Probability and Random Variables

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Lecture - 31
Cyclostationary Processes

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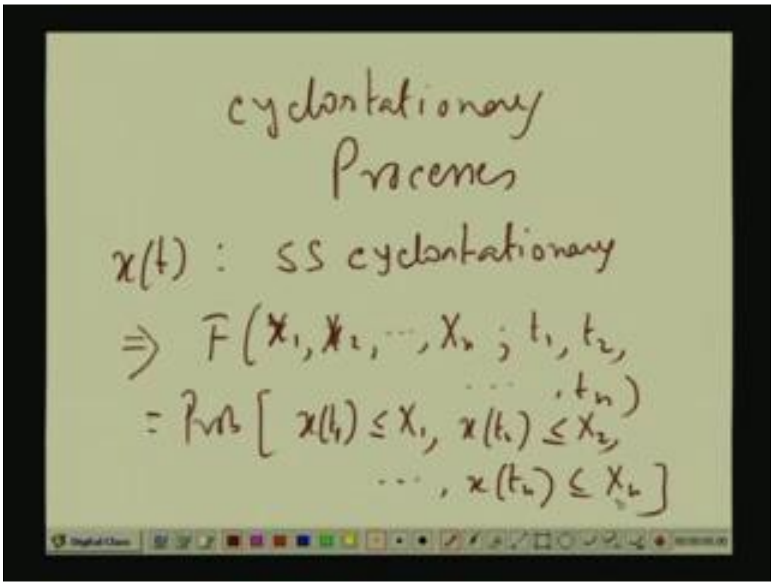

So, we start today important topic that is Cyclostationary Processes. It is a special kind of random process, which is not fully stationary, but it does exhibit some sort of stationary and occurs frequently in many applications especially, in digital communications also called periodically stationary processes. Actually, these processes are such that their statistics they vary periodically over a constant type period. That is if I talk of a Cyclostationary Process which is strict sense Cyclostationary. That is if xt is SS; SS is stands for strict sense Cyclostationary.

So, this would imply that if you take the probability distribution of F that is let me, make it capital. By the way, what does this mean though we have already done it, but still there is nothing wrong in just restating what you already know. This is the probability joint, probability distribution function of the of the random process xt. That is it is a joint probability of, that is by definition what is this probability of…

So, what x at t1 lying less than equal to x1 x at t2 lying less than equal to X2 dot dot dot x at tn lying less than equal to Xn. These are joint probability, this is called the probability distribution. And this number n this the number of sampling points you have chosen that is purely up to you, You can take it from 0 to up to infinity. Now, if the processes is stationary, we know if the processes stationary Strict Sense stationery not Cyclostationary.

But Strict Sense stationary that time you have seen that these joint distribution for any n that you choose that does not depend on this exact values of t1, t2 to tn. Rather, if all of them are given a constant shift or constant delay. Then that is if t1 is replaced by say t1 plus c t2 is replaced by t2 plus c the same c dot dot dot tn is replaced by the tn plus c. In other words, if the time origin is shifted by the amount c.

Then, still you get the same distribution function. That it is the statistic does not depend on the exact location of time. There is the relative positions of t1, t2 to tn that is what is important. Then, that is Strict Sense stationary In this case if it is Cyclostationary then this would imply that this is equal to this should imply that this is equal to…

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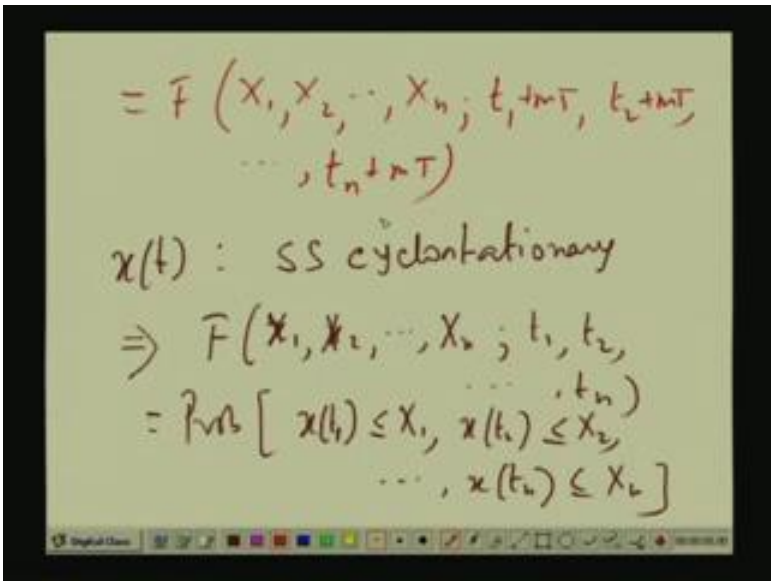

If instead of t1 I get a constant shift which is the integral multiple of a basic time period T. So, t1 plus either t or 2t or 3t like that say mT; M is an integer. Then, again t2 plus mT m m is an integer dot dot dot just a minute. Then, dot dot dot tn plus mT if they are same. So, that is that joint density joint probability distribution function and also therefore, density whatever it is value may at this time points t1, t2 up to tn; you will get the same thing.

If you move over either move forward or backward by T for each of the indices or move 2t or 3t or minus t, minus 2t, minus 3t like that. But in other words, if the time origin is given a shift either to the right or to the left by integral multiple of T, then and only then it will repeat.

So, it is not fully stationary it is not that you know these 2 distributions are same for any shift that you get, but the shift is only m into T is not c. I mean is not true for any c, but the constant c that is the shift of the time origin that is only some integral multiple of a basic period T. Then this is called a Strict Sense Cyclostationary Process or also periodically stationary process. Obviously, this means that suppose you take.

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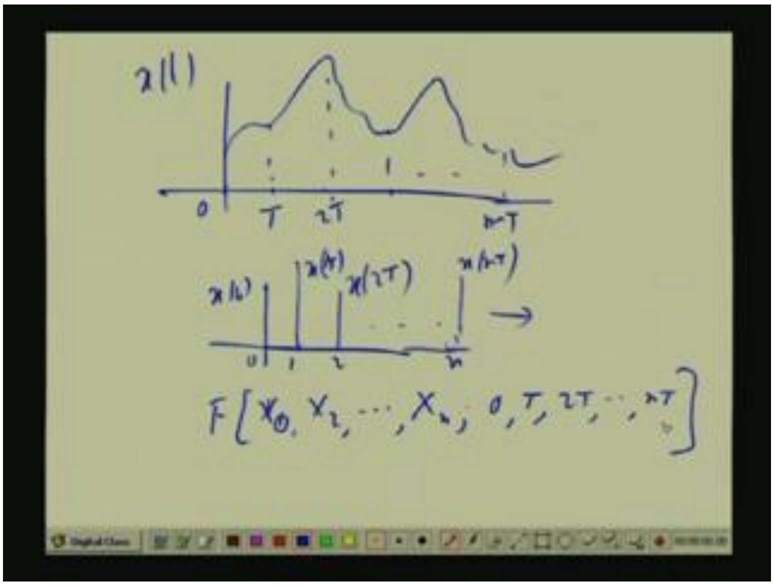

Suppose this your xt what you have at you take at 0 t equal to 0 then at T at 2T dot dot dot you take these sample at mT. Now, if you just take them what does that mean that, if you take they are this sequence, the sequence that is x0. Then, the first sample second sample is x1 then x that is xt then x2 T at 0'th, 1, 2 like that. Then, you can easily see that this sequence becomes actually a stationary sequence.

