Probability and Random Variables
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Lecture - 21
Sequences of Random Variables

So, today we will discuss an important topic that is we start this topic sequence of random variables.

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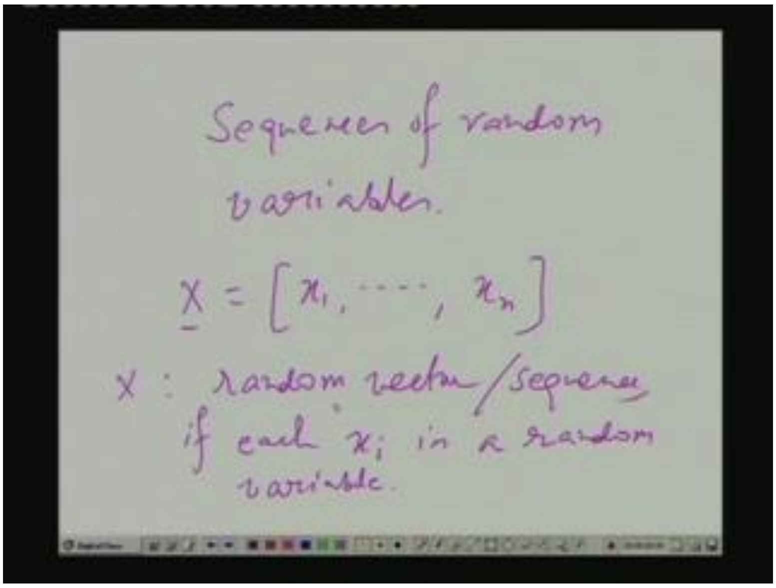

You see we started with first a single random variable, right and we considered a function of single random variable. Then, we generalized it to two. I mean two random variables. So, you know you can then say that it is a sequence just of two. I mean elements, two random variables, two elements sequence, and we consider function of two random variables and functions of two random variables and all that, right. The whole treatment will now be generalized where we will be considering a vector say X as x 1 dot dot dot dot x n. This X will be called a random vector. If each element x 1 to x n is a random variable that is X will be called a random vector, or say sequence if each xi is a random variable.

So, there you see earlier we considered two random variables. So, we had a vector of two elements x 1 and x 2. We call it xy. This is a defined notation, but now it is more general in such random variables, right. So, for the hindsight, you can see that if you talk of the joint density or joint probability distribution of x, it will be basically a function of n variables, right x 1 to x n.

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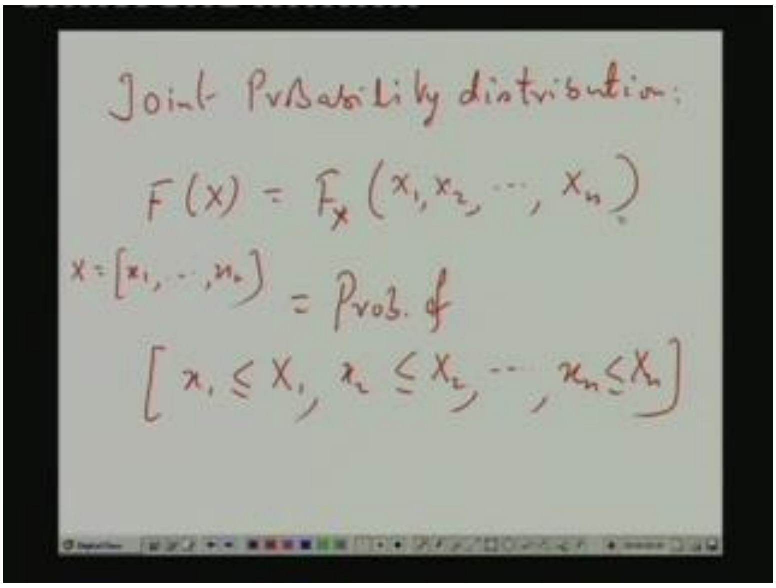

So, we can define joint probability distribution. Actually x would write like Fx for x say takes values x 1, this specific value x 2 dot dot dot x n, and this capital X, actually the vector consisting of the variables small x 1 dot dot dot small x n. Here capital X 1 is a particular value for small x 1. X 1 is variable. Capital X 2 is a particular value for the variables small x 2 and likewise. What does this mean? It means the probability of this event that x 1 is less than equal to capital X 1, x 2 less than equal capital X 2 dot dot dot x n less than equal capital X n.

This would occur jointly. There are n joint events that is one is small x 1 less than equal capital X 1. Another is small x 2 less than equal capital X 2 so on and so forth. This should occur jointly simultaneously. That is why it is called joint distribution, right and that is denoted by Fx x 1 up to x n. Sometimes when I do not need, I may not I may skip the subscript x and I will put x 1 to x n. I think you can really understand sometimes, but when there is confusion, when suppose I am dealing with more than two such distribution functions, then to differentiate between the functions, I will put the subscript. Now, for the joint probability distribution, but as a same token you can then define, and you can easily see that is now a function of n variables x 1 to x n. By the same way, I can next define the joint probability density function.

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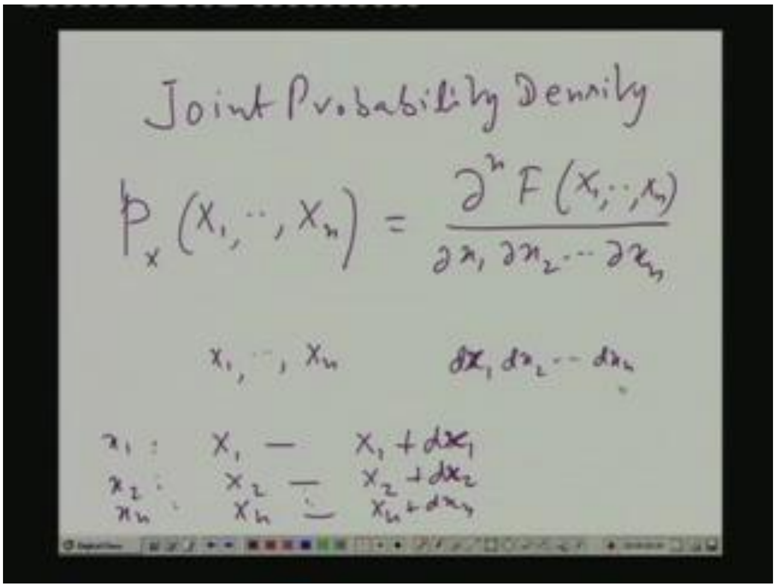

Joint Probability Density-Recently you know that we have to differentiate. When you differentiate the joint probability distribution function, so you define like this Px X 1 up to Xn is nothing but del n n times, and such skipping the subscript here X 1 dot dot dot Xn with respect to del x 1 del x 2 dot dot dot del x n. This is the joint probability density. What does it mean actually? It means that if you now consider n dimensional space whose axis are x 1, x 2 up to x n and there if you take an infinitesimally small volume i mean mass, in fact as far as region whose value is say dx 1, dx 2 up to dx n, that is along x x x 1 axis. That is suppose in n dimensional space, you are located at a point up to x n.

