet time power spectral density phi e to the power j omega omega is just an angle radiant. Question is xn is xa nT. So, that is the time domain relation between the 2 processes. So, what is the relation between the 2 frequency domain functions? One is the analog another is discreet time version of the power spectral density. Analog power spectral density version is discreet time power spectral density.

So, we just derived the real a star a and I thought I think that I did it in hurry, because it was running short of time. So, may be it would not be bad if I just spend some more time on it not much time, but just little more time and do it once again. So, that you know that important thing is kind very well understood. So, what I do we know that xn is equal to the inverse Fourier transform relation omega is a variable of integration.

So, you can put it omega omega prime theta alpha beta anything, but n is your choice coming from outside x of n. That is you want to find out the sequence at n so that, n is put in here inside the integral This relation we know, but at the same time we know that xn is nothing but, xa nT and xat is a inverse analog Fourier transform of phi a j omega evaluated at time T. So, there instead of time T, if you write n into T, then we will get back xa nT.

So, that is analog inverse analog Fourier transform phi a j omega e to the power j omega not t, but for a specific value of small t that is n into T. That is I am evaluating the integral at a time n into T. That will give me the value of the function at a value of the function x a t at a time n into T and that is nothing but equivalent to xn. Then my purpose will be to manipulate thing. So, that his integral from minus infinity to infinity if this you know I mean make to its if looks identical to or it make to look identical to the previous integral from minus phi to phi.

That is a 2 integrals I want to converge in the same form therefore, by comparing terms I can find out the relation. That is my goal. So, here what I do there is an infinite integral. So, integral is formed over the real x is Omega from minus infinity to infinity. So, this whole integral I may write as an infinite sum of some finite integrals. I will be dividing inter real line into small non overlapping segments. I will carry out the integral over all those segments. And my segments will start from minus infinity and go at plus infinity. If I do that, it will be like this it will be like this; this is my omega axis.

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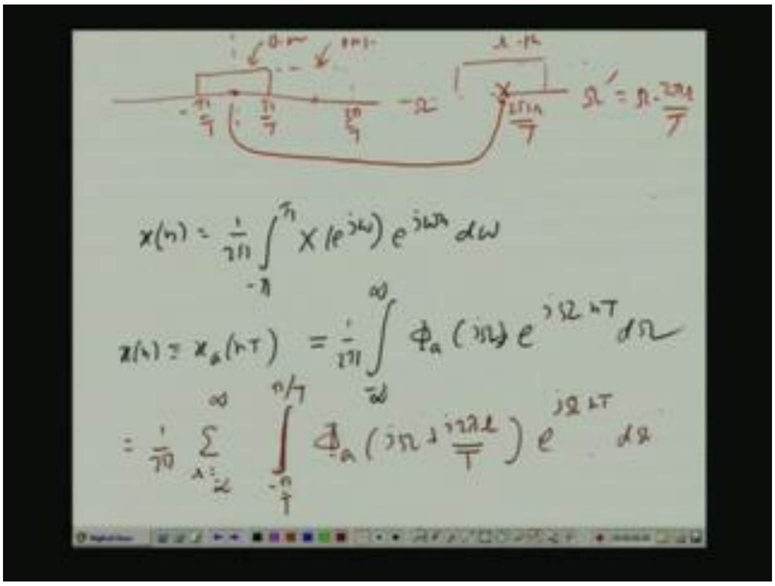

This is my origin. So, I will have 1 slot or 1 sector from minus phi by T to phi by T this I call it zeroth. Then phi by T to 3 phi by T dot dot dot dot r th will be where. If you for zeroth means the sector is from minus phi by 2 to minus phi by T center is 0. Then the first 1 first is here here the center is at 2 phi by T. And we go phi by from 2 phi by T we go phi by T to the right and phi by T to the left to go to meet to boundaries this.

So, for the r th case also the center point will be 2 phi r by T and from there I go to the right by phi by T and to the left by phi by T. So, I meet the 2 boundaries. So, what I m doing this entire integral I will write as a summation of integral over this zeroth sector then first sector second sector dot dot dot dot r'th sector. And this I will go on starting from the minus infinity site t the plus infinity site. And if I do that; that means, this integral I am writing as a summation over r.

R'th if write the integral for the r'th slot and r'th sector then r will be from 0 1 2 3 dot dot dot and on the other hand minus 1 minus 2 minus 3 like that. So, the r th case will be what 2 phi r by r T 2 r by T minus phi by T. So, 2 phi may be I write as 2 r minus 1 phi by T 2 r plus 1 phi by T and rest is same rest is same. Then I m going to double summation 1 is a discreet summation over r another is just called integral. And both assuming that both exist they can be inter changed unfortunately we cannot inter change here. Because I mean the if if I inter change the outer summation integral, but its limits contain r where as inner summation is over r.

So, once the summation is done or disappears, but at the same time r exist in the limits of this integral. So; that means, I have to do something. So, that the. So, that r disappear r is made to disappear from the limits of the integral. Then I can do the inter change of the 2 summations. For that matter what I do I now introducing new variable I just shift the origin in this figure from here I shift that into here. That is I introduce a variable omega prime and later I will call it as omega again, but for the time needs omega prime as omega minus 2 phi r by T.

If I do that then you see d omega and d omega prime as same. Firstly, omega is omega prime plus 2 phi r by T d omega d omega may are same they are 2 limits of the integral. When omega is 2 r plus 1 into phi by T omega prime takes the value phi by T. And when omega; omega is 2 r minus 1 into phi by T omega prime takes the value minus phi by T. So, that means, here only I do some change, because I do not have space to take on 1 more step minus phi by t.

So, you see r has disappeared from the integral limits and what I have got here phi a not j omega, because it is now in terms of omega prime. So, j omega prime plus j 2 phi r by T into the interesting thing is e to the power, you see omega is to be replaced by omega prime plus 2 phi r by T. 1 term will give rise to e to the power j omega prime nT; the other term e to the power j 2 phi r by T and then outside n into T. So, T and T cancels and you get e to the power j 2 phi r into n R and n both are integer. And remember e to the power j 2 phi into any integer is always 1.

So, that is why I do not write that factor here. And d omega d omega prime are same. So, I can write as d omega prime. And then you can ask I can take a permission to convert omega prime into omega again, because I have probably assigned a local variable, variable of integration. We are more used to handling the variable omega rather than omega prime. So, why carry the notation omega prime again bring back the same notation omega it is not the same omega as used earlier. It is actually the omega prime some is the previous omega. But, I am now forgetting entire thing I m just calling it omega without any relation to the previous omega. At the 2 summations now can inter changed. So, that means, this outer summation can be brought in inside. One more thing I do now this Omega T this Omega T you know I introduce a variables omega as Omega T.