The Strict Sense stationary sequence obviously, because if you want take the joint density here. If you take say few samples, if you take up to the few sample. So, 0 up to n n'th sample xn T. And you take the joint density that is basically, the joint density here of the sample. Now, if you give a shift there for any amount the shift is actually in integral multiple of T that is if you shift.

So, from 0 you get to 1 from 1 you get to 2 and from 2 you get to 3. Basically, in time axis you are getting shifted by either T or 2T or 3T like that. So, by from the Cyclostationary T you should get the same joint distribution function right. So, if you sample it at T, T2, 3T and all that. Then, the sequence that you get that is Strict Sense stationary this must be obvious.

Because, what is this after all I mean the what is the joint distribution. This is nothing but x1 x2 dot dot dot Xn at time 0 T, 2T dot dot dot say nT you started x 0 say. And then if you sample if you give a shift to this sequence. So, 0 is sample moves to fast and like that. That means basically you are giving a time shift by T, but from Cyclostationary of the original process you should get back the same probability distribution function. So that means, there is some relation between or before coming to that.

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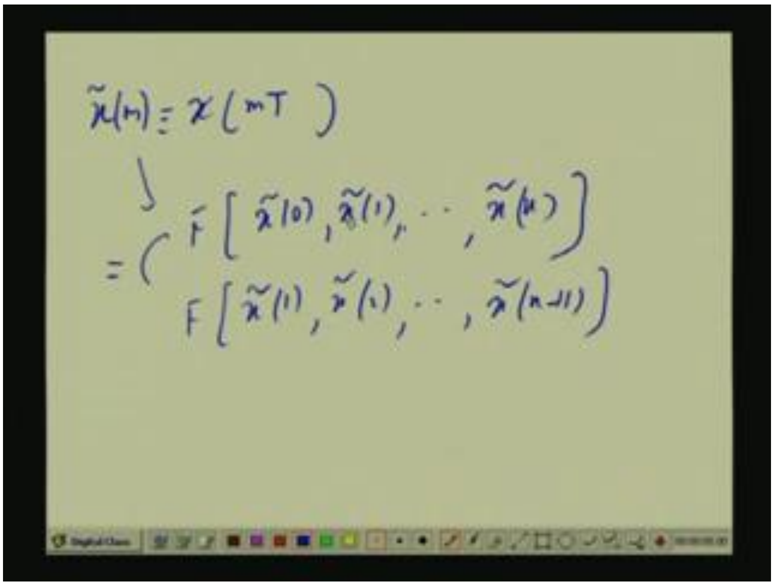

This means that if you really take if you follow the discrete process mT say some sequence xm as mT. Then, this sequence becomes strictly stationary. Because, here suppose you take as I you take the joint density, joint distribution x tilde 0 x tilde 1 dot dot dot x tilde say anything you can takes a k k sample. If state if you give a shift here, this is state you take x tilde this instead of 0 you jump by 1.

So, 1 x tilde 2 dot dot dot say x tilde k plus 1 these 2 are same that, because comes from the Cyclostationary of the original analog process. Because, this samples corresponds to where, which time indices 0 T up to kT. And here, instead of T instead 0 you are taking advancing by T instead of T you are advancing by 2T. Advancing by T instead of making it 2T and likewise. So, the sample what is and what you sampling at we have to get T that becomes a Strict Sense stationary process. In this context, some relation very much useful 1 is this.

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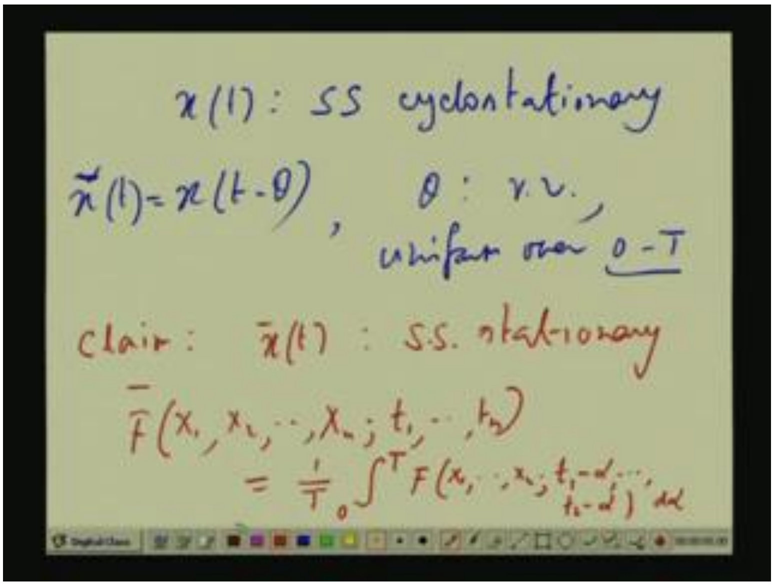

That suppose, xt is given to be SS Strict Sense Cyclostationary and instead of xt you suppose shift it by theta. So, you get another process we call it x bar t when theta is random, theta random variable uniform that is uniformly distributed over 0 to T, which T the same T over which, this is Cyclostationary xt is Cyclostationary and the basic period then is T. So, you are shifting it by a theta, but theta is uniformly distributed over 0 to T.

So, you get another random process xt in that but then our claim: x bar t is Strict Sense stationary strict stationary. And if you take F bar stands for the distribution function, joint distribution for this new process x bar t. We take that is equal to this is our claim 1 by T integral 0 to T original distribution function xn. And t1 minus may be alpha dot dot dot tn minus alpha d alpha. So, we can prove this first. So, first we consider this what is the meaning of this distribution function.

