Probability and Random Variables

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Lecture - 2
Axioms of Probability

So, in the previous class, we had first discussed, we have presented this outline of this course, a detailed outline was given as to what all will be covered and then I tried to develop the motivation for this theory of probability in very general terms. At that time, I mentioned that this theory will be developed using some solid mathematical foundation of set theory, classical set theory. And then I discussed the basic set theory motions, definitions and operations at length.

Today, I will be using those concepts of set theory to first introduce you to the topic of the subject of probability space and we will be discussing the properties of probability space. And using this definition and properties, I will then go into what is called axiomatic definition of probability. We will enlist certain axioms. And then we will get in to the consequences of those axioms and then I will try to relate them to real life world because after all the axioms are totally mathematical.

And then once such axioms are given, are described, then I will get into some further sophistication in the form of something called field of sets. In fact, a particular class of field, particular type of field called borel field. And in this borel fields only mathematically we can apply this set theory axioms, probabilistic axioms correctly. So, I will go up to that. We will also consider some examples. So, first I start with probability space.

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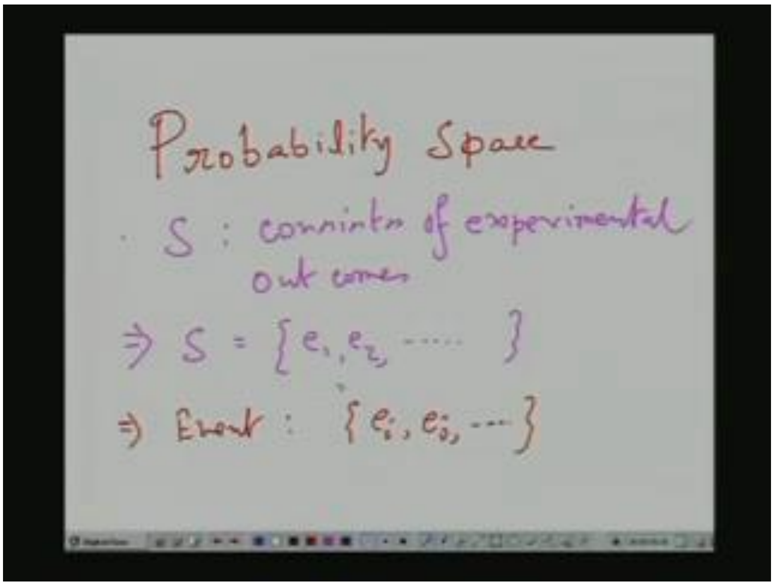

First let there be a set S. This set can be finite, countable, or infinite; finite and countable are the most you know the simplest of all cases; finite that is total number of entries is finite and countable. Though it can infinite, but countable like you know I mean do you know we can you give an example of such set where the set is infinite, but countable say set of integers. It is countable. It is called countably infinite or say it can be itself non countably infinite where say real line; it can be anything. But for our treatment, I will try to first take those examples where S is finite and countable, and then we will try to get…

In fact, if you really try to generalize it to those infinite sets, then only I have to get into that what I said field of sets and borel field, and all that. So, this set S, it consists of, it consist of experimental out comes I will explain. Meaning S is something like e 1, e 2, dot dot dot dot, the way I am writing, sorry the way I am writing you know, it is giving you an impression that it is countable, but it may not be…

Each of this elements e 1, e 2, each is an experimental outcome like that means, some experiment is going on or some observation is being made; either you observe e 1 some something or you observe e 2 and likewise. Collection of those basic experimental outcomes, all possible outcomes I would say you; collect all of them; that is the set S. Then comes event.

An event is any subset of S which means e 1 itself could be an event because e 1 is a set consists of e 1 only; that is only an experimental outcome. e 1 could be a subset of S which means that is an event, e 2 itself is an event, and likewise. Then an empty set phi, empty set phi that is also an element of S; that also is an event. What it means and all that we will see later. And not only that, you can now have things like this, you know, for instance say ei sorry ej dot dot dot some combination; that is an event. This event means actually that either observation was ei or could be ej or could be something else.

For instance, suppose you are I mean you are tossing a dice, say. So, there are 6 faces. So, actually there have been six elements here, 1, 2, 3, 4, 5, 6 whichever comes on top. But you can now look for those cases, where only even valued face comes up. So, 2,4,6 that could be one event. It does not mean that all three will come simultaneously; that is not happening; is it not? But either this or this or another one, just they are a collection that it denotes an event. In that case, the event means that dice showing up as an event because only the even faces, even number faces are showing up on top; that is an event.

So, that way, subsets of this consist of, I mean constitute, event, but here one thing I tell you; I am not making the statement that you should include all possible subsets. I am rather silent here. Subsets of these constitute event; sometimes all possible subsets are okay, sometimes not.

In fact, when you come to this infinite set, when it is infinite set, all possible subsets are not allowed. They do not constitute legal, I would say in our terms or valid events. It is there that the concept of borel field will come to define what could constitute at the event and what not. So, this is an event.

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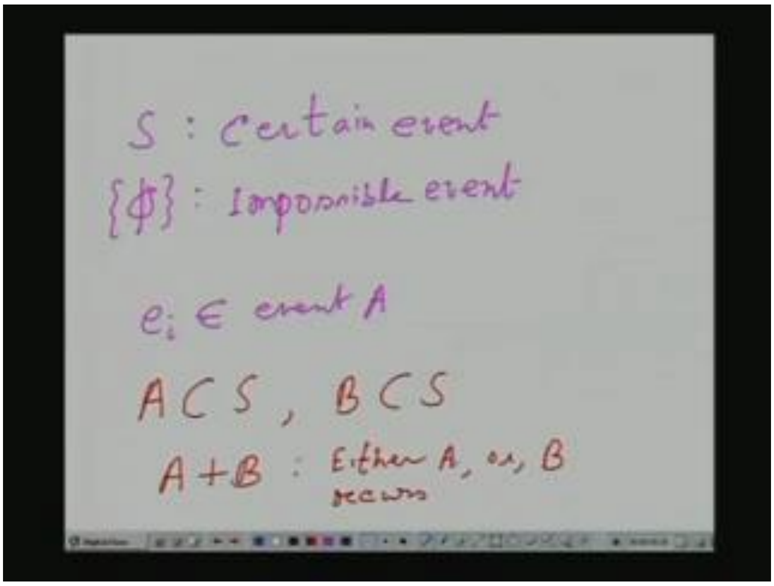