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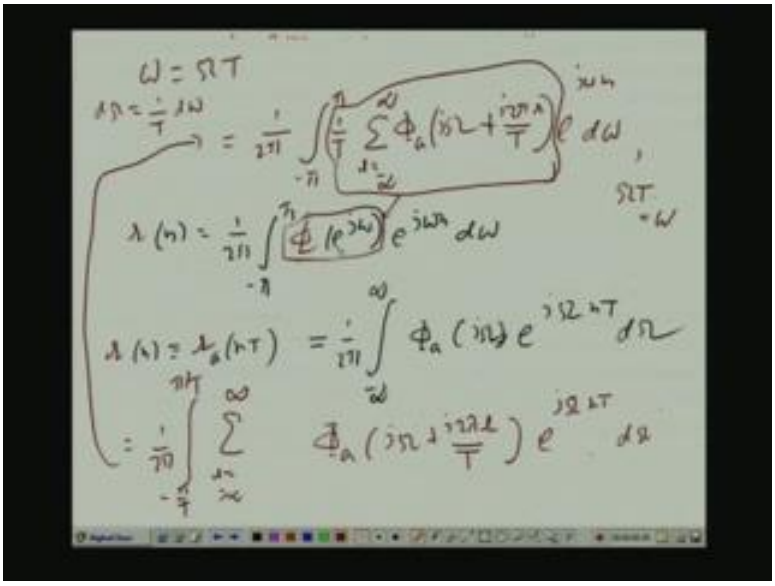

Capital omega its unit is radiant per second. Unit of time T is second. So, omega into T is just an angle radiant. So; that means, if I make the substitute in the integral then first you see when Omega becomes phi by T omega becomes phi. And when Omega becomes minus phi by T omega becomes minus phi. So, limits now of the limits of the integral now are from minus phi to phi.

So, what you have in the integral… This integral now becomes equal to 1 by 2 phi minus by 2 phi summation. Rest I will write later. What is d capital omega; D Omega is 1 by T d omega. So that means, I will have 1 by T here and to the extreme right I will have d small omega. Then phi a j you can write Omega you can substitute by omega by T. I still what I my style is to still write it as capital omega. I will tell you what I do I do not write in terms of omega here and then this remains same as before 2 phi r by T into now e to power j Omega nT.

Now, Omega T is omega small omega. So, it is just e to power j omega n. And after doing this I mention separately what is this Omega. This Omega is such that Omega T is omega, where omega is used in the integral So, actually either you can replace Omega by omega by T directly here or you can still retain the notation capital omega, but give the meaning of Omega separately. That Omega is nothing but, omega by T I prefer this style.

Now, if you compare the 2 integrals you see left hand side in both cases are xn and right hand side also the 2 integrals are exactly alike. You have got d omega in both you have e to the power j omega in both minus phi to phi both the integrals are 1 by 2 phi and all that. So, by comparing you can easily see that x e to power j o omega is this quantity this quantity and this quantity they are same. You can compare these 2 are same. So, this is not x I made 1 mistake.

And this not x this actually r and this is phi, because we are dealing with auto correlation not the exact not the actual sequence, but its auto correlation function analog and discreet time. So, these 2 functions are same. So, this gives rise a relation between the analog and discreet time psd power spectral densities. We just take its meaning what it is?

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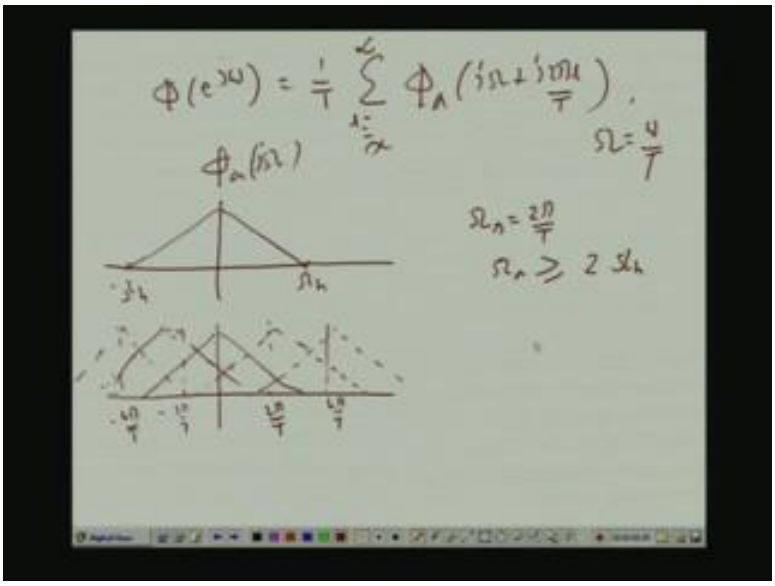

So, I write down the relation separately again. And then you substitute this as things.

What it means that suppose phi a omega as been like this. This highest frequency this is actually typical figure of band limited psd. So, this is the band upper this freq after omega h H for high this power spectral that is transferred to be 0 and and minus omega h. Now, suppose, but what I am saying now as got nothing to do with band limited nature I just took a fair like this. Suppose this is given to you phi a omega or phi a j omega if you call does not matter, because this j is at present is a real function.

That means, if you chose a digital frequency omega equal to Omega into T. And you want to find out the discreet time power spectral density phi e to the j omega and that digital frequency small omega. How will compute? You have to carry out the right hand sum. So, what you have to do is this first you plot as a right hand side as a function of Omega. Then only I will transfer Omega change Omega to small omega.

So, right hand side summation I do not count for the 1 by T factor, because it is just a scale factor right hand note summation say for r equal to 0 you have got this only phi a j omega. Then that equal to 1 equal to minus 1 phi a j within bracket omega minus 2 phi T. So, this entire thing will be shifted to the right by 2 phi by T. So, if 2 phi T is here then you will have another function here and for r equal to plus j omega plus 2 phi by T. So, if 2 phi by T is here minus 2 phi by T you can take T will be shifted here, then at 4 phi by T at minus 4 phi by T.

Now we have to superimpose. You understand that if the overlapping this between all thus shifted versions. Than; obviously, resulting the resulting function on after the total superimposition will no longer resemble the original phi a j omega. So, they will be distortion we call it aliasing. For aliasing distortion not to take place this 2 phi by T should be where? If 2 phi T is and what is 2 phi by T by the by 1 by T is the sampling frequency. Two phi T is analog sampling frequency in radian per second T is a sampling period.

So, 1 by T is the sampling frequency in hertz 2 phi T is sampling frequency radian per second we call it omega s. So, if omega is suppose omega s is greater than equal to twice the band limiting frequency omega h. Then what will happen?