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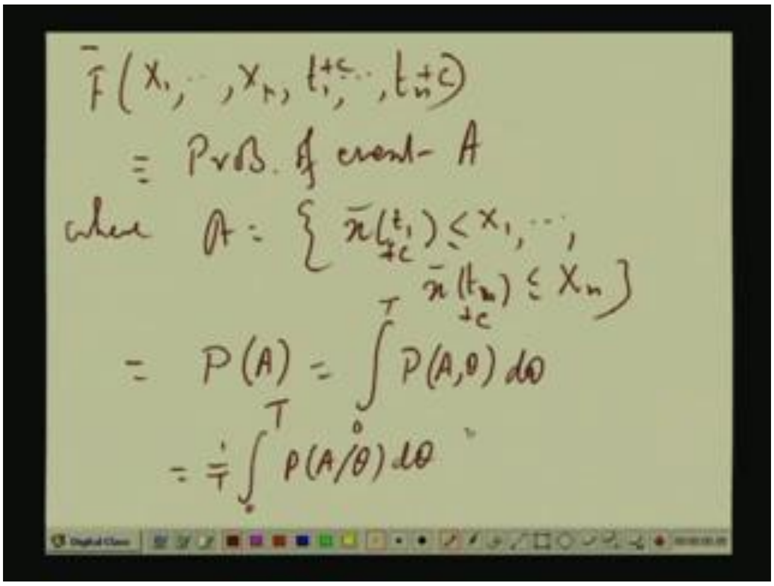

When I said F bar x1 dot dot xn comma t1 dot dot dot tn. What does it mean It means, it is equivalent to probability of event A where A is what this new process x bar t1 less than equal to x1 dot dot dot x bar t2 less than equal to x2 Xn, this a probability got it. Now, we have to show that this process x bar t is Strict Sense stationary. If it is strict sense stationary that means, instead of t1, if I give a constant shift c.

So, it is t1 plus c then t2 plus c dot dot dot tn plus c. And then this distribution becomes equivalent to probability of event A, Where event A is simply this that a time t1 plus c x bar within bracket t1 plus c less than equal to x1 dot dot dot x bar tn plus c again, less than equal to Xn this is the event. So, instead of t1 t2 up to tn I am just giving a constant shift a constant delay or advancement.

Whatever, you know to the time points by the same I am just shifting all of them by constant amount c. I have to show that, this probability distribution is independent of c. And then only it will be proved that this x bar t is a Strict Sense stationary process. That is whatever be the shift you give to the time origin on the time axis the probability distribution, the joint probability distribution does not change.

So, we consider this event A which is this that is the process. And value at time point t1 plus c is value taking less than equal to x1 comma dot dot dot it is value again, at time point tn plus c taking values up less than equal to Xn. The joint this is joint event you know these this means, so probability of this event is actually this distribution. Now, 1 thing x bar t1 plus c.

So, within that actually what is x bar t1 plus c after all It is x t1 c plus minus theta by definition of x bar. And therefore in this process x bar there are 2 random components: 1

is coming from the original process x bar t. So, every time you observe you get some data, but again that is delayed by another variable theta which is also random. So, there are 2 random you know 2 random phenomenon, that are occurring simultaneously, 1 is from this process xt, another the dealing variable theta.

So that means, PA is nothing but PA comma theta d theta this is joint density of the event A and theta taken some value. And theta is taking values over the range 0 to T. So, this is PA theta as we know is nothing but PA by theta into P theta and P theta is what theta is given to be uniform from 0 to T. That means, with within interval 0 to T it is probability density is constant and that is equal to 1 by T. So, that area is 1 so that 1 by T is P theta 1 by T comes out of the integral and the 0 to T d theta.

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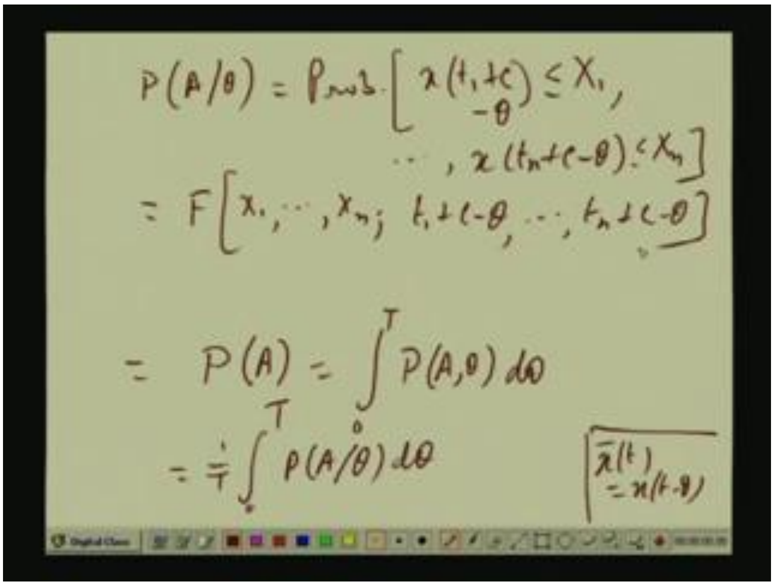

That means, that implies what, but what is PA by theta We know the event this is simply joint probability of what. For a given theta what is the value that, the process at time T1 plus T takes values less than equal to x1 dot dot dot it is time tn plus c that is x bar within bracket tn plus c less than equal to xn. But we know that, we know these thing that x bar T is xt minus theta and in this context theta is constant that is given a particular theta.

So, theta is no longer random. So that means, when I say that x bar t1 plus c less than equal to X1 this is equivalent to saying x less than equal to x1 dot dot dot x tn plus c minus theta less than equal to Xn. This is using our notation this is nothing but probability distribution of the original process x1 dot dot Xn integral. What are the time points t1 plus c minus theta dot dot tn plus c minus theta.

So, we have to substitute this PA by theta here in this integral and get that probability. That will be the joint distribution of the delayed process x bar t. And that will show, that is independent of this constant c. So, if you really remember this you are going to substitute for PA by theta by this expression multiply by theta integrate from 0 to T and multiply by 1 by T. That means, if I erase this part.