At this point you are considering an infinitesimally or infinitely small or infinitesimally small, I mean region or cell whose sides are we call it actually hyper cube, where actually real life cube means it has got only three sides, but in n dimensional space, I will call it n dimensional hyper cube whose sides are dx 1, dx 2 up to dxn. So, volume is this. So, basically it defines an area where x goes x. The variable x 1 goes from capital X 1 to capital X 1 plus dx 1, capital X 2 goes for that is this zone. This is for variable x 1. Therefore, variable x 2 again this is the range dot dot dot. So, that means, at this point capital X 1 to capital Xn at that point, I am considering a small cell, a small region.

In fact, this hyper cube I would say whose sides are of length dx 1, dx 2, the dx 1 along that x is x 1, this variable small x 1 dx 2 along the axis small x 2 and so and so forth. So, I basically cover a volume of value dx 1, dx 2 times dxn, and then the probability that the random variable x 1, x 2 up to x n, these variables lie within that hyper cube located at this point capital X 1 up to capital Xn and of this volume, that is small x 1 lie within this range, small x 2 lie within this range, small x n lie within this range simultaneously. I repeat again, this means that I am looking for probability of these elements X 1 to Xn lying within the small hyper cube located where at the point capital X 1 Pxn.

In this manner, that is starts at X 1 along the smaller x 1 axis goes up to capital X 1 plus dx 1 to side. It covers the side of length dx 1. Then, along the small x 2 axis, it starts at x 2 and goes up to x 2 plus dx 2, so covers a side of distance dx 2 and likewise. So, at this point capital X 1 to capital Xn a small hyper cube is formed infinitely. Hyper cube is formed because volume is this dx 1, dx 2 dot dot dot dxn. The probability of this variables lying within that hyper cube together will be given by nothing but just minute. That will be given by I am dropping the subscript this times dx 1, dx 2 dot dot dot dxn.

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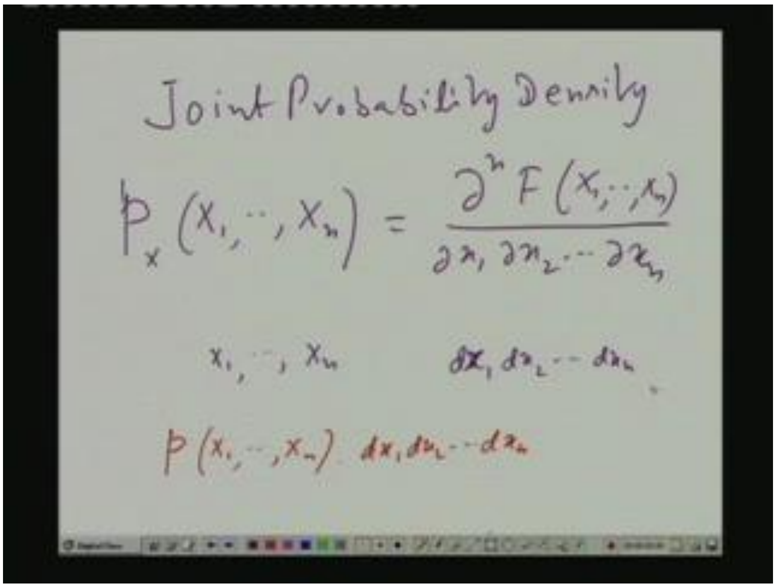

This image actually follows from this formula that basically this is nothing but the derivative, partial derivative of distribution function. So, if you multiply this probability density with dx 1, dx 2 up to dxn, then that means nothing but the probability of the variables X 1 to Xn lying within the hyper cube located at capital X 1 capital Xn of sides dx 1, dx 2 up to dxn.

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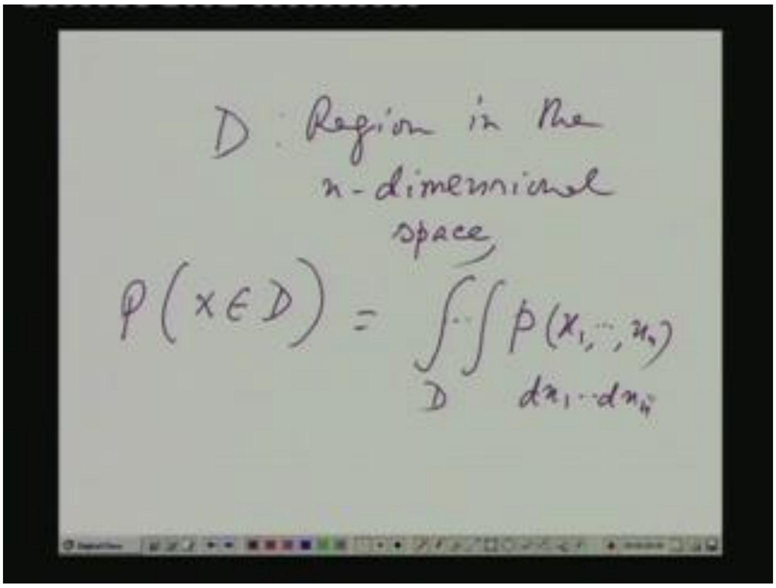

This means that if I consider a region D, the arbitrary region in the n dimensional space, then probability of this vector X falling within D, that region is nothing but integral. Actually it will be n-dimensional integral. I will call it what D p x dx x is a vector. You got to say x 1 up to if you even want you can write it like this. You can see that is nothing but just generalization of our previous concepts. It is generalization of our previous concepts from two variables, two in the general case of n variables. Certainly certain more things also you can see some other things.

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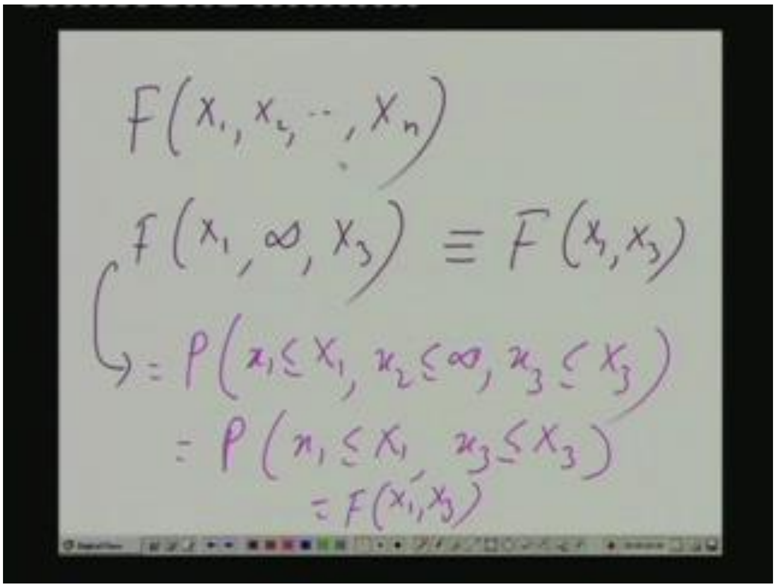

For instance, if you consider the distribution function F. Suppose here some of the values I put as infinity for examples suppose I have got X 1 infinity X 3, then my claim is this will be nothing but the joint distribution for just X 1, X 3. Why? After all what is this? This is nothing but probability of x 1 less than this, x 2 less than equal to infinity, x 3 less than equal to x 3. Now, x 2 less than equal to infinity means all possible values of x 2 are covered which effectively means that I am basically looking at the probability just of this event. I am not selecting a particular choice of x 2 because all possible x 2's are covered. X 1 should be less than equal to capital X 2, X 1, but all cases of x 1 will be considered for which x 2 can take value from minus infinity to infinity.

