Probability and Random Variables
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Lecture - 30
Stationary Processes

So, in the last class, we have given introduction to what is called stochastic process. Earlier we used to consider random variables. Maybe our experiment and depending on the outcome of the experiment, a variable takes a value. Since outcomes are random, the variable takes values in the random way and we call it random variable. So, we studied single random variable, we studied double random variable and functions of random variable and all that their probability densities and probability distribution functions.

In the case of process, it was extended to function that is depending on a particular experiment, you got I mean just a function. Function could be continues, first function could be discrete, but a function discrete in time, a function emerges and the function is random because depending on the particular outcome at hand, you get just to one function, right. So, that set of all functions is called the process. This phenomenon is called the process. The set of all possible such random function is called the ensemble.

You can just stick to your particular time point to be t or t 1, say on the time axis. So, at that point, every time you have the experimental outcome. You get one value because the function takes a particular value. Next time the value changes. So, the value that this function takes if we call the function x of t, then x of t 1 for a fixed t 1 itself is a random variable and that is true for all such t 1s on the time axis. So, basically a random process is nothing, but an infinite collection of random variables which takes different values depending on the experimental outcome at hand. So, basically it is random functions.

Then, in the last class we also discussed the meaning of you know joint probability density joint probability distribution functions, given a stochastic process or random process and then correlation covariance, and related things also are defined. Today we will be considering the particular class of random process called stationary process which is very important. You see very general random processes are fine theoretically, but for practical purposes you know they are very difficult to be made use of. So, we need to enforce some additional assumptions or a random process, so that you know I mean their statistical properties or statistical characteristic, they become simpler for handling and we know in practice. Stationary process is one such notion actually.

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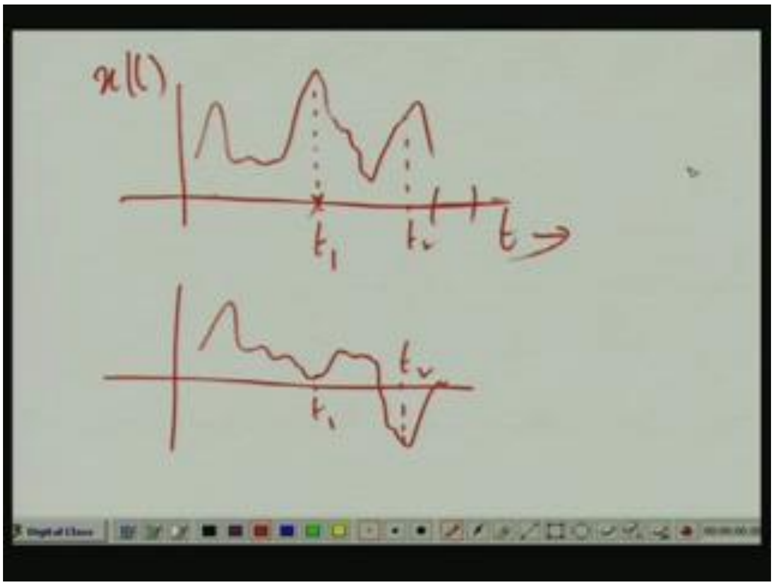

Suppose you are conducting an experiment and depending on the experimental outcome, we have been observing some waveforms. So, waveforms are random, every time you get a random waveform and this whole phenomenon is called the random process. So, maybe we have something like this and you fix a time point t 1. So, this time you find if you call the random process xt, this is time axis. So, these are function of time also. We have taken continuous functions of time. So, at t 1, you get some value this time and may be at t 2, we get some value, but next time when you observe the same waveform, maybe at t 1 you get only this much and here you get negative.

Next time may be at t 2, we get very high value and like that this is possible. Now, in the case of stationary process, we assume that this randomness. What is randomness? This tendency of this waveform to fluctuate around this time axis, this randomness that I mean with this time, we get one waveform. Next time the waveform thoroughly changes, next time it changes again. So, this tendency to fluctuate from experiment to experiment fluctuates around this axis. This is not having any bias over some zones of the time axis. This randomness is rather homogeneously distributed on the t axis. This means that at point t 1 whatever be the statistics, you have for the random variable x of t 1. Same statistics should hold at another time point say t 2. That is as we move along the time axis to the right or to the left, the randomness, the properties of the randomness, properties of this process, they do not change.

For a particular waveform, it may so happen that you get a high value here and low value here. Next time the value changes and all that, but when you talk in terms of their statistics like a probability density or probability distribution functions, then whatever you have at one time point, same thing you should have another time point. That is true for all the time points on this axis. Then, this process is called strict sense stationary. Not only stationary, strict sense stationary, this is the ideal stationary situation that the statistics does not change. Statistics of the random variable, say x of t for a particular t, it does not change. If you change if you move t from minus infinity to infinity at all points, the statistics remains same.

So, for that matter, say probability density of this random variable x of t 1 because that is a random variable and it has its own probability density. You should have the same value same thing at t 2, at t 3. There is randomness. You just try to visualize the randomness. It does not really look at this time axis. It is just you know randomness is just homogeneously distributed along this time axis. It is not bias on some sectors of the time axis. That is not that around t equal 0, maybe in a zone you know it is got some kind of property and as you move further down from the origin, maybe another sector on the time axis. Maybe here we have got you know some other kind of statistical properties.

The statistical properties do not change while these are very qualitative statements. I will quantify it. I will use you know the notions of probability density probability distribution functions to really I mean give a proper definition of this, but I am just trying to say that randomness for this process is really uniform along this axis. This process is as much random if I may put it as much random say at origin as it is at t 1 as it is t 2. This randomness property has given by the statistics, there is a probability of densities is same whether you are seeing it at origin or at point t 1 or t 2 or any other point. Then, it is called strict sense stationary process.

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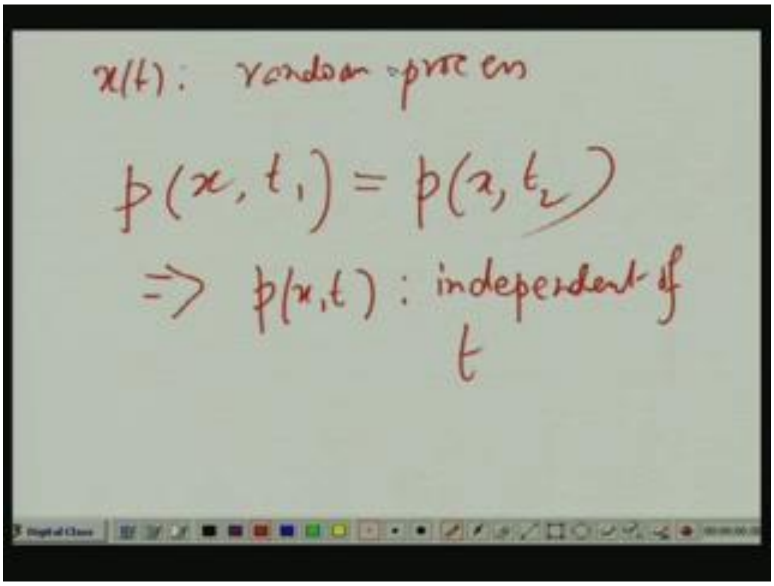

Suppose x of t corresponds to a random process. That means, there is an experiment every time you observe a waveform, you get some waveform. You observe even outcome, experimental outcome. You find some observations; some waveform xt waveforms are random. A particular xt is a sample function. This is just reputations of what early we have discussed and collection of all possible xt is called the ensemble for this random process.