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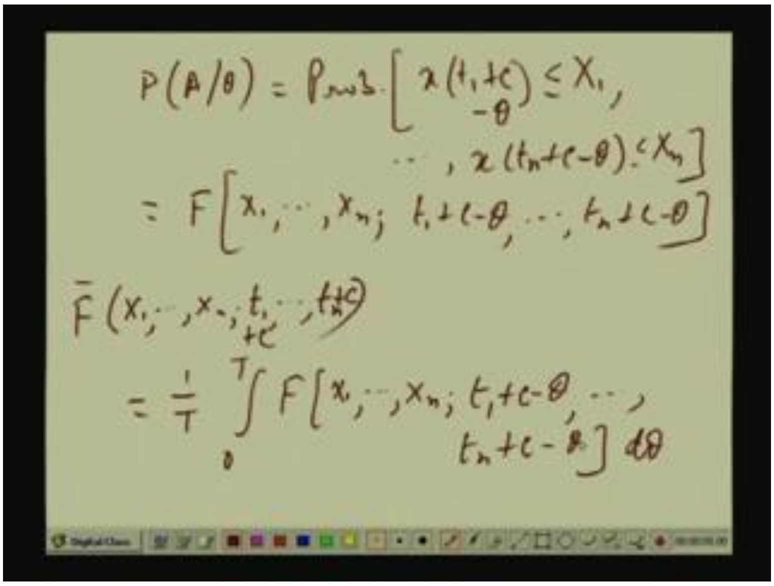

That means, our starting point was this was the starting point t1 plus c tn plus c This is equal to this you have seen PA by theta d theta integral 0 to T multiplied by 1 by T. So, I simply substitute with our expression F actually, from this only using your intuition you can I mean you can make this infinite And say loop theta is integral variable and you are integrating for 1 period, because this distribution is periodic.

If theta jumps over by an if theta is replaced by theta plus T, that is theta increases by it jumps by a factor T. You should get back the same probability distribution functions simply, because of the Cyclostationary t. So, this distribution is actually periodic in theta, because if a theta is replaced by theta plus T, you would get the same thing. Because, theta plus T means this net thing net time index for all the points they get I mean they get subtracted I mean T get subtracted.

So, that is minus 1 times T gets added. So, from Cyclostationary t you would get the same probability distribution function. So that means, this is in the periodic in theta and you are integrating this over 1 period. So, what value you should get that should not get depend on your t 1 plus c. That is it should not depend on your this constant c. Anyway, this comes from intuition we can easily prove it. We can easily get, what this expression would be. This is actually, simple mathematics now.

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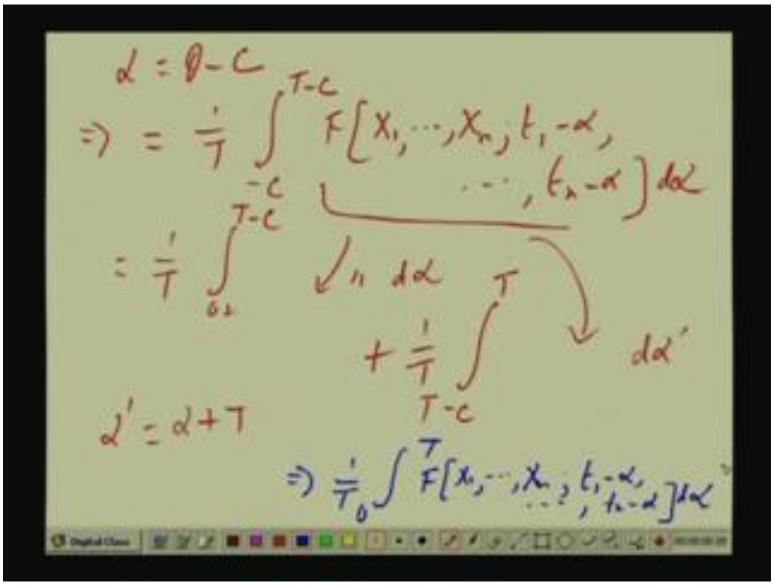

Suppose theta minus c. I say alpha equal to theta minus c. That means, it is integral will be what when theta equal to 0 alpha is minus c. So, from minus c to T minus c. That means, it integral will become equal to 1 by T d alpha. And now, maybe I can erase this part. Then, this integral 1 part it is from 0 to T minus c if you what you can call it 0 plus the same thing comes here d alpha.

Then, again 1 by T in other case it is from minus c to 0 minus the same thing d alpha. Let us consider this second integral is if suppose alpha is replaced by alpha prime plus t. Suppose, I bring a variable alpha prime which is alpha plus t alpha plus t, then what happens when alpha is minus c this becomes T minus c and when alpha is 0 this becomes T.

So, integral limit is changes to this will become T minus c this is alpha prime and this becomes T. And within this we have got this probability distribution function, but alpha to be replaced by alpha prime minus T. So, we have got instead of T1 minus alpha now we have got t1 minus alpha prime minus t tn minus alpha prime minus t. So, everywhere I am going to the constant shift by T and from the Cyclostationary t of the original process. You should get back, the same distribution function as given by this.

Because, going forward or backward by T or twice T or 3T would not change anything right. So, we get back the same thing, and then you replace again alpha prime by alpha by these 2 different variables. So, 2 integrals can be brought together. So, 0 to T minus C at T minus to T, so you basically would get finally, you will get these thing 1 by T 0 to T F.

So, we see this entire thing is independent of that original shift parameter c. So, this the expression we wrote that time. So, this proves that this process x bar t is a Strict Sense stationary process. Actually, this is of an important you know, because the communication sometimes I mean this is based by the way from that you transmit that.

Basically, you choose a pulse and then pulse is multiplied by the symbols and you get a pulse strain each pulse is modulated by a vertical symbol. And that say symbols are coming randomly then entire things becomes and continuous value random process that will becomes Cyclostationary.

But then often that process gets delayed while this in the receiver. And the delay variable theta or phase shift whatever, now that is often found to be uniform as a result the delayed process turns automatically strictly stationary. That is why this context is example is important. Then, if you have x Strict Sense stationary, obviously you have also Wide sense Cyclostationary.

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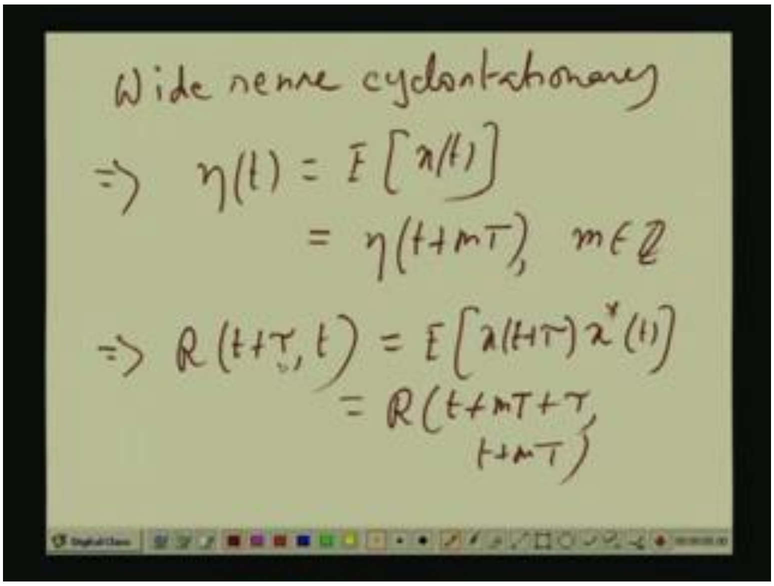

Also called WS Cyclostationary here we should imply then if you take the mean at any particular time say xt. It is no longer like in the case, of stationary process we have seen when you come down to w wss that time it is no longer the joint distribution that we consider we just take the mean and auto and correlation here also. We take the mean and if we call this mean t.