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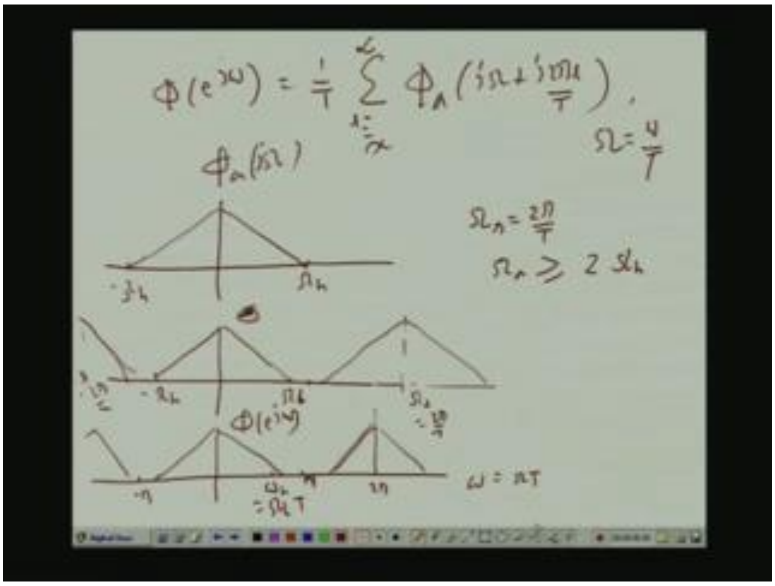

Suppose, this is your omega h I am drawing in that same scale, because I do not have much space to my left here. Some very narrower suppose this is a thing and omega h 2 phi by T;

that is omega s is here. So, when this entire thing is shifted there is no overlap similarly minus 2 phi T is here. So, there is no overlap. There is no aliasing distortion and now if you convert it into omega axis you always see 1 thing. How will you convert? Omega is Omega into T.

So, therefore, what was earlier Omega h that will become omega h; omega h is equal to Omega h into T. Similar on this side shape will remain same. Remember 1 thing Omega s that is 2 phi by T. Whatever may be T 2 phi by T that radiant per second that that is analog sampling frequency what omega does it give rise to… If you multiply it by capital T here T and T cancels it will always give rise 2 phi; no longer T is no longer present. So, analog sampling frequency is always you know always give rise to omega equal to 2 phi.

So, this will map to 2 phi and half sampling frequency which is here it will at phi. So, something like this. Now, my claim is that suppose this is given that the analog power spectral density is given to be band limited up to some frequency Omega h. And we follow this Nyquist rate of sampling that is sampling frequency Omega s is greater than equal to twice of the band limit frequency omega h. In that case, if you take the discreet power spectral density phi, phi e to the power j omega there will be no aliasing.

So, what about you see from minus phi to phi that is between in the range of half sampling frequency. On either side that is an exact replica of the original analog power spectral density. Only this is from here from this discreet time power spectral density if you have to go back to analog 1. Only thing we have to do is from omega you have to go to Omega by these transformations, omega equal to Omega into T. And there is of course, 1 by T factor there is a scale factor that we have to observe. You get back the original power spectral density.

If that be, so then to measure the power spectral density of any analog random process real real life analog random process, what we have to do is this We must sample it maintaining the Nyquist rate and once I have got that discreet sequence I develop algorithms which will work out the discreet sequence, because algorithms means I will be using a computer that can work only on discreet data not on continuous data. So, my algorithm will work on the discreet data. And it will be a good algorithm.

So, the power spectral density discreet type power spectral density will be estimated to a good amount of accuracy. And once that is know from that I will obtain the analog power spectral density just by transforming from omega going for small to capital omega. So, hence forth our all efforts to estimate the power spectral density will be concentrated on the digital side. Thus, estimation the discreet power discreet time power spectral density by using by developing some algorithm which work on discreet data.

Assuming that band limited analog power spectral density was band limited and Nyquist rate of sampling was Nyquist condition of sampling was maintain. I have I am I have nothing to lose from the discreet type power spectral density. I can get back the analog power spectral density just by the transformations omega equal to Omega T.

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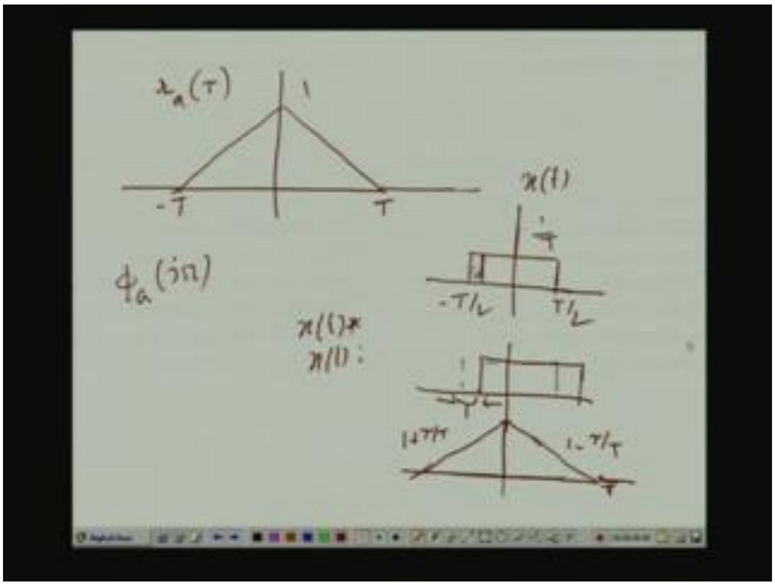

I can just quickly take an example. Suppose, for an analog signal is given like this 1 some T this is not sampling period, this is just T and minus T. Question is what it is phi I j omega or phi a omega? You can see 1 thing how do we get this functions you know If you have a rectangular function from T by 2 minus T by 2. And area is 1 by T I height is 1 by T the area is 1. If you convolve this function you can call this function xt.

If you convolve this with itself that is if you do what you get. You first reverse and you have to go on shifting it. If you reverse it and do not shift that is a 2 functions come 1 below the other then we have maximum overlap. You just multiple the 2 functions and calculate the overlapping area that overlap may be in this case will be 1. So, that will be the value at 0 shift that is at T equal to 0. Then you shift to the right say by T; that means, this much area if you shift it by tau. Then how much area goes out? So, tau into 1 by T.

So, net area previous area that is equal to 1 minus tau by T. So, as tau increases tau by T also increases. So, the correlation decreases, so it is like a actually it falls down linearly like this 1 minus tau by T. And T equal to at tau equal to T this touches 0. After that there is no overlap 1 rectangle as gone purely I mean out of the domain of the other integral. So, there will be no overlap and value will be 0. Similarly, on this side also if you shift it to the left it will fall in a linear manner like this. The function here will be tau by T, because tau takes negative values and you get back what it is?