Now, the stationarity means that suppose we are considering the statistics. Either probability density or distribution maybe we stick to the probability density say at a particular time say t 1. What does it mean? It means that this wave form x of t if you observe it at time point t equal to t 1, then we have got a random variable x of t 1. It is the probability density of that random variable. So, that random variable lying between x and x plus dx is given by this density times dx. Now, if it is that is the general definition, but if it is stationary, then whether you observe this probability density at time point t 1, that is for the random variable x of t 1 or at some other time, point t 2. That is for the random variable x of t 2, you should get the same thing.

If it is strict sense stationary that is it should be same as px, p 2 and so on and so forth. You can take px t 3, px t 4 and like that you should have the same thing. This is for individual densities this means px, t independent of t if the random process xt is given and you find out for a for any point t, the random variable x of t. If you look at its probability density, that should be independent of t that you have chosen, but this is only individual densities, but you see this does not define. This does not give the entire statistics. We have to consider all joint densities. It does not give us any you know idea about the joint variation, but when it is strictly stationary, it is not only the individual statistics is independent. I mean of time that is you know you observe the same thing at various time points, but this joint statistics also should be time invariant.

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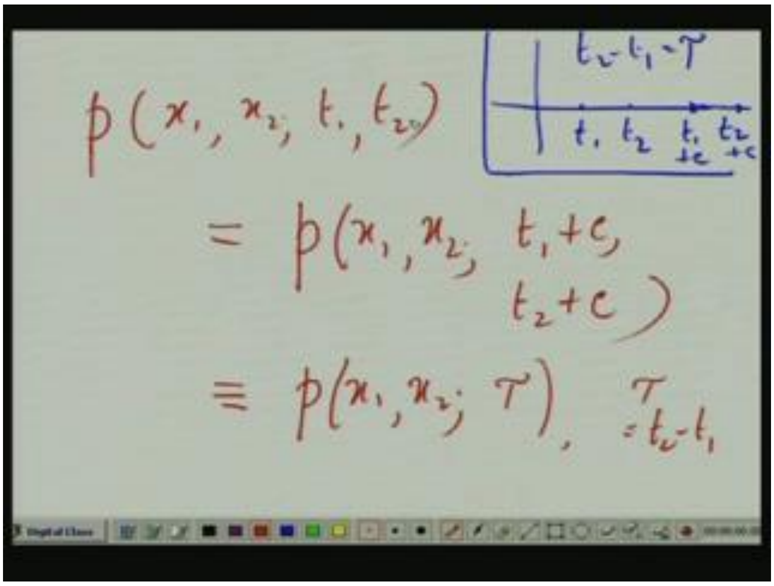

That means further matter suppose you have got now px 1 x 2; t 1 t 2. What does it mean? Actually there was a random process x of t. You are fixing two particular time point t 1 and t 2. At t 1 you have got the random variable x of t 1, at t 2 you have got the random variable x of t 2. What is the joint probability density of x of t 1 and x of t 2? That is given by this. So, the joint probability of x of t 1 lying between x 1 and x 1 plus dx 1, and at the same x of t 2 lying between x 2 and x 2 plus dx 2. That is given this function times dx 1 dx 2. I am not writing all these because this is by now pretty well known to all of you because this is just the definition of joint density.

So, I repeat again x of t 1 and x of t 2 are two random variables. This is a joint density. It means the joint probability of x of t 1 lying between this x 1 and x 1 plus dx 1 and at the same time x of t 2 lying between x 2 and x 2 plus dx 2, that is given by these density function times dx 1 dx 2. That is the meaning of the joint density, but if it is strictly stationary, then what should happen that I fix up 2 to 2 time points t 1 t 2 and based on that I am getting this probability density. If it is strictly stationary, I should get the same thing. If instead of t 1 I say t 1 plus some constant c and instead of t 2, t 2 plus c that is whether I choose two time points, t 1 and t 2. I am drawing a figure here. This is a time axis whether I choose t 1 and t 2 here and see the joint statistics of the two random variables x of t 1 and x of t 2, or I see the same thing at two other points, t 1 plus some constant c, t 2 plus some constant c. Just delay both the points by the same amount. That is important, that is joint density between two variables which are situated at a gap of t 2 minus t 1.

So, whether you see them at t 1 and t 2 that is if you call it t 2 minus t 1 is equal to suppose tau. So, actually it amounts saying that joint density of two random variables which are situated at a gap of tau. So, whether you see them at t 1 and t 2 or at t 1 plus c and t 2 plus c, as long as the gap remains same, you should see the same density. This is in the case of joint statistics involving two variables. That means this should be equal to just px 1 x 2. This is a semicolon and neither t 1 nor t 2. You can say tau where tau is equal to t 2 minus t 1. So, it should depend on neither t 1 nor t 2, but this density would depend only on the gap between the two, but again this is only joint statistics involving two variables.

Strict sense stationary applies to joint densities involving any countable number of variables x 1 x 2, x 1 x 2 x 3, x 1 x 2 x 3 x 4 so on and so forth. So, in general, from this you can now generalize the thing that if it is strict sense stationary, maybe I write it out here.

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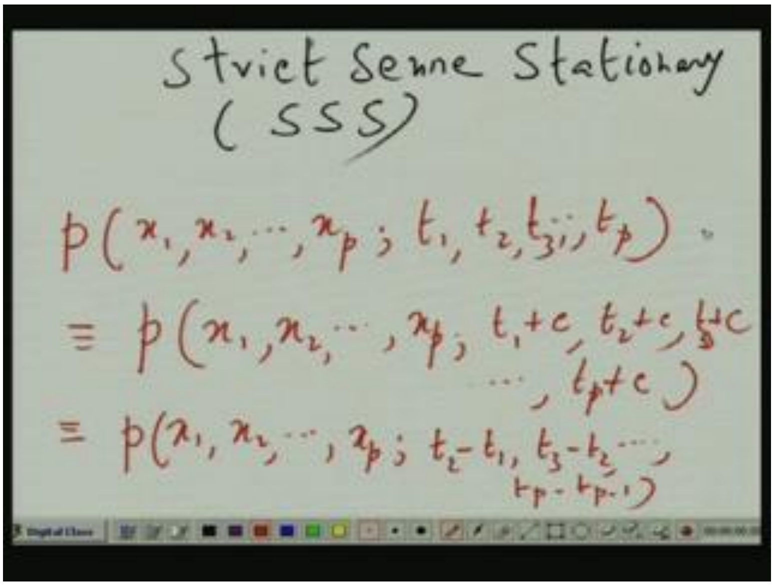