Then I would say that S itself which is the probability space, this S itself is an event, but it is a certain event. It is an event after all because what is an event? It is a collection of, it is a subset that is a collection of all those experimental outcomes. Now, the collection where all the outcomes are included, that also is a collection, but that is nothing, but S. So, that is an event. But that has to be a certain event because one of the outcomes has to take place; there is no other possibility. So, S it is called the certain event and phi you should always write like this. phi alone is not the set. It is a set with phi being the element which is in fact, null set. This corresponds to an impossible event.

Now, some more definitions or and discussions also. Suppose there is no particular outcome. ei here is the outcome. It is a part of, it is an element of event A. Event A is what? It is some collection of those experimental outcomes; ei is one of those outcomes; it is part of event A. And now, when you conduct the experiment, suppose outcome turns out to be ei only, now this particular outcome, then we will say that event A has taken place.

Because event A could have contained ei or may be ej, may be something else el, em, one of them has taken place; means event A has taken place. That also means that whenever you conduct the experiment, certain event always takes place. There is certain event that always takes place. Because any outcome, after all there has to be some outcome, but any outcome is an element of that certain event S. So, that always takes place. Certain event always takes place.

Then, suppose I have got one event A element subset of S, another event A, B sorry again subset of S, then A plus B, in our notation plus actually stands for union, but instead of using that U sign, I discussed in the previous class, since I have to go along with this book, I use the plus sign. In fact, which is more like or… So, A plus B, this also constitutes an event. What event it constitutes? That either the outcomes contained in A, they take place, or contained in B they take place; there could be overlap between A and B.

There have to be some experimental outcomes which are both part of A and part of B, but some could be exclusively for A, some could be exclusively for B. A plus B means either A takes place or B; one of them has to take place at least. So, this is the bigger event. In fact, if B and A are both non empty then A plus B actually corresponds to a bigger event because it has more number of outcomes in general. Of course, B could be

A subset of A. So, A plus B is A itself, but in general this is a bigger event that either A or B. So, that means, I can also write both; both means they overlap or both occurs.

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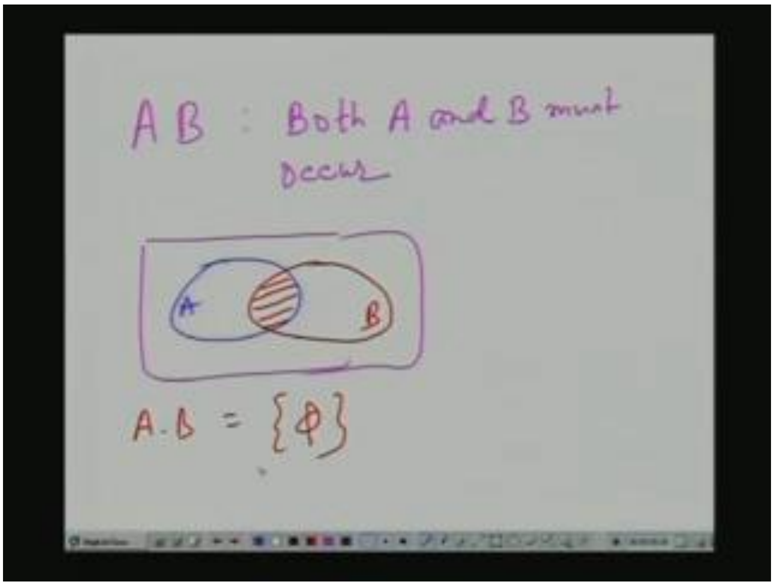

On other hand, consider A B. This is actually A intersection B, but again going by the notation of the book, we reserve this product sign as if it is A and B. A B also is an event, but it means that both A and B must occur. That means suppose if I consider the Venn diagram, this is A and suppose this is your B, if both A and B have to occur, that means, outcome should consist of observe I mean experimental outcomes that occur that lie here; that is where A and B intersect; is it not? Then only both A and B can take place simultaneously.

If A and B are such that they are mutually exclusive, that is if A B is nothing, but this, that means, there is no overlap between the two. In that case, the events A and B cannot take place simultaneously. If A takes place B cannot and if B takes place A cannot. For instance, when you just take the set S, you have got e 1, e 2. They are the basic outcomes. If you just take e 1, a set with e 1 only or a set with e 2 only, they are mutually exclusive because either e 1 has to take place or e 2; both cannot exist; both cannot take place simultaneously. Let me now get into the axioms; axioms of probability.

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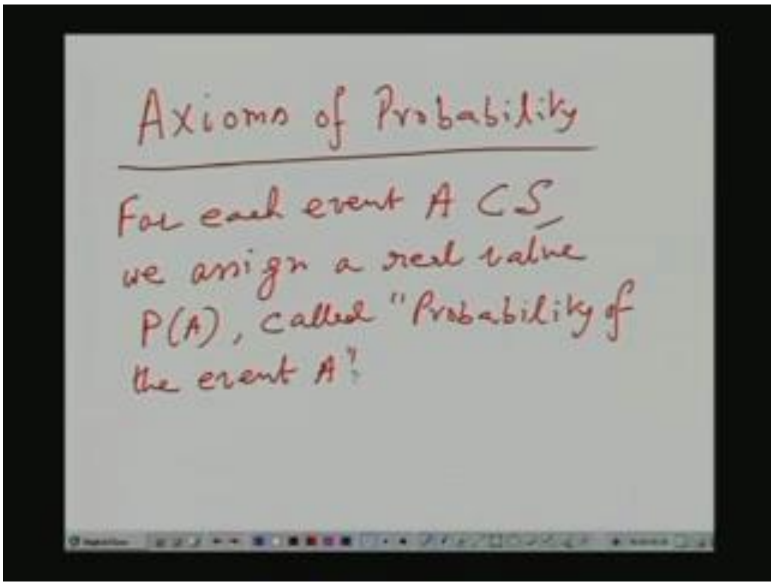

Actually, let me tell you something about what is meant by an axiomatic treatment. Often you know in physics or engineering, we have some physical concepts and we try to associate some kind of calculation, some kind you know I mean some I would say rough mathematics with that. But when mathematicians step in, they will look into the mathematical aspects of the variables present and all that. They do not go into the physical parts and all that so easily.