So, you can see r a tau is a convolutions between 1 rectangle and itself. And rectangle is what height 1 by T and duration from minus T by 2 to T by 2. So that means, what will be the analog Fourier transform of r a that will be nothing but, square of the fourier transform of the rectangle function. That is we now seeing function though it will c square, because you know convolution time domain is equal to product in the frequency domain. So, thus the x is convolute with itself; that means, the power I mean power spectral with these Fourier transform xt as to be multiplied by itself.

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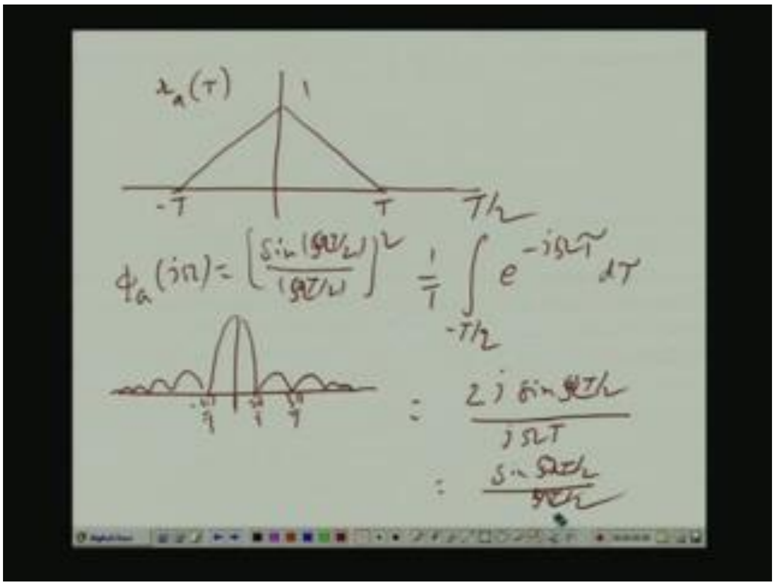

Then you will get back this. So, what is the for a rectangle what is this. If you take the rectangle what is the Fourier transform that is seen, but what is the exact equation is very simple to do. That is height is 1 by T e to power minus j omega tau j Omega d tau. And that is equal to 2 j and j will cancel, because j tau will come below. You calculate and tell me what it will be 2 j sin T and this equal to actually sin omega T by 2 divided by omega T by 2.

So, this phi a j omega will be square of this function sin omega T by 2 divided by omega T by 2 square this Omega am making a mistake every time. If you plot it there is sin square. So, it will be always positive it will be like this. What are 0 crossing frequencies? Whenever numerator becomes 0 that is omega T by 2 is equal to say phi or 2 phi or 2 phi like this. So, 2 phi by T minus 2 phi T 3 phi by T so on and so forth.

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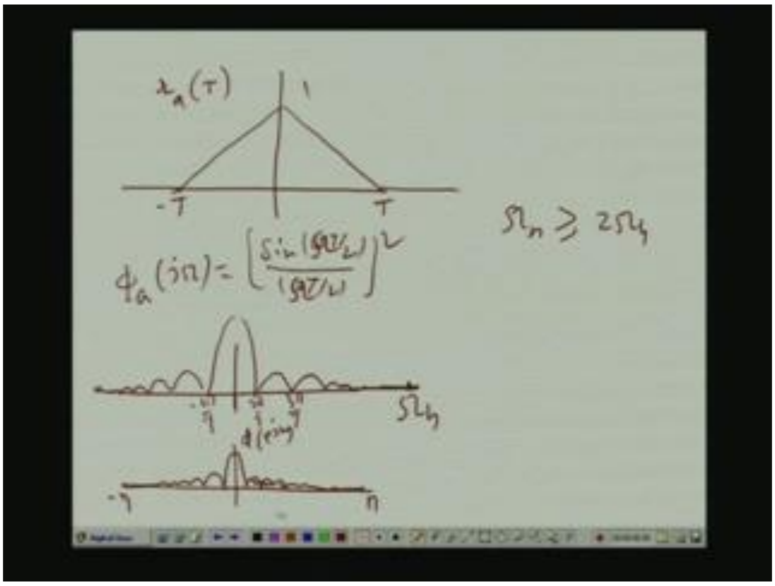

And, then is it band limited theoretical no, but practically yes, because you see this function is phi a j omega rapidly falls down to 0. So, may be a when you are here or here it is already 0. So, you can call this omega h. So, if you derive omega s equal to I mean following this omega h and sample this random process to get a sequence say xn. There is a corresponding discreet time auto correlation function what are the discreet time power spectral density phi to the power j omega will be a replica of this. You go up to phi and minus phi it will be no aliasing.

So, if I now take that sequence and estimate this power spectral density from that I can easily construct my analog power spectral density this is an example. Now, so far we did this spectral analysis we define what is you know we gave the full all the properties of we define power spectral density both for analog process and discrete time process. We gave all the physical meanings and we carried out some examples. Now, from now onwards for this lecture, and next lecture and may be part of next lecture I am not sure depends on how much time we have.

I will be spending some time on methods of estimating the power spectral density. Again you know power spectral estimation is a full 1 semester course. So, obviously in 1 or 2 lectures I cannot cover much I will be covering some very basic things. There are actually 2 approaches 2 main approaches for estimating power spectral power spectral density: 1 is called parametric another is called non parametric. In a parametric method whole estimation of power spectral density can be it basically boils down to estimating just a few parameters of some model.

That is also called model based estimation; that is actually very powerful method that will be considering after this. In a non parametric method what you have to do you are just given a data record of that approaches and discreet data record, using that you have to do estimate the power spectrum. Without taking recourse any model or you know without mapping the entire problem to the estimation to the problem of estimating instead of parameters.

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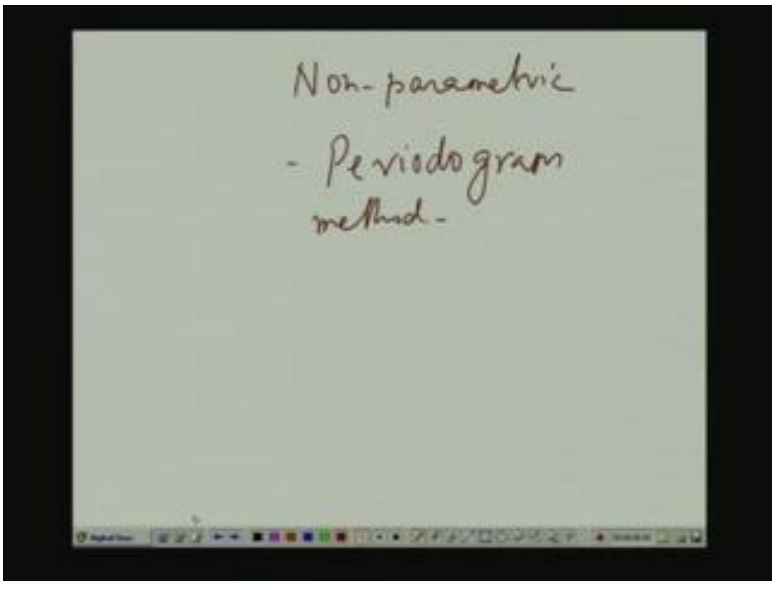

So, this non parametric method the most popular case here is that of Periodogram. Here, you are given a data record that is some samples. However large be the record; suppose, you are given the data record from 0 up to N minus 1 its only like this.