Strict because I did not write anywhere strict sense is called SSS. So, in general if you have x 1 x 2 dot dot dot up to xp colon, t 1 t 2 dot dot dot tp. What does it mean? It is very simple. X of t was the random process. So, we are fixing some points t 1 t 2 tp. X of t 1 is a random variable, x of t 2 is a random variable dot dot dot x of tp is a random variable. It is a joint density of them, so that the joint probability of x of t 1 lying between x 1, x 1 plus dx 1. Similarly, x 2 lying between x 2 and x 2 plus dx 2 dot dot x 3 laying between x tp laying between xp and xp plus dxp. That is given by these density function multiplied by dx 1, dx 2 dot dot dot dxp.

So, again here it should not matter if I advance the points, all the points. If I just shift instead of t 1, I call it t 1 plus c. Delay it or advance it that is move it rightward. Move the function x of t rightward or backward. That is instead of observing this joint density between variables which are located at t 1 t 2 dot dot dot t p. If I give a constant shift to all of them, that is may be t 1 plus c, t 2 c dot dot dot tp plus c, I should have the same thing. That means it should never depend on t 1 or t 2 or say t 3 dot dot dot tp. It depends only on the gap between t 2 to t 1 and t 3 to t 1 and likewise because that gap limits invariant. If you give the same shift c to t 1, if you at the same c c to t 1 t 2 t 3 may be put a t 3 here dot dot dot is t 3. If you add the same constant c, you get different time points t 1 plus c, t 2 plus c, t 3 plus c dot dot dot dot, but strict sense stationary implies that you should get the same density function.

So, this only means that what is important is not the absolute positions of t 1, t 2, t 3 etcetera, but only the gap. The difference between t 2 to t 1 and t 3 to t 2 and t 4 to t 3, that is what is important because that limits invariant whether you have t 1 plus c and t 2 plus c or t 1 and t 2 are similarly whether you have t 2, t 3 or t 2 plus c t 3 plus c. So, that means under strict sense stationary, this becomes a function of t 2 minus t 1. If you call it tau 2, then t 3 minus t 2 dot dot dot dot in general tp minus the last one last, but one tp minus 1 variable.

Earlier it was p variables, but the gaps result in p minus 1 variable. You can call it tau 1, tau 2 dot dot dot tau p minus 1. Then, you can even generalize this concept may be there are two random processes x of t and y of t and they are jointly related. So, I can define jointly I mean strict sense stationary for joint processes, that is jointly strict sense stationary jointly SSS x of t and y of t.

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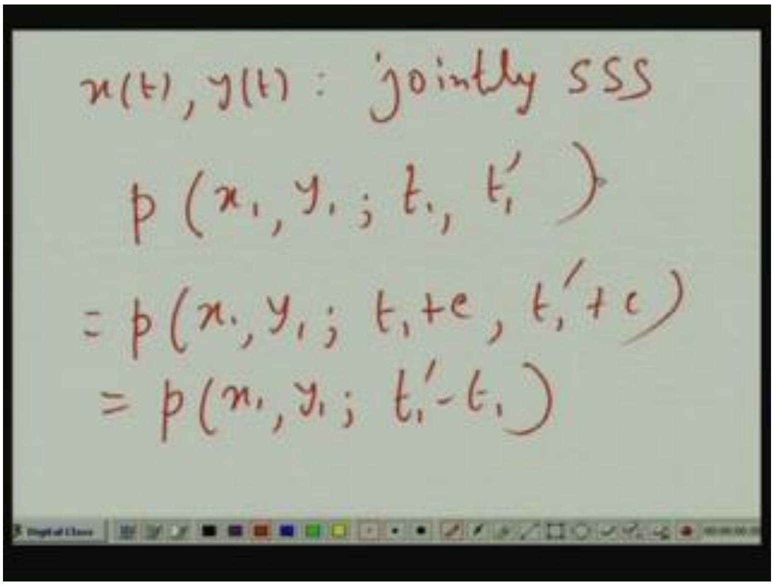

This also has been that if you are looking for now joint density between the x and y, maybe you just start with one variable only. It is easily generalizable. So, we will take 1. Suppose it is given joint density x 1 y 1; t 1 t 1 prime. Prime strands apply to y and non prime quantities apply to x. What does it mean? It means this is joint density, so that x of t 1 joint probability of x of t 1 lying between x 1 and x 1 plus d dx 1 and that at the same time, y 1 lying between y, t 1 prime lying between y 1 and y 1 plus d y 1 is nothing, but this density multiplied by dx 1 dy 1. See this is a pretty standard step. By now we have been dealing with this definition of probability density joint and single. I am not writing them.

Now, if it is jointly strictly stationary, it should depend only on the gap, that is this way semi colon you should have the same thing. If you advance them by some constant which means it depends only on the gap between that two difference between t 1 prime and t 1 and not the absolute values of t 1 prime and t 1. Similarly, if you have x 1 x 2, y 1 y 2 and t 1 t 2, t 1 prime t 2 prime, we can define like that. It is a very simple generalization.

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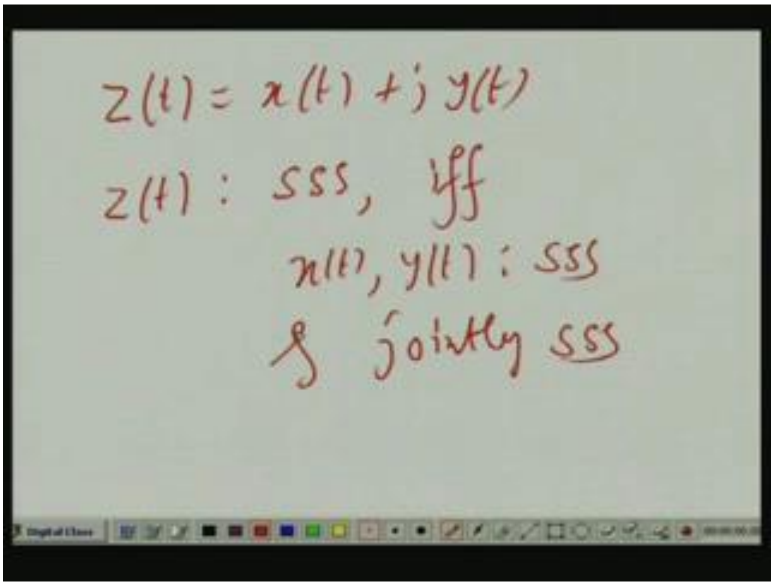

Just for definition sake if you have got a process zt which is a complex process. Zt will be strictly stationary if may be if and only if xt yt both are SS and jointly SS individually, they are stationary process and they are also jointly stationary both in the strict sense. This strict sense stationary you know it is more theoretical in the sense that it is an absolute notion of stationary. That is why you talk in terms of that absolute statistics is probability density as you know probability density define everything. If you know all the probability densities, individual and all joint densities of the process, then the entire statistics of the process is characterized.