They try to find out the basic mathematical properties that those variables or the existing mathematical structure must satisfy those basic, you know minimum number of properties that they satisfy. And then, based on that, develop a theory; those basic properties are called actual axioms. Like idea of probability was existing earlier also, but when mathematicians stepped in, they tried to find out if there is something called probability in practice. Like you know the other day I told you something about what we say frequency of frequency based interpretation.

That if suppose capital N number of trials are taken; among the N, small n number of outcomes or a particular kind take place. Then approximately small n by N is you know is a chance of occurrence which you call probability on that event. So, with those kinds of definitions and how definitions already present, when mathematicians looked into them, they found out that they should satisfy some basic properties. These are those probabilities.

And by using this, you can develop further I mean you can establish other properties with which also people in science or engineering familiar. But then now you are forming a solid foundation. Those basic properties are called axioms.

Here, you are seeing that with every event, with every event, we assign a value and that value we will just term probability. What is its physical meaning and all we are not bothered at the moment. So, if that is for each event A we assign a real value PA called probability of the event A, what we are doing? We are just looking at all possible events that are available and to each I am assigning some value; value is real, not complex.

Again it is abstract. I am not at the moment bothered by what is the physical significance of those values and all that. I am only bothered about the properties that this value should satisfy; basic mathematical properties which you cannot beat. If you valued these properties, the whole thing crashes. So, those values are called probabilities. I denote like this P of A, is a probability of A. It should satisfy some properties. These properties are sorry…

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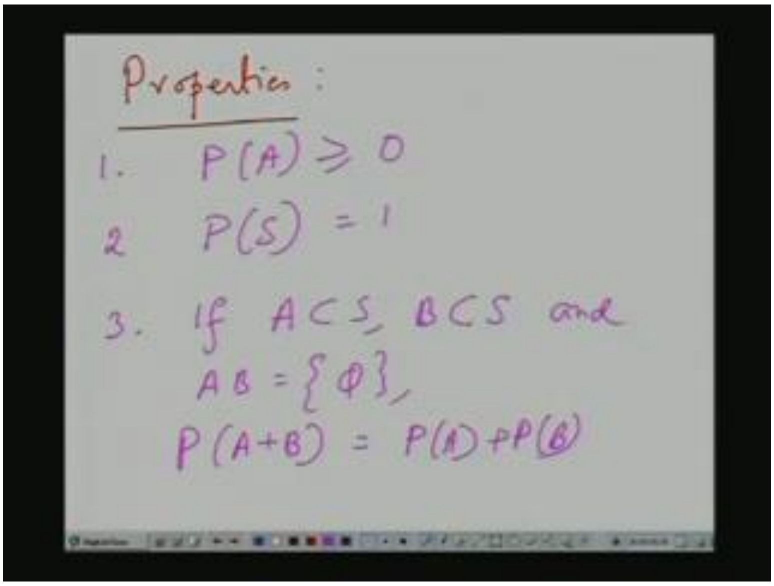

Properties: Number one: PA any probability that I assign, any probability value, that must be non-negative; it should be greater than equal to zero. Then, probability of the total event that is the certain event, that must be 1. See, we may know we will not be imposing so many conditions, so many properties; that is the beauty here. Some just three basic properties will be taken up and many other you know properties with which we are familiar, they will follow from this; this is the beauty here.

The mathematicians find out what is the basic minimum condition that are required to be satisfied by this measure P of A. Actually this value P of A sometimes called measure, probability measure. That is how an established topic called measure theory in mathematics. In fact, which is intimately connected to probability theory, but I will not get in to that. It is called probability measure.

Now, what are the basic properties, minimum basic properties that this should satisfy? These two. And the third one is if A is an event of S, B another event, and they are mutually exclusive, that is A B is phi mutually exclusive, then P A B sorry P A plus B. What is A plus B? That is firstly, A and B they are mutually exclusive. So, A consists of some experimental outcomes; B consists of some other outcomes. There is no overlap because they are mutually exclusive.

And now, P A plus B means you now consider the bigger event where outcomes of both A and B are present. Any one occurring will be enough. We can say that the event A plus B has taken place. Probability of that bigger event should be equal to P of A plus p of B; that is all. Now, let us see what consequences follow from this.