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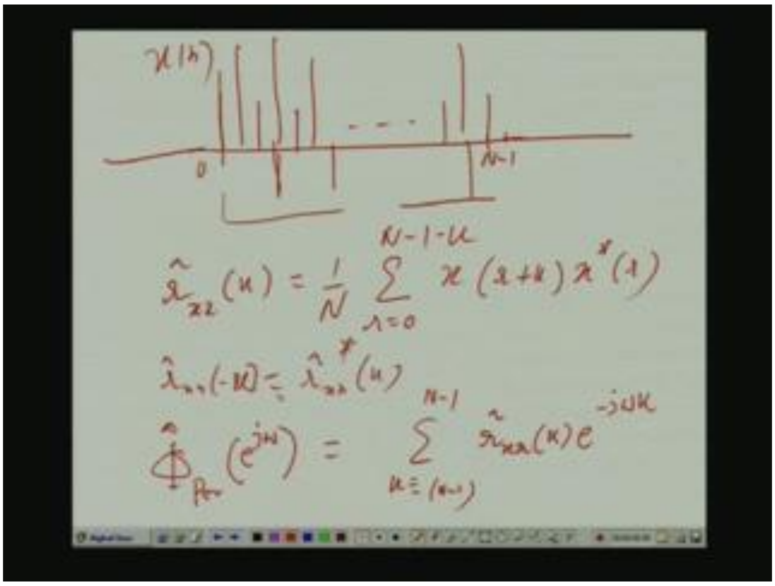

And then you have nothing you have 0, actually this is not 0, but you do not have the data you have data only over this period. So, using this data what we first do is that we find out by some sample averaging technique find out some estimation of the correlations. That is firstly, how to find out the variance you take each sample take its mod values square up square up the erased sample square up the first sample square the second sample dot dot dot up to the N minus 1 sample. Add all them divide by N if N is large it will be a good estimation. This is called sample estimate.

So, if N is large it statistics can be used I mean it can shown that this estimate tends to a good estimate for the variance. Then how to find out the auto correlation for it lag 1? Take the zeroth same and first sample what is the lag 1. So, x 0 into x 1 then take x 1 into x 2 then x 2 into x 3 then x 3 into x 4 dot dot dot go up to x N minus 1 x N minus x N minus 2 into N x 1 you cannot go beyond that. Add all of them again divide by N.You get an estimate for auto correlation with lag 1 then for lag 2 lag four lag 3 lag 4 so on dot dot so and so on.

Now, you know what happens is this in most of the practical cases you know the correlation is actually located mostly amongst an adjacent samples. That is for low values of lag you have got higher values of correlation that as lag or the gap between samples increases. Naturally the mutual influencing capability decreases if the samples are far apart. And therefore, there correlation decreases. So, correlation as actually die out as the value of lag that integer k r of k that integer k increases in either direction or magnitude.

Therefore, if I have got a sufficiently large data record and I get estimate the auto correlation for say lag 0, lag 1, lag 2 up to some finite number of lags. It is enough I can assume that since the auto correlation is a dying function. And I have already estimated sufficient number of auto correlation figures for lag 0 lag 1 up to some lag say Lag say L. I can simply assume that subsequent values of the auto correlation are approximately equal to 0 or can be neglected.

So, I can rely on this take them and use them to compound the discreet time Fourier transform. And that will some idea of some estimate of the power spectral density that is the preview of basic preview of Period gram. So, what I do here you first estimate this, this if for xn k how you estimate this way not k here. So, what I am doing first take k equal to 0 that is the variance that mean I am taking the summation from 0 to N minus 1. And what are the values x say r equal to 0. So, x 0 x star that is mod x 0 square then x 1 into x star 1 with their mod x 1 square dot dot dot.

So, each sample I am taking its absolute value squaring up and adding and then dividing by N. So, that is obvious give me the best estimate I mean the good estimate of the variance. Then consider lag 1 what I am doing I am taking say x 1 for r equal to 0 x 1 into x star 0 and x 2 into x star 1, x 3 into x star 2 dot dot dot up to what I am going I am going up to N minus 2. And k equal to 1. So, r plus will finally, become N minus 1, when I equal to N minus 1 minus k for a given k this r plus k becomes N minus 1.

So, this is N minus 1. So, in this case it is x N minus 1 and x star N minus 2. So, that means, I am multiplying 2 samples at a lag 1 after taking conjugate of the second sample and adding and then averaging. So, that will again give me a good estimate of auto correlation value at lag 1. Similarly, for lag 2 so for a general case of k th lag what I am doing xk into extra 0 for r equal to 0. Then next k plus 1 x star 1 x k plus 2 x star 2 dot dot dot up to when r equal to this N minus 1 minus k r plus k becomes N minus.

So, xN minus 1 into x star N minus 1 minus k so again lag k. So, there added and divided that gives an estimate of auto correlation function with a lag k. So, all I have remember since I have got just a causal data record from 0 to N minus 1 and I find out rxx 0 rxx I mean r cap cap is for estimate r cap xx 0 r cap xx 1 dot dot dot. Then I use the fact that r cap xx minus 1 is minus k is. So, I get the other left hand side also, because I have to make it symmetric. And once I do that then what is happening once I do that then I take this discreet time Fourier transform.

I take this discreet time Fourier transforms and that will be my periodogram that is just give me a minute. So, I have got now auto correlation sequence from minus within bracket N minus 1 to plus within bracket N minus 1. And if I take this discreet time Fourier transform I get that periodic periodogram power spectral density. Phi cap, because this is these are estimates periodogram, periodogram e to the power j omega. What is that?

This is say k equal to minus of N minus 1 to plus of N minus 1. Obviously and you know, because of this relation you can easily see that what I get here that would be a real function that will be non negative function all this can be seen. So, this is called a periodogram estimate. So, here actually computationally there is something they first from computation angle point of view. What we have to do? We have to first use the data to calculate the auto correlation values estimates.

Then use the estimates in the dtft computation, but there is 1 way were you do not you can skip this intermediate step of calculation of the auto correlation sequence. And you can directly obtain the same periodic this periodogram. We will go to that next; if you look at this expression forget out the 1 by N factor this part. If you look at this part this you know you can write also as a convolution of what, these 2 sequences.