So, when I define stationary strict sense in terms of this or when I define stationary in terms of these statistics remaining invariant on the choice of time points, then this really is an absolute notion, but for practical purposes this is not. So, you know we can take a special case of this strict sense stationary which is called wide sense stationary (WSS) which is really very useful for all practical purposes especially in communication control signal processing and others estimation detection and all. This is called wide sense stationary, where we go only up to second order and talk in terms of correlation and mean not in terms of probability density. So, I will come to that now.

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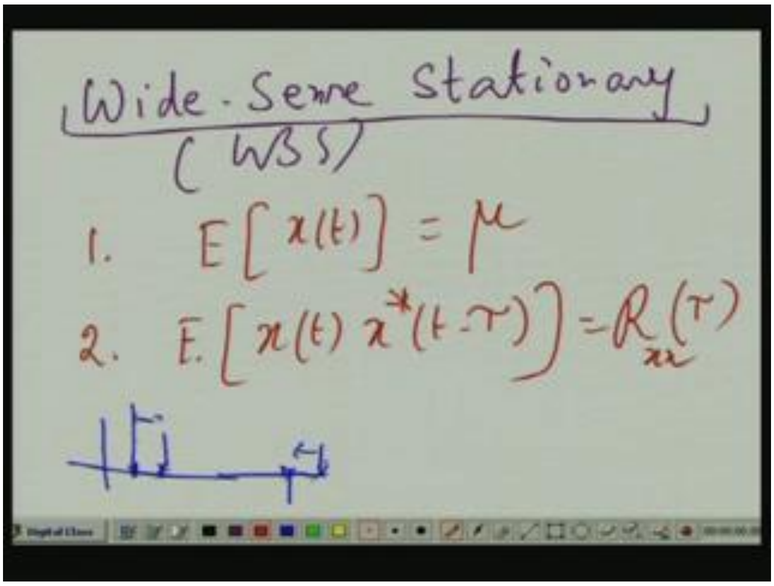

It is also called WSS. In wide sense stationary, I say that we will be considering only moments, but moments only up to second order, that is first order moment means and second order moment is correlation. There we will be enforcing some notion of stationary and that is all and with that only we remain happy. Those processes which satisfy them will be called by us as WSS processes (wide sense stationary processes). So, here when it comes to the first order moment, we say that given a random process x of t if you take for a particular time t a, random variable x of t and if you take its mean, then that mean should be invariant. Mean should not depend on t whether you observe the mean at a particular point t 1 or t 2 or t 3, you should get the same thing. Then, you can call it just mu. It does not depend on t.

See I tell you for a particular waveform; it may so happen that you can get very high value at t 1 and very low value at t 2, but next time it can change. If it is stationary may be at t 1, you get very low value and t 2, it is high value. So, when you average out, you get the same mean at both t 1 and t 2. This happens if the randomness does not have any bias on the time axis. It is not that it gives high value. You know high values to the process at t 1 and very low range of values to the process at t 2. It is very not blind. You know it is very fair. It is just homogeneously or uniformly distributed. The randomness I am talking. I am not talking about the probability, the randomness or the fluctuating tendency or these inner properties of fluctuate that is just uniformly distributed along the time axis.

So, at any point you get the mean, at some other point also you will get the same mean. There is for first order thing is. In the case of second order thing, we know the correlation and I am considering the complex case, complex value processes to be in most general. So, there we know what definition of correlation xt is. It is xt minus tau. This correlation should also depend not on what is t and what is t minus tau, but only on the gap between those two samples. That is the degree to which two samples are correlated is a random process. That should not depend on where their absolute locations are, but how far they are separated.

So, whether you are observing two samples, you know like whether you are observing two samples here and here or observing them here and here. As long as the gap is same, these two gaps are same; you should get the same correlation. You can call autocorrelation. So, the absolute time points, these two points or these points, they do not give matter because again this randomness is uniformly distributed along this axis. So, the property of correlation, the correlation characteristics of this process is how to what extent two samples are correlated that characteristics does not change whether you are between two points here or whether you are saying it between two points here as long as that gap between that two remains unchanged. This means that then this should be a function of only gap tau.

You can also in some cases I can put Rxx especially when two random processes are involved xt and yt. Then, xx means actually you are taking the correlation between x and x. So, Rxx is autocorrelation. If it is x and y, Rxy like that, but loosely it is especially when I do not have any other process involved, I will drop this subscript of simplicity and it will become just r of tau.

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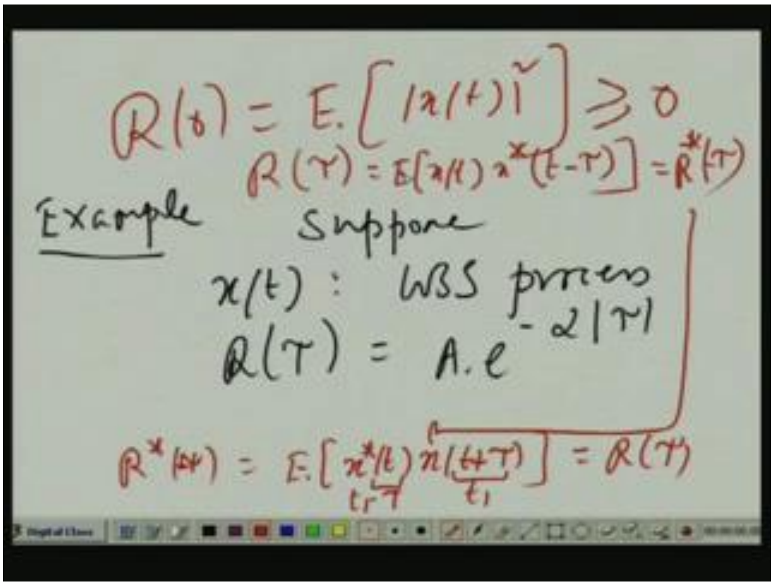

What is R of 0? We know it is nothing, but e of x xt xt minus tau, that is zero star. So, that is xt into xt star. That is mod of xt square which is nothing, but the average power. That is for any instant you are taking the mod square, where the instantaneous power and their averaging, because of stationary, you get a constant value independent of t and that is the average power. Obviously, this is greater than or equal to 0. Power can never be negative. Just with one example, suppose x of t is a WSS process and it is given that therefore, its autocorrelation R tau is some constant a times e to the power minus alpha say mod tau.