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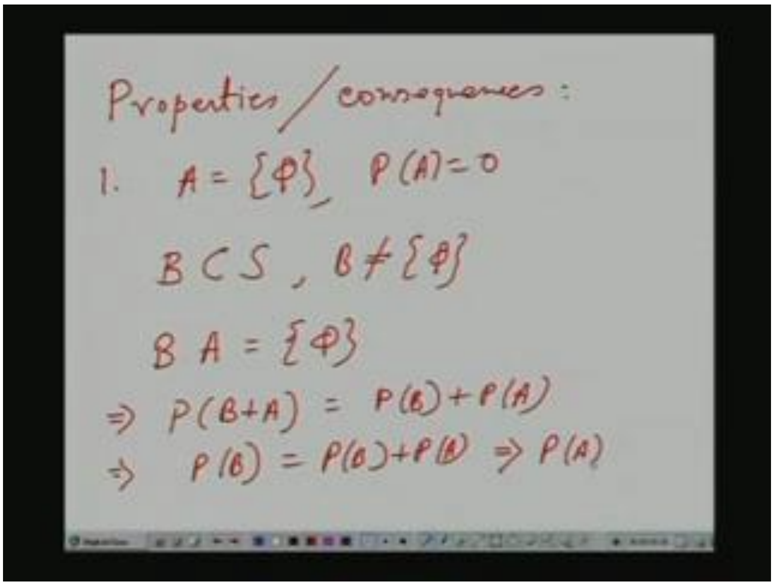

Let me call it those three things. Actually instead of calling them properties, I should I mean it is better to call them axioms. Then I call it properties. Here, actually properties or consequences. First if you consider the empty set, that is an impossible event because no outcome; empty set means there is no outcome. That means however much you try the experiment again and again, you cannot get that thing; that is an impossible event.

So, if A is phi, then P A equal to zero. How it happens? I will just use those three axioms only. Now, you see, take any event B, element of S and B is not this. That is I take any non-empty subset and subset which actually is given to be an event, a valid event; call it B. And obviously, B and A where is the intersection? phi only; A is phi empty set and B is any non-empty subset. So, there intersection is phi. In that case, that means, P of B plus A that is B union A will be… But B union is B only because A is empty set. So, that means, P B and you have P B here plus P A which means P A equal to zero. So, the impossible event has probability zero; total event has probability 1.

I will also show that as the set expands from smaller event as you get in to bigger and bigger event, probability value increases. So, when you go to the final biggest event which is the total event S, then it finally gets the value 1. So, this is how we go close to practice because that is what happens in practice. Is it not? But you see, we have only those three axioms. Only those three are necessary to develop the entire concept with which we are familiar. This is one consequence.

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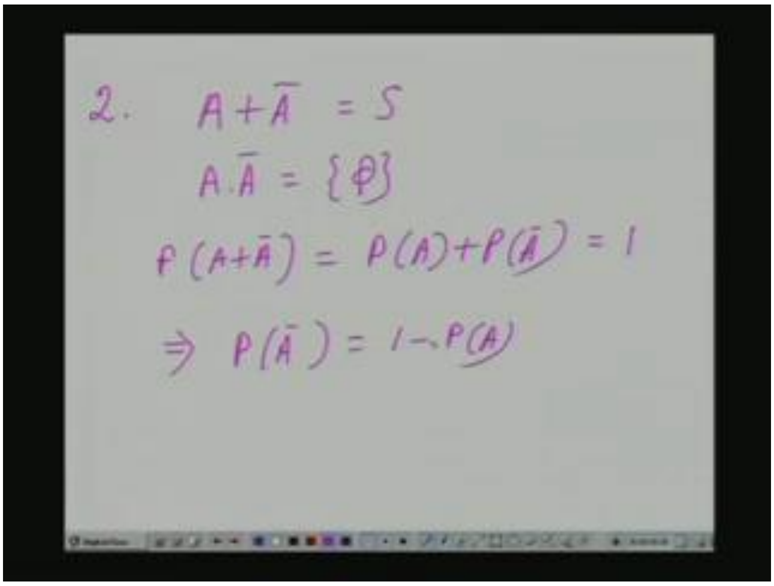

The number 2: since A plus A bar, A bar is the compliment of A, A union A bar that is equal total set S, and A and A bar, they are mutually exclusive. That means their intersection is phi. That means P of A plus A bar; since both are mutually exclusive, I can write using one of the axioms, axiom number three; this is three. But A union A bar is nothing, but total set S, and P of S going by Axiom number 2 is 1 which means P of A bar is 1 minus P A. So, when event is expressing some probability P, then the probability for the event not to take place is 1 minus P.

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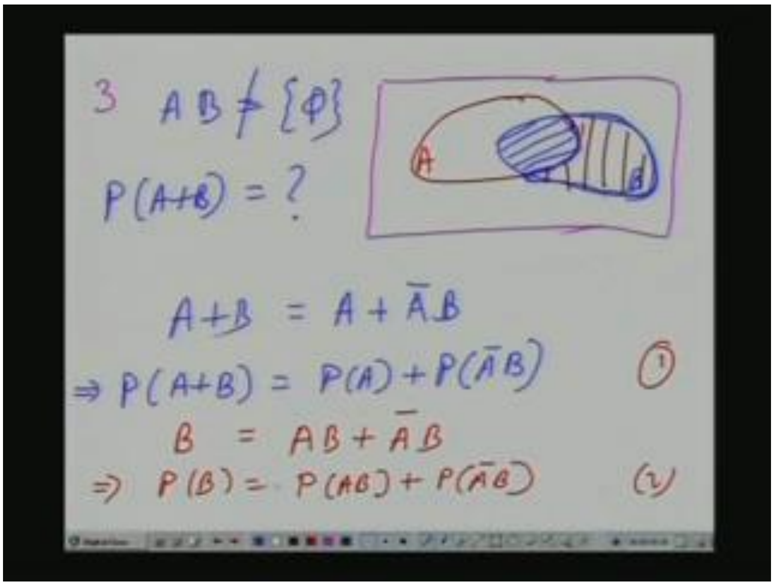

Now, suppose A and B are two subsets, two events which are not mutually exclusive, so I cannot write P of A plus B is P of A plus P of B. Than what can I write? That is the question. That is now I have something like this property 3. Actually, we have got something like this. It is A; this is B; this area is your A B; total is A plus B. So, here, A and B are not mutually exclusive. That is given A B; it is not this is given. In that case question is what is P A plus B equal to what? This is the question. So, this is one formula. We have to derive first. Let us see one thing. It can be derived in various ways. Consider A plus B, that is this entire set; is it not? I can write it, I want to write it as the union of two disjoint subsets.