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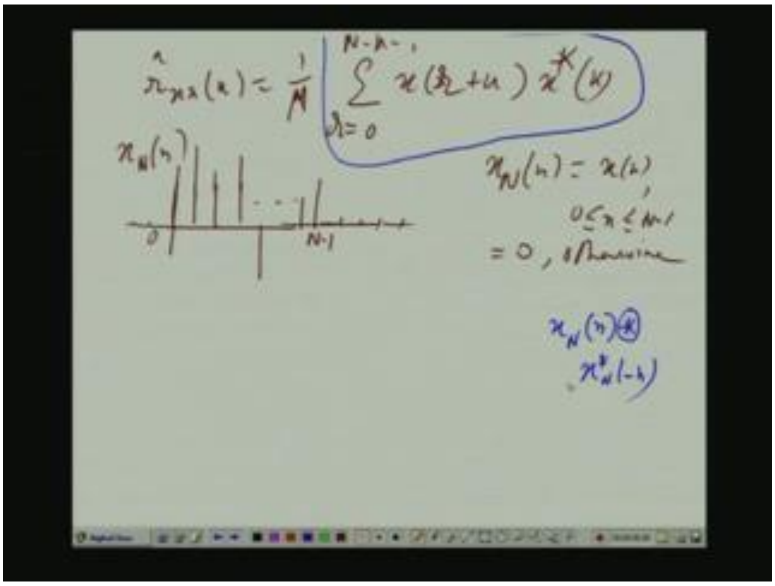

Firstly, just a minute, I rewrite everything here, because I need more space now. So, that this was according to us this now you define if you define 2 6 1 sequence x N n. That is the window sequence actually and then 0; 0 is nothing but I am taking the zeroth sample of xn first sample second sample up to N minus 1 sample. To the right of that N minus 1 is sample and to the left of zeroth sample I am putting zeroes. So, basically I am multiplying the original sequence by a rectangular window from 0 to N minus 1. Mathematically x N minus n is equal to xn.

For n less than equal to N minus 1 greater than equal to 0 and 0 otherwise. My question is if you convolve these 2 sequences. If you carry out suppose x this xN n convolve with xN minus n what do you get you see that we get nothing but, this term. So, for that we have to see were how it is. What is xN? There will be a this is a convulsions and there will be star this star is actually. So, let let me put a circle here this circle star is convolution and this star is conjugate.

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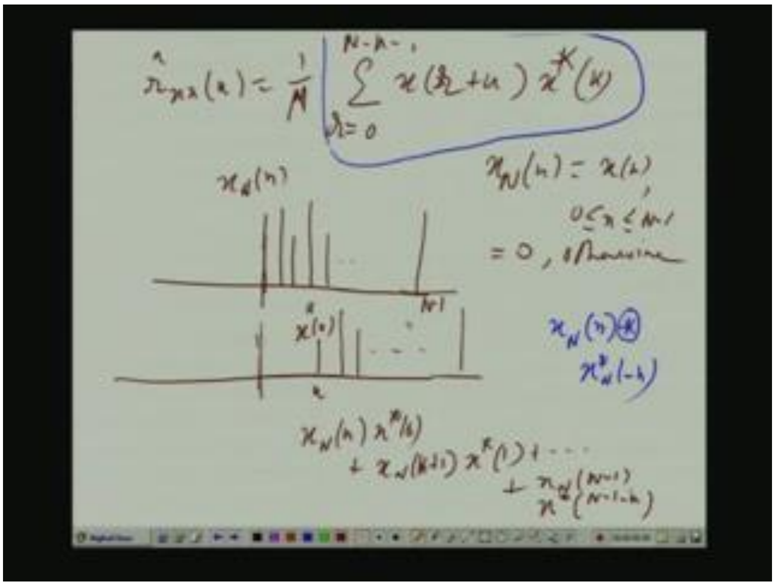

Now, what is x N minus n I need some space let me delete this part. This is your xN what is xN minus N forget about the star we will assume that we are conjugating the sample that not important xN minus n is the reverse version of that. So, just if you start at 0 and go up to minus of capital N minus 1. These 2 sequences if you want to convolve what you have to you have to again reverse 1 of the sequences. So, if you reverse the second sequence you get back same 1. I am no I am skipping those step XN minus n is what?

That is the reverse version of that which start at 0 and goes up to minus of N minus 1. That is 1 sequence and another is xN n this 2 are 2 convolve for convolving what we have to do hold 1 sequence as it is. So, first sequence I hold as it is second sequence I have to again reverse.

So, I get back what I it was earlier. And then you have to shift it to the right or left by an amount at where which by a by an integral say N or by an integer k for which you want to find out the convolution. Now, suppose I reverse it and now shift it say by k

So; that means, at k th point I have got x 0 like this. And if you now x convolve 0 here. In fact, I should say now x star. X star 0 it multiples xn k, because these are k th point. So, you will get xn k x star 0. Then xn n plus 1 k plus 1 x star 1 dot dot dot so on. So, and then last 1 will be, because from this first sequence I can go only up to n minus 1 th point. So, xn n minus 1 into x star the lag is k. So, n minus 1 minus k, this is precisely what we are within the sum then 1 by n of course.

Now, if you if you this is, so as we can see then of course, you can shift it not only to the right by you know positive k you shift it to left also by minus k and you get both side it. So, carry on the total convolution total convolution we will get a correlation sequence conjugate symmetric. If you now take that will give you r cp xxk and you have to take the dtft of that as I said discreet time Fourier transform to obtain the discreet time power spectral density which the periodogram estimate to power spectral density.

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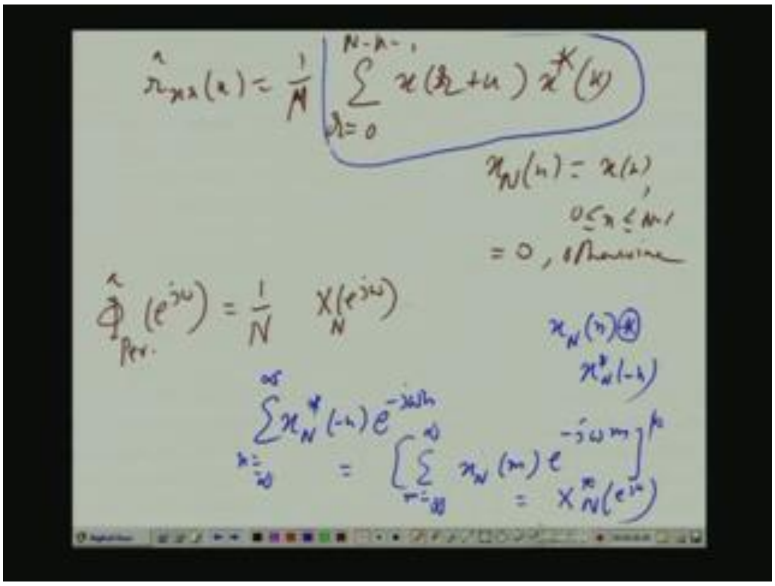

There you can now see this summation is a convolution between x and n with xn star minus n. So, discreet time fourier transform will be what convolution in time domain is prod prod multiplication in frequency domain. So, that means, phi cap period e to the power j omega which is the dtf t of discreet time Fourier transform r cap x as k. That will be what 1 by N, because 1 by N is as it is and this summation. In time domain it was a convolution between the 2 sequences.