In the case of stationary process, this was this become independent of t, but if is Cyclostationary that is in the case of stationary process if you take any indices t1 t2 t3 you should get the same mean. Here it is not sure, you would get the same means provided instead of t you either go forward or backward by T or twice or 3t like that.

That is this is equal to when m is an integer that is m is from z. This is number 1 I mean, in the case of correlation that is say R t plus tau and t.

There is 2 integers the 2 time points are taking 1 is the t plus tau another t. This is same as say if you consider real value process, this is same as or in there is a complex value this is this. If it is Cyclostationary then this correlation even that the function of in the case of Strict Sense stationary what happened. These 2 time indices t1 and t2 that is t plus tau and t they did not mater, but matter was the separation tau.

In the case of Cyclostationary process this is not so for all t, but for a specific values of t. That is either, as t is replaced by t plus T here and here or t plus 2t that time this gap the separation which is always tau that should come into play. That is it should become of a function tau not all not at all t, but at the time points that are separated by integral multiples of T. T is a basic period or it is a Cyclostationary.

So, that is if t is replaced by t plus m into T M being an integral plus tau and t plus mT. Only then these 2 correlation values will be same that is not that the auto this correlation function becomes a, this correlation becomes a function of only their gap or lag that is tau. For any t, but these indeed becomes a function of tau only at certain choices of t. That is either t or t plus mT t plus T t plus 2t t plus at this 2 points the gap is again tau between these 2 t of time points time indices, but is not true for any t. So, this is the definition of Wide Sense Cyclostationary t and obviously, you can see that if the process is given to a Strict Sense stationary. It is also Cyclostationary, Wide Sense Cyclostationary are not vice-versa.

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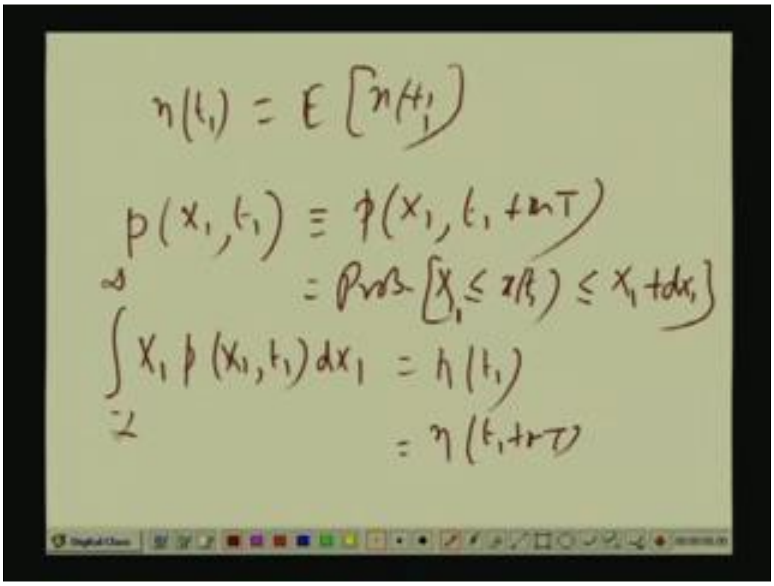

Obviously, I mean like in the case of in the case of, we will take the expectation; that means you in the process given to be say strictly say SS Cyclostationary. Then, we know this density is same as t1 plus any mT. What does this density means, that is probability that xt x t1 or t1 plus mT takes values within this range between x 1 we have to come to density now not distribution dx 1 .And what is eta t; eta t you simply multiply x1 by this eta t1 say that will give you eta t1.

Similarly, if you want to find out that t1 plus mT instead of t1 you make it t1 plus mT. But since, it is Strict Sense Cyclostationary. The 2 densities are same and you get the same result. Suppose, it is Strict Sense Cyclostationary it is also Wide Sense Cyclostationary at least for the mean we have seen same we can see for the correlation also.

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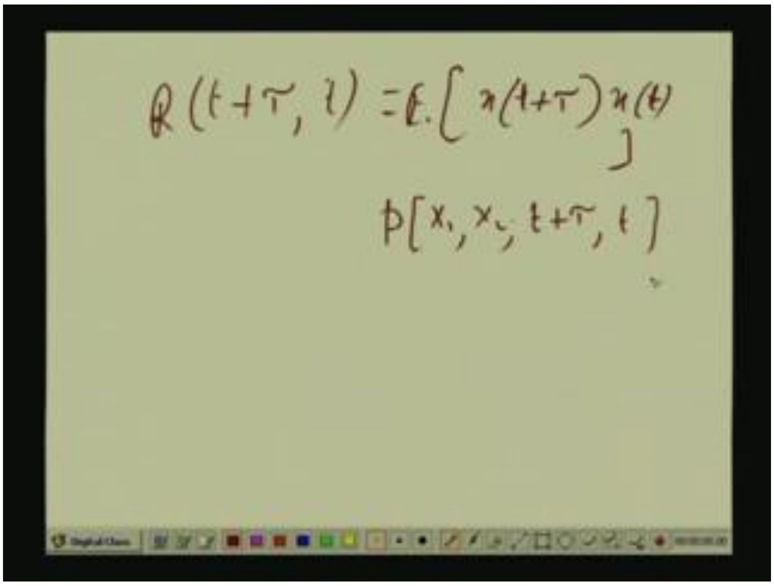

That is what is Rt plus tau consider a real valued case. That is how, will you calculate it you multiply these 2 by the joint density. What is joint density? That is p x1, x2 at t plus tau and t meaning the joint probability that at xx at t plus tau xt plus tau lies between x 1 and x 1 plus dx 1 and xt lies between x2 and x2 plus dx2. We multiply these 2 functions x1 and you multiply this by x1, x2 integrate over x1, x2 over the entire range. Then, you will get this. If t is replaced by t plus mT here you would get the same integral is, because from the Strict Sense stationary t the 2 distributions or densities will be same. So, that follows obviously, immediately. So, we do not spend more time on it. So, similar in this case also we can show that.