Disjoint means their intersection should be phi. So, one possibility is this. Take this side as one and rest as it is. Are you following me? That is take A as it is and do not take entire B, but take only this part; this part of B that is this side. That means, A plus B can be written as A union this side. Now, what is this side? Firstly, using this Venn diagram, things can be written very easily. This is A. So, outside A is A bar; is it not? Outside A is A bar. This entire thing, all these places; this is A bar including this. This entire thing is A. Outside this is whatever you have, that is A bar; that A bar consists of these open spaces plus this space.

With that A bar, if you take intersection with B, actually if you have the intersection between A bar and B, what do you have? B is this entire thing, but out of that, this part falls under A. So, that does not come under intersection. This part falls purely under A bar because A bar means outside this area. So, this part is nothing, but here and they are mutually exclusive. That means, I can write by axiom number 3 now, P A plus and then this value is coming. This is a problem. A bar B is this area.

Now, consider B. B has got two parts: one is this part which is A B and this is B bar. Another is this part; A bar B. If from this is followed, similarly consider B now. You call one. Consider B now. What is B? B is this entire thing. It has got two parts say I want whatever parts I have, I want them to be mutually exclusive because axiom number 3 works only for mutually exclusive subsets. So, I break B into two mutually exclusive parts. One is this; another is this. This part is we already know A bar B and what is this part? A B; A intersection with B. So, you have got A B plus A bar B.

As a matter of fact, if you now apply odd logic and all that, B can be common and A plus A bar is 1. But let us follow this Venn diagram thing. Let us not bring the logic variables things here. This is your B; that means, P B is equal to what? These two are mutually exclusive; is it not? This and this, that means, I can I apply axioms 3 again, but then it becomes P A B plus P A bar B. So, this value I take from here. What is P A bar B? That is P B minus this and that I replace here and that gives you the formula.

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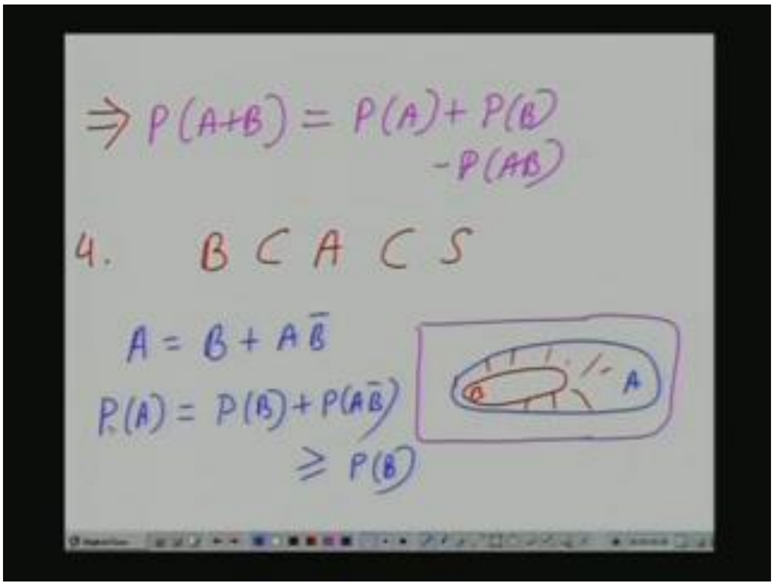

In this case, P A plus B, P A plus… You will get this; these are result. Finally, this was property number 2 or 3? 3.

So, another thing we see number 4. Suppose I have an event B that is a smaller event. There is a bigger event A. That event is going from B. I get into say A both B and A are events; they are part of S; subset of S, but B is contained in A. Then, we will show probability for B is less than the probability of A. How to show that?

Once again, using Venn diagram, you can take the Venn diagram. This is your A and this is your say B; B contained in A. So, A can be written as I again want to write A as a sum of two disjoint subsets. One is B; another is outside portion of B. That part which lies outside B, but inside A; that will be what? Firstly, take B. What is B bar? B bar means entire zone outside except for the area given by B; the entire area outside B; that is B bar.

If you take intersection between that and A, then we will get what? After all intersection between A and that so all areas of A have to be included, but not this because this is not part of B bar. So, only this part will be…So, that means, you can write A as B plus A B bar and these two are disjoint, which means P A is by axiom 3 P B plus and since both are I mean non-zero, this is clearly greater than equal to P B. If this is empty set, then only these two are same. When can this be empty set? When B is B itself is A; there is nothing no area here. Then only P and P B can be same. But as long as there is some non-empty subset in the form of A B prime, probability of the smaller event B becomes less than the probability of the bigger event A.

Now, this has been done axiomatically. I have not related to you know real life examples, but let us consider those frequency interpretation cases and see that this actually these axioms and the consequent properties, they satisfy those, you know I mean, physical things; physical observations.

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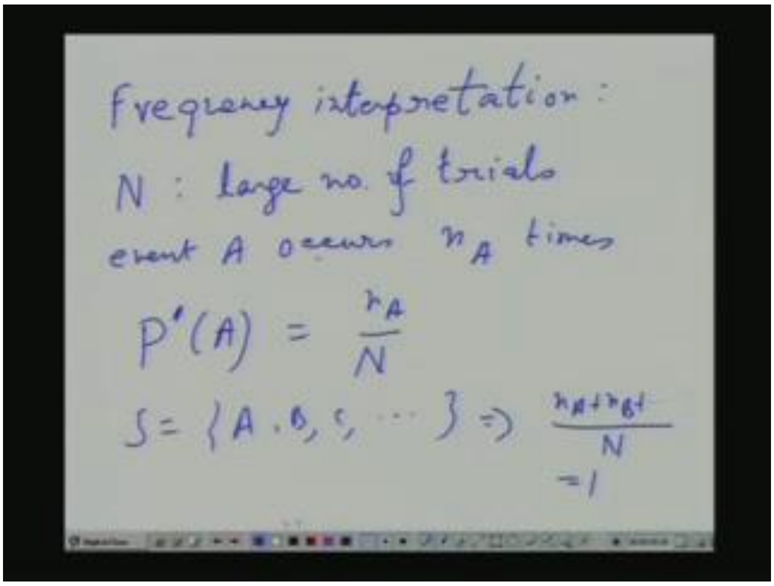

Suppose we are conducting an experiment out of which say I am saying that frequency interpretation, I am coming to this. Suppose, we are conducting an experiment; we have taken a very large number of trials say capital N; large number of trials out of which one particular event A occurs n A times. Then classically, we know the approximate value of the chance of occurrence of A is what? n A by N.