So, this is nothing but this F. I guess you understand this. I repeat again in case there is confusion. X1 less than equal to capital X 2, capital X 1, x 2 less than equal to infinity. I am considering the joint events at x still less than equal to capital X 3. This is equivalent to this joint event x 1 less than equal to capital X 1 still less than equal to capital X 3 because this have to consider all possible cases for which x 2 can take some finite value, some other value sign another value up to infinity up to minus infinity. So, there the whole range of x 2 is absorbed in this joint event.

So, these two joint events are equivalent, but in this case I am getting the probability distribution for just x 2, x 1 to x 3 which means in general if this probability distribution functions of n random variables. If you put infinity for some, then what I get is nothing but the joint probability distribution of rest of the variables is most important, right, rest of the variables. Then, you define with respect to those variables, those remaining variables; you get the corresponding joint probability density, that is just by the normal definition.

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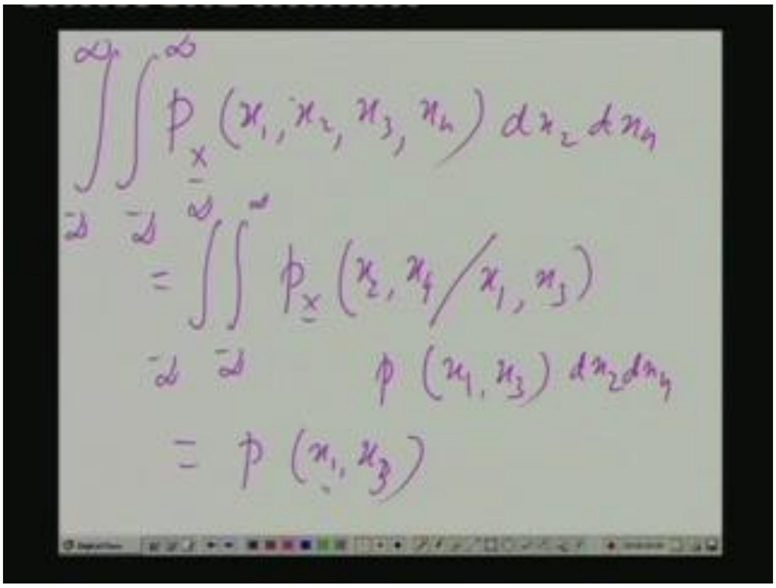

Similarly, suppose this probability density function say an example, take any example say x 1, x 2, x 3, x 4. Suppose I integrate it with respect to say one or two variables, not all of them because if I integrate with respect to all of them I get one of course, but total probability is 1, but suppose I integrate it with respect to say two variables, say dx 2 dx 4 minus infinity to infinity. This is my X vector for x vector is x 1, x 2 up to x 4. I put a subscript here purposely. What does it mean? You can easily see you can write this as conditional probability of given some x 3, x 4 times P x 3 x 4 dx 2 dx 4. Sorry there will be some change here which is x 1 x 3.

Now, P x 1 x 3 go outside. This integration is with respect to x 2 x and x 4, and if you integrate it, integrate the spot with respect to x 2 x 4, you have minus infinity to infinity. Obviously, that will be 1, because subject to this given condition, I mean sorry that x 1 x 3 are given. Probability of x 2 falling within minus infinity to infinity and x 4 falling within minus infinity to infinity, that has to be 1 because it covers all possible values. So, you get nothing but P x 1 x 3. That means, if the probability density functions of all these random variables is integrated with respect to some of the random variables, then what we get is the joint probability density function for the rest of the variables. Then, you remember when you considered two random variables; there we moved to function of random variables. We considered two functions of two random variables, right.

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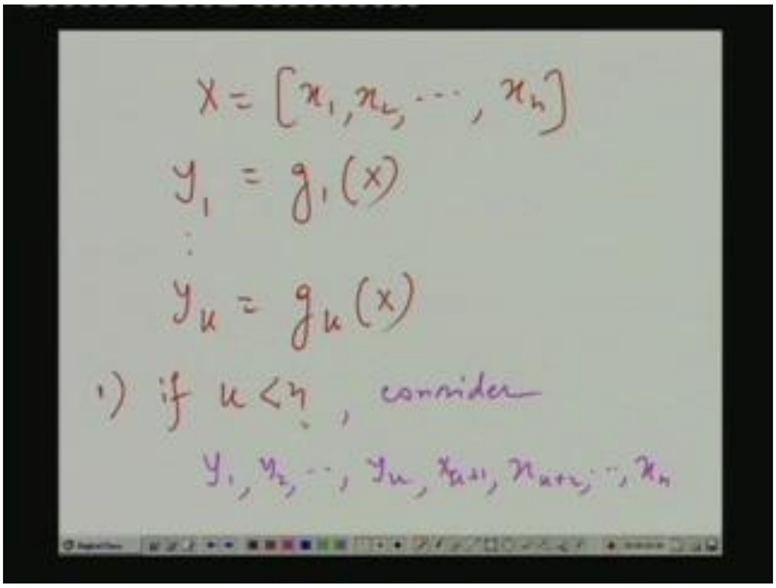

Here what you are given is this X as we consider some functions y 1 yk. Our purpose will be to find out the joint statistics, that is the joint distribution or joint density. I would say I mean both are equivalent. You can obtain one from the other. Joint density say of this random variables y 1 to yk given the joint density or distribution of the random variables x 1 to x n. We have already tackled this problem when was two, that is for the case of two random variables, we have already tackled this case. That time we took two functions g 1 and g 2, but now it is general case we have k functions.

Obviously, there are three choices possible. K can be less than n, k can be equal to n and k can be greater than n, but we will finally consider only k equal to n case because the other two cases can be easily handled. If you know the result for k equal to n, let us first see how that is possible, and then we will come to k equal to n case and find out the expression for the joint density of the random variables y 1 to yk. First if k is less than n, then we will consider y 1, y 2 up to yk, and then to make the total number of such variables equal to n, I simply add x k plus 1 x k plus 2 dot dot dot x n. That is from this random vector, I take out the last n minus k entries.

Now, y 1 is g 1 x, y 2 is g 2 x, yk is gkx and xk plus 1 also, you can say it is a function of x where given x you only pick up one particular element that is xk plus 1. Xk plus 2 is also a function of a x after all given the entire vector x, you just take out one particular element that is xk plus 2 and same for x n. So, then the length becomes n. We already know joint density of x or this random variable vector x. Then, for this case, the length becomes the length of this modified random vector if you call it k prime k prime is equal to n, and we have already told that for the case where the total number of random variables or total number of functions is same as the total number of random variables x, that is n, we will consider that case soon. That theory can be applied here.

What I mean is that we have already made this statement that we will state the result for the joint density of the random variables y 1 to yk for the case, where k equal n. So, that result should be applicable to this modified random vector. So, when k is less than n, we simply can append some elements from the vector x to make it length n, and then apply the theory which we have to discuss soon to find the joint density of this entire stuff. That should not be difficult. If we look at that theory, we will come back to this later as an application of that theory when total number of random variable is same as total number of these functions g 1 to gk, that is k is same as the total number of random variables x 1 to x n that is n, but when k greater than n, then some problem comes.