So, in frequency domain that will give rise to product of the corresponding discreet time Fourier transform. First 1 is say giving rise to what after all what is xn n that maybe I can write x N it is by j omega. What is that? That is the discreet time Fourier transform of that finite duration sequence x N n. That is the sequence which starts at 0 at x 0 x 1 goes up to x capital N minus 1 and then 0 to the right and 0 to the left of origin. For that sequence that dtft discreet time Fourier transform is xN e to the power j omega. So that means, from the first term if that be so. What is the Fourier transform of the next 1?

This is actually a basic you know basic dsp fact, but in case we do not know I have to it do it again XN star minus, what is the dtft of that is a rough calculation I making I will erase it. Replace minus n n by minus n n in place minus n by m. So, when n is minus infinity n will be plus infinity and when m plus infinity m will be minus infinity. So, it will be from plus infinity to minus infinity, but summation can be always whether you take the summation from plus infinite and go to minus infinity or from minus infinity to plus infinity 1, 1 of the same thing. So, once again you get back m m and minus n was m. So, e to power j omega m you know forget take the star out from here put a minus here. And star this is nothing but conjugate of this xn e to the power j omega.

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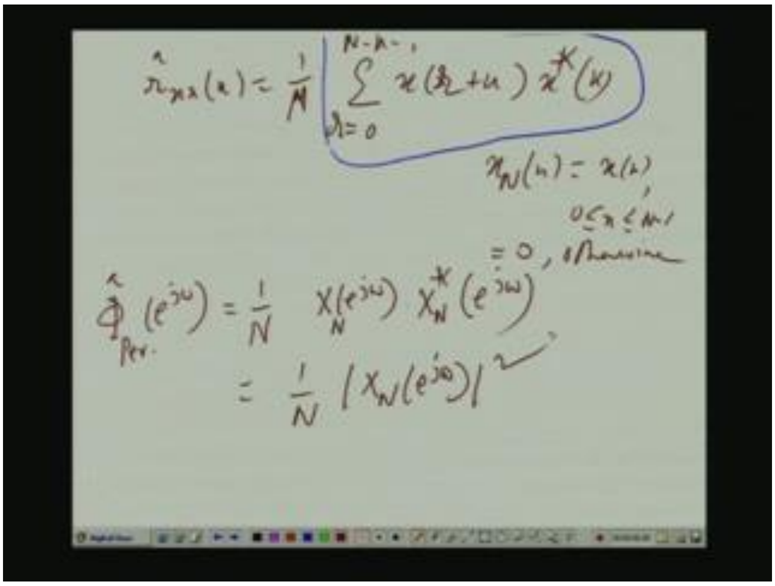

So; that means, you can see this is a real function, but non negative function and all that.

This is the periodogram estimate of power spectral density. This is quit hand in practice only problem is I mean this actually often give rise to biased estimate. The other version the other not version the other approach is that parametric approach.

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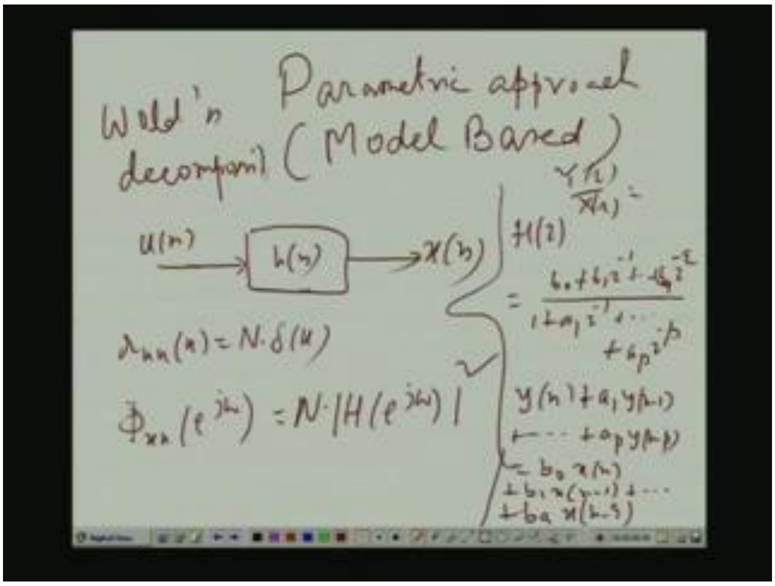

Model based here the entire problem of estimation of the power spectral density can be made to boil down to an equivalent problem of estimating estimation of a few parameters for the model. And very sophisticate algorithms exist in literature. Now, before I going into that just consider this view. That suppose there is a lti system linear shift and linear system of impulse response hn. It is excited by a white process un 0 been white process output is xn.

So, since I do not have much time in few to few minutes time I will just give a prelude to that and in the next class I will consider it. So, un is a white process un is a white process that is 0 mean white process. Just ruu k is some N into N is some positive integer delta k. So, if you take this dtft it is flat N. So, this is the power spectral density. What is phi xx e to the power j omega? By our relation it is N phi uu, because this is equal to N this is equal to N.

So, you can write N times this. N is in the constant it does not change the shape of the power spectral density it only is this gain factor. So, essentially to obtain the power spectral density of the process xn we have to just; if you know, H e to power j omega which generate this process then we know the estimate. We know the power spectral density. Now, what we do here in parametric modeling we go for some rational modeling of HN rational approximation. Rational approximation means in general Hz which is the z transform of Hz Hz.

It be not be having a close form expression. Firstly, and even if it is a close form expression it does not have to be rational function. Rational function is normally a ratio of 2 polynomials in z 1 is numerator polynomial like b 1 z inverse plus dot dot dot b q z inverse q. And here it is 1 plus a 1 z inverse plus dot dot dot dot ap z inverse. So, this is called rational model. We assume that Hz is rational like this which is a very special assumption we are making. And there is estimation for it which will consider detail in next class.