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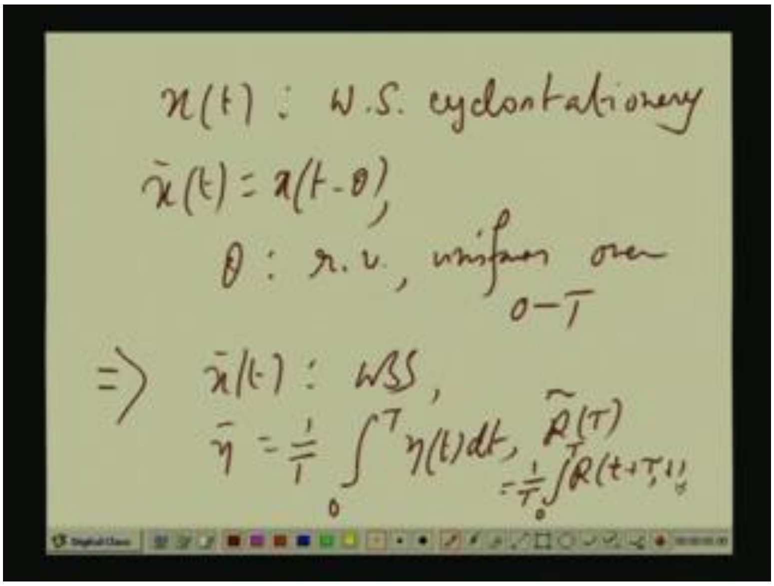

Suppose, xt is Wide Sense and you generate x bar t as where theta is a random variable uniform over 0 to t. Then, our claim is x bar t will be WSS with it is mean mu will be 1 by T and correlation tau will be 1 by T integral 0 to T R Y plus tau comma t dt we prove this.

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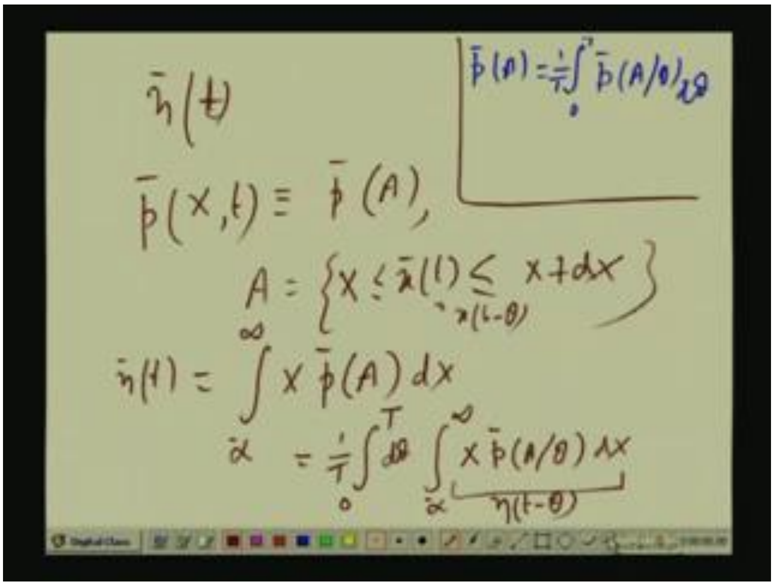

Now, how do we find out this quantity for that matter, we define these density p bar xt this is same as p bar A where is this event x bar t at this time point t takes values within this zone. At time t this process x bar t takes value within this zone. In that is case, simply xp bar A dx, but as I told you earlier p bar A is nothing but p bar A by theta into p theta. And p theta is 1 by T p bar A is what, seen earlier is p bar A comma theta d theta over 0 to t.

What is the entire range of theta, but p bar A comma theta we have seen earlier is nothing but p bar A by theta into p theta that is 1 by T. Because, p theta that is uniformly distributed over 0 to T and that 1 by T goes outside and d theta. And here theta is fixed. So, what we get here if you replace p bar A by this expression and inter change the 2 integrals what you get is 1 by T 0 to T.

Then, this integral this T theta comes here this integral minus infinity to infinity x p bar A by theta dx And what is this theta is fixed here. So that means, x bar t taking value x bar t which is also equal to we know x t minus theta and theta is fixed. So, p bar A by theta is same as the probability of x taking value x t minus theta taking value. That is the process xt at time point t minus theta taking value between x2 x plus dx.

So, this will give rise to expected value of the process at t minus theta. I repeat theta is fixed p bar A by theta means for a given theta. So, once theta is fixed look at this event A x bar t is same as xt minus theta. So, probability of A subject to fixed theta is what the probability of x t minus theta this random variable taking values between X to x plus dx. That multiplied by x integrated, that will give rise to the expected value of this random variable x of t minus theta which is eta t of t minus theta right.

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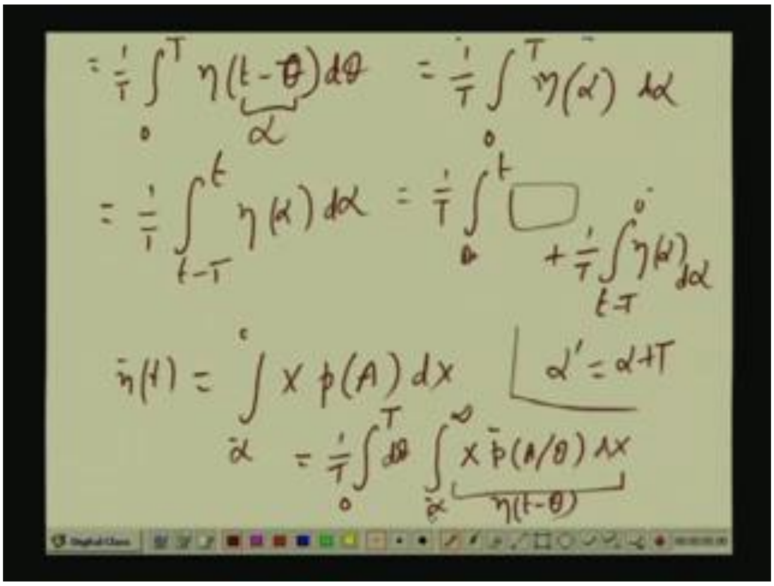

Then, it is very simple you have this integral 1 by T t minus theta. Once again, you know this eta t, because the Cyclostationary t we know this is periodic over a period T so eta t a. Similarly, if theta is replace by theta plus 2 theta plus T or theta minus T will do the same thing, because of the Cyclostationary t. Because, the date argument get shifted either by plus or minus T or plus or minus 2t and so on and so forth.

Or we have integrating this over a entire period of theta. So, what mean eta So, this eta if you plot it as function of argument then basically you are integrating 1 period integrating this function eta over it is 1 period and averaging. So, that should not depend on whether I to I put t here or minus theta here I can call it I can with any variable eta of say alpha d alpha 0 to T 1 by T we can always put like this.

So, this comes physically and again you if we still want to verify it. If you still want to verify this, it is not very difficult you can call this alpha. So that means, when theta is 0 alpha is t and when theta is T is t minus t d theta, because minus d alpha. So, that 2 integral limit is were interchanged essentially, you get 1 by T. As I told you t minus 0. So, it was t lower limit was t upper limit was t minus T, but that gets interchanged.

So, t minus T and then t alpha d alpha and again, this you break in this form we have already done it from 0 plus to t the same the integral. And another, integral is again 1 by T from t minus t to 0 minus same integral. And rest is very simple you can as before you bring in alpha prime as alpha plus t. So, when alpha is t minus T alpha prime is t and when alpha is 0 alpha prime is T and then the 2 integrals is just from under 1 number and you get that result. That means we prove this part.

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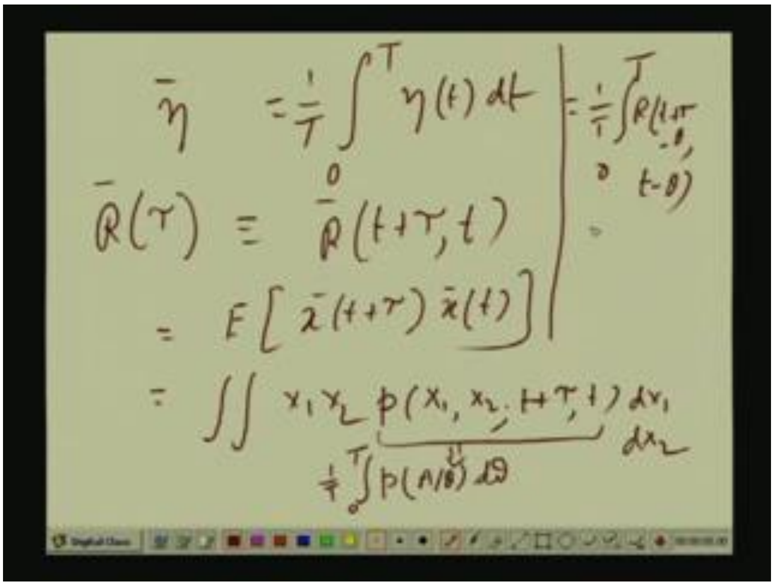

We have proved this part that I call it alpha now again i am calling it T it does not matter it is an integration variable. In the same manner, R bar we wanted to prove that is R bar is basically function for tau l1. Because, it is Strict Sense stationary actually, this should be I should remove this dependence t now. Because this entire thing is a constant you see.

If you replace t by if you just substitute t by I mean this is just a formula this is just a results which is constant and that is eta bar. Similarly, my claim is that this delayed process is a strict sense stationary process I have not yet proved it. I have proved it by it is mean, but the correlation; that means, I have to show that it the correlation becomes a function of tau l1.

So, this will show, but for that, so I am just writing as function of tau, but I have not yet proved it i keep that in mind. We will show we will get it is value, this actually is what this is a general expression. And what is that, as before E of x bar I am assuming real otherwise, you put another star here. And how to carry out this integral, you have to find out that is this should be this bring this joint density p x1, x2 t plus tau and t x1, x2, d x1, d x2.

But this joint density I can write as a conditional density that, it if it that is this is subject to theta multiplied by p theta integral 1 by. And p theta is integral 0 to T and p theta that is this part I will write as p we call this event A A by theta into 1 by T to 0 to T theta this part. And then what happens you take the integral with respect to theta outside inside you have got x1, x2 multiplied by this joint density

What is the meaning of joint density, that is x bar t plus tau taking value between x1 to x1 plus d x1 and x bar t taking values between x2 and x2 plus d x2, but x bar t plus tau for a fixed theta now x bar theta is fix here for a x bar t plus tau is same as x t plus tau minus theta. And x bar t is same as xt minus theta. Theta is fixed here mind you, because this is now conditional density. So, if you carry out the integral from the previous study. It is then nothing but this will that outer integral was there equal to T and inside you will have original R at T plus tau minus theta comma t minus theta d theta. So, write in other way here, so what I get may be I go to the next page, what I get is this.

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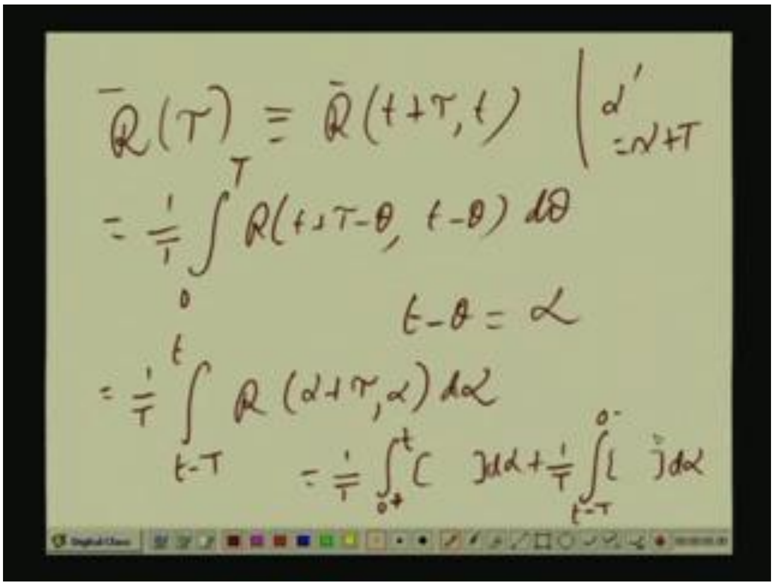

R bar tau we have to be prove that it is integral function of tau alone. So, still I writing doing before end. So, actual it is as I told you R bar t plus tau comma t this we have shown. It is nothing but this integral original R t plus tau minus theta comma t minus theta d theta. And then this remaining part is very simple if you call if you call theta minus tau to be if you call theta minus tau to be some variable say tau 1 or alpha.

Then, it is integral becomes what same thing I am doing again. In the next class, I will take a just an example of Cyclostationary process, because then we will get a practical feeling. When, theta is 0 alpha is minus tau. So, from minus tau to t minus tau and this becomes R of t minus alpha just a minute it should instead of tau I should take this theta minus t to be alpha.

So, it is when theta is 0 it is T minus t and when theta is T this T minus t. And then R may be I can make a still better still more convenient for you forget about this. Suppose, I call t minus theta which is common as some variable say alpha. That means, this becomes 1 by T when theta is 0 this is t when theta is T it is t minus T or and then d theta becomes minus d alpha. So, this integral limit is will be changed and minus will comes.

So, this becomes this and this remains, because d theta and it is minus d alpha. So, minus comes and the 2 integral limit is are interchanged. So, minus sign goes and this is your alpha plus tau comma alpha. From this again it is obvious, because the entire thing is periodic about alpha over a period at T still if you are not convinced you can break the integral in 2 halves: 1 is from 0 to t that is 0 plus 2t same thing, another is from t minus T to 0 minus same thing dt.