That is the physical probability actually with which you are familiar; physically that is for us that is probability, provided capital N is large, but you see that satisfy this properties. Firstly, P prime A say I am using a prime here. It is because P of A was used for you know within the axiomatic definition of the probability of A event A, whereas P prime A relates to the probability with which we are familiar. That is based on frequency interpretation.

So, this P prime A is nothing, but as we know as we know n A by N, but this is always a non-negative number. Minimum value could be zero; that is you try capital N, N number of times and out of this time, N number of observations also you do not get the event ever. So, in that case, nA is zero and minimum value is zero, but you cannot get it negative. So, this satisfies axiom number 1. Then axiom number 2 is what? That probability of the total event is 1. Now, you see, out out of capital N number of trails on n A occasion, you get event A; on n B occasion you get event B and so on and So forth. But total event means say event A, then event B, event C dot dot dot dot, but total this total thing should be equal to S. What is the probability of S out of capital N number of trails, how many times I get the total thing? N A times here, n B times here, n C times and all that. So, total is n A plus n B plus n C dot dot dot dot that by capital N. That is by capital N.

But this is equal to capital N itself. After all when you try capital N number of times, on certain occasions you get A; on certain occasion you get B; on certain occasions you get C and all that. But if you add all of them, then that equals N. The capital N is large so that I mean so large that all the events will take place adequately. A also has taken place on sufficient number of occasions; n A occasions; B also has taken place; C also has taken place and all that. Only thing is that if you add up n A, n B, n C and all that, you get capital N.

So, I mean, so since capital N is large, you can take that entire happening that n A times capital A n B times B and all that, that entire thing amounts to observing the total set. So, in that case, that probability will be 1 because number of occasions on which you get the total thing is after all n A plus n B plus n C plus dot dot dot dot which is n divided by N which is equal to 1. Actually this is trivial. Axiom 1 and axiom 2, they are satisfied trivially.

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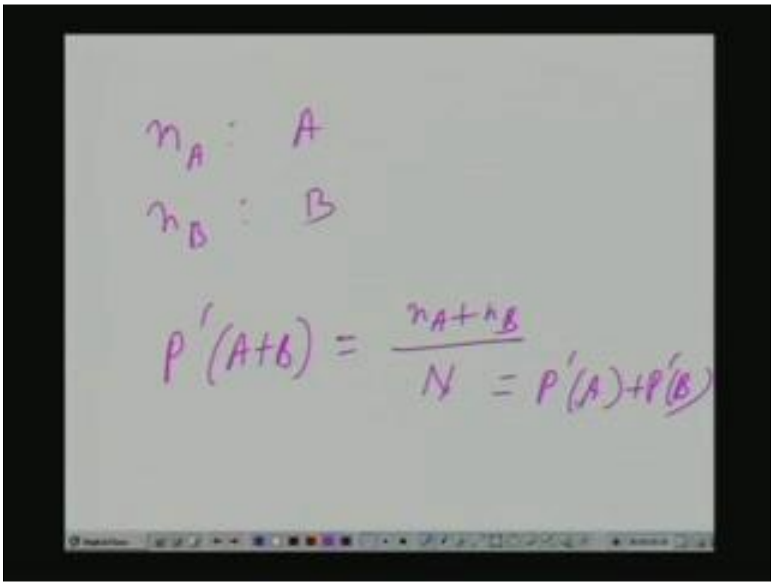

And axiom 3 suppose out of capital N, out of trials n A occasions, you get on n A occasions you get A; n B occasions you get B. A is an event, B is an event, and suppose they are mutually exclusive, that is event A may involve some outcomes; either this outcomes or that outcome or that outcome like that same for B, but there is no overlap.

In that case, if you try capital N number of times and on n A occasions A occurs, n B occasions B occurs, you are assured that on each of the n A occasions where A is occurring, B is not occurring because they are mutually exclusive. There is nothing common; no common outcome between the two. In that case, if I now look for the bigger event A plus B, if I now look for the bigger event A plus B, that means, all outcomes of A or all outcomes of B, any of them occurring, what will be the probability?

Out of capital N number of trials, n A times you are getting outcomes from A; n B times outcome from b and no overlap. So, n A plus n B times you are getting outcomes of A plus B. That means P prime A plus B will be what? Not 1. n A plus n B by N; try to understand this. A and B are mutually exclusive; A occurs n A times. In each of these occurrences, B does not occur. There is nothing common and same for B. B occurs n B times and each of these occurrences, A cannot occur. So, on n A, n B occasions what do you get clearly? Nothing less than this; that is I am saying it has 2 n A plus n B.

On n A plus n B occasions what do you get? We get either events from A either A or B; that is either some outcome from A or some outcome from B; nothing less than n A plus n B because they are mutually exclusive.

So, what is the probability that divided by N and now that you write as n A by N plus n B i B by N which is nothing, but P prime A plus P prime B. So, those axioms really satisfy the physical facts.