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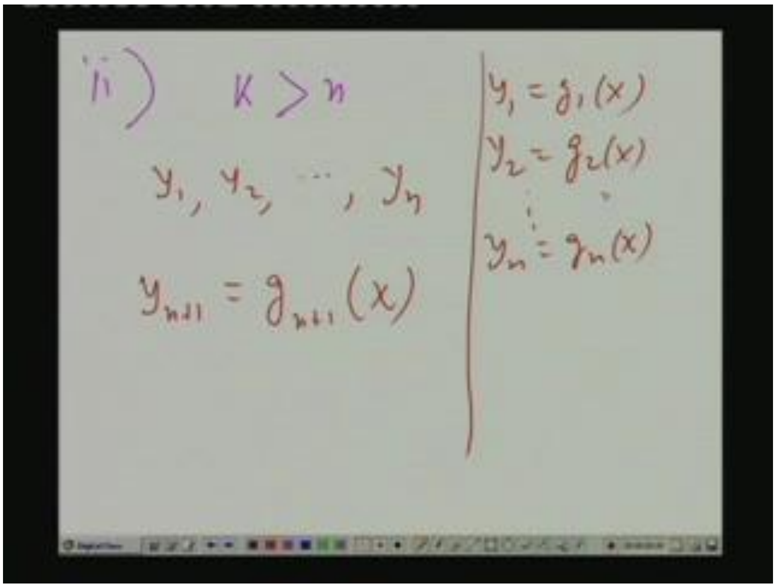

Here, you can see one thing. We consider y 1, y 2 up to yn. So, the length is up to this is same as the total number of random variables x, that is x 1 to x n. So, that functions y 1 after all is function g 1 x, y 2 is a function which is g 2 x. So, the total number of functions up to this is same as the total number of random variables, it is n, k is greater than n. So, you have got k minus n extra functions. How about that because I have to actually find out the joint density of this k function y 1 to yk, but I have only considered a part of it y 1 to yn. How about the rest?

Now, if you consider for that matter yn plus 1 which is nothing but say gn plus 1 x. Now, my claim, my statement is that most of n, you can express this x in terms of that is the vector x. That is the elements of x, x 1 to x n in terms of y 1 to yn. How? That is we know that y 1 is g 1 x, y 2 is g 2 x. For example, suppose these are all linear functions, some linear combinations of the elements of x. Suppose y 1 is twice x 1 plus thrice x 2 plus dot dot dot plus say v 10 x n similarly another. So, that means to linear equations. Similarly, suppose you get 1, the linear equations that another here. So, you can solve them and get a solution for y in terms of x.

In general case, you can get more than one solutions. That is fine, but what it means is this you can replace x by this y, either one possible y or several possible y's. Given random variables x, x 1 to x n, you can replace them by y 1 to yn, right which means yn plus 1, yn plus up 2 up to yk, this extra function, they can be expressed in terms of y 1 to yn. Then, my claim is joint density of y 1, y 2, yn up to yk can be expressed just by the joint density of y 1 to yn. If I find out the joint density of y 1 to yn, then that is enough for me. With that I can find out the joint density of y 1 to yk. Now, how to explain that I just considered the example? Forget all these for the time being.

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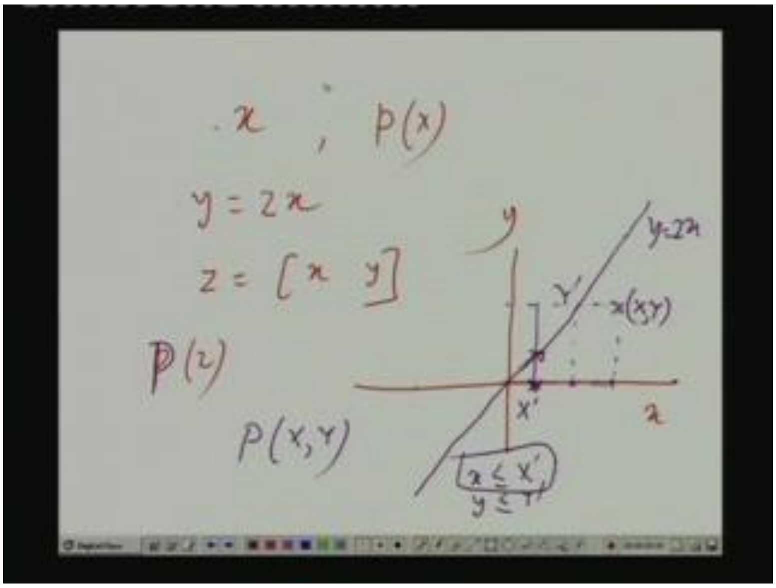

Suppose x is a random variable and you got this probability density functions say px, and you define another variable y as 2 x. Now, we have to find out and now you follow vector say z as xy. You have to find out P z. My claim is since y is expressible in terms of x to find out the joint density of xy, this vector is enough. If you know the joint probability density of x, Pz can be easily written in terms of that. This is not very difficult. What it means is this suppose x and y with a plane and this is the line y equal to 2 x.

We start with probability distribution for that matter because that is easier to explain and suppose you got say other any point here, some x as corresponding y y. In this case, what is the probability? Suppose you call it capital X capital Y this point. So, where is a joint distribution Pxy? That means, what is the probability that small x is less than equal to capital X and small y is less than equal capital Y. Remember that for any given x y lies precisely on this line and nowhere else.

So, if you got at this point now capital X, the probability that small y lies below this point is 0. If you go a little left, the probability that small y lies again below this height again 0 because if you are at this x corresponding y is here on this line. If you are at this x corresponding y is here, but if you are looking for small y taking value less than this height, so obviously this point is not on this line. So, probability is 0, but if you go up to here call it say x prime, then there is no problem. I mean x prime what to the left of it, there is no problem because there is a probability.

What is the probability of say x less than equal to x prime, y less than equal to y prime? What will that? For instance, suppose you are here. Probability that small x is less than equal this and small y is less than equal to this, what will that be. Obviously, that will not be 0 because if small x is here, y will be here. So, that is less than this, but then what is the probability. Then, simply the probability distribution of x is up to this point. This is because y is purely determined by x. If small x is up to this point, y has to be here, so that the joint density x will transfer to mean.

There is a probability, the joint distribution transfer to mean the simple probability distribution of x. I repeat again suppose you are now here x double prime. Probably you call this xy prime. Forget this. Call it y prime. Suppose x prime is here and this is y prime. So, now the probability of x taking values less than equal to x prime and y taking value less than equal to capital Y, see the probability of x here x taking value x prime and y less than equal to y prime, that is not 0. If you go backward, same thing occurs that is because for any choice of given x be it x prime, the corresponding y is here that is below this height.

So, probability is not 0, and then what is probability of then x lying I mean, what is the probability of x lying to the left of say x prime and y lying x less than equal to x prime and y less than equal to y prime, what is the probability? It is simply the probability of x less than equal to x prime. So, we see if that is satisfied, these are all satisfied because y is equal to 2 x. So, if small x is less than equal to x prime, y has to be less than equal capital Y prime because y will be on this line and that is below this height. That is automatically satisfied. So, this is enough to have only this. If this is satisfied, these are always satisfied. So, basically this is the event or distributions are same. I mean this is the event, right. This is not that these two events are same, but this is the event if it is satisfied, the other is automatically satisfied.

So, the joint probability is nothing but joint probability of these two events occurring together. Simply the probability of this event alone means for such cases, where one variable is expressible in terms of the other. The joint density of the vector is obtainable simply from the joint distribution of the original vector is obtainable simply from the original probability distribution, and then by the same logic, it apply I mean and by extension, the same logic applies to the probability density function also. So, this explains why.