Consider this integral second 1 here again bring this variable alpha prime is alpha plus T same way. So, d alpha and d alpha prime are same when this integrals are with respect to alphas d alpha. So, d alpha and d alpha prime are same and when alpha becomes t minus T alpha prime becomes t. When alpha becomes 0 alpha primes becomes T and the 2 integrals can be brought under same 1 and you get the result, that result that is a function of tau value.

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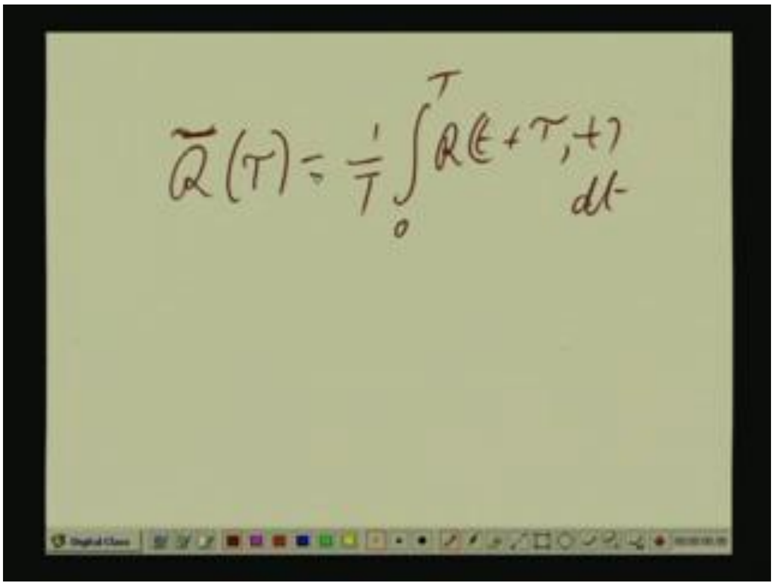

What you get is R you can call it alpha or you can bring back t again, say does not matter, because that is an integration variable T dt. So, you can call it alpha as well instead of t 0 to T 1 by T this is your R bar tau. So, this process is strictly is a Strict Sense stationary. These are some applications in communication we will take 1 example. The example of pulse amplitude modulation was formed PAM, there will be a practical feeling and you understand why we spend some time on it.

After that, we will move to systems that is processing of random processes by some linear systems or non-linear systems time in variance systems and all that. And that will take us to finally, spectral analysis, power spectral density and things like. So, that is all for today.

Thank you very much,

Today, we discuss systems with random processes input.

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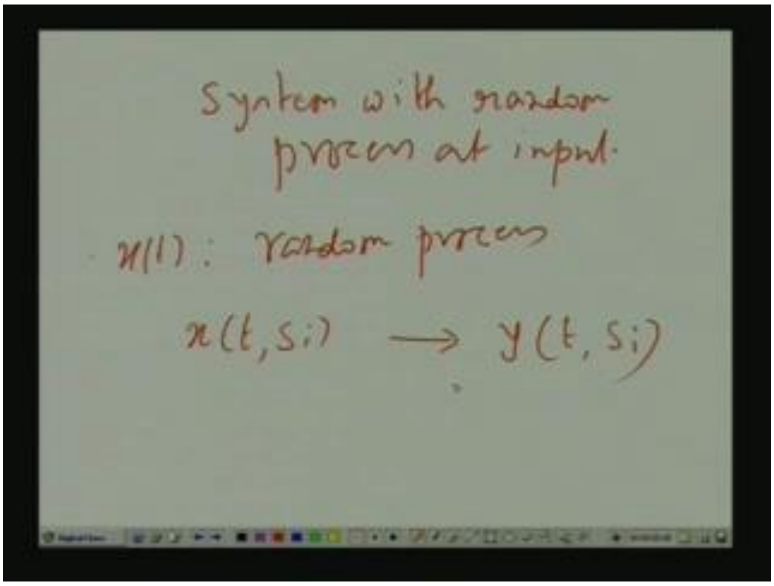

Suppose, xt is a random process; that means, you conduct experiment and with a every time you have an experiment you know we observe 1 wave form right. So, may be 1 particular wave from we denote as before as this, where si denotes the i'th outcome. So, in the i'th outcome you get an wave form as a function of time. So, this is represented by xt comma si. So, is actually function of time si just denotes the fact is a just stands for the fact that it is related to the i'th outcome a particular waveform.

Now, suppose we have some kind of some a rule. We develop or we have in which some kind of rule by which given particular waveform we generate another waveform by a rule some waveform we call it y. And obviously, that wave form also is associated with the i'th outcome of the experiment.

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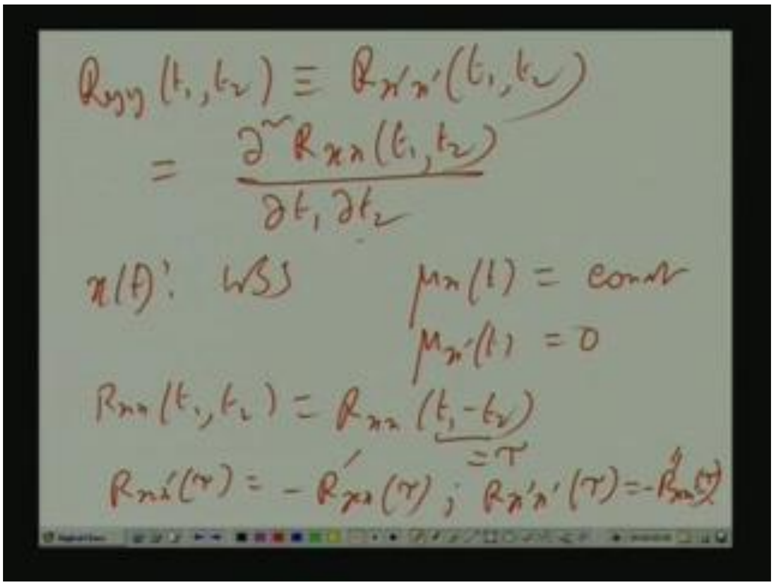

So; that means we can write it as y t si for example, you can a for example 1t 2. Now, here we have got Rx t1 comma t2 is a function of actually Rxx t 1 minus t2 t 1 minus 2 equal to say tau. So, it is function of tau. So, here but i do in 1 minute here if you take these derivative del Rxx t1 minus t2. So, t1, t2 is tau we differentiate with t2 first and t1 first I limit as an exercise to you can easily verify these 2 things. Rxx tau Rxx prime tau you can easily verify will be Rxx tau first order derivative and R x prime x prime tau will be second order derivative. So, that is all for today.

Thank you very much.