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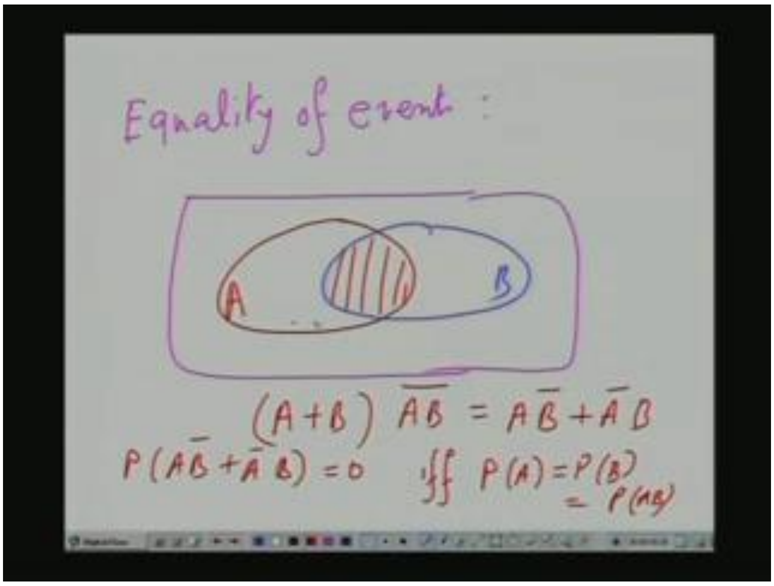

Then, another definition: Suppose two events are given, A and B. We would say that they are equal. As per one definition they are equal if the outcomes containing A they are same as those contained in B. This is a very strong definition of equality of events that they are identical. But in probability theory, many things are presented using the notion of probability. I would say I have given alternate definition also. That says that two events A and B, they are equal if you consider this. You consider the Venn diagram first.

This is your A B. If the probability of either this part, this event, or this event taking place is zero. Now, probability of an event is 0 if it is an empty set. So, indirectly it means this half and this half, they are empty; that means, they overlap entirely which means A and B are identical. But you see there is a mathematical difference. Physically it appears to be, so but in terms of conditions there is a difference actually.

I am not saying they are equal. I am using those axioms and using that I am presenting the equality of set two events in this way; that probability of this event, what event? What is this portion? This part and this part, this is actually A plus B. What is A plus B? This is this total thing is A plus B and intersection of this. A B is this. Its compliment is the entire area except for this; that when intersects with this A plus B, only this half goes; this portion goes; both this and this remains. Using this De Morgan's law and all that, you can even write this as actually exclusive, exclusive bar as they are mutually disjoint.

We would say that A and B, these two events are equal if probability of this is 0 which means this is an empty set. The difference is that you know you see again it is purely using mathematics. You know I mean what mathematicians say I said if the set is empty, then the P of empty set is 0; that we proved using those axioms. But I have not proved that if probability of a set is zero, the set is an empty set. There we are silent. We are now I can also say that you are not trying to force some strong condition there; you are silent there.

That is that is what we have in this definition. In one definition we are very strong. That is a strong condition that two sets are, two events are equal if they are identical. They have same elements and all that. But in the other case, I am putting it in an indirect way that probability of this portion, this event, union with this event which are mutually exclusive, that should be 0 which means if indeed it is that probability of an event zero means the event is impossible event. Then, it is obvious that would have meant this part and these are empty sets. A and B, they overlap entirely here, but this is more general than that you follow the distinction.

Now, for this two happen that is if the probability of this is 0, then this is satisfied; that is P of this part A B bar plus A bar B this is equal to 0 if and only if P A equal to P B equal to P A B. Now, if this is satisfied, this will be satisfied; that I will show, but if this is given, then show that this is true; that I will leave as an exercise. But I need this Venn diagram. So, may be I have to draw again or before I since I do not want to draw it again, let me tell beforehand, what I want to do.

A I will write as this part which is what? A B bar. A B bar; this is A; B bar as I told you the entire plan except for this B. Intersection between that and A will be only this; not this part. So, A can be written as sum of this; that is A B bar and A B. I will use that.

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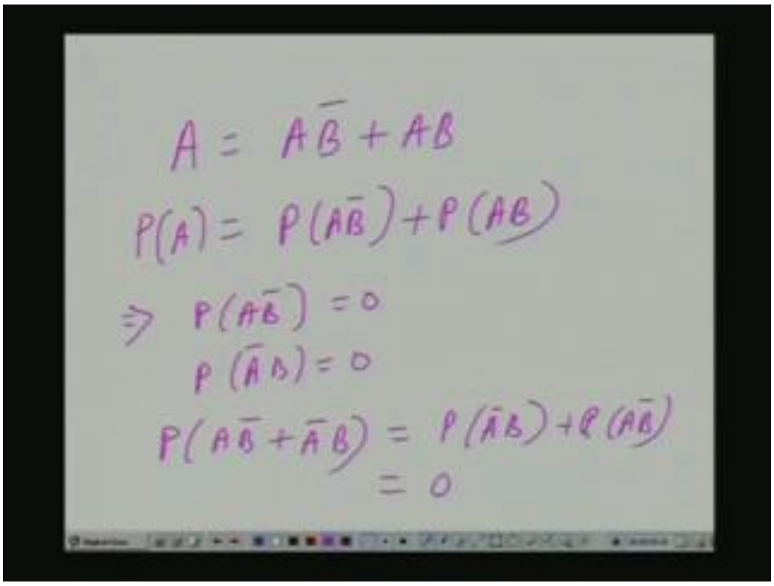

That is A is logically it is very obvious. You can take A common and Bi, B bar; B bar is 1 and all that; that means, and these two are mutually exclusive. So, I apply that axiom. But this is given to be this. P A equal to P B equal to P A B. I am assuming that is given; that means, this and this are same which means P A B bar equal to 0.

In a similar way, I can show that P A bar B equal to 0. This time I took A. Next time I will put it B; replace A by B, B by A, like that which means I have to prove this know, but this two are mutually exclusive. A B bar one half, A will be another half, which is same as is equal to 0, but both are 0.

So, given that condition that P A equal to P B equal P A B, you prove this. You obtain this, but I will leave it as an exercise for you to show that given this, show that it amounts to P A equal to P B equal to P A B.

Now, I get into fields of sets. Firstly, fields of sets.