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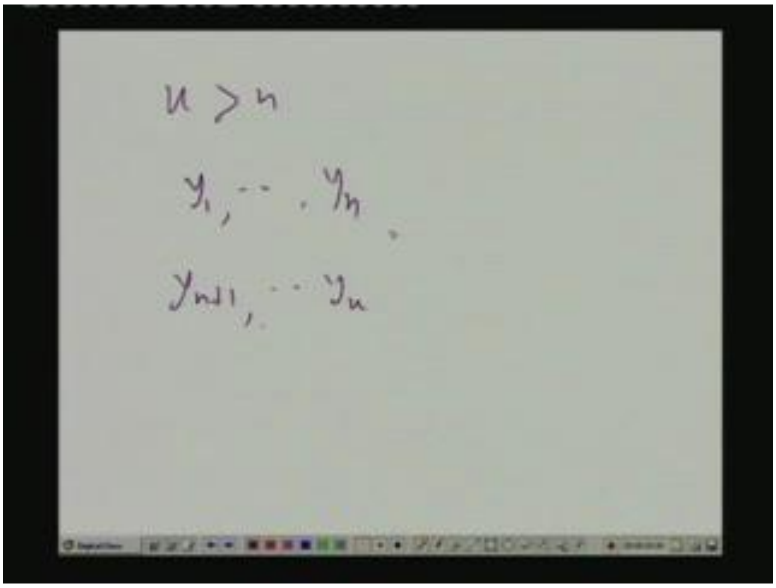

For the case k greater than n, we can just happily take up to y 1 to yn and find out the probability density of this because I have told you yn plus 1, 2 yk. Each of them can be expressed I mean in terms of this. How? Because each of them y 1 is g 1, x y 2 is g 2 x, yn is gnx. If you solve them, then all the elements of the x vector x 1 to x n, they can be expressed in terms of these variables, and by that process yn plus can be written in terms of these variables, yn plus 2 also can be in terms of these variables y 1 to yn like that. As I told you if yn plus 1 is expressible in terms of these variables or y here up to yk, so yk also is expressible in terms of these variables. The overall joint distribution can be written down just in terms of the joint distribution of y 1 up to yn.

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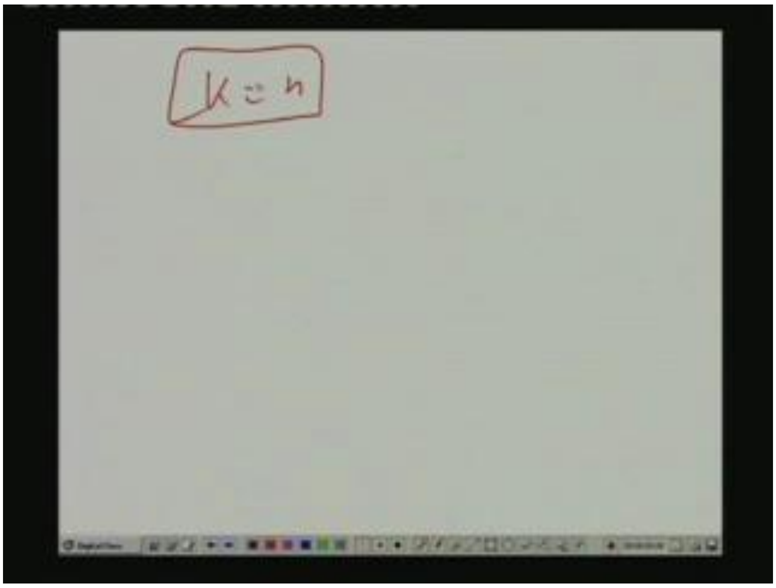

So, this means we can simply take K equal n. So, if there are n random variables, we take n functions y 1 to yn. I repeat x 1 to x n, there are n random variables and we take these functions. I mean y 1 as g 1 x, y 2 g 2 x up to yn gnx. In this case, what is the joint density of y 1 to yn?

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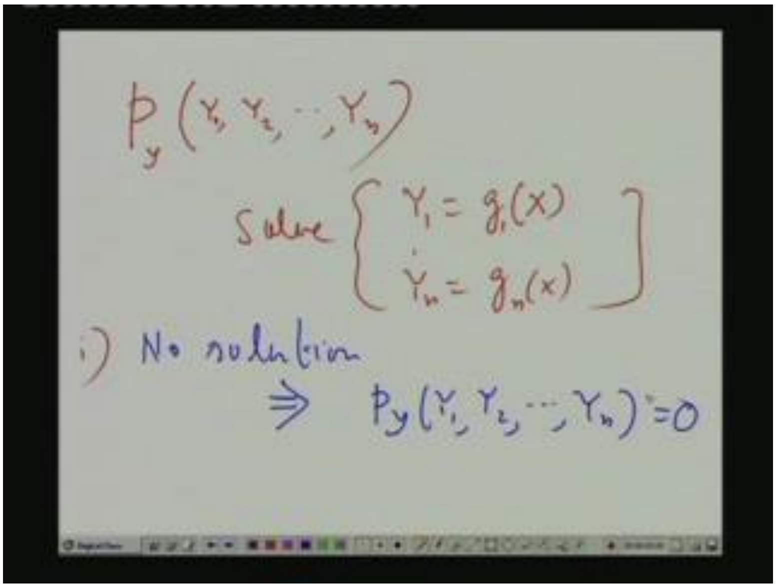

That is what is you could now put a subscript say y. Some values are given the joint density at these values. Capital Y 1 for small y 1, capital Y 2 for small y 2. This is specific values and small y 1 is the variable, small y 2 is variable and like that. What is that you have to find out? For that first we solve these equations. First, this we solve by solving each component of the vector x, that is x 1 up to x n will be written in terms y 1 to yn. As I told before that now first possibility is no solution. Suppose no solution exists, then the theory says that this would be 0. It is 0.

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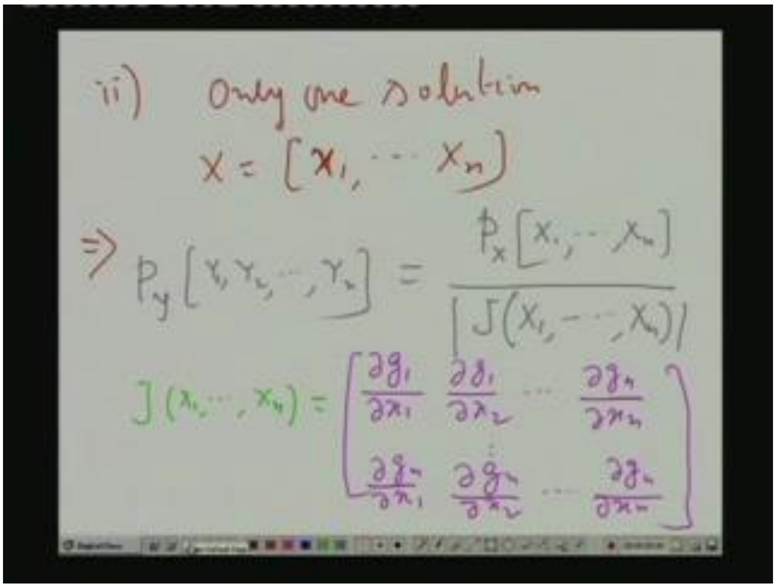

Then, suppose only one solution is in this case is by the theory you replace, put the solutions here. So, capital X 1 for small x 1, capital X 2 for small x 2 up to capital Xn for small x n. Remember each of the variables capital X, each of these elements capital X 1 to 2 Capital Xn. They are basically I mean expressed, they are basically obtained in terms of y 1 to yn. So, this becomes actually a function of y 1 2 yn. This is divided by determinant of a matrix. The matrix is called Jacobian. Again it is evaluated at this.