One thing I forgot to mention that R tau may be I mention here another property. I forgot to mention. I will come back to the example soon. R tau which is same as e of xt x star, this is a very important property. I mean I just forgot to mention. This is same as my claim is R conjugate minus tau. Why? If you take R conjugate, i am working out here and then, I will erase it. What is R? It is star tau by this definition conjugate applied inside. So, it becomes E of x star tau and this becomes xt minus tau and then, you are making minus tau. So, minus tau means it becomes t plus tau. T plus tau you can call as may be t 1. So, t becomes t 1 minus tau.

So, this nothing, but e of xt 1 into x star t 1 minus tau which is nothing, but autocorrelation at a gap of tau. So, is nothing, but R tau. So, that proves it. So, in the case of complex valued processes, autocorrelation sequence actually is a Hermitian sequence that is if you take the negative of the lag tau is called the negative of the lag, and then put a star here. Then, R star minus tau becomes sigma star R R of tau. In the case of real valued process, the conjugate has no meaning. So, it is just an even function R tau and R minus tau. They are the same.

Now, I first erase this part. Now, this is just an example. Xt is a WSS process. Its R tau is given. I do not know its mean though. You can assume zero mean. Mean will not way of any use in this example. R tau as you can see is e to the power minus alpha mod tau. This mod is important. Mod tau makes sure that R of tau Rr of minus tau, they are same, because it is only mod of tau that is important.

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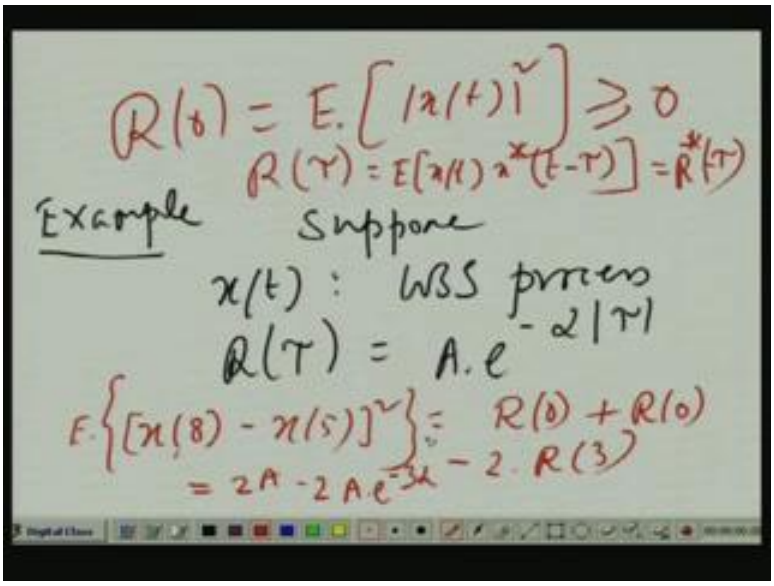

If this is given, then this example says that suppose you have to find out x of 8, that is t equal to 8 minus x of 5. This difference just for some reason we want to find out the expected value of this difference. How much will that be? When you square it up, you get e of x square 8, but x square 8 expected value of that should not depend on the time whether it is 8 or whether it is 4 or 5 or it is x square t. So, it will be basically R 0. I repeat again. First term is e of x square 8 that is x 8 into x 8. That is correlation with lag

0 whether it is 8 or 80 or 800, it does not matter x 8 x square 8 means x of 8 into x of 8t. So, two samples with lag zero for expected value of that product is nothing, but correlation of lag zero.

Similarly, there will be an x square five terms that will give rise to another R 0 minus twice e of 8 x 5. So, that is a correlation term between two samples. One is at 8, another 5 lag is 3. So, that will give rise to twice R 3 and now you know r 0 is how much A e to the power minus alpha 0 is 1. So, R 0 and R 0, so twice A minus twice A into e to the power minus 3 alpha, tau is 3. This is just an example because correlation and there is an equally important concept called covariance.

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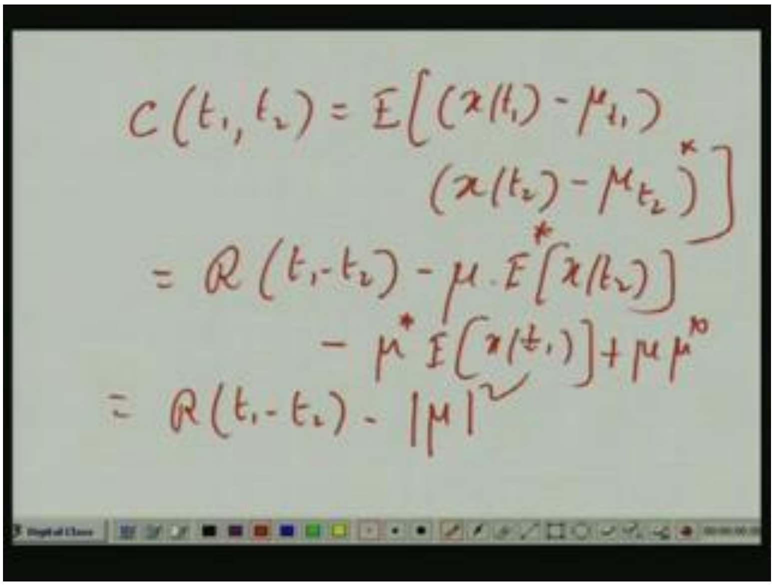

So, t 1 t 2, it means e of minus mu at t 1. This is general definition x of t 2 minus mu t 2 star. Now, if you expand it and we know it is essentially what kind of process is WSS. So, this cross term gives rise to what e of xt 1 x star t 2. So, lag is t 1 minus t 2 and that will give rise to R t 1 minus t 2. Then, mu t 1 is constant. Expected value of x t 2 star which is same as conjugate of expected value of xt 2, but expected value of xt 2 is the mean at t 2, but since it is WSS mean at t 2 or mean at t 1, they are all same. Mu t 1 and mu t 2, they are all same. You can just call them mu. So, this is mu into may be for your sake, I write like this.

Another cross term again mu star and then, these are constants the last one. So, no expectation operator mu and mu star. This only means what is e of x t 2 and that is mu independent of t 2 or invert t 1. It is independent of t because this is WSS. So, we get another mu star mu mu star. Similarly, e of xt 1, you still get mu not mu of t 1 or not mu of t 2 because it is independent of the time t. So, again another mu star mu or mu mu star and mu mu star is a positive sign. So, effectively you get R. Then, there is some definition of correlation covariance coefficient.