So, if you know that that which the entire model is describe by a few parameter b 0 up to bq and a 1 up to ap. If you know the parameter you know what is Hz you know what is H e to power j omega fine? And then you know, what is the power spectral density? You can estimate capital N even if you do not know it is just a scale factor it does not change the shape. This particular model it has both zeros and pole it is called the pole 0 model or something it is called ARMA model autoregressive moving-average model.

If only the numerator is present no denominator then this as only zeros then it is called only moving average or ma model. And if it has I mean no 0 may be at constant v naught only and only poles then it Is called all pole model or autologous. Ar model of this 3 ar model has been very successful in practice; it is 1 of the most celebrated thing in statistical signal processing and we will spend our effort mostly for ar model estimation. Remember in time domain this equation means what if you call it Hz equal to yz by xz its nothing but yn plus a 1 yn minus 1 dot dot dot ap y m minus p is equal to b 0 xn plus b 1 xn minus 1 plus dot dot dot plus bq xn minus p.

So, we consider from the given data record of xn we will consider methods of estimating in the model parameters by some sophisticated algorithms. And we will be considering mostly ar model, but before that I will give a justification for choosing such rational model. There is a solid theoretical justification which comes from something called wold’s theorem; wold wold’s decomposition theorem So, this will consider in the next class.

Thank you very much.

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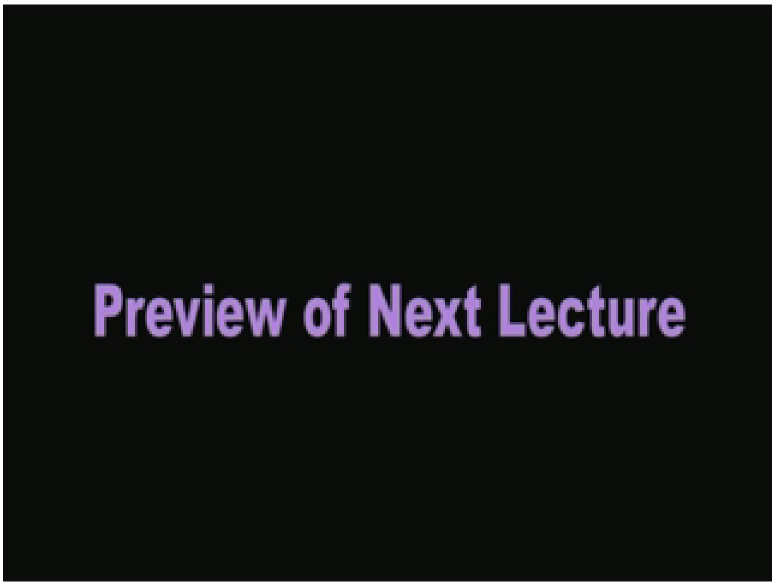

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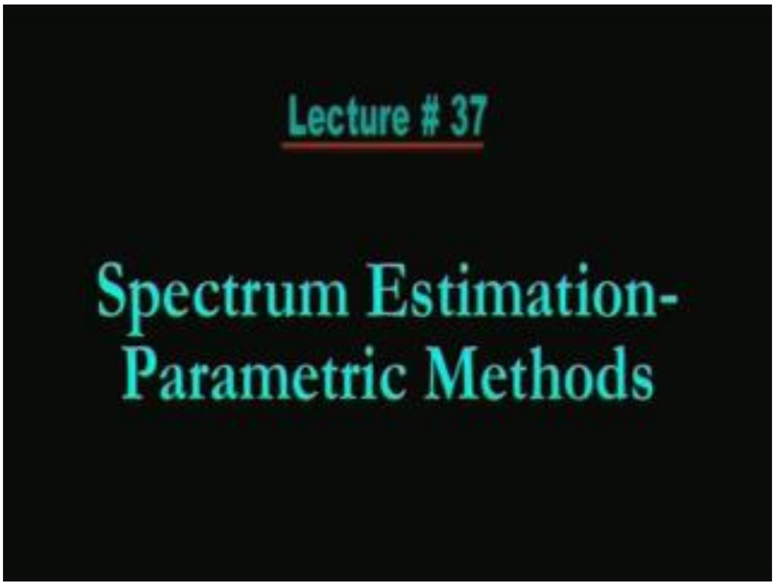

So, today we will be considering as I told you in the last part of yesterday's lecturer. I will be considering parametric method of spectrum estimation that is spectrum estimation by modeling. And I have already told you the elementary ideas of modeling that is you assume that, the process is generated by passing a white sequence through a rational lti system of transfer function saying z which can be all pole 0 or pole 0 both. And therefore the power spectral density of the received sequence would be nothing but mod square of h e to the power j omega h e to the power j omega being the transfer function.

So, mod h e to the power j omega square into some constant the constant denotes the input power spectral density now input is white. So, input power spectral density is flat which is equal to some constant. So, in this case, the entire spectral density estimation works down to identifying that model or a estimating the parameters of those model. You know the coefficients that occur in the numerator and denominator polynomial exist. Now, before I go further into that I mean I told you yesterday also that there is the justification for you know making these assumption and going by this method.

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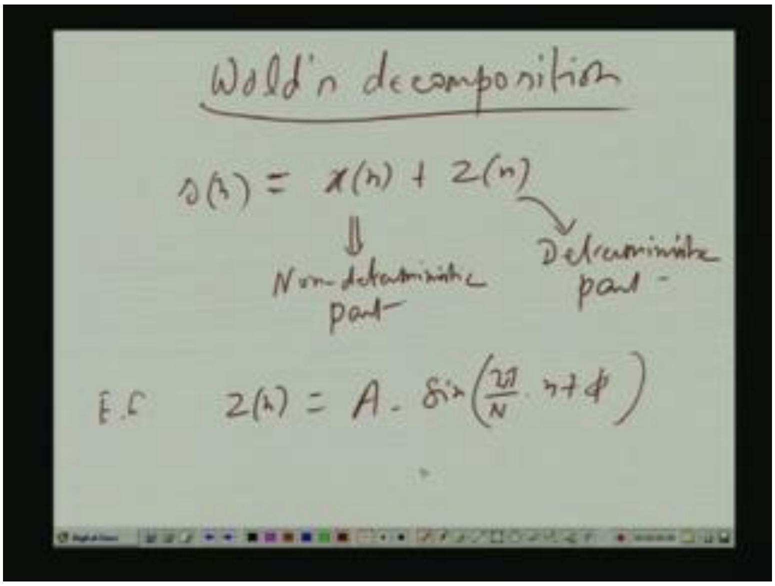

And that comes from what is called wold's decomposition of famous theorem by a great statistician statistics person, this was wold's is called wold's. This says that given any random sequence sn you can always decompose it into 2 parts. One is xn and other is say zn, where xn is called non deterministic part and this is called deterministic part. And these two are mutually uncorrelated.