What is Jacobian? This partial derivative, but that evaluated at this solution point. They are not general expressions del x 1 del g 2 del g 1 by del x 2 dot dot del gn up to del gn. So, if these partial derivatives are evaluated at x 1 equal capital X 1 small x 2 equal to capital X 2 and dot dot dot small x n equal to capital Xn, then this matrix you compute the determinant and ratio of this will be the probability density. As I told you each of this capital X 1 to capital Xn, they are actually functions of, they are already expressed by our solution in terms of y 1 to yn.

So, right hand side actually becomes a function of y 1 to yn and if you have more than one solution, then you will have such terms just added. Suppose there is one solution x 1 up to x n, there is another solution x 1 prime up to x n prime and call it capital X prime. In that case, a total probability density will be this plus again Px within bracket X 1 prime dot dot dot x n prime divided by the Jacobian determinants of the Jacobian evaluated at x 1 prime to x n prime and so and so forth, that is the theory.

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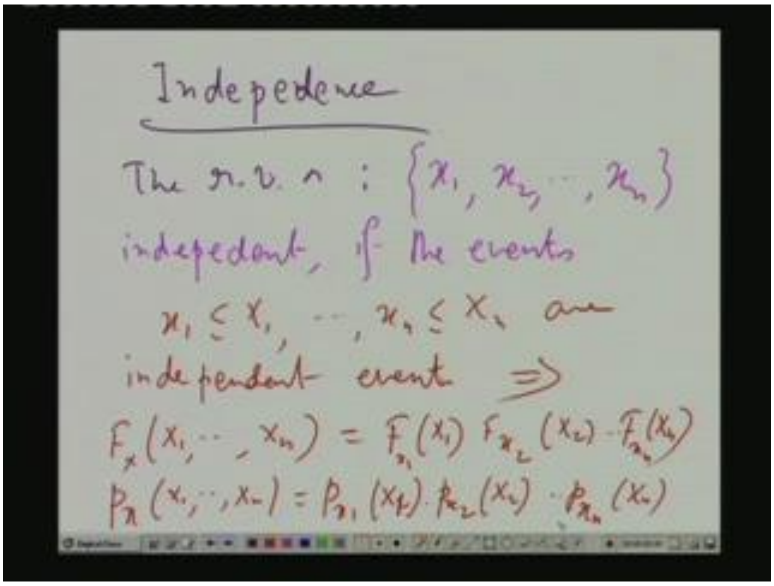

The random variables, there is a random variable and they are called independent is just extension of the concept of statically independent. Earlier we have independents of two random variables, the case of n random variables nothing else. They are called statically independent. If you consider the joint event and if the events are independent events meaning the joint probability of these events together, basically the joint probability which is nothing but the distribution will be nothing but probability of x 1 less than equal to capital X 1 multiplied by probability of x 2 less than equal capital X 2 dot dot dot dot.

As a result if you want to find out the joint density, then as by your theory you have to define say this with respect to x 1 up to x n, but if they are statically independent, the distribution is a product of the individual distributions. So, when you differentiate the right hand side with respect to x 1, x 2 and x n, then obviously the density becomes where you differentiate this with respect to x 1, only this guy is a function of x 1. So, this is it gets differentiated with respect to x 1 which gives rise to the probability density of x 1. Similarly, it is probability density of x 2 so and so forth. So, it becomes sorry x 1. Just consider an example as to how to apply the theory.

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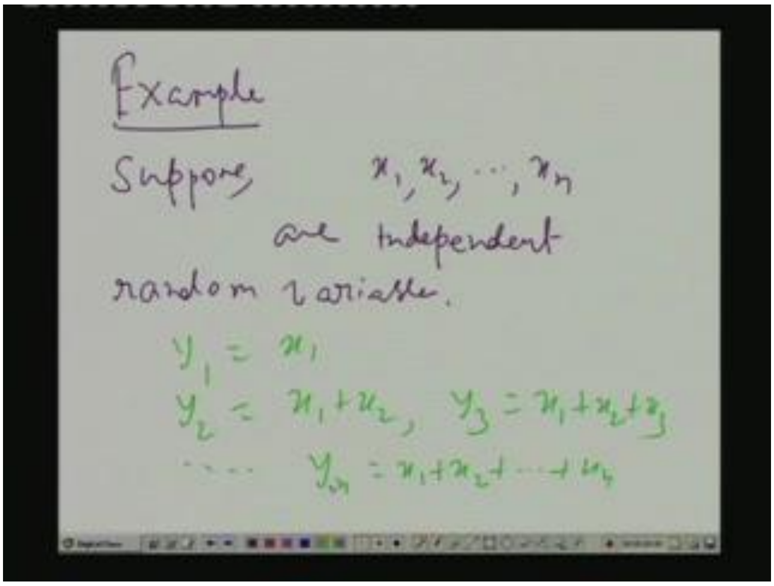

Suppose x 1, x 2 dot dot dot x n are independent and you form y 1 as x 1, y 2, x 1 plus x 2, y 3 x 1 plus x 2 plus x 3 dot dot dot up to yn up to x n, so essentially we have got some linear functions here. Y 1, y 2 up to yn, there all functions of x 1 up to x n, but they are some linear functions of this specific time. Now, we are given the fact that x 1 to x n, they are independent and we know their joint density because we know the individual densities. This is given. We have to find out the joint density of y 1 to yn by the previous theory. So, first to do that we fix up, that is we have to find out.

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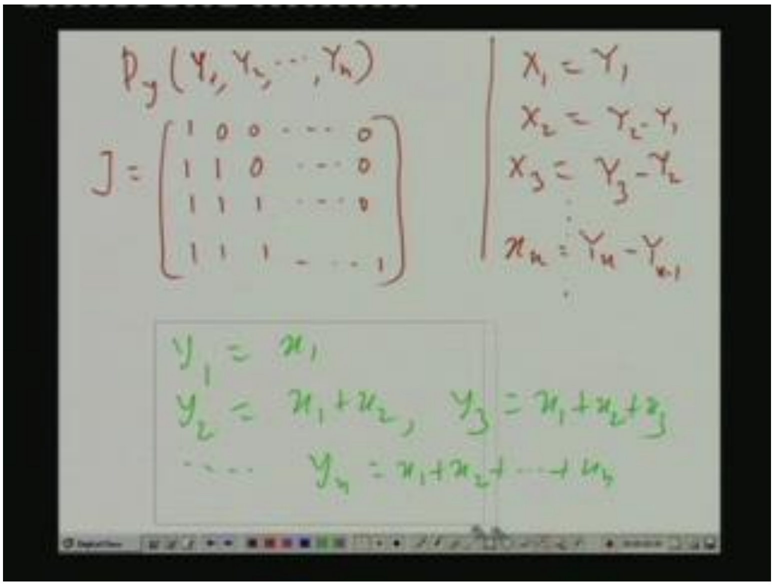

We have to find out Py capital Y 1. Some specific value capital Y 2 dot dot dot say Yn, where you find out there is a way to find out the joint probability density of y at a specific value say capital Y 1 capital Y n. This will be chosen arbitrarily. So, you will get the above all functions. So, as per our theory, we first solve for x 1 2 x n given this Y 1 to Yn. So, you can easily see the solution that x 1 is nothing but from this first equation Y 1 itself X 2 is nothing but Y 2 minus x 1 and x 1 is Y 1, so Y 2 minus Y 1. X 3 is nothing but you can easily see Y 3. If you put Y 3 here, what is x 3? It is y 3 minus x 1 minus x 2 which is y which means Y 3 minus Y 2 so on and so forth.

Actually in general, xk is nothing but Yk minus Yk minus 1 this way. So, the solution is unique. So, it is not the solution. Solution does exist, but only one solution. No other solution. It is probably a linear equation. This is number one and how about the Jacobian to form the Jacobian plus del Y 1 with respect to del x 1. That is 1 because del x 1 with del x 1 that is 1 and del y 1 with respect to del y 2 del y 1 by del x del x 2 that is 0 because it is not present here. It is same for other entries. Then, del y 2 del x 1 that is one del y 2 del x 2, again 1 and then 0, then 1, 1, 1, 0, 1, 1, 1, dot dot dot 1.

You can easily know, you can easily see determinant of this matrix is 1 because 1 times whatever be a square matrix, then 0 times raised they all go. Again apply the same thing here. Finally, 1 into 1 into 1 into 1 into 1 is 1. So, Jacobian, all they are has determinant equal to 1. So, now, the theory is very simple.

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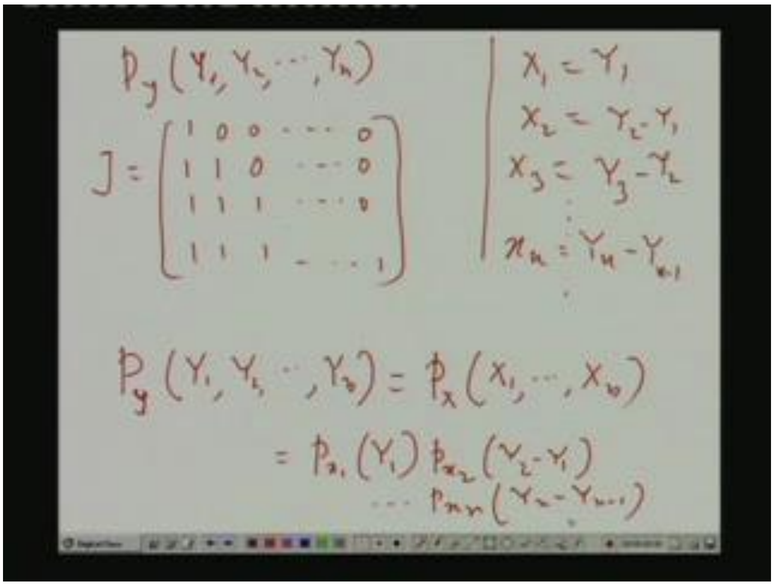

You can easily see it will be nothing but px at this choice X 1 up to X 10 divided by the determinant that is 1, and we are given the fact that the random variables x 1 up to x n, they are independent. So, this joint density is nothing but the product of the individual densities. So, Px 1 and capital X 1 is Y 1 px 2 which is Y 2 minus Y 1 dot dot dot Pxn Yn minus Yn minus 1. So, you can easily see that the concepts involved here, they are nothing new. They are simple generalization of the concepts that our valid or those were introduced rather in the case of two random variables. That is why I am not giving you the proof and all that. You can argue about this on your own.

So, I stop here today. In the next class, I consider this issue further and going to things like you know mean, covariance, correlation. In fact, we will have a correlation matrix, now covariance matrix and things. Again the characteristic function issue as relevant here and that takes us to a very important theorem called central limit theorem. So, that is all for today. Thank you very much.

Preview of the next lecture.

Probability and Random Variables

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Lecture - 22
Sequences of Random Variable (contd.)

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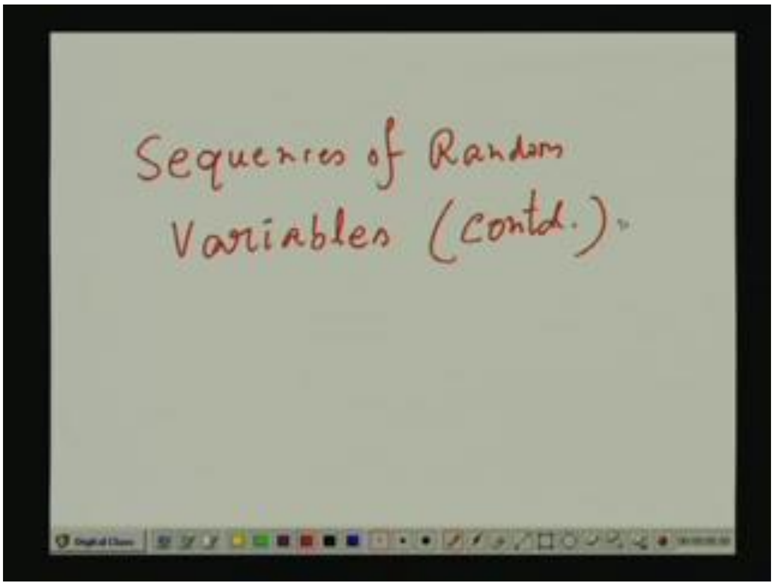

So, you know we have been discussing this issue of random sequences. That is just for a recap. We initially started with just one random variable and function of one random variable, then two random variables, then one function involving two random variables and two functions involving two random variables. Here, we tried to generalize that to a set of n random variables which are ordered as sequences x 1, x 2 up to x n, and we first considered one function of such n random variables, and then n functions of such n random variables, right.

So, there are several issues which we did not discuss last time. There are all you know just analogous to those issues which we considered in the case of two random variables. So, we will just continue from where we left last time, but just one remark. We have so long used real value random variables, but random variables could be complex value also. Now, suppose we have got this.

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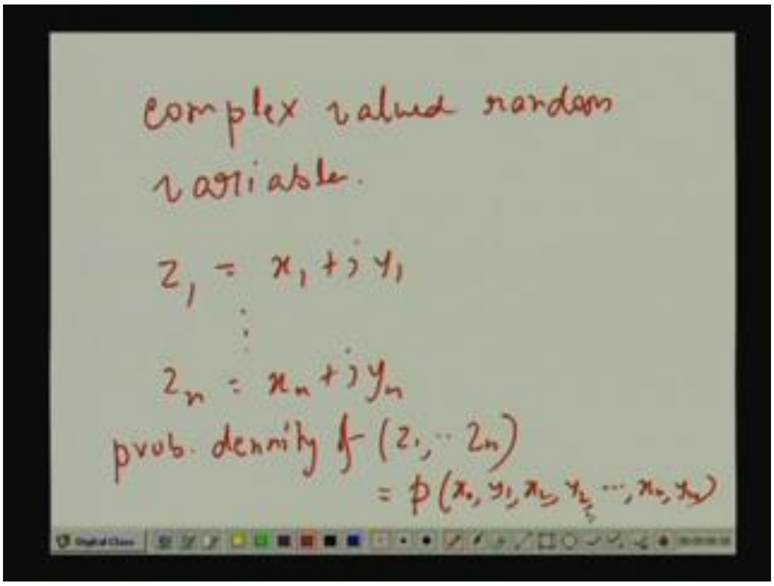

Suppose z 1 is x 1 plus jy 1 dot dot dot up to say zn x n plus jyn. So, z 1 to zn, we have got set of n complex random variables, but remember each of these z 1 to zn, they have got two components. One is the real component and another imaginary component x 1 y 1, and now x 1 also is random variable, y 1 also is random variable. Similarly, x 2 is a random variable; y 2 is a random variable. So, a set of n complex random variable equivalent being is a set of twice n real value random variables which means probability density of zn is same as actually probability function probability density function of two n variables x 1 y 1 x 2 y 2 dot dot dot x n yn.

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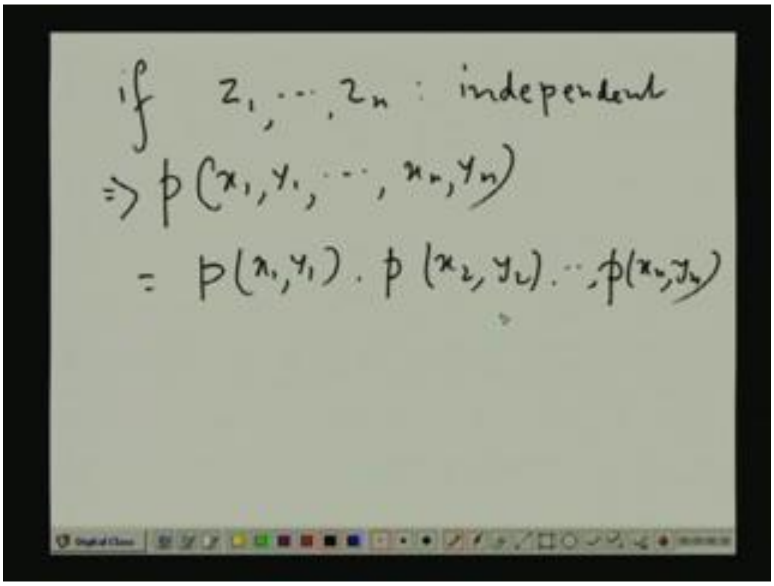

Further if z 1 dot dot dot zn, they are independent, then it means that p x 1 y 1 dot dot dot x n yn, that will be like this which stands for the probability density for z 1 because z 1 has two random variables, real value random variables x 1 y 1 in it, and then p x 2 y 2 dot dot dot p x n yn.

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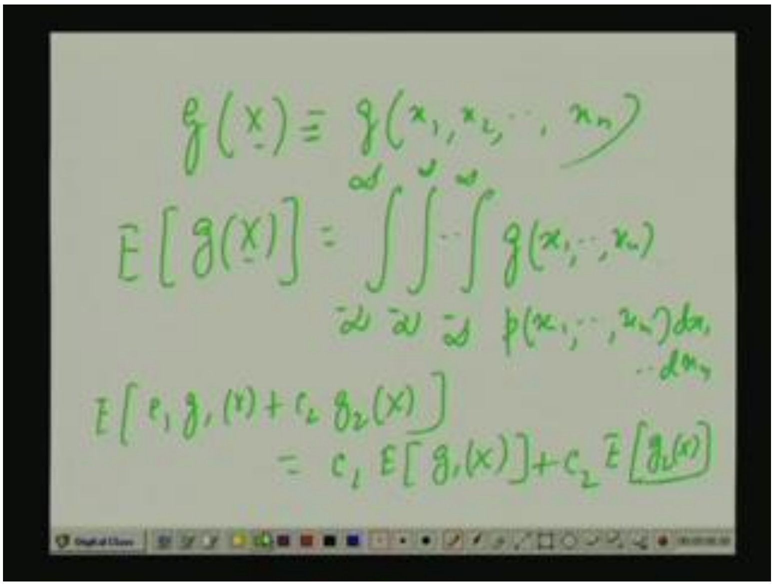

Then, again suppose we have got single function g x. X is vector. Basically x means actually is a function of n random variables x 1 x 2 dot dot dot x n. Now, by extending our previous argument, we can then say that expected value of g x is nothing but times the corresponding probability density function dx 1 dot dot dot up to dxn. Obviously, if instead of these real variables x 1 x 2 up to x n, we are a set of complex n number of complex valued random variables z 1, z 2 up to zn, then instead of having n fold multiple integral, you would have had twice n fold multiple integral, and the probability density function would have been I mean this is g z 1 up to zn and probability density function as we proved have been p of x 1 y 1, x 2 y 2 dot dot dot x n yn.

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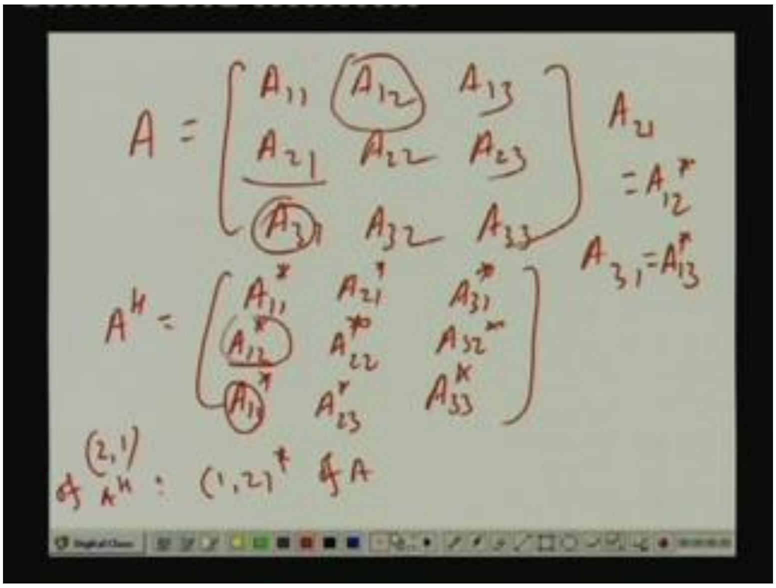

Then, A will be called a Hermitian matrix. That means, firstly the diagonal elements A 1 1 must be A 1 1 start which means, it must be real valued element. So, the diagonal similarly for A 2 2, similarly for A 3 3. So, for a Hermitian matrix, diagonal entries are real. Other entries are conjugates symmetric of each other like 2 1th element of AH, that is second row first column. This is nothing but 1 2th element of there is 2 1 8 of AH is same as 1 2th star of A.

So, 2 1 th element, it is this at 1 2 th element of A is this. If you put a star here, you get back this element. That means, AH this matrix it is i jth element is nothing but A matrix. It is j ith element star, this is Hermitian transposition. Now, if A, and AH are same, that means, A 2 1 and this that is we should have A 2 1 is equal A 2 1 should be just a conjugate of this A 1 2 star. A 3 1 should be A 1 3 star because this and this are to be same likewise. So, the transfusion, the Hermitian matrix i jth element means I think I will continue from here in the next class because some more properties are to be discussed because Hermitian matrices, they give rise to what is called correlation and covariance matrices. So, that is all for today. We continue for another because time is up.

Thank you very much.