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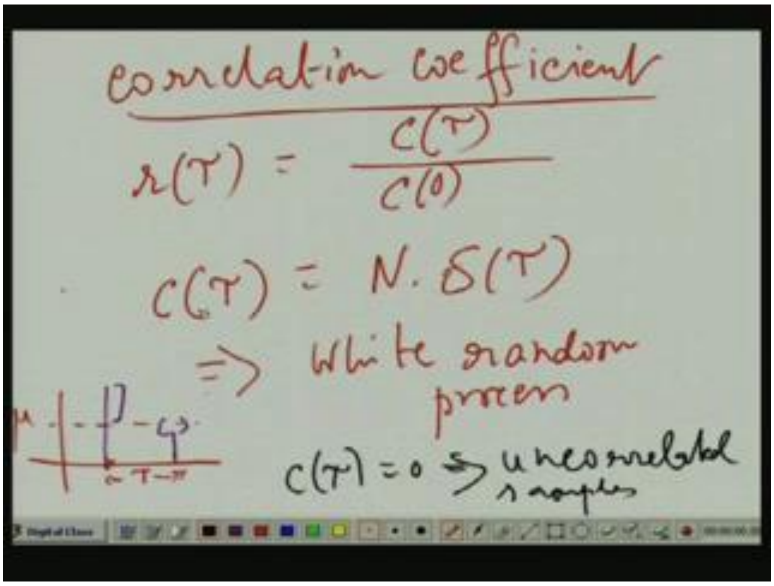

Correlation coefficient may be you can call it R tau is nothing, but the correlation. So, you understand one thing that correlation there is the covariance. Then, for WSS process the covariance is also the function of the lag, the gap between the two time points chosen t 1 and t 2. It is not a function of t 1 and t 2. It is a function of t 1 minus t 2 that is tau. So, in this case, the definition is original definition of correlation coefficient is c of t 1, t 2. That is the co covariance between t 1 and t 2. The two samples are t 1 and t 2 divided by square root of c t 1, t 1 into c t 2, c 2, but c of t 1, t 1 or c of t 2, t 2, they are now same because they won't be the same thing, because it is a function of the lag. T 1 and t 1 lag is 0, t 2 and t 2 lag is 0. So, square and square root, they cancel. So, you get basically c of lag zero. This is the correlation coefficient.

A particular case where c tau, the covariance c tau is some constant N times delta tau a. Delta tau is the impulse function, unit impulse function or also called derived delta function. Then, this is called white random process. Why white? Some of you already may know, but we will discuss all these when you consider power spectral density and all later. What does N into delta tau means? It is covariance. What does covariance mean actually? There is mean mu here; there is mean mu between two random variables.

Suppose you are taking two random variables at t 1 and t 2, the gap is tau. This is random this is mean mu. Sometimes it is taking value above this and maybe it is here. Covariance means you take only the difference that is the deviation across the mean between the two multiply and add multiply and average. Now, if these two variables are really not having any correlation between them, they are very much independent. Then, sometimes this difference can be positive, but this can be negative. Sometimes this difference can be negative, this can be positive. Sometimes both positive, sometimes both negative because there is no relation between them. They are independent. They are sort of you know free of each other. In that case if you average out the product, you should get 0. So, this c tau c tau 0 implies uncorrelated samples.

Now, for this white random process, c tau is given to be N into delta tau. We know that delta tau is 0 at all points except that t equal to 0. So, for any non-zero lag, c tau is 0 for a white process that is if you really take a white random process, take any two sample which are not located at the same time point, then they are uncorrelated because covariance is 0.This is the meaning of white random process. I will give an exercise for you. We will take it up in the next class.

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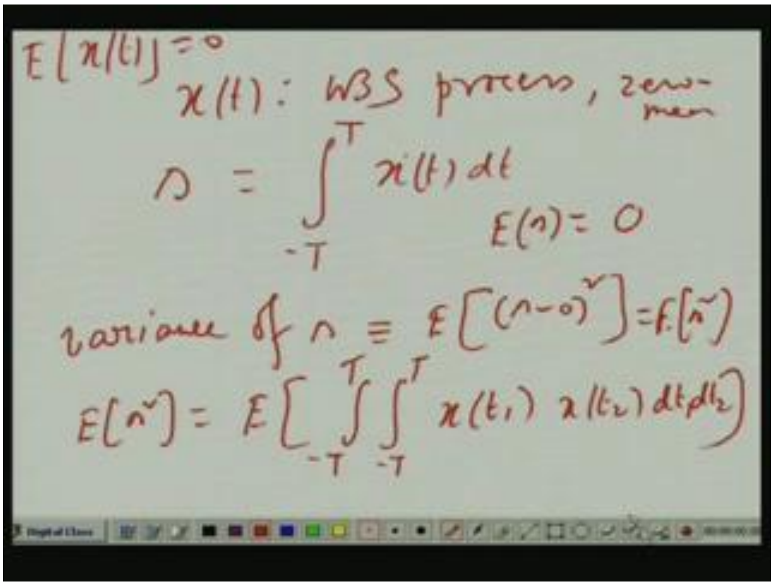

Suppose it is given that xt is a WSS process and you define one random variable s by integrating that is every time you observe an xt. We integrate it within the range minus t to t. So, whatever you get that also is a random quantity because it changes from experiment to experiment. So, s is random variable. If you want to find out and also we can assume that is a zero mean process. Zero mean means E of xt which is independent of t. Suppose that constant is 0. So, we have to find out the variance of s which is nothing, but E of s minus mean whole square. I am not taking any mod because everything is real. X is a real value process here. Mean is 0.

What is the expected value of s? What is E of s? That is E of this. Integral E is a linear operator. Integral is a linear operator. So, you get take E inside the integral, and E of xt is given to be 0. So, E of s is also 0. So, what is the variance of s? It is s minus 0. Its mean square mean is 0, so E of s square. Now, given s at given xt to be the WSS process with some covariance function or correlation function auto-covariance or autocorrelation function. What is E of s square? That means s into s. So, we integrate twice minus T to T minus T to T. Once you have got may be variable xt 1 dt 1 that will remove s. Again xt 2 dt 2 that also will give you s. So, dt 1 dt 2 and again as I said integrals are linear operators, we can bring expectation also the linear operator. So, they can be interchanged. So, you can bring E directly over this product x of t 1 into x of t 2. I erase this part.

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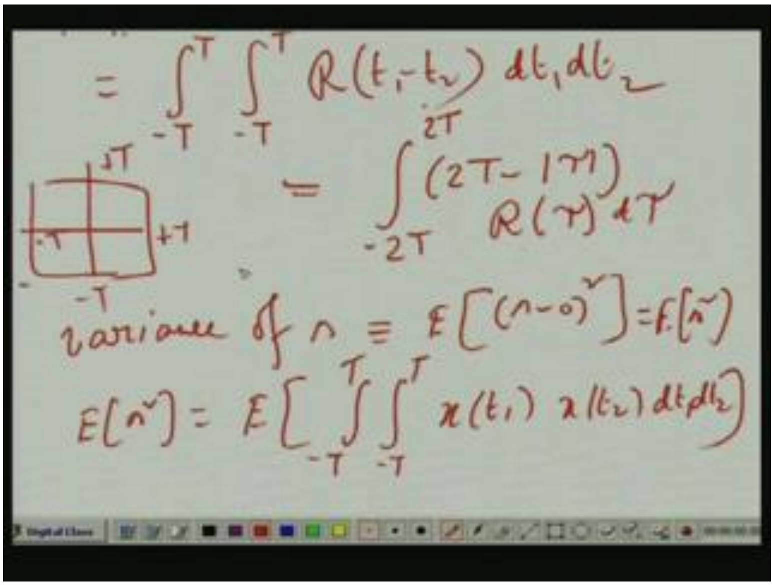

So, this gives rise to if you integrate, if you apply the E on this, then what you get is R. You can call it c also because it is zero mean process. So, for zero mean process autocorrelation and auto-covariance or correlation covariance have been same because mean is 0. This integral dt 1 dt 2 up to this is fine, but since this is a function of neither t 1 or nor t 2, but this is function of only the gap. It is function of neither t 1 nor t 2. T 1 and t 2, they are varying, but whenever t 1 and t 2, their difference is constant, you get the same value. T 1 is going from minus t 2 plus t, t 2 is going from minus t 2 plus t.

So, there is a box. There is square around the origin. One side I mean going from there is square like this you know minus t minus t 2 plus t again plus t minus t. The integral is done within this, but on those points where t 1 minus t 2 is constant are those lines or those points. This value remains same because this is a function of neither t 1 nor t 2, but it is a gap is the function of the gap. So, you can identify those points and since on those points R is constant, you can take out R as constant and carry out the integral by moving those points across this entire square. Actually I am trying to give you hint, but by following this kind of procedure, you show that this gives raise to this integral can be written as a single integral. This I will not do today. It is 2 t minus mod tau and R tau d tau, because R is a function of tau. Really it is not a function of t 1 and t 2. It is function of the gap. So, that is double integral can be written like this.

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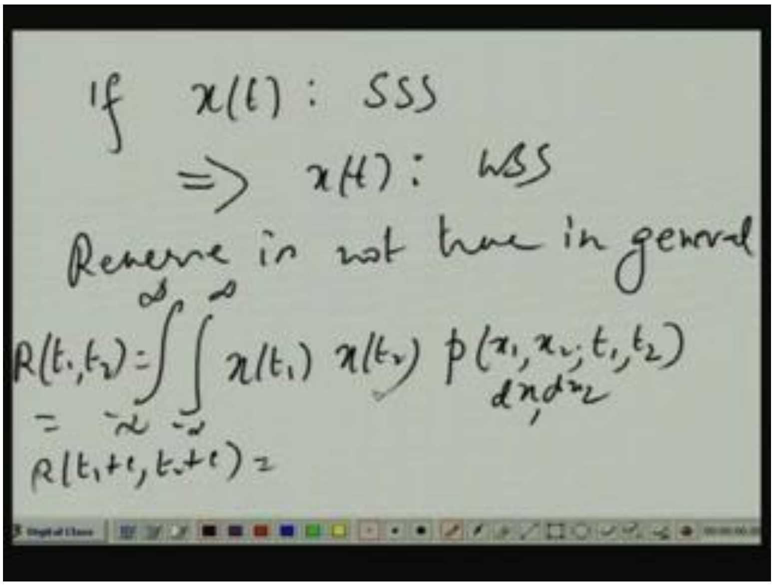

Then, it automatically follows xt also is WSS, but reverse it is not true. In general, reverse is not true. Why? How this follows that if xt is strict sense stationary, it is also wide sense stationary. Obviously, it is strict sense stationary. Now, we want to find out the correlation between two points. You know xt 1 and xt 2. Obviously, we will be multiplying it by the joint density t 1 t 2 dx 1 dx 2, right. This will give you the correlation R t 1, t 2. What is R t 1 plus c t 2 plus c? You can just replace x of t 1 plus c t 2 plus c.

Now, since it is strict sense stationary p of x 1, x 2 and t 1 plus c t 2 plus c is same as p of x 1 x 2 t 1 t 2, and it does not matter whether there is x of t 1 or t 1 plus c t 2 plus c because these variables are I am taking over the entire range minus infinity to infinity. This will give the same thing. So, this will be equal to this. Are you following this? What I am saying is this. If you move t 1 to t 1 plus c t 2 to t plus c, probability density still remains same. You get the same density function, and x of whether it is x of t 1 plus c and x of t 2 plus c or x of t 1 and x of t 2 does not matter because this is the variable of integration. In fact, instead of calling it x of t 1 and x of t 2, I should rather change them to this.

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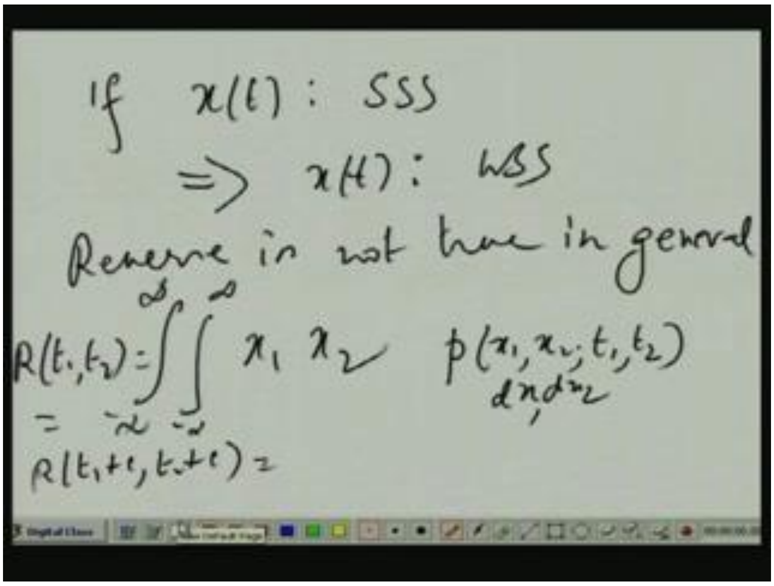

So, obviously these two are same applies for the mean easily, but you can easily see that the reverse is not true. So, this topic is very important, this stationary process because especially WSS here we are bothered about only correlation and mean. Correlation becomes easier functions mean becomes let us say a constant, and correlation becomes a function of only one variable that is lag variable. This you know has got tremendous application in filtering communication signal processing control.

So, in the next class, we will continue from here. We will get into some kind of you know some examples of WSS processes, and a particular process which is very important in many communication examples for that matter it is called cyclo stationary process. So, we will be considering them. Then, we will consider linear systems, linear time invariant systems whose input is given by a random process which is WSS. What happens to the output? This kind of studies will make afterwards. So, that is all for today. Thank you very much.

Preview of next lecture.

Probability and Random Variables

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Lecture - 31

Cyclostationary Processes

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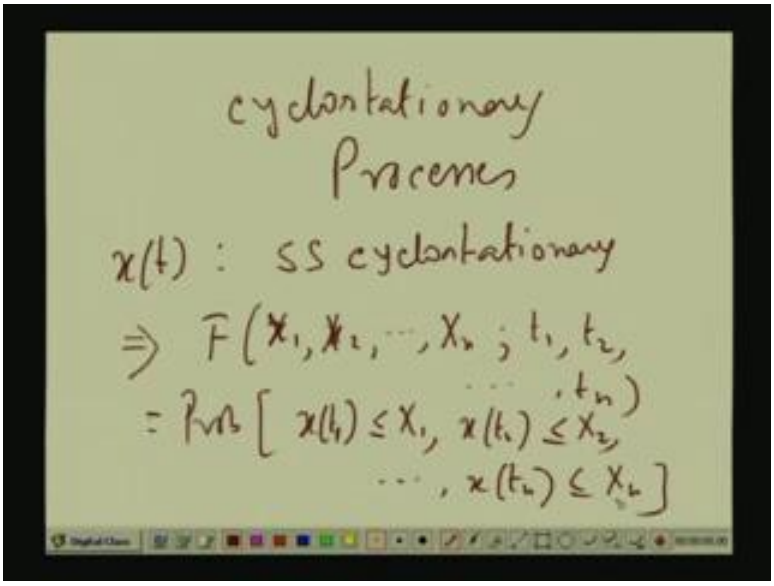

So, we start today important topic that is cyclostationary processes. It is a special kind of random process which is not fully stationary, but it thus exhibit some sort of stationarity and occurs frequently in many applications especially in digital communications and also called periodically stationary process. Actually these processes are such that their statistics vary periodically over a constant time period. If I talk of cyclostationary process which is strict sense cyclostationary that is if xt is SS, SS stands for strict sense cyclostationary. This would imply that if you take the probability distribution of F, that is let me make it capital. What does this mean? Do we have only we done it, but still there is nothing wrong in just restating what you already know.

This is the joint probability distribution function of any random process xt. It is a joint probability of that is by definition what is this probability of what x at t 1 lying less than equal to x 1 x 2 at t lying less than equal to x 2 dot dot dot x at tn lying less than equal to capital Xn. These are joint probability. This is called the probability distribution, and this number n is the number of sampling points. You are choosing that is purely up to you. You can take it from 0 to up to infinity because d theta and its minus d alpha.

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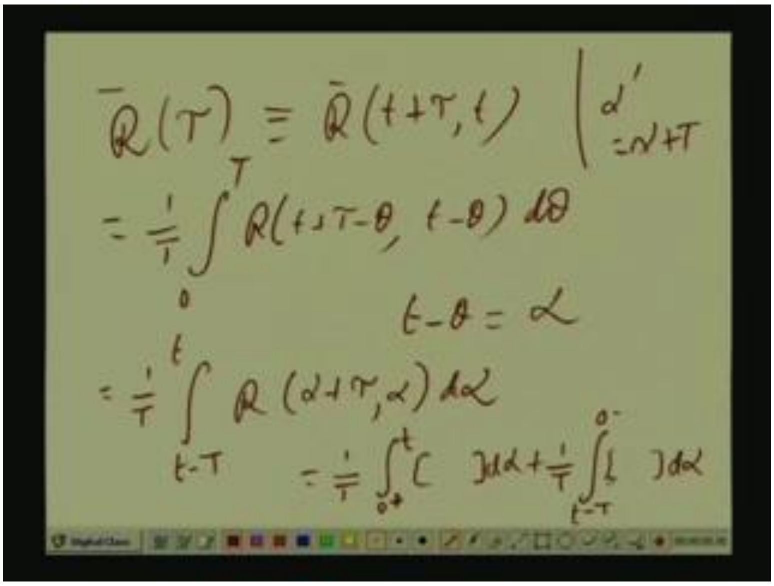

So, minus comes and the two integral limits are interchanged. So, minus sign goes and this is your alpha plus tau, alpha. From this again it is obvious because the entire thing is periodic about alpha over a period at capital T. Still if you are not convinced, you can break the integral in two halves. One is from 0 to t that is 0 plus 2 t. Another is from t minus T to 0 minus same thing dt. Consider now this integral. Second one here again bring this variable alpha prime is alpha plus T same way. So, d alpha and d alpha prime are same when these integrals are with respect to alphas, d alpha. So, d alpha and d alpha prime are same and when alpha becomes t minus capital T alpha prime becomes t. When alpha becomes 0, alpha primes becomes capital T and the two integrals can be brought under same one and you get the result. That result is a function of tau alone. What you get?

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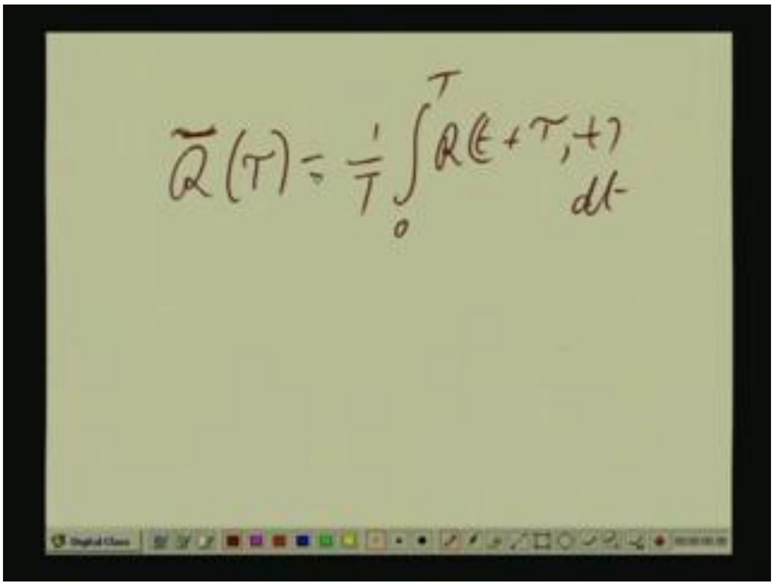

What you get is R. You can call it alpha or you can bring back t again, say does not matter, because that is an integration variable t dt. So, you can call it alpha as well instead of t 0 to T 1 by T. This is your R R bar tau. So, this process is strict sense stationary. These are some applications in communication. We will take one example. The example of power amplitude modulation alpha (PAM) will give you a practical feeling and you understand why we spend some time on it. After that we will move to systems, that is processing of random processes by some linear systems or non-linear systems time invariance systems and all that. That will take us to finally spectral analyses power, spectral density and things like. So, that is all for today.

Thank you very much.