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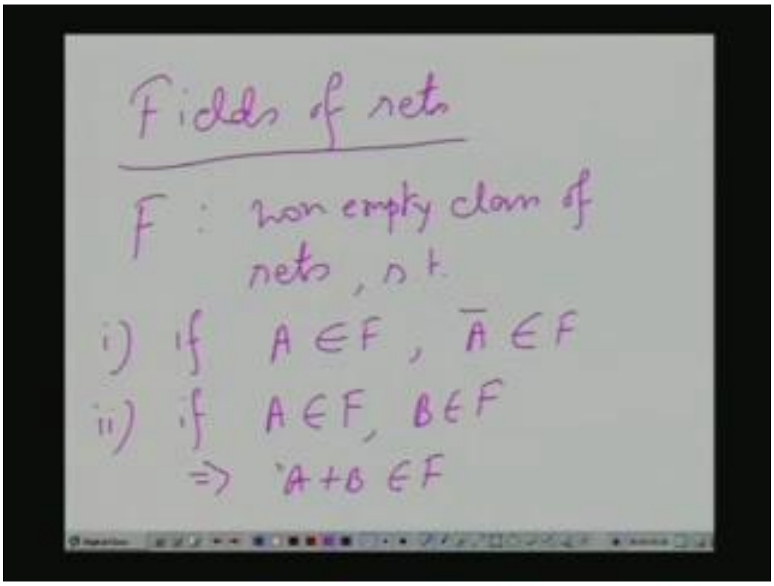

Actually, I said that given any probability space S, what is that event? Event is some subset of those outcomes. But that time I said that it is not that all subsets will qualify for events. As far as a finite countable set is concerned there is no problem, but problem comes in infinitely long sets; sets which are infinite, there problems can come up. Not all subsets qualify for events. These are mathematical problem. I will not get into that. I mean it can be shown that if you consider all possible subsets of an infinite set and consider the subsets to be events, the axioms do not work perfectly on all subsets. That is we have to use some another concept called fields. That is something called borel field. But before I get into borel field, I will quickly just define some basic axioms of the fields.

A field F is a non-empty class. Actually when you define set, we say it is a set is a collection of objects and all that. But when you have a collection of sets, then we use the terminology class. So, it is a non-empty class of sets, so non-empty. So, F may consist of, F may consist of empty set, but that is not alone. It will consist of some other sets also.

It should satisfy two properties. That is so that number 1, if A is part of F, then compliment of sorry this also must exist in the field. This is one requirement. If A is part of F, A compliment also should be part of F.

A may be taken from that event from the probability space S. From there I took A, but then I know what is the A compliment there. Now, I am constructing a field. If A is present in that field, A bar must also be present. And number 2 is if this should be actually element because these are now, sets are now elements of the class; class is what? A collection of sets. So, each set is an element of the class. So, it should be element; not subset.

Then these two; using these we will show that this applies true also for two the intersection. Using these two, we will develop some properties. We will show that if A is an element of F and B is an element of F than A B is also an element of F.

So, in the next class, I will take from here and I get into something called borel field and there I give the more exact definitions actually. Borel field, I will tell you why borel fields are required. Because when you have got that infinite set, all subsets do not qualify for events. Only those which give rise to borel fields, they give rise to they qualify for events.

After having done that, I will get into again more practical things that is conditional probability and all that. We will solve some problems also. So, that is all for today.

Thank you.

Preview of the next lecture

Axioms of Probability (Contd.)

Okay. So, in the last class, we discussed the axioms of probability, their properties and then we discussed something called field of sets, and in that connection we mentioned borel set, borel field.

I will just touch up on those things today to start with and also explain the motivation for considering these, but these are again only for you know mathematical correctness. But having just done that, I will go over to the notion of conditional probability. That I will be explaining first notionally and through examples. Then, using this conditional probability, I will find out its properties.

You know, we will discuss something called total probability and Bayer's theorem. Again we will take up some examples and then we will go for statistical independence of two events, and we of course, try to give physical interpretation of these.

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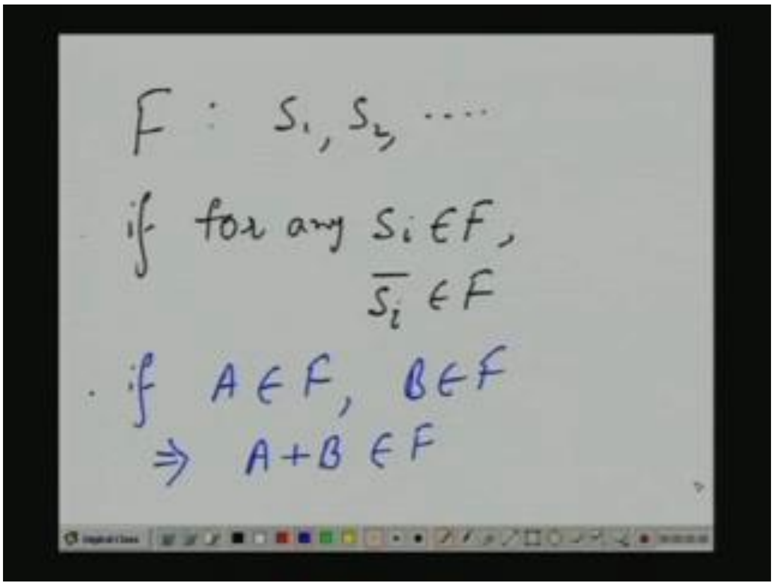

So, as I mentioned in the last class, I will consider a field. I will consider a field of sets F. It will consist of suppose sets S 1, S 2 and dot dot dot so on and so forth.

Now, this will be a field if for any S i, I would say element because this is a field means it is a class of sets. So, each set is an element of this class F, we should have Si bar also member of F. That is if a set belongs to this field, then its complement also must belong to the field number 1. And number 2, number 2 is if A is an element of F and B is an element of F, then union also is an element of F. These two conditions must be satisfied. There are some properties which follow this which we will be now considering sorry ah.

Thank you.