Probability and Random Variables
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Lecture - 32
System with Random Process at Input

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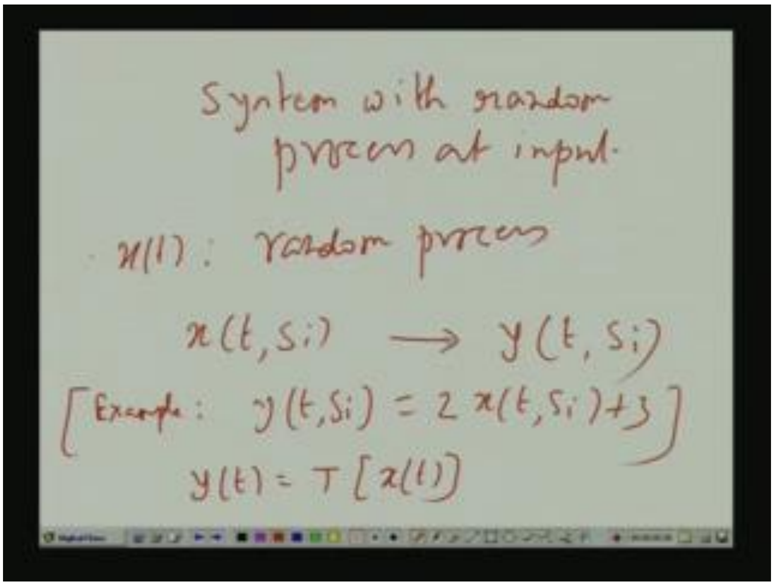

Today, we discuss system with random process input, suppose xt is a random process; that means you conduct experiment and with a every time you have an experiment you know you observe 1 waveform. So, may be 1 particular waveform we denote as before like this; where si denotes the i'th out come. So, in the i'th out come you get a waveform as a function of time, so this is represented by xt comma si. So, this actually a function of time, si just denotes the fact, si just stands for the fact that it is related to the i'th out come a particular waveform. Now, suppose we have some kind of some rule, we develop or we have in which some kind of rule by which given the particular waveform we generate another waveform by a rule.

Some waveform we call it y and obviously that waveform was is associated with the i'th outcome of the experiment. So, that which we can write it as y t si; for example, you can have a sa very simple rule is nothing but say twice xt si say plus 3, which means whenever you observe a waveform say what the i'th outcome you have got a particular waveform this is xt comma si. Now, take that waveform multiplied by 2 at 3 to each at t whatever waveform you get that is what in this case is yt comma si. So, that means, for the same outcome si, we have a waveform for x and we did not have an waveform for y.

So; that means, we can say that we can generate a random process yt whose waveform for the si it'h outcome is giving by yt comma si. In other words given, xt we say that we generate a random process as T working on xt, where t stands for operator or that is the rule or the way of mapping they given waveform xt comma si we mapped it to another waveform by some rule. So, that mapping or that rule that is contained in this operator T; So, T takes or each waveform of this random process xt and maps it to another waveform yt. So, y they there by you generate a random process yt; Obviously, because with each outcome we are not only getting the waveform for xt; I am also getting a waveform for yt and therefore, you can also say that yt is such a random process whose waveform for the si i'th outcome is yt comma si.

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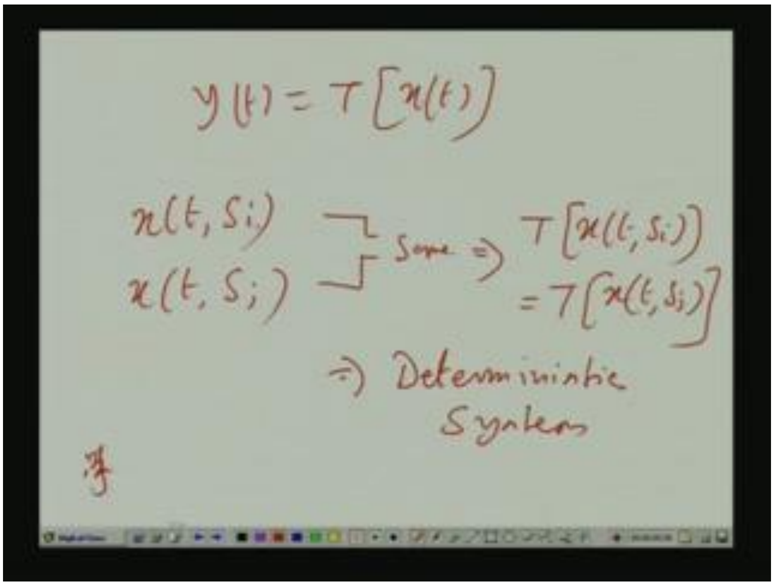

So, I repeat again we have this;, so this T which is actually a rule by which you map each waveform of xt into a waveform of yt, this rule actually in a bigger sense is called system. So, T stands for the system operator or the mapping rule, but we normally call we normally preferred to use the term system. So; that means, a system takes xt and maps it to some waveform of yt. Now, if it, so happens that giving this waveform xt comma si when t works on it. If T that the system works only on this as a function of time and takes si as parameter, then we say it is a deterministic system. That is suppose si and sj are 2 different outcomes, but the waveforms are same. If, so happens then 2 waveforms are same. In that case, if these 2 are same, implies T of xt comma si should be same as T xt, sj. That is the system only you look at the time t; it does not look at the experimental outcome, it does not look at the s values.

So, whether this time you observe a waveform or after some time for another outcome you observe the same waveform in both cases, your output will be same, because a system only looks at the function as a function of time. It does not look at the particular experimental out come with which is it associated. If that be, so then is called deterministic system, but if it, so happens that a system not only treats this as a function of time, but also looks at the particular experimental out come. That is; even if, after getting this waveform xt comma si for another outcome xj sj you have the waveform. Suppose if the system acts on that waveform differently that is if not all it changes its behavior from outcome. Then the system that is chi, now system is changing is property with time from outcome to outcome.

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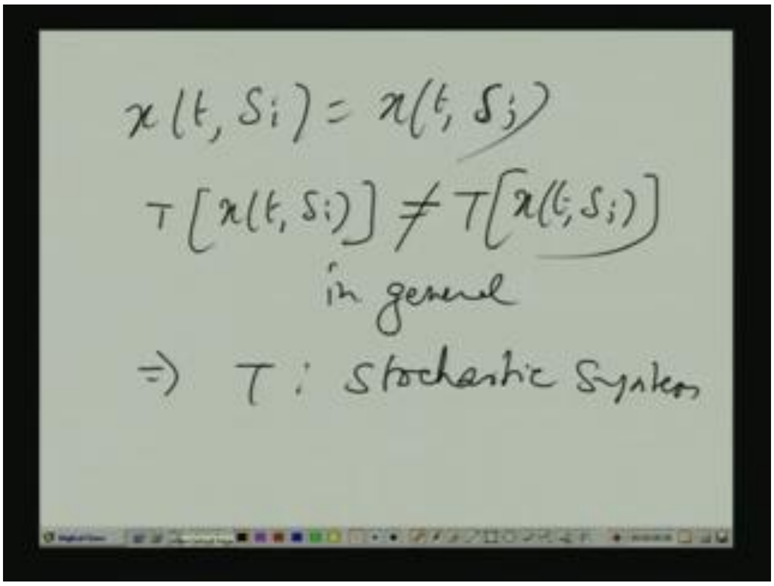

So, in that case system is called stochastic system. If that is, suppose it is given that is for 2 outcomes Si and Sj. The 2 wave forms stand out to be same, but if the system is stochastic it will not give the same output, because it is not only look it is not looking at this waveform just as function of time. It is also considering a particular out come So, for a are are out come si its behavior will be different from it is the action when the outcome is sj. So, in general in general t of not equal to in general, this leads to T as stochastic system. That is, if T such a state whose individual parameters built in parameters.

So, internal parameters they vary randomly that is they vary they undergo change from experimental outcome to outcome; obviously, the action of the system will also change in that cases system is stochastic. So, its behavior on this xt comma si and xt comma xj even if though if the waveforms are same will be different, because you know with experimental change in experimental outcome system parameters are change. So, system behavior has changed, but in this course we will be considering mostly system that is deterministic not stochastic. A particular class of a particular case of deterministic system is what is called memory less system.

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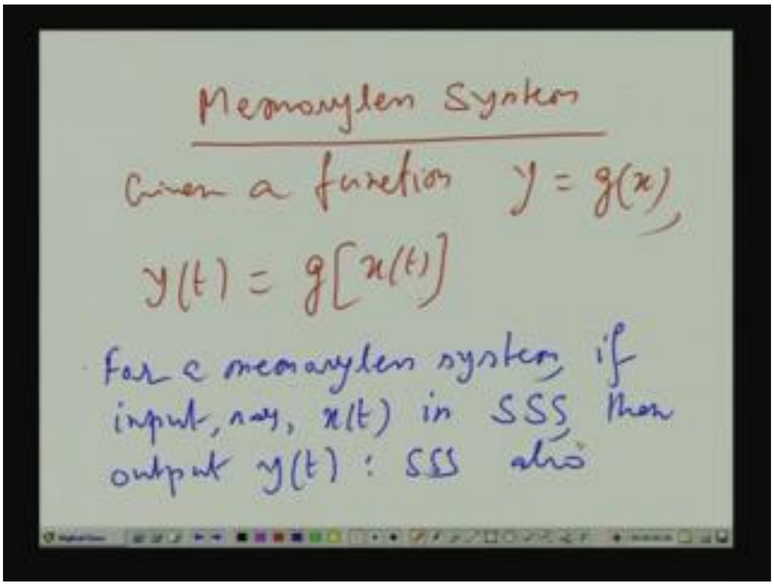

That is suppose, given a function y equal to gx, that is give any value for x we find out y. This memory less system will be given by yt as g va of xt; That is xt that is for each experimental outcome you got some waveform for xt. What g does at each t, it takes the value and evaluates this function g of x that is g of that value and gives you the corresponding y. So; obviously, here when g works on that value it does not look at the past or future of xt as a result we call it memory less system. I repeat again, what it does we know this function that giving any value of x, how to work out g x and what is y that we know. Then this system is constructed like this; that this random process xt is the input g work on it and gives you yt.

Its meaning is take any experimental outcome say xt comma si that is i'th outcome. Therefore each, so you get an waveform, in that waveform for each time t you have a value for that waveform that is amplitude or whatever g works on that s as for this given function and gives you a value that value is assigned to yt. So, here as you see it will work on the instantaneous value of this waveform at a particular time it will not it will neither look at future not look at past. So, this will be called memory less system. A memory less system, if the system is memory less than in the input is stationary. If it a strict sense stationary SSS then output also SSS that we can see.

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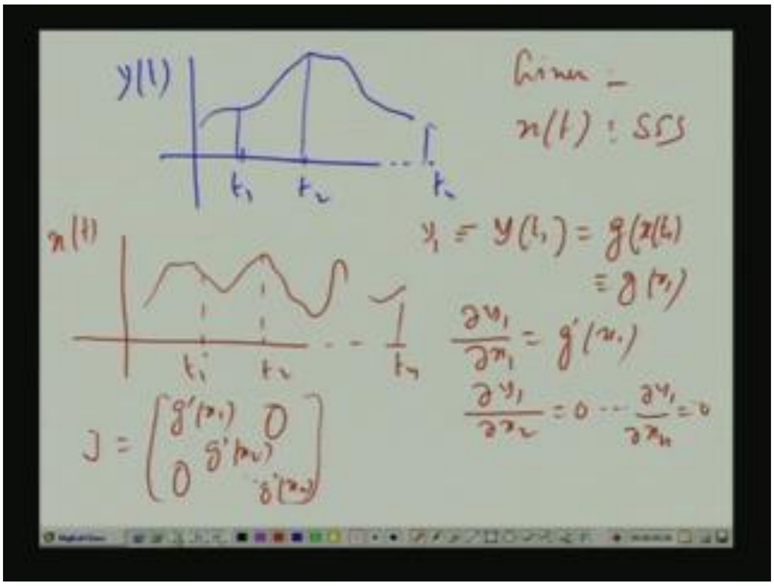

Let us let me write down first, will prove it first this is yt if I note time t1 t2 dot dot dot dot dot say tn. We will find the joint statistics joint probability density or distribution any 1 will do. For the samples of y at these values t1 t2, and then if that joint density does not undergo any change if a constant shift is giving to all these time points that is instead of t1 I go t1 plus c t2 plus c tn plus c. If you still get the same joint probably statistics then by the definition its strict sense stationary. We will check suppose it is giving that input is strict sense stationary, that is x t if you take a waveform given xt given is SSS. Now, you remember we did some study on this.

You know joint statistics of a group of random variables and that is given joint density for x that is x t1 x t2 dot dot dot up to x tn. If I take those variables and if, I take those random variables take that joint density, how to obtain the joint density for y. That is how to obtain the joint density for y t1 y t2 and y tn dot dot dot dot dot y tn. So, there is a formula that involved the joint density for x that is xt 1 xt 2 up to xtn and the Jacobean what also Jacobean. Jacobean was in this that Jacobean you remember in this case for the Jacobean will turn out to be this excuse me is this. First this y t1 it is a function of just x t1.

It is not a function of the other variable x t2 x t3 x tn, that is y t1; obviously, del y, if you call y t1, if you denote y t1 as y 1 and xt 1 as x 1 this is the x1; obviously del y1 del x1 is g prime x1, but del y1 del x2 is 0 dot dot dot del y1 del xn is 0. What is x2, x 2 is x t2; what is xn that is x of tn it is a it is a short notation. Similarly, when you consider y at t2 and call it y2 then del y2 del x1 that is 0. Because y2, that is y t2 is just g of x t2 that is g of x2. So, we can under we will see the Jacobean becomes a diagonal matrix, g prime x1 g prime x2 dot dot dot g prime xn and 0 here 0 here and that expression involved the determinant of this matrix. So, determinant is obviously, product of this mod of determinants, so mod of the product of the diagonal entries.

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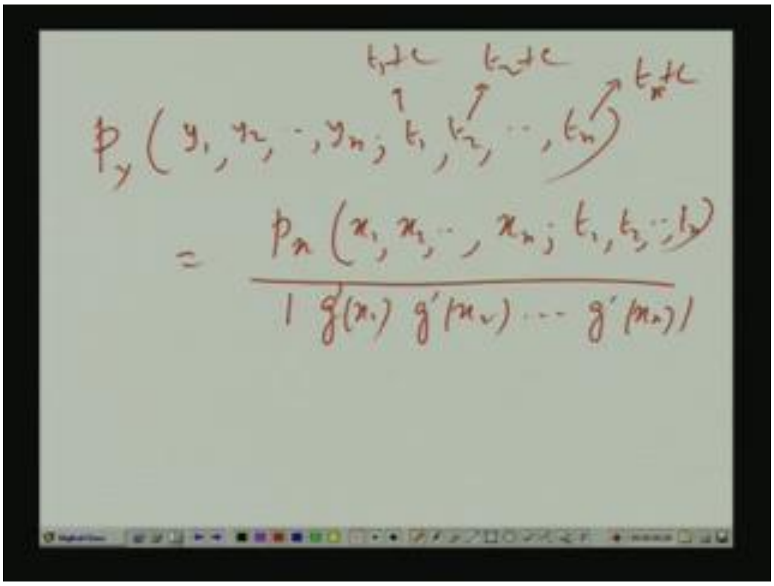

So, if you do that what happens to the joint density that is; this y stands for the fact that you know, I am looking at the joint density for the y variables at say y1 y2 dot dot dot yn at what values t1 t2 dot dot dot tn that will be Px by that formula divided by the Jacobean expression and Jacobean you have seen it will be mod of g prime x1 g prime x2 dot dot dot g prime xn. Now, we are we are giving the fact that, input is strict sense stationary

So, you see is it true for output? For that if I give a constant shift if t1 is changed to say t1 plus c, t2 is changed to say t2 plus c, tn is changed to say tn plus c.

Then here also, by this formula t1 should be changed to t1 c t2 should be changed to t2 plus c tn should be changed to tn plus c, but since input is strict sense stationary this will not produce change. I will get the because the shift is constant, because of strict sense stationary, I will get the same expression. So, it does not make any change here and we see denominator this is independent of time. So, it does not get changed by this shift in t.

So; that means, there is no change in the lhs expression, so whether you give keep t1 t2 up to t n or give each of the values a constant shift by c which; obviously, means that output was a strict sense stationary and if strict sense stationary it is WSS also. We now, consider a particular case of systems called linear system, which is not necessary memory less system.

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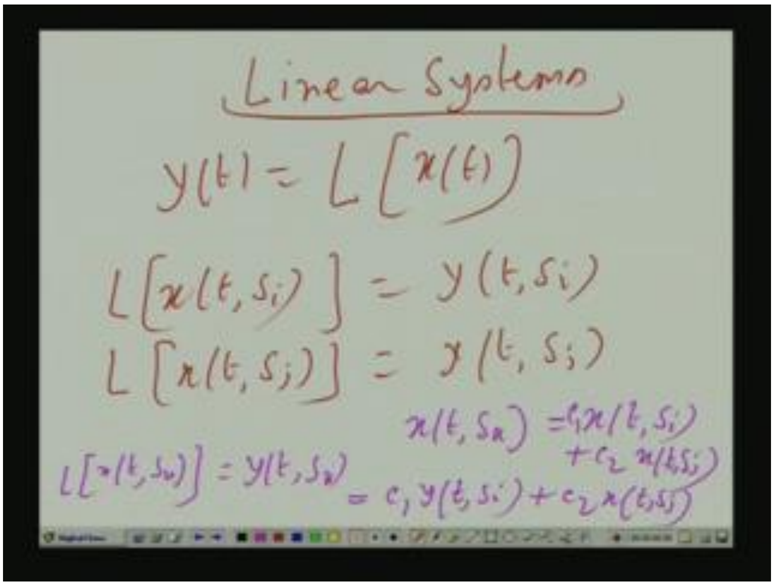

This linear system that is another important system linear system here, we use the notation L to denote for that stand for that operator system. Because as L stands for linearity system is linear if suppose you give 1 waveform for a particular outcome x.

Suppose xt si L of that is some waveform yt si, again xt sj another waveform yt sj; now suppose you have an experimental outcome where the waveform is actually a linear combination of these 2 waveforms. That is suppose, you have got a situation where xt sk is nothing but, linear combination of these waveforms. Whatever you had for this c1 times that plus c2 times xt sj.

Now, if it is linear then L working on this guy, it would generate; obviously, yt comma xk the waveform for y for the same outcome xk. If it is linear, if these become same as the response c1 time, the response due to this first waveform that is xt comma si plus c2 times the response due to the second waveform xt comma sj. That is, if you it is transferred to be c1times; yt comma si this particular waveform plus c2 times xt comma sj; which means principle of super position should be valued. That is if you have 1 waveform xt comma si you yt comma si. Similarly, another waveform xt comma sj get yt comma sj; now, if you linearly combine them as c1 xt comma si plus 2 xt comma j 2 waveforms are linearly combined.

The corresponding response should be simply c1 times, the response due to xt comma si that is yt comma si pus 2 times, the response due to xt comma sj; that is, c2 times y this t comma sj. So, the principle of super position should be valued. If that be, so then it is called linear system. A particular class of linear system is linear time invariant system, I will come to that later, but before that let us do some let us review recall something from system theory that is how, to describe a continuous a continuous time system. Now, you all are familiar with this thing this direct delta function or unit impulse function delta

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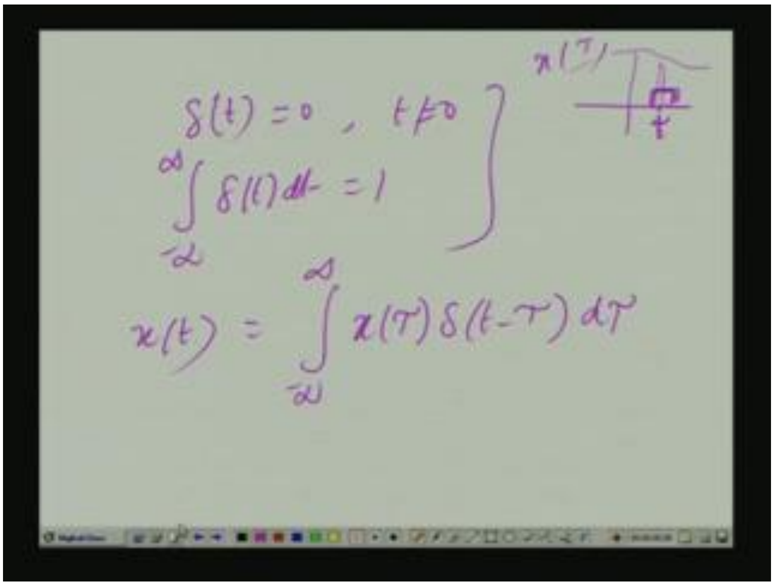

What is delta? Delta t such that is 0, if t not equal to 0 and integral of delta t dt all of this is direct delta function or unit impulse function. You all know this, that we start with box function whose area is 1 start compressing the bow, but keeping the area 1. So, the height increases final in the limiting case it becomes that impales area remains same as 1, but since outside originate is 0; at originate putting up to infinity that kind of function. Then since it is not a class on system theory, I cannot go into those details, but any waveform any functions say xt.

For the time being for convenience I am dropping this second parameter that is earlier I was writing xt comma si or xt comma sj. For time been, I will just write it has x t meaning waveform that is a function of t, but actually since there are coming from random process, I should also mention si or sj indicate the particular outcome with which 0 waveform is associated, but for the timing just for covariance I am dropping that a notation, I mean dropping that symbol. So, now given any xt; this there in systems theory you can verify also you can write it as this.

You obviously, get xt, because again this delta as far as this delta t minus tau delta t minus tau is a impulse shifted at tau. That is you are doing an approximation of the impulse first by taking a new box of any area 1, you have shifted into a tau earlier it was it sent at origin and now you start compressing it here. And finally, it in the limiting it becomes and impales at tau. Now, what is happening is you are not only taking the box multiplying the box by a value x tau. So, area is not 1, but x tau and now, start from compressing it. So, area will still given x tau, actually we are multiplying with your multiplying delta first your approximating delta t minus tau by x to box of area 1 and there is a function x tau, this x is tau you are multiplying it by tau.

You have shifted in to t actually, because this all these are function of tau t is constant t is your choice from outside. So, t is fixed;, so there is a box approximate in the delta there is function of tau. It is center is t remembers delta t minus tau or delta tau minus t they are same. Delta t such a function whether t you replace by t minus t or keep t it does not make any change. So, actually delta t minus tau means delta tau shifted by t. So, centre is at t; you are multiplying it by x tau, when you multiply the 2; these are only in this area only where the box is located you get a multiplication otherwise everywhere else is this function x tau is multiply by 0.

Now, you start compressing the box keeping the area 1. So, in the limiting case what happens is this, you get a value xt, because if the box is sufficiently narrow, then what the area is of box not just 1, but multiplied by the value of the function. Now, if the box is sufficiently the narrow that function its infinitely small width then that function as actually approximately value equal to extreme that is the value at the centre point that is the x t time in the box area that is x t. You still compress it further, so area will be you will get finally, and you’re integrating. So, you get actually that is x t details have given in system stage book I just this is just to kind of recap, so that you know we do not get really confused.

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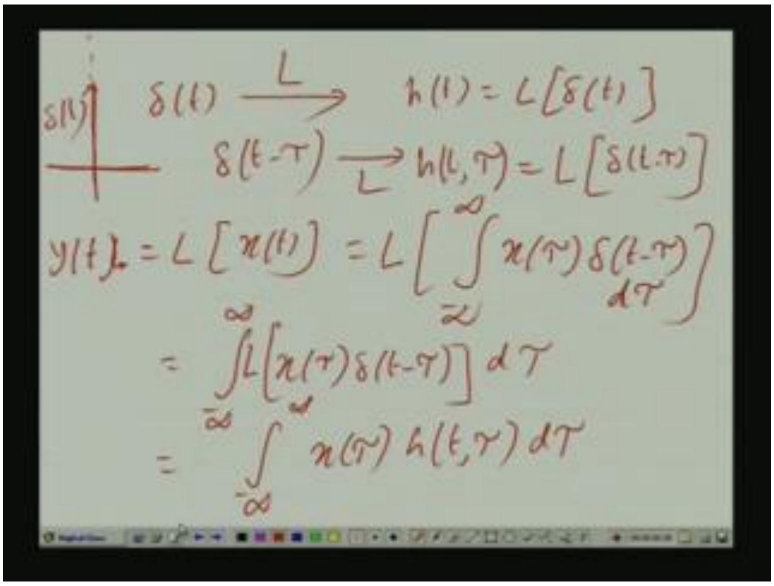

Now, if such a system is pass through a linear system what happens; in general delta t that is an impulse at origin we denoted it by this delta t. If delta t is passed through a linear system output is called the impulse response ht as L of delta t. If instead of delta t, I give delta t minus tau this is more general case. So, in the general case the output here will depend on what, the 2 indices if the function of t I agree, but it also depend on that shift parameter tau by which that delta has been shifted. If the system is shift invariant or time invariant then it should only depend, it should not depend on it should only depend; I mean the waveform that you get here will be same as ht just ht will be delayed by same amount tau.

That is without any shift or without any delay in delta t you got ht; now, you delay the delta t by some tau. So, corresponding response also should be ht minus tau. That is, its previous response delayed by the same amount, but in general, if that happens when the system is time invariant. That is, there was a gap of tau in giving the input delta t that is why, I am giving now, input delta t minus tau within that time period tau the system has not change its property system is time invariant and therefore, the response now also will be in nature same as the previous response ht, but if you plot it on the same scale
It is now again delayed by the same amount tau.

So, on the same scale if you plot it, it will be not ht, but ht minus tau that happens in the time in variant case, but in the more general case, system may change its property within the period tau. So, output for delta t minus tau will not only will depend on 2 things

It will be a function of t, but it will also a function of tau, if instead of tau you delay the impulse further by say tau prime. The corresponding impulse response might be different, because system is undergoing change in property with time. That is why, in the general case, it is a function of 2 things ht comma tau.

Only when it is time invariant it is not ht comma tau, but it is just t minus tau. That is same ht delayed by the same amount tau, because input is delayed by same amount, but in general if is not system time invariant that is if property also changes with time. Then impulse as a function of time should also depend on by, how much this parameter tau by which the input is shifted. If it is shifted further may be I will get some other kind waveform, not only just the delayed version of ht, but even the nature will change. So, let us take this;, so in this case yt, we have already express xt in terms of like this in previous slide and L now, it is a super position, because of integral and system is linear, integral only mean summation, but if it the system is linear super position principle works.

So, if you apply the system after the summation you should get the same thing; if you apply the system before that is L over this integral is same as integral over L working on these inputs. Remember system response over a some input same as some of the individual input in responses. That is, because of linearity you should have L working on this integrals is only a summation that is individual responses summed. Earlier I was taking the response of by first summing the various input by the integral then apply to the input, because of linearity that would be same as first applying individual input x tau times delta t minus tau getting the corresponding response and then summing the individual responses by an integral and remember here is function of time on delta x tau is kind of amplitude.

So, L will work only on this, so this is x tau and I know L working on delta t minus tau, as I said earlier it will be function of 2 variables in the general case ht comma tau remember this result. Now, using this result we will prove some result some other results.

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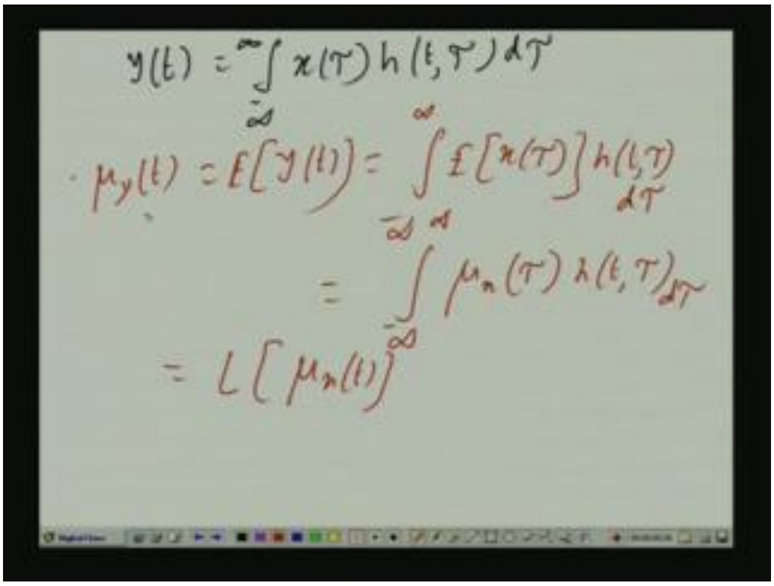

So, I write again first if I take the mean of this mu y t at particular time t that is E of yt that is E of the right hand side, but right hand side is a summation and remember E was, E that expectation operator is linear, that is E expected value of sum of random variable is same as some of the expected value of the individual response random variable. Only thing is here you have got a continuous sum in the form of integral, but still the linearity property of E operator prevails. So; that means, this is same as you can put push E inside the integral and h is not random.

So, E will work only on x that is mu x tau ht comma tau d tau and that is same as L working on this function. Remember when L was working on any function xt x tau ht comma tau the tau integral, but instead of xt now other function mu xt. That is why mu x tau ht coma tau d tau;, so as though the mean function that is if you take that mean or expected value of x you again get a waveform. It is not a random waveform, but waveform if you pass it through the system whatever output you get that is what you get, for the mean waveform of the output this is number 1 result.

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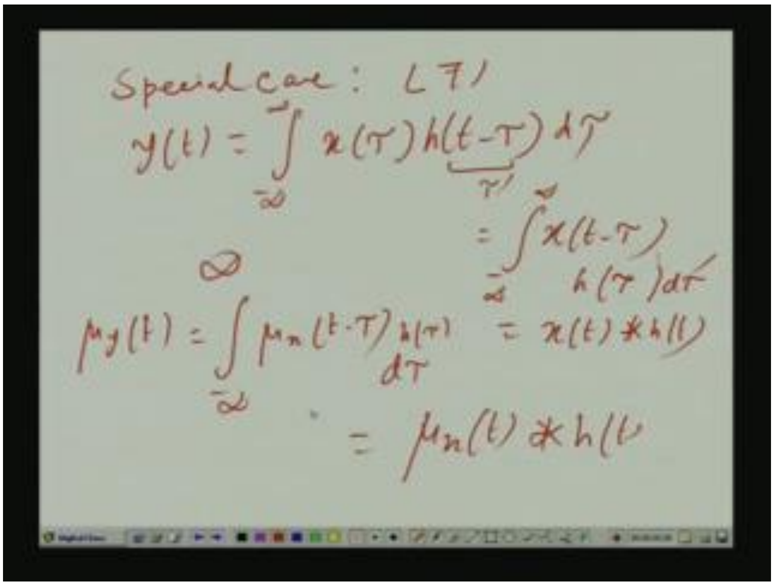

In the special case, when it is LTI linear and time invariant LTI. In that case, we know that yt is not t comma tau, but it is actually really a delayed percent by tau. If giving delta as input you got ht as impulses response now, delta d shifted by tau. So, ht also will be shifted by the same amount tau. That happens where when it is not only linear it is linear an time invariant that a special case. And they if you replace t by minus tau y tau prime So, tau is t minus tau prime and then you see d tau is minus of d tau prime and when tau is minus infinity tau prime is plus infinity. So, plus infinity here minus infinity here d tau means minus d tau prime prior minus has come you reveres the 2 limits again.

So, minus infinity another minus comes. So, it becomes plus and you get back x tau prime h you get back this xt minus tau prime h tau prime d tau prime. I repeat again if you replace t minus tau by tau prime. So, d tau prime is minus d tau, so d tau is minus d tau primes. So, minus is coming you look at the limits tau becoming minus infinity means; tau prime plus infinity and here, minus infinity. So, plus infinity to minus infinity Reverse the limits minus infinity to plus infinity. So, another minus sign, so 2 minuses cancel each other and you get back this relation. And no point in carrying out tau prime you can again bring back your good old tau.

In such case mu yt is what and this is call convolution between 2 sequences xt and ht this is the formula. Now, in this case what is mu it, mu I t is apply E put the E inside E works on ht minus tau that is mu xt minus tau multiplied by h tau d tau.That is again convolution between these 2 functions mu xt and ht this a formula then let us find out this that.

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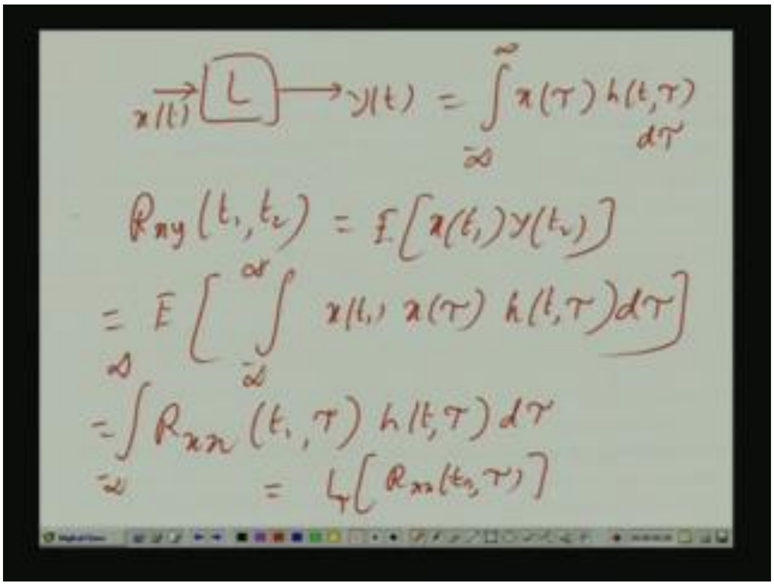

There is system input is xt there is system linear system not this a time invariance output is yt. We know yt equal to remember only when it was time invariant it was it becomes a function of t minus tau and there I could replace t minus tau by tau prime and all that. I could write it in the reverse way, that instead of writing x tau ht minus tau could also write h tau x t minus tau, but when this is not time invariant, I cannot proceed any further from here. So, I have to stick to this to most general I still keep the linearity only I do not bring in time invariance unless a necessary.

So, for the linear case; suppose, you want to find out 1 function on cross correlation function Rxy say t1 t2 that is expected value of x t1 y t2, what is it? Now, you t2 you know, you put in the formula that is x tau h t2 minus t2 comma tau d tau. And x t1 multiplying that;, so we bring that x t1 within the integral. So, x t1 brought within the integral y t2 for that x tau. Now, expected value can be brought inside, because expected values of summation are summation of the expected values h is deterministic. So, E will work only on this x t1 into xt tau. So, what you get is Rxx t1 comma tau ht coma tau d tau.

So; that means, what is it, that if you take it as a function of Rxx t1 comma tau you just forget t1 for the time being take it as a function of tau then isn't it. That it is the same thing. If I will get, if you give the function of tau, forget t1 is 1 function of tau to the system that is input. Now, output; obvious, will be this function multiplied by ht coma tau d tau that is why I am getting here just to indicate that here L means, the integral will be with respect to tau, I denote tau here, I put tau here; that means, you take an integral multiply Rxx t comma tau into ht coma tau integral with respect to tau. Remember if the system works on any function on any quantity.

So, function of tau here that is why this tau is indicated L tau tau is a function. then what does is same do you multiply this by ht coma tau, integrate with respect to tau. So, that will give you the output at t. Here it is t 1, I will it will be better if I general if I simplified further for the LTI case that will be better, because actually this is this should be y x t1 y t2 this will t2, because this should be t2 . So, when I say L tau means you give this input or Rxx t1 in case my handwriting is not perfectly legible, you give this function Rxx t1 comma tau as a function of tau to the input to the system input were system evaluates the output at a time t2.

So; that means, this functions should be multiplied by h t2 comma tau integral with a tau that will give you the output at t2, that output will be function of t2 and also this t1 will carry, because t1 is given as a parameter from the beginning. So, it will be a function of t 1 comma t. In the special case, where it is LTI its time invariant also let us see what happens.

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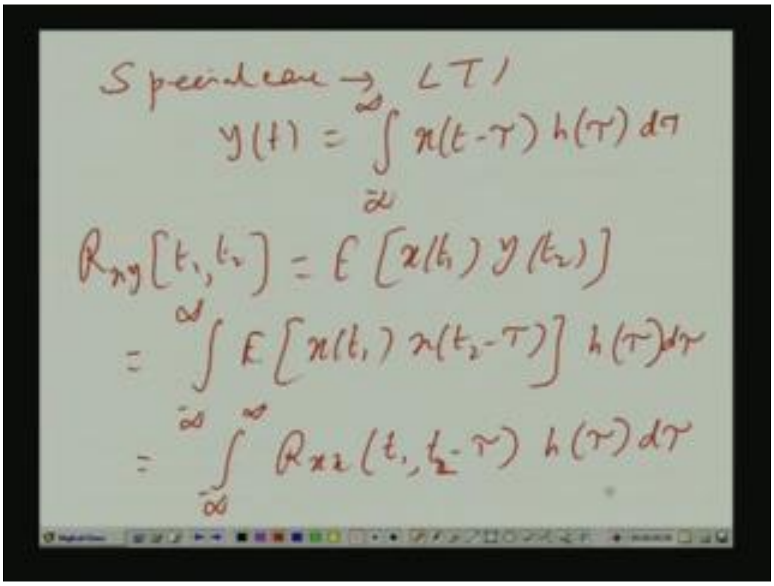

Special case LTI in such case, I know that y t is, so what is Rxy t1 comma t2 that is E of x t1 y2. So, for y t2 put t equal to t2; here, multiply I mean push this parameter x t1 from outside the integral t2 inside integral and then bring in push this E operator inside the integral again. So, what you get there is this E working on x t1, that has come from outside the integral then x say its y t2 it should be x t2 minus tau h remains separately, because it is not sophistic and what is this function this is Rxx.

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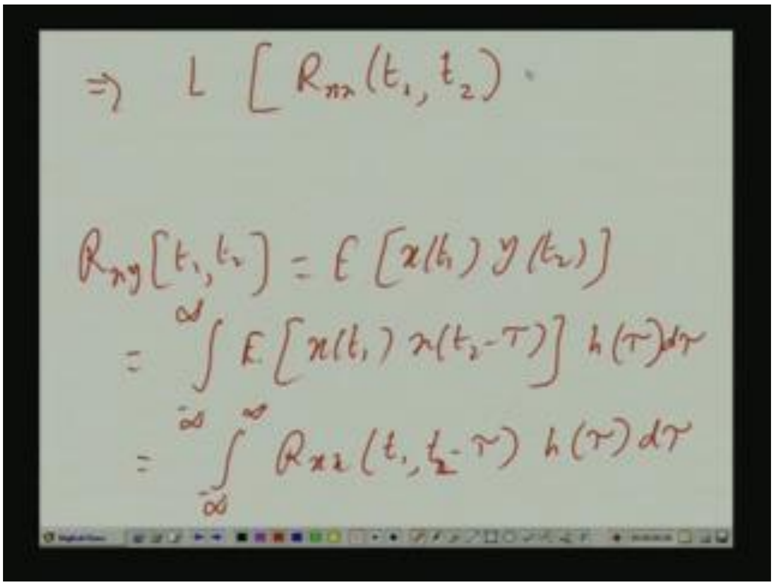

t1 t2 minus tau that means, we can write now this is equal to L Rxx t2 and I write t2 here.What does it mean this will be treated as a function of t2 only; if the just to indicate that I put t2 here. If that will be this function is given as a function of t2 as a input to the system. So, what is the output and the LTL case. Simply, this will be replaced by t, I mean here instead of t2; I will have t2 minus tau that is what is happening here multiplied by h tau integral. These are cross correlation between input and output and what is the auto correlation in output.

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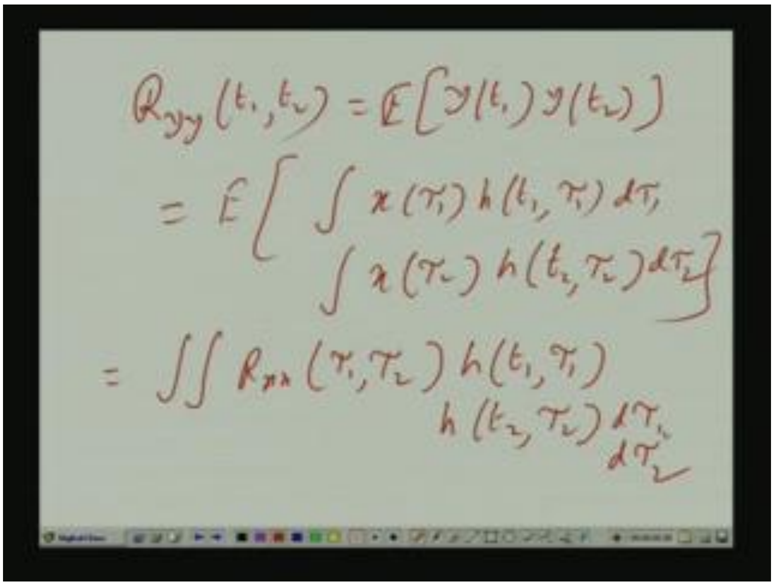

That is what is your Ryy t1, t2; this more interesting here simply replace by those expressions y t1 was in the again in the just linear case not necessary time invariant. So, y t1 for that we will have x tau 1 and then here x now there variable tau 2; take the E inside the integral and on x into x others are not stochastics. So, you get Rxx here it will be better if I take the time invariant case.

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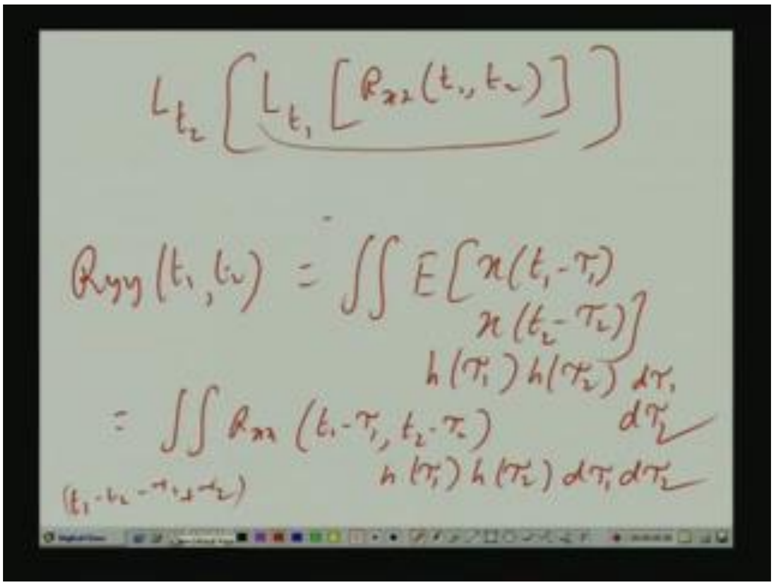

If it is time invariant there instead of Special case we consider now, LTI here. So, Ryy t1 t2 will be what E now, I am doing first Ry means E y t 1 into y t2. So, E of x t1 minus tau 1 x t2 minus tau 2 h tau 1 h tau 2; that is, Rxx what is the physical interpretation of this very interesting this means what. If we just consider this inner integral Rxx t1 minus tau, tau 1 t2 minus tau 2 multiplied by say h t 1 h tau 1. So, that times it is the function of just tau 1.That means, first you are giving this guy suppose I do this; first t1 t2 and then I will Lt 1 first what does it mean; it will give rise to an output, what output Rxx t1 minus tau ,t2 h tau 1 delta 1 integral.

Apply here, so again it is it was a function of t2. So, now instead of t 2; it will become t2 minus tau 2 and h tau 2 will come again another integral. So; that means, this input auto correlation function is pass through the linear system, where it first exceed as a function of t1. Then the corresponding output which is the function of t2 is again pass through the same system this time it takes this intermediate output as a function of t2. A particular thing you see if input is the wide sense stationary; If x is WSS then; obviously, Rxx t1 minus tau 1, t 2 minus tau 2 it will be the function of just t1 minus t2 minus tau 1 plus tau 2; tau 1 and tau 2 their integration variables. So, once you carry out the integration there will be no trace of tau 1 and tau 2 there will be.

So, entire thing will be function of just t1 minus t2;, so that means, Ryy t1, t2 will be just function of t1 minus t2. So, it is not individual t1 or t2 will matter, but the gap which means output also will be WSS. I mean in term of correlation in term of mean also it will be WS we have seen earlier. How, because you have seen that if input is a if input mean function you take mu xt then output is mu xt convolute with ht. So, if mu xt, because of stationarity it is a constant thing. Then the convolution will also yield a constant function 1 thing does not need any proof, we just make a statement we make a statement that if a system is linear.

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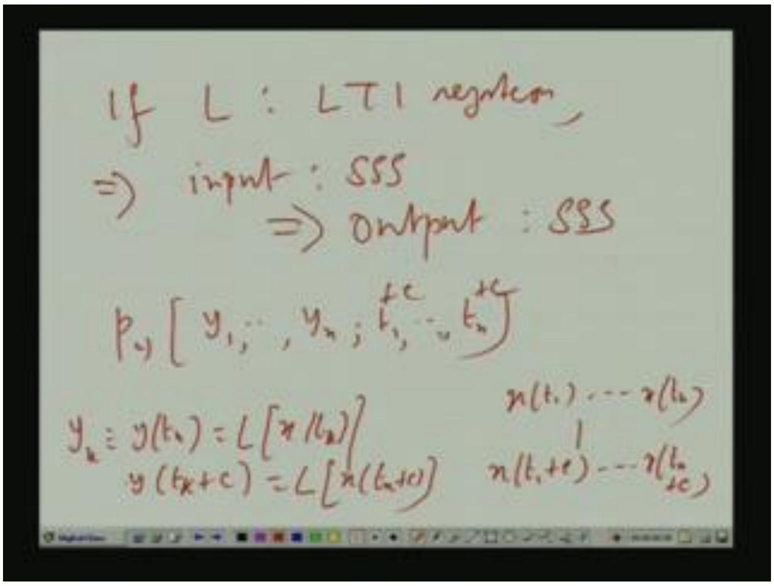

If L denotes an LTI system then input SSS means, output also SSS. Now, start with difficult output say you are tried to find out this y1 that is y of t1 dot dot dot say yn , t1 dot dot dot tn. What is y1? That y at t1.What is y at t1? That is L of x t1. Remember I just say yk is what is equivalent to y of tk that is equal to L of xtt; suppose, you have somehow value of this joint density. Now, instead of t1 or tn suppose I give a shift plus c plus c ; that means, that time what will be yk it will L of x tk plus c. Now, you remember you understand that input this; if in the earlier case when there was no such c added the joint density will become actually function of the input statistics, because after all y1 y2 yn they are coming from input;y1 is L of t1 L of x t1 y2 is L of x t2 dot dot dot.

So; obviously, the joint density will depend on input statistics that is input joint density. Now, if I want to find out the same joint density not a t1, but t1 plus c tn plus 2 plus c tn plus c. It should depend on input device statistics for what samples x of t1 plus c x of 2 plus c dot dot dot x of tn plus c, but you see input is stationary strict sense stationary. So, whether you have the variables x of t 1 dot dot dot up to x of tn or you have got statistics wouldn't change, because input is given to strict sense stationary. What the LTI thing comes, because we said that, if y1 is L of x t1 that is a y of t1 y1 is y of t1. If y of t1 is L of x t1, then because of time in variance y of t1 plus c is L of nothing but x of t1 plus c. That is, y of then tk plus c, because of time invariance it is…

So, when you have t1 plus c, dot dot dot tn plus c the corresponding joint density again will depend on the joint density of x of t1 c, dot dot dot x of tn plus c, but, because of strict sense stationary of the input, that will that joint density will be that statistics will not the different from the earlier 1. When this constant shift c was not applied; obviously in the output case also 2 density will be same and you get strict sense stationary. As an example of this linearity you consider a differentiator.

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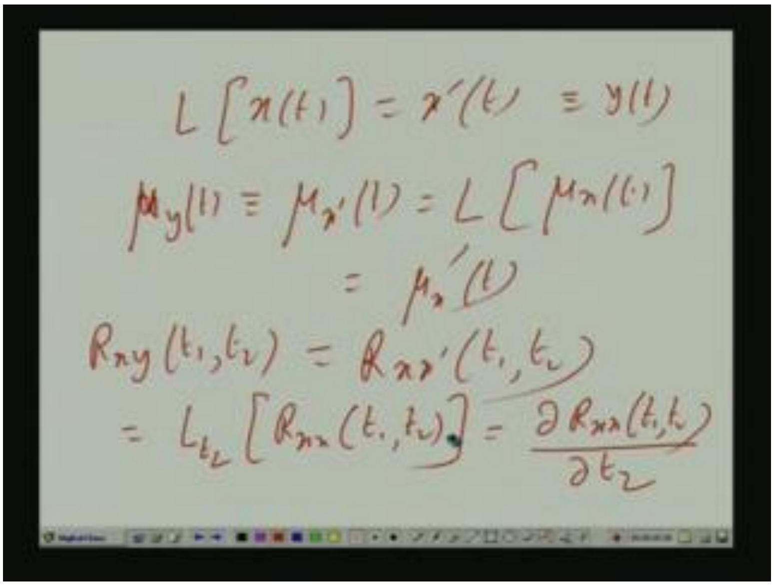

A system which takes xt and gives you first order derivative; it is obviously linear, because we know the derivation is linear derivation is a linear operator. So, what is x prime t actually is stands for yt here;, so what is mu yt also you can denote as mu x prime t. That is by our definition that will be like L working on mu xt and L working on this means what you will have to differentiate it with respect to t. Then Rxy t1, t 2 which is also you can write as Rx x prime t1, t2; that we have seen L work on t2; we have seen earlier of what this input auto correlation function. Let us take it as a function of t2, and then let the system work on it, but system work on it means then it will be differentiate it again with respect to what t2. So, it will be del and just a couple of minutes and I will go ending up.

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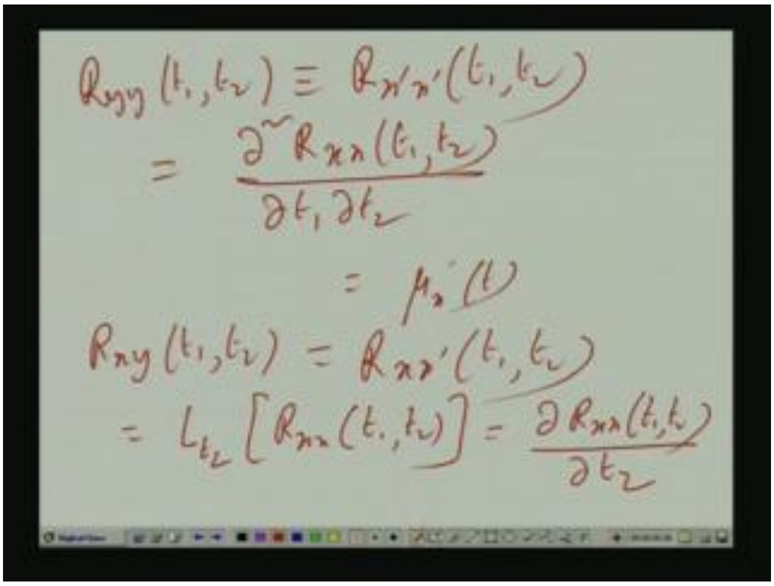

The output Ryy which you can also denote as, that will be what; obviously, you have to apply this Rxx t1,t2 first with respect to say t1, I mean you to say Rxx t1,t2 will it pass into the system passed. As function of say t2 then output will you past through the system, but that time is function of t1 or vise versa that you have seen. So, if you do that that will be; obviously give rise to the double derivative first with respect to t1 then with respect to t2 or it does not matter it can be first del t and del t1 and then del del t2 or vise versa.

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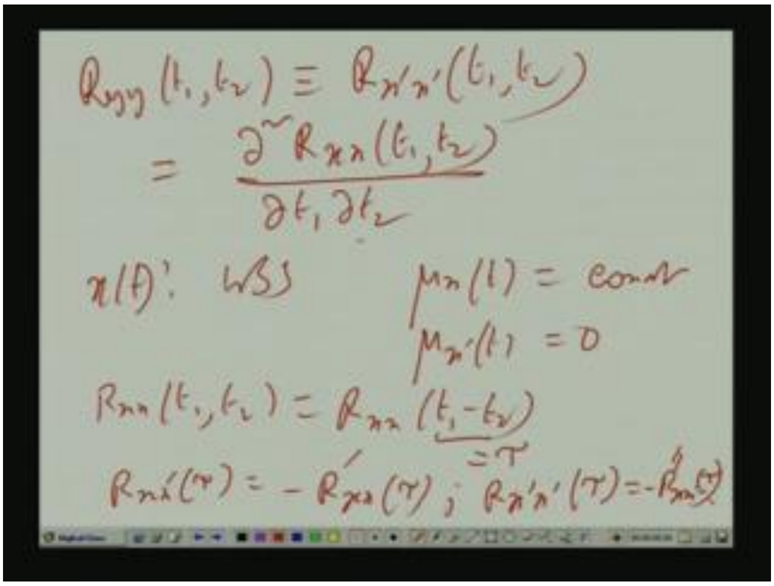

Suppose xt WSS; that means, mu xt is constant then mu x prime t which is derivative of this is 0. So, again in stationary mean wise you can easily have it verified here, that is for the output also mu is constant, then how about auto correlation that is how about this Ryy t1, t2. Now, here we have got Rxx t1, t2 is a function of actually Rxx t1 minus t2; t1 minus t2 equal to say tau, so its function of tau. So, here, but I know its 1 minute here, if you take this derivative del Rxx t1 minus t2; t1 minus t2 is tau we differentiate with respect to t2. First and t1 first I limit as an exercise to you; you can easily verify these 2 things. Rxx tau, Rxx prime tau you can easily verify will be Rxx tau first order derivative and Rx prime x prime tau will be second order derivative. So, that is all for today.

Thank